$$Q(t) = (\Phi^*_{(t)}, \Phi(t))$$

$$Q'(t) = (\Phi'^*, \Phi')$$

$$\frac{-\Phi^* \Phi + \Phi \Phi}{\dot{Q}\,\dot{Q}}$$

$$(Q \cdot \dot{Q}' - Q' \cdot \dot{Q})$$

$$\dot{\rho}(Q_f, Q_f', t) = \iint \mathcal{D}[Q(t)]\,\mathcal{D}[Q'(t)]\, e^{Q_f' \cdot (Q - Q') + Q \cdot (Q_f - Q_f')}$$ ← write the phases

$$-\frac{i}{\hbar} \cdot (H_0(Q(t)) - H_0(Q'(t))) \cdot \rho'(Q, Q', t_i)$$

$$- e^{\int_0^t dt_2 \int_0^{t_2} V^x(Q, Q', t_2) \cdot \left(\sum_k c_k^R e^{-i\nu_k^i (t_2 - t_1)} \cdot V^x[Q, Q', t_1] - \sum_{ck} c_k' e^{-i\nu_k^i (t_2 - t_1)} \cdot V^\circ(Q, Q', t_1)\right)}$$

$$i \int_0^t \underbrace{V^x(Q, Q' t)}_{\rho_J} \cdot \underbrace{\left(\sum_k c_k^R e^{i\nu_k^R (t - t_1)} \cdot V^x(Q, Q\, t_1) - \sum c_k^I e^{-i\nu_k^I (t - t_1)} V^\circ(Q Q, t_1)\right)}_{\rho_T}$$

$$\mathring{\varrho}_1(Q,Q',t) = \int D[Q]\,D[Q']\Big[e^{Q'_f\cdot(Q-Q')+Q\cdot(Q_f-Q'_f)}$$

$$-\frac{i}{\hbar}\cdot\left(H_0(Q(t))-H_0(Q'(t))\right)\cdot\varrho_1'(Q,Q',t_0)$$

$$-\left(e^{\int_0^t dt_2\int_0^{t_2}V^\times(Q,Q',t_2)\cdot\left(\sum_k c_k^R e^{-i\nu_k^R(t_2-t_1)}\cdot V^\times[Q,Q',t_1]-\sum_k c_k^I e^{-i\nu_k^I(t_2-t_1)}\cdot V^\circ(Q,Q',t_1)\right)}\right)$$

$$\Bigg(\Bigg(\cdot\left(\int_0^t dt_1\sum_k c_k^R e^{-i\nu_k^R(t-t_1)}\,V^\times[QQ',t_1]-i\sum_k c_k^I e^{-i\nu_k^I(t-t_1)}\,V^\circ(QQ't_1)\right)$$

$$\cdot V^\times(QQ,t)\cdot\int_0^t\sum_k c_k^R e^{-i\nu_k^R(t-t_1)}\,V^\times[QQ',t_1]-i\sum_{c_k} c_k^I e^{-i\nu_k^I(t-t_1)}\,V^\circ(QQ't_1)$$

$$\cdot\varrho_1(Q,Q',t_0)\Bigg)$$

$$+\left(\int_0^t\left(-i\sum_k c_k^R\,e^{-i\nu_k^R(t-t_1)}\cdot\nu_k^R\,V^\times(t_1)-\sum c_k^I\,e^{-i(t-t_1)}\,\nu_k^I\,V^\circ(Q,Q',t_1)\right.\right.$$

$$\left.\cdot\,\varrho(Q,Q,t_0)\right)$$

$$+\left(\left(\sum_k c_k^R\,V^+(Q,Q',t)-i\,c_k^I\,V^\circ(Q,Q',t)\cdot\varrho(Q,Q',t_0)\right)\right]$$

$$Q \quad Q$$

$$\rho_s(Q, Q', t) = \int D[Q]\, D[Q'] \Big[e^{Q_f'\cdot(Q-Q') + Q\cdot(Q_f - Q_f')}$$

$$- \frac{i}{\hbar} \cdot \big(H_0(Q(t)) - H_0(Q'(t))\big) \cdot \rho'(Q, Q', t_0)$$

$$\Bigg(\Bigg(\Bigg(\int_0^t dt_1 \sum_k c_k^R e^{-i v_k^R (t-\tau)} V^\times[Q Q', \tau) - i \sum_{c_k} c_k^I e^{-i v_k^I (t-\tau)} V^\circ(Q, Q' \tau)\Bigg)$$

$$\cdot V^\times(Q Q', t) \cdot \int_0^t \sum_k c_k^R e^{-i v_k^R (t-\tau)} V^\times[Q Q', \tau) - i \sum_{c_k} c_k^I e^{-i v_k^I (t-\tau)} V^\circ(Q, Q' t_1)$$

$$+ \Bigg(\int_0^t \Big(-i \sum_k c_k^R e^{-i v_k^R (t-\tau)} \cdot v_k^R V^\times(t_1) - \sum c_k^I e^{-i(t-\tau)} v_k^I V^\circ(Q, Q', \tau)$$

$$+ \Bigg(\Bigg(\sum_k c_k^R V^+(Q, Q', t) - i\, c_k^I V^\circ(Q, Q', \tau) \cdot \rho(Q, Q', t_0)\Bigg)\Bigg]$$

$$\cdot \Bigg(e^{\int_0^t dt_2 \int_0^{t_2} V^\times(Q, Q', t_2) \cdot \left(\sum_k c_k^R e^{-i v_k^R (t_2 - t_1)} \cdot V^\times[Q, Q', t_1] - \sum_k c_k^I e^{-i v_k^I (t_2 - t_1)} \cdot V^\circ(Q, Q', t_1)\right)}\Bigg)$$

$$\cdot \rho_s(Q, Q', t_0)$$

$$\dot{\rho}_1(Q_f, Q_f', t) = \int D[Q]\,D[Q']\Big[e^{\overset{Q\cdot\ddot{Q}(t)}{Q_f'\cdot(Q-Q') + Q\cdot(Q_f - Q_f')}}$$

$$- \frac{i}{\hbar}\cdot\big(H_0(Q(t)) - H_0(Q'(t))\big)\cdot \rho_1'(Q, Q', t_0)$$

$$\left(e^{\int_0^t dt_2 \int_0^{t_2} V^\times(Q,Q',t_2)\cdot\left(\sum_k c_k^R e^{-i\nu_k'(t_2-t_1)} \cdot V^\times[Q,Q',t_1] - \sum_k c_k^I e^{-i\nu_k'(t_2-t_1)} \cdot V^\circ(Q,Q',t_1)\right)}\right)$$

$$\left(\left(\cdot\left(\int_0^t d\tau \sum_k c_k^R e^{-i\nu_k^R(t-\tau)} V^\times[QQ',\tau) - i\sum_k c_k^I e^{-i\nu_k^I(t-\tau)} V^\circ(QQ',\tau)\right)\right.\right.$$

$$\cdot \underline{\int_0^t \sum_{k'}} V^\times(QQ',t)\cdot c_k^R e^{-i\nu_k^R(t-\tau)} V^\times[QQ',\tau) - i\sum_{c_k} c_k^I e^{-i\nu_k^I(t-\tau)} V^\circ(QQ't_1)$$

$$\left.\cdot\, \rho_0(Q, Q', t_0)\right)$$

$$+\left(\int_0^t \left(-i\sum_k c_k^R e^{-i\nu_k^R(t-\tau)} \oint_{\cdot\,\nu_k} {}^R V^\times(\tau) - \sum c_k^I e^{-i(t-\tau)} \nu_k^I V^\circ(QQ't_1)\right.\right.$$

$$\left.\cdot\, \rho_0(Q, Q, t_0)\right)$$

$$+\left(\left(\underline{\sum_k c_k^R V^+(Q,Q',t) - i\, c_k^I V^\circ(Q,Q',t)\cdot\rho(Q,Q',t_0)}\right)\right)\Big]$$

$$\dot{\rho}_1 = \mathcal{L}_2(\rho_2) + \mathcal{L}_0(\rho_0) + \boxed{}$$

$\mathcal{L}V^x$

$\mathcal{L}_0 \Rightarrow H$

$\rho(Q_0', Q_0, t_0)$

$\rho(Q_e, Q_e', t)$

$\rho_1 = \boxed{}$

$\dot{\rho}_1 = A + B + C$

$A =$ ____

$B =$ ____

$C =$ ____

$$\dot{\rho}_j = \sum_k \dot{\rho}_{j,k} = \sum_j \gamma_k d_k \rho_{j,k}$$

$$= \gamma d(\rho)$$

$$\sum \rho_{j,k} = \rho_j$$

$$\rightarrow \sum_k \gamma_k \rho_{j,k} \overset{?}{=} \gamma \rho_j$$

$$1 + \cancel{2} + 3 + 4 = 10$$

$$\rho_1 \quad \rho_2 \quad \rho_3 \quad \rho_4 = \rho$$

$$\sum_{k=1}^{4} \rho_k = \rho$$

$$\left(\sum_{k=1}^{4} \gamma_k \rho_k \overset{?}{=} \gamma \rho \right)$$

$$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 4$$

$$= 1 + 4 + 9 + 16 = \gamma \rho$$

$$\partial \dot{\rho}_j = \mathcal{L}(\rho_j) + \dots$$

$$\partial_t \sum_k \rho_{jk} = \sum_k \dot{\rho}_{jk} = \sum_k \gamma_k \mathcal{L}_k(\rho_{j,k})$$

$$\sum_j \partial_t \rho_{j,k} = \sum_j \gamma_k \mathcal{L}(\rho_{j,k}) \neq \gamma \mathcal{L}(\rho_j)$$

$$\prod \rho_1 \; F(\dot{\rho}_{1,1}, \dot{\rho}_{1,2}, \bar{\rho}_{1,3}, \rho_{1,n})$$

$$\partial_t \rho_{1,1} = \delta_k(\rho_{2,1})$$

$$\rho_2 = (\quad)$$

$$Q(t) = (\Phi^*(t), \Phi(t)), \quad Q'(t) = (\Phi^{*\prime}(t), \Phi'(t)), \quad V^{o}(Q, Q', t) = V(Q(t)) + V(Q'(t))$$

$$V^{x}(Q, Q', t) = V(Q(t)) - V(Q'(t))$$

$$\dot{\rho}(\Phi_f^*, \Phi_f, t) = \int D[\Phi^*_{(t)}, \Phi_{(t)}] \int D[\Phi'^*_{(t)}, \Phi'_{(t)}] \; e^{-\Phi_f^* \Phi_f + \Phi_f^* \Phi} \cdot \Big[\frac{i}{\hbar} \big(H_S(Q,t) - H_S(Q',t) \big)$$

$$\cdot \rho(\Phi^*, \Phi', t_0) - i V^{x}(Q_f, Q_f', t) \cdot e^{\int_0^t dt_2 \int_0^{t_2} dt_1 \, V^{x}(Q, Q, t_2) \left(\left(c_{k_1}^R e^{-i(t_2 - t_1) v_{k_1}^R} + c_{k_2}^R e^{-i(t_2 - t_1) v_{k_2}^R} \right) \cdot V^{x}(Q, Q', t) \right.}$$

$$\left. - c_{k_1}^I e^{-i(t_2 - t_1) v_{k_1}} + c_{k_2}^I e^{-i(t_2 - t_1) v_{k_2}^I} \right) \cdot V^{o}(Q, Q', t) \Big)$$

$$\int_0^t dt \left(c_{k_1}^R e^{-i(t - t_1) v_{k_1}^R} + c_{k_2}^R e^{-i(t - t_1) v_{k_2}^R} \right) V^{x}(Q, Q', t_2) - i \left(c_{k_1}^I e^{-i(t - t_1) v_k^{I_1}} + c_{k_1}^R e^{-i(t - t_1) v_{k_1}^R} \right) V^{o}(Q, Q', t_1)$$

$$\cdot \rho(\Phi^*, \Phi', t_0) \Big] \; e^{-\Phi_f'^* \Phi_f' + \Phi_f'^* \Phi'}$$

$$\tilde{\mathcal{F}}(Q,Q',t) = e^{\int_0^t dt_2 \int_0^{t_2} dt_1 \, V^{\times}(Q,Q,t_2)\left(\left(c^R_{k_1} e^{-i(t_2-t_1)\nu^R_{k_1}} + c^R_{k_2} e^{-i(t_2-t_1)\nu^R_{k_2}}\right)\cdot V^{\times}(Q,Q',t) - c^I_{k_1} e^{-i(t_2-t_1)\nu_{k_1}} + c^I_{k_2} e^{-i(t_2-t_1)\nu^I_{k_2}}\right)\cdot V^{\circ}(Q,Q',t)\right)}$$

$$\rho_1(\Phi^*_f,\Phi_f,t) = \int D[\Phi^*_{(t)},\Phi_{(t)}]\int D[\Phi'^*_{(t)},\Phi'_{(t)}]\cdot\Big[\mathcal{F}(Q,Q,t)\,\cdot$$

$$\int_0^t dt \left(c^R_{k_1} e^{-i(t-t_1)\nu^R_{k_1}} + c^R_{k_2} e^{-i(t-t_1)\nu^R_{k_2}}\right) V^{\times}(Q,Q',t_2) - i\left(c^I_{k_1} e^{-i(t-t_1)\nu^I_{k_1}} + c^R_{k_1} e^{-i(t-t_1)\nu^R_{k_1}}\right) V^{\circ}(Q,Q',t)$$

$$\cdot\,\rho(\Phi^*,\Phi',t_0)\Big]\, e^{-\Phi'^*_f\Phi'_f + \Phi'^*_f\Phi'}$$

$$|\psi(t_f)\rangle = e^{\frac{i}{\hbar}\cdots H(t_f - t_i)} |\Phi_f\rangle = \iint D[\Phi_f^*, \Phi_f] |\Phi_f\rangle e^{-\Phi_f^*\Phi_f} \langle \Phi_f | \psi(t_f) \rangle$$

$$\rho_A(t_f) = |\psi_f\rangle\langle\psi_f| = \int d\Phi_f^* \, d\Phi \; |\Phi_f\rangle \rho_A'(\Phi_f, \Phi') \langle \Phi_f' |$$

$$[\, H(Q) - H(Q) \,]$$

$$\dot{\rho}_1(\Phi_f^*, \Phi_f, t) = \int D[\Phi^*_{(t)}, \Phi_{(t)}] \int D[\Phi'^*_{(t)}, \Phi'_{(t)}]\ e^{-\Phi_f^* \Phi_f + \Phi_f^* \Phi} \Big[g(Q,Q,t_0) e^{S_s[Q] - S_s[Q']} \cdot F[Q,Q',t]$$

$$\cdot V^x(Q_f, Q_f', t) \left(\int_0^t dt \left(C^R_{k_1} e^{-i(t-t_1)V^R_{k_1}} + C^R_{k_2} e^{-i(t-t_1)V^R_{k_2}} \right) V^x(Q, Q', t_2) - i \left(C^I_{k_1} e^{-i(t-t_1)V^{I}_{k_1}} + C^R_{k_1} e^{-i(t-t_1)V^R_{k_1}} \right) V^0(Q, Q', t_1) \right)^2 \Bigg)$$

$$- i \int_0^t d_1 \Big(\Big(-i\, C^R_{k_1} e^{-i(t-\tau_1) V^R_{k_1}} V^R_{k_1} - i\, C^R_{k_2} e^{-i(t-\tau_1) V^R_{k_2}} V^R_{k_2} \Big) V^x(Q, Q', t_1)$$

$$- i \Big(-i\, C^I_{k_1} e^{-i(t-\tau_1) V^I_{k_1}} V^I_{k_1} - i\, C_{k_2} e^{-i(t-\tau_1) V^I_{k_2}} V^I_{k_2} \Big) V^0(Q, Q', t_1) \Big)$$

$$+ \left(C_1^R + C_2^R \right) V^x(Q_f', Q_f, t) - i \left(C_1^I + C_2^I \right) V^0(Q_f, Q_f', t) \Big) \Big) \Big]$$

$$+ \left(\int_0^t dt \left(C^R_{k_1} e^{-i(t-t_1)V^R_{k_1}} + C^R_{k_2} e^{-i(t-t_1)V^R_{k_2}} \right) V^x(Q, Q', t_2) - i \left(C^I_{k_1} e^{-i(t-t_1)V^{I}_{k_1}} + C^R_{k_1} e^{-i(t-t_1)V^R_{k_1}} \right) V^0(Q, Q', t_1) \right) \underbrace{\partial_t g(Q,Q,t_0) e^{S_s[Q] - S_s[Q']}}\ \cdot$$

$$\cdot\, e^{-\Phi_f'^* \Phi_f' + \Phi_f'^* \Phi'}$$