Agent Technology Practical - Final Report
Customer Utility and Turnover Rate within Hotelling's Law

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1 Introduction

1.1 Subject

Hotelling's Law, also referred to as the principle of minimum differentiation, observes that in almost all markets it is rational for producers to make their products as similar as possible (Hotelling, 1929). In his original paper Hotelling shows that this law upholds for stores in one dimension, e.g. on a street or a beach. Stonedahl and Wilensky replicate and extend this law, in their NetLogo(Wilensky, 1999) sample model, by allowing the stores to freely move on a two dimensional plane (Ottino, Stonedahl, & Wilensky, 2009).

The sample model models phenomena within the world of retail, such as the tendency for similar stores to clump together (e.g. usually KFC and McDonalds are right next to each other (Talwalkar, 2012), Groningen illustrates this example as next to every KFC there is a McDonalds) or the tendency for outlets to have very similar products on offer (e.g. every supermarket has similar stock as every other supermarket). However, the model can also be used as an analogy for phenomena outside of retail, such as the tendency for political parties to adjust their views and policy to be closer to their competitors.

1.2 New idea

This paper extends the NetLogo model to be able to analyse the effects of Hotelling's law on the utility of the customers. We define a customer's utility as the sum of the inverses of the price it has to pay and the distance it has to walk, such that the more it has to pay and the further it has to walk the lower it's utility will be. Formally:

$$u = \frac{1}{p} + \frac{1}{d+1} \tag{1}$$

where **u** is the customer utility, **p** is the price of the store that a customer has chosen and **d** is the distance to that store.

The new model will more specifically be used to analyse the effect of turnover rate on customer utility. We define the turnover rate as a variable that is affected by the probability of a new store emerging and the energy it expends each step (i.e. the cost of existing, which affects how often stores go bankrupt). This paper will try to answer what combination of start up probability and cost of existing will produce the largest utility for customers. In other words we are going to see if the *implicitly* defined turnover rate and customer utility are correlated.

As an analogy to the world of retail, this model could then be used for policy makers to decide whether more funding should go to beginning entrepreneurs, as well as whether perhaps taxes should be increased such that more stores go bankrupt. Outside of retail it could show voters whether it is smart to shift their votes away from established parties to newer parties. However, the limitations of a model like this should be kept in mind before letting it influence real world behaviour.

2 Methods

2.1 Conceptual model

2.1.1 Ottino, Stonedahl and Wilensky's model in a nutshell

In their original model Ottino, Stonedahl and Wilensky simulate a two dimensional plane with a predetermined number of stores. Each coordinate, or patch, represents a customer. The plane consists of 40 by 40 customers and does not wrap around. The stores try to find their biggest revenue possible by separately adjusting their location and price on every tick. The revenue is defined as the number of customers multiplied by the asking price. First a location is chosen and afterwards the best price is determined.

The location is chosen by weighing the possible revenues for one step in each possible cardinal direction, the possibility of not moving is only taken into consideration when the store already has at least some market share. Thus, each tick every store considers four or five different locations and checks in which location there would be the most willing customers.

The pricing is then chosen for the new location (if the store did indeed move), by considering whether the same price, an increment of one or a decrement of one would give the most revenue. If for all possible pricings the revenue is zero, then the store will automatically decrement its price.

Notice that the stores only communicate with the customers when considering their next move. The stores only check whether a customer would actually attend their store when changing location or pricing, they do not consider what their competition is doing.

To facilitate finding an answer to the research question, the model has had to be extended in several ways.

2.1.2 Start up probability and cost of living

In the original model the number of stores was determined at the start and remained static for the rest of the simulation. The new model extends on this with a variable start up probability and a variable cost of living, these will be used as the independent variables in our experiment.

Start up probability is a random chance for a new store to pop up on each tick and can range from 0 to 100%. Thus in a run where the start up probability is 20%, we'd expect a new store to arrive around every five ticks. As this addition can cause the amount of stores to grow indefinitely, a limit for a maximum number of stores is added as well.

Each store has been given a certain number of assets (or energy in more familiar agent-technology terms). If the amount of assets drops below zero, the store goes bankrupt and disappears. Each tick a store increases it's assets with its current revenue (it's price multiplied by the amount of customers) and subtracts the global variable 'cost of living'. Thus for higher cost of living, less successful stores will tend to go bankrupt more often.

The amount of assets that the stores initialize with, is determined randomly by picking a number from a Pareto distribution. The Pareto distribution was chosen as wealth in the real world tends to follow a Pareto distribution (Tomić, 2018). The formula for a random number on a Pareto distribution is:

$$\frac{m}{x^{\frac{1}{\alpha}}} \tag{2}$$

Where x is a random float between 0.0001 and 1, while α determines the steepness of the distribution. The 'm' determines the minimum amount of assets that each store has.

2.1.3 Smarter customers

When stores can actually out compete one another, the chance of one store gaining a monopoly arises. This in itself is not a problem, however, the store can rack up its price indefinitely, as the customers in the original model had infinite pockets, which caused an unrealistic competitive advantage as a single store could quickly rack up it's assets in the time it had alone, rendering it virtually immortal. To counter this we limited the utility each customer was willing to expend, i.e. a maximum distance and price was implemented for the customers, which is determined at setup.

The maximum distance is randomly determined for each customer by picking a random value from a normal distribution. A normal distribution was chosen as we did not have any literature available on the distribution of laziness, however, many human traits tend toward a bell curve, thus we would expect that laziness is no exception (Long, 2020). The logic behind this is that, for example, if two exceptionally lazy parents produce a child, the laziness of the child will have to regress towards the mean as otherwise laziness would quickly diverge with each passing generation, as groups of lazy people will produce lazier and lazier offspring while groups of non-lazy people will do the opposite, which is not what we seem to observe in the real world.

The maximum price each customer is willing to pay is determined by picking a random value from a Pareto-distribution (equation 2) in the same way that the initial assets for each store are determined.

The customers select their preferred store by looping over all stores and then checking which of the stores that falls within their preferences will give them the most utility.

2.1.4 Smarter stores

As the market for the stores has changed to become harsher, the capacity to plan for the stores has been updated as well. Instead of only analysing in-/decrements of one, the stores now also consider what would happen if they in-/decrease by two or three. If all of the checked prices return a revenue of zero, the store will drop its price by three.

As negative prices of a price of zero is unrealistic, the stores will never go below a price of one. This also makes sure that there are no division by zero errors when calculating customer utility.

2.2 Implementation details

As stated earlier, the model was written in NetLogo and extends on the the original sample model 'Hotelling's Law' by Ottino, Stonedahl and Wilensky (Ottino et al., 2009).

The full model is downloadable at github here: `https://github.com/SanderKaatee/ATP`. All new lines of code implemented on top of the original Ottino model are marked with three semicolons (;;;). A total of 135 lines were added (including comments) while 25 lines were removed.

The full experiment, as described in the next section, can also be found attached to this model, with in the Behaviour Space, named 'experiment_2'.

2.3 Experiments and analyses

This paper tries to answer what the effect of turnover rate is on customer utility, therefore we will be varying start up probability and cost of living and we will be watching the global average customer utility. The start up probability will range from 0 to 100% and will be incremented with 4% for each run. The cost of living will range from 0 to 10 000 per tick, and will be incremented with 400 for each run. Each run will be 750 ticks, as the customer utility varies per tick thus we will take the average global customer utility per run as our output.

2.3.1 Fixed variables

As the model contains multiple variables other than the variables we are interested in, we will keep these variables fixed such that the experiment stays orderly and clear. For some we will explain our reasoning as to why we choose a certain value, for others the value might have less of a basis: they just ensure that there is some activity during the runs. All variables could be explored in future research, however, they are outside the scope of this paper.

For each run of the simulation the initial number of stores will be kept fixed at three, during the runs themselves the number of stores will vary depending on the combination of our independent variables. The maximum number of stores will be set to 15.

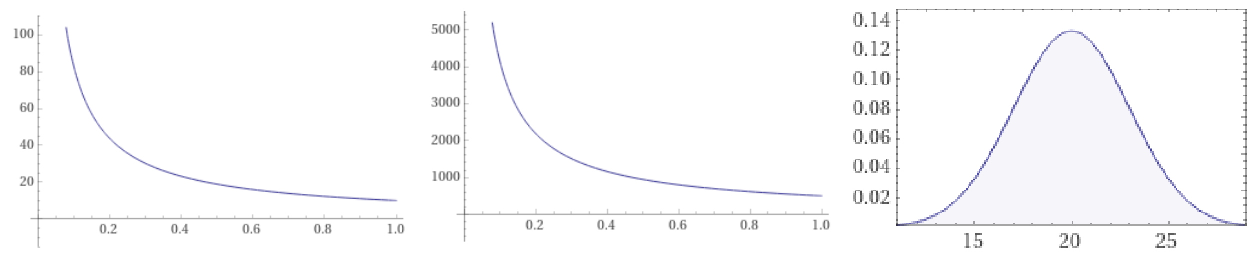
For all Pareto distributions we use in this experiment, thus the store assets and the consumer funds, we set the alpha to 1.09132916932, which resembles a distribution where the richest 20% own 90% of all wealth, as that might most closely resemble the distribution of wealth in the real world (Tomić, 2018).

The minimum amount of funds for the consumers will be set to 10 (Figure 1a), the minimum amount of assets for each store will be set to 500 (Figure 1b).

The normal distribution from which the travel distance of the consumers is chosen will have a mean of 20 and a standard deviation of 3 (Figure 1c).

Figure 1: The distributions from which random numbers are pulled

(a) Consumer funds (b) Starting assets (c) Travel distance



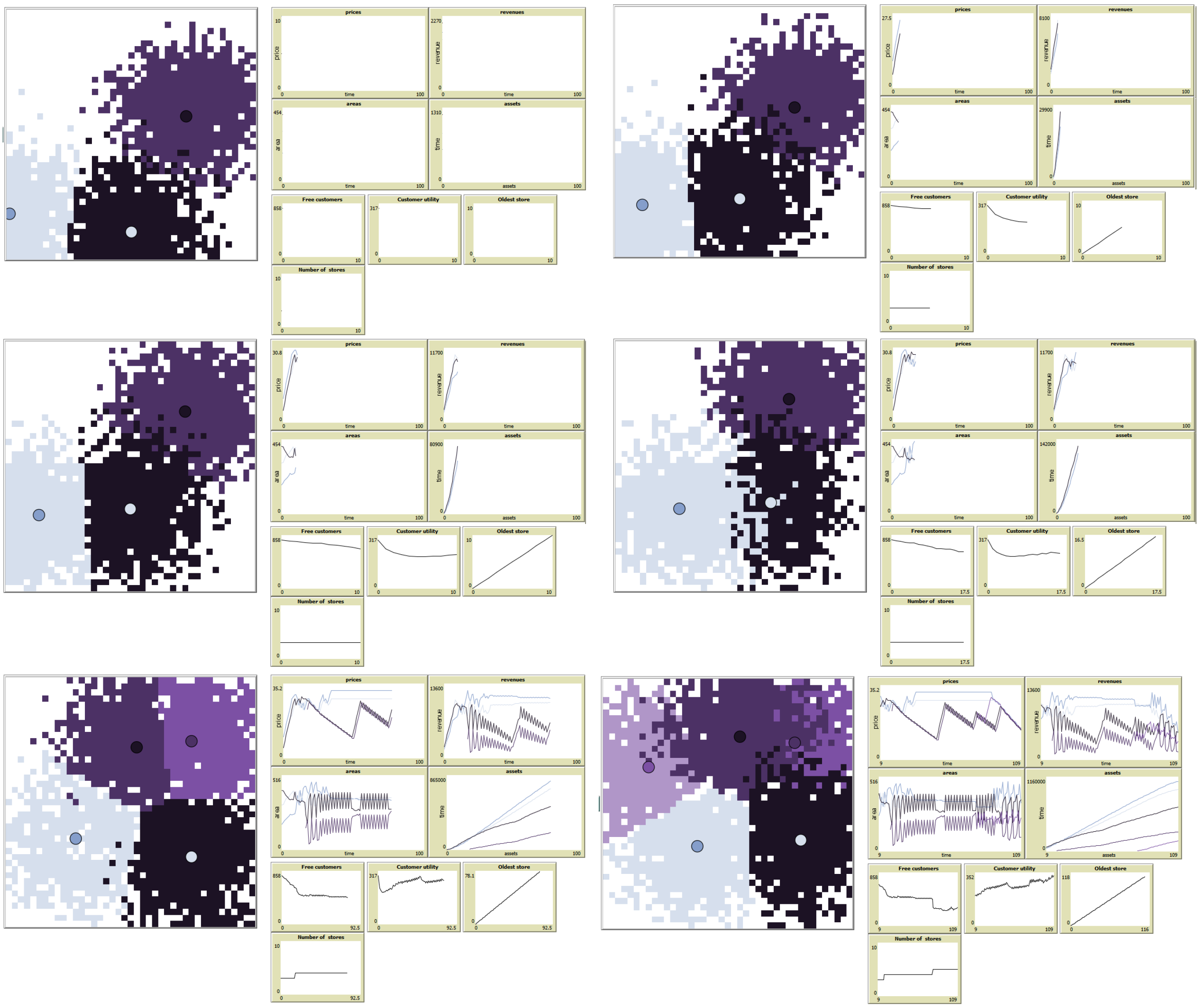
3 Results

3.1 Example model run

When we set up our model the model works initially as an improved version of Hotelling's law. The stores move closer to each other until they find a more or less stable position with their prices and movement. It gets into a loop where the prices and positions repeat. The movement of the stores, their price movement and the cloud of customers is fuzzy. This is because the probabilistic initialization of the price that the customers are willing to pay (normal distribution) and the Pareto distribution of the price the customers are willing to pay.

When new store is introduced based on the *start-up-probability* the store either dies out or survives based on *cost-of-living*. The store survives when it adjusts its price and location where it can generate the biggest revenue. The choice of decision per store is made by the procedures *new-location-task* and *new-price-task*. When a choice has been made by a store its up to the patches to choose the best store. The best store is chosen by the procedure *choose-store*.

Figure 2: Screenshots from running the model over time. Stores were added during run time. The first 4 screenshots are taken 5 ticks apart, the rest are taken 100 and 150 ticks in.



3.2 Other results

When aggregating for cost of living and start up probability, the following line graphs are produced; both showing the effects of cost of living and start up probability on customer utility.

Figure 3: Average customer utility as a function of the cost of living

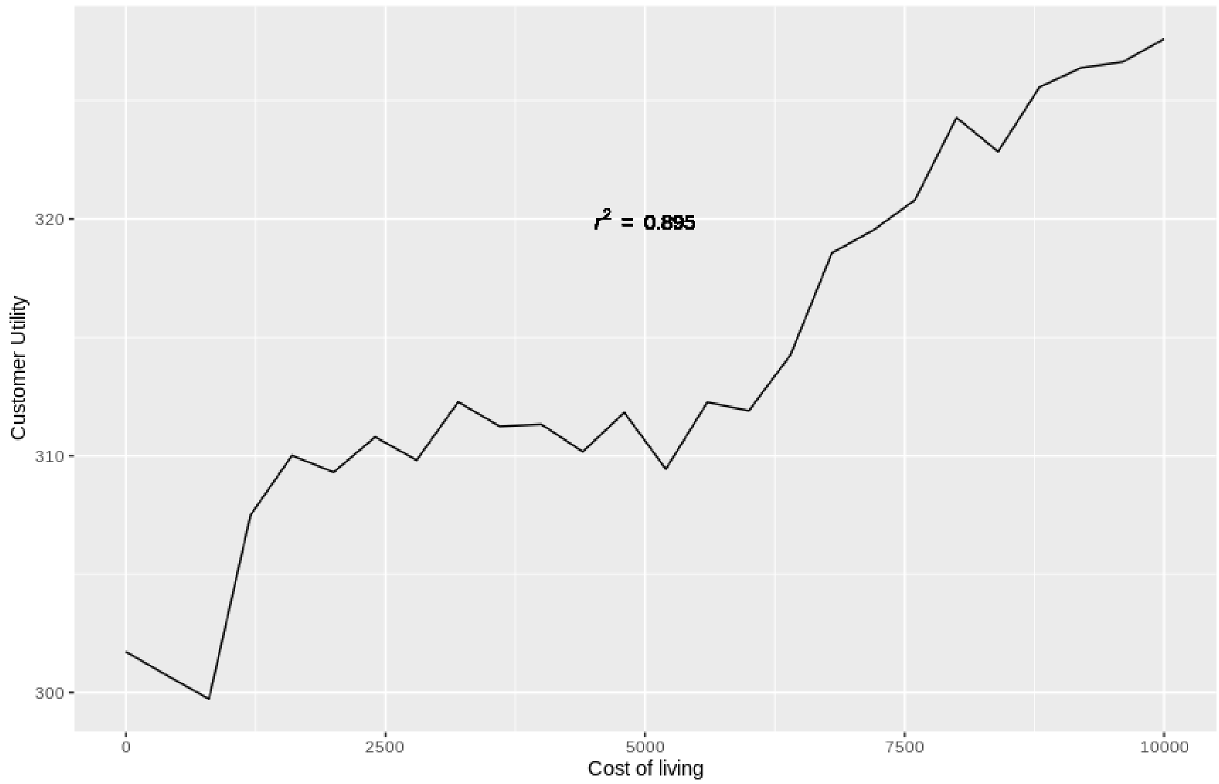


Figure 4: Average customer utility as a function of the start up probability

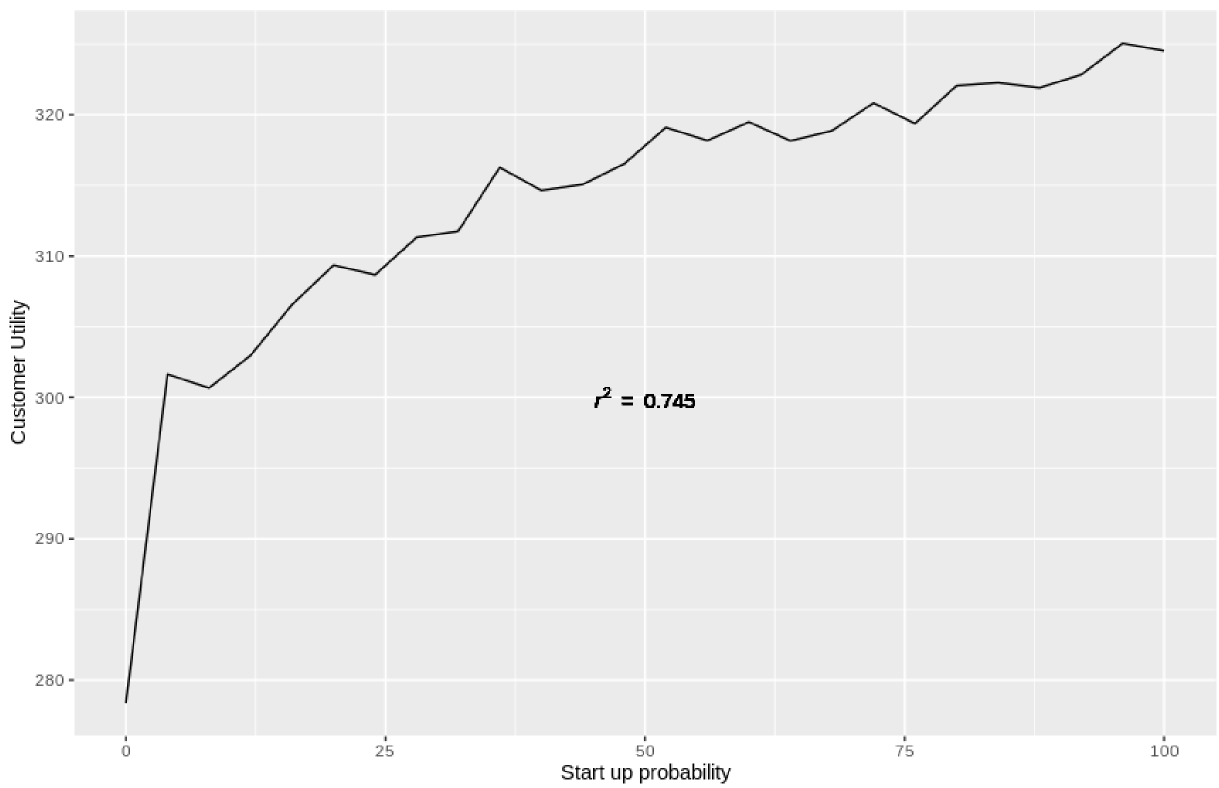
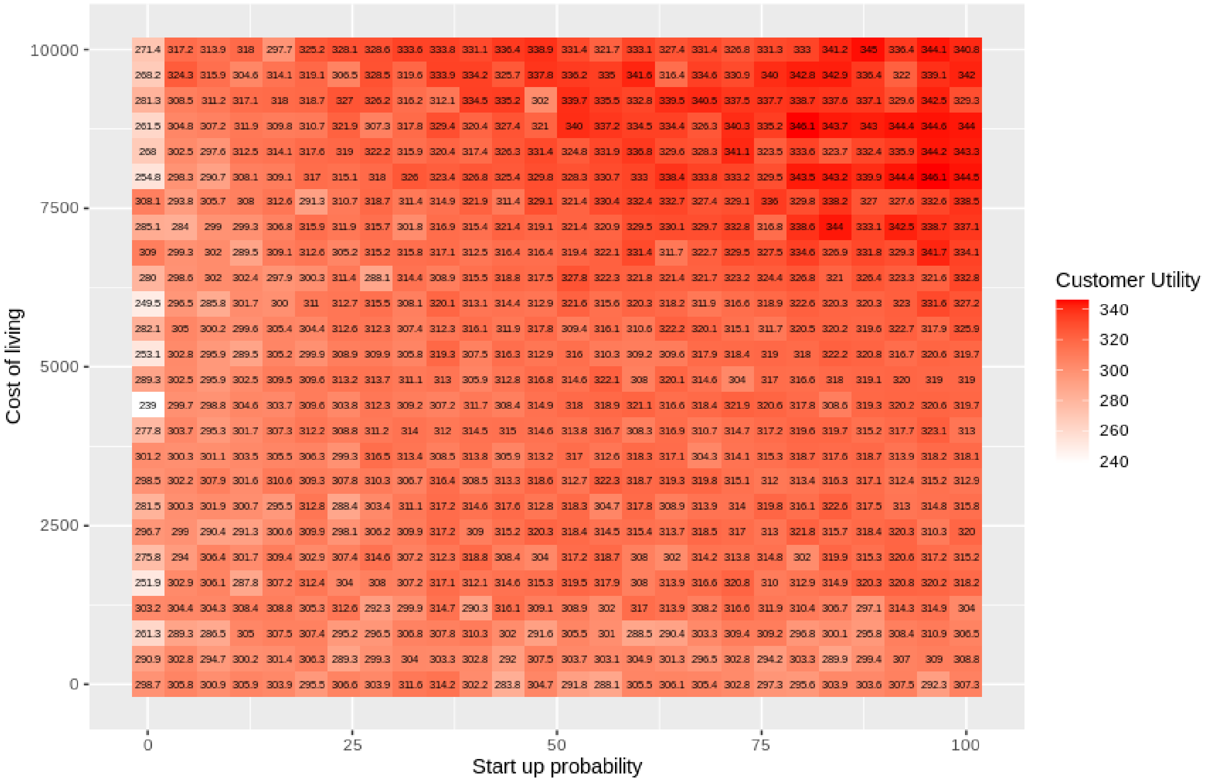


Figure 5: Average customer utility as a heat map projected on cost of living and start up probability



4 Discussion

4.1 Conclusions

Both cost of living and start up probability are highly correlated with customer utility, having an R^2 of 0.895 and 0.745 respectively. This is reflected again in the heat map (Figure 5), where we can see that customer utility increases as the combination of start up probability and cost of living increase as well. The heat map shows that the highest customer utility is achieved when combining high values of both start up probability and cost of living.

4.2 Discussion

One of the limitations of the model is how the stores do not take the competition into account when considering their next move. All though this is simulated in some sense through the communication between all stores and all customers, the model might become more realistic when stores would directly access the prices of their competitors.

5 Division of labor

It is no longer entirely clear how *exactly* the labour was divided as there was a lot of brainstorming and back and forth of ideas and different implementation between the group members. Quite a bit of effort was put into researching, brainstorming, and trial and error testing of what our project could be. Another big sink of our effort was learning the Netlogo language, i.e. adjusting functions and seeing if they work or not. Both are hard to measure afterwards as there is no clear result of this effort in the final product. However, all four of us are content with the amount of work each put into the project. In general the division of work went something along the following lines:

5.1 Group member 1: Anya van der Burg

Report: Introduction and Methods, Division of labor; Coding (Netlogo procedures and fixing Netlogo Plots)

5.2 Group member 2: Daria Levchuk

Report: Results and Conclusion, References ; Coding (Netlogo Methods and R)

5.3 Group member 3: Lazar Popov

Coding (Netlogo procedures and fixing Netlogo Plots), searching for suitable project, proofreading and adjusting the report

5.4 Group member 4: Sander Kaatee

Coding (Netlogo and R), searching for suitable project, proofreading and adjusting the report

6 References

Hotelling, H. (1929). Stability in competition. *The Economic Journal*, *39*(153), 41–57. Retrieved from `http://www.jstor.org/stable/2224214`

Long, B. (2020, Feb). *Long, b., 2020. human traits follow statistical rules. [blog] medium, available at: [accessed 26 march 2021].* Retrieved from `https://medium.com/swlh/human-traits-follow-statistical-rules-b78df3e34569`

Ottino, B., Stonedahl, F., & Wilensky, U. (2009). Netlogo hotelling's law model. *Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL. Retrieved from http://ccl. north western. edu/netlogo/models/HotellingsLaw.*

Talwalkar, P. (2012). *Why are McDonald's and Burger King usually located near each other? fast food location game theory.* `https://mindyourdecisions.com/blog/2012/10/23/why-are-mcdonalds-and-burger-king-usually-located-near-each-other-fast-food-location-game-theory/`.

Tomić, Z. P. (2018). The pareto distribution of world's gdp. *KnE Social Sciences*, 16–32.

Wilensky, U. (1999). *Netlogo. evanston, il: Center for connected learning and computer-based modeling, northwestern university.*