# Lindblad

Projekt

nature12016

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# 1 Introduction

The most important question in thermodynamics is, how to convert thermal energy into work. For such tasks it exist many classical engines, as example the steam-machine. To quantify heat-engines, its commen to look at the ergotropy. In my master-project I will quantify the a three level maser. The three-level maser is a Quantum heat engine (short: QHE). The work extraction from a classicle heat engine is a moving piston. But in this case it is a driving field.. Experimentally we need two different reservoir. The high-temperature reservoir can be realized by a gas noise lamp and the filter by a wave guide cutting off the lower frequencies.

# 2 Theory

## 2.1 Model

The Basic of our system is a three-level quantum system. This three level system is driven by a hot bath and a cold bath. Those have the temperature $T_c$ and $T_h$. This quantum system lives in a cavity. In my calculation, the thermal bath is constant, so i can use the Lindblad-masterequation. The efficiency is given by the formula:

$$\eta_M = \frac{\nu_s}{\nu_p} \tag{1}$$

The master equation is:

$$\dot{\rho}(t) = \frac{1}{i\hbar}[H, \rho] + \mathcal{L}\rho \tag{2}$$

The first part is the vonNeuman equation. The von Neumanequation is the analog of the Schrödinger-equation but for density matrices. This part of the equation is unitary and therefore the process is reversible. The non-unitary part of the equation $\mathcal{L}$ have three parts. $\mathcal{L}_h$describe the interaction with the hot bath. $\mathcal{L}_c$ Describe the interaction with the cold bath. $\mathcal{L}_{cav}$Describe the Photons which leaves the cavity. so if we have a small $\kappa$ means less photons will leave and stay in the cavity. we see that in the Fockplott.

The interaction Hamiltonian or Jaynes-Cummings Hamiltonian is:

$$H_{int} = \hbar g(\sigma_{12}a^\dagger + \sigma_{21}a) \tag{3}$$

The Hamiltonian of the photons is, which describes the phonons in the cavity:

$$H_{free} = \sum_{i=1}^{3} \hbar\omega_i |i\rangle\langle i| + \hbar\omega_f a^\dagger a \tag{4}$$

The total Hamiltonian

$$H = H_{free} + H_{int} \tag{5}$$

The interaction with the various environmental heat baths is described by the Liouvillian:

$$\mathcal{L}\hat{\rho} = \frac{\gamma_h}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_h}{k_b T_h}] - 1} + 1\right] \cdot \left(2\sigma_{13} \cdot \rho \cdot \sigma_{13}^\dagger - \sigma_{13}^\dagger \sigma_{13}\rho - \rho\sigma_{13}^\dagger \sigma_{13}\right)$$

$$+ \frac{\gamma_h}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_h}{k_b T_H}] - 1}\right] \cdot \left(2\sigma_{31} \cdot \rho \cdot \sigma_{31}^\dagger - \sigma_{31}^\dagger \sigma_{31}\rho - \rho\sigma_{31}^\dagger \sigma_{31}\right)$$

$$+ \frac{\gamma_c}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_c}{k_b T_c}] - 1} + 1\right] \cdot \left(2\sigma_{23} \cdot \rho \cdot \sigma_{23}^\dagger - \delta_{23}^\dagger \sigma_{23}\rho - \rho\sigma_{23}^\dagger \sigma_{23}\right)$$

$$+ \frac{\gamma_c}{2}\left[\frac{1}{\exp[\frac{\hbar\omega_c}{k_b T_c}] - 1}\right] \cdot \left(2\sigma_{32} \cdot \rho \cdot \sigma_{32}^\dagger - \sigma_{32}^\dagger \sigma_{32}\rho - \rho\sigma_{32}^\dagger \sigma_{32}\right)$$

$$\kappa\left[\frac{1}{\exp[\frac{\hbar w_f}{k_b T_f}] - 1} + 1\right] \cdot \left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

$$\kappa \left[ \frac{1}{\exp[\frac{\hbar w_f}{k_b T_f}] - 1} \right] \cdot \left( 2a^\dagger \rho a - aa^\dagger \rho - \rho aa^\dagger \right)$$

$$(6)$$

When we work with density matrices, its commen Since we are working with density matrices here, it is easy to work with expectation values with $< A >= Tr[A * \rho]$.

# 3 Calculation

In our case only $\omega_f$ interact with the light.

## 3.1 Software

For the hole implementation of the tree-level-system in a cavity I used qutip. Qutip is library in python, which allows to solve Masterequation pretty fast.