Lindblad

Projekt

nature12016

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1 Introduction

The most important question in thermodynamics is, how to convert thermal energy into work. For such tasks it exist many classical engines, as example the steam-machine. To quantify heat-engines, its commen to look at the ergotropy. In my master-project I will quantify the a three level maser. The three-level maser is a Quantum heat engine (short: QHE). The work extraction from a classicle heat engine is a moving piston. But in this case it is a driving field.. Experimentally we need two different reservoir. The high-temperature reservoir can be realized by a gas noise lamp and the filter by a wave guide cutting off the lower frequencies.

2 Theory

2.1 Model

The Basic of this work is a three-level quantum system in a cavity. This three level system is driven by a hot bath and a cold bath. Those have the temperature T_c and T_h. This quantum system lives in a cavity. In my calculation, the thermal bath is constant, so i can use the Lindblad-masterequation to derive the density matrix ρ.

We can estimate the density matrices with the following differential equation. The master equation is:

$$\dot{\rho}(t) = \frac{1}{i\hbar}[H, \rho] + \mathcal{L}\rho \tag{1}$$

This equation The first part is the vonNeuman equation. The von Neumanequation is the analog of the Schrödinger-equation but for density matrices. This part of the equation is unitary and therefore the process is reversible. So the unitary evolution is described by the Neuman equation.

The interaction Hamiltonian or Jaynes-Cummings Hamiltonian is:

$$H_{int} = \hbar g(\sigma_{12} a^\dagger + \sigma_{21} a) \tag{2}$$

The Hamiltonian of the photons is, which describes the phonons in the cavity:

$$H_{free} = \sum_{i=1}^{3} \hbar\omega_i \left|i\right\rangle \left\langle i\right| + \hbar\omega_f a^\dagger a \tag{3}$$

The total Hamiltonian

$$H = H_{free} + H_{int} \tag{4}$$

The non-unitary part of the equation is depended from the coupling with the thermal bath. $\mathcal{L}$ have three parts. $\mathcal{L}_h$describe the interaction with the hot bath. $\mathcal{L}_c$ Describe the interaction with the cold bath. $\mathcal{L}_{cav}$Describe the Photons which leaves the cavity. so if we have a small κ means less photons will leave and stay in the cavity. we see that in the Fockplott. The interaction with the various environmental heat baths is described by the Liouvillian:

$$\mathcal{L}\hat{\rho} = \frac{\gamma_h}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_h}{k_b T_h}] - 1} + 1\right] \cdot \left(2\sigma_{13} \cdot \rho \cdot \sigma_{13}^\dagger - \sigma_{13}^\dagger \sigma_{13} \rho - \rho \sigma_{13}^\dagger \sigma_{13}\right)$$

$$+\frac{\gamma_h}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_h}{k_b T_H}] - 1}\right] \cdot \left(2\sigma_{31} \cdot \rho \cdot \sigma_{31}^\dagger - \sigma_{31}^\dagger \sigma_{31} \rho - \rho \sigma_{31}^\dagger \sigma_{31}\right)$$

$$+\frac{\gamma_c}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_c}{k_b T_c}] - 1} + 1\right] \cdot \left(2\sigma_{23} \cdot \rho \cdot \sigma_{23}^\dagger - \delta_{23}^\dagger \sigma_{23} \rho - \rho \sigma_{23}^\dagger \sigma_{23}\right)$$

$$+\frac{\gamma_c}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_c}{k_b T_c}] - 1}\right] \cdot \left(2\sigma_{32} \cdot \rho \cdot \sigma_{32}^\dagger - \sigma_{32}^\dagger \sigma_{32} \rho - \rho \sigma_{32}^\dagger \sigma_{32}\right)$$

$$\kappa\left[\frac{1}{\exp[\frac{\hbar w_f}{k_b T_f}]-1}+1\right]\cdot\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

$$\kappa\left[\frac{1}{\exp[\frac{\hbar w_f}{k_b T_f}]-1}\right]\cdot\left(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger\right)$$

(5)

When we work with density matrices, its commen Since we are working with density matrices here, it is easy to work with expectation values with $< A >= Tr[A * \rho]$.

The efficiency is given by the formula:

$$\eta_M = \frac{\nu_s}{\nu_p} \tag{6}$$

3 Calculation

In our case only ω_f interact with the light.

3.1 Software

For the hole implementation of the tree-level-system in a cavity I used qutip. Qutip is library in python, which allows to solve Masterequation pretty fast.