

$$Q(t) = (\Phi_{(t)}^*, \Phi(t))$$

$$Q'(t) = (\Phi_{(t)}'^*, \Phi'(t))$$

$$\frac{-\Phi^* \Phi + \Phi \Phi^*}{Q \cdot Q}$$

$$(Q \cdot \dot{Q}' - Q' \cdot \dot{Q})$$

$$g'(Q, Q', t) = \iint \mathcal{D}[Q(t)] \mathcal{D}[Q'(t)] e^{Q' \cdot (Q - Q') + Q \cdot (Q' - Q')} \leftarrow \text{write the phases}$$

$$- \frac{i}{\hbar} \cdot (H_0(Q(t)) - H_0(Q'(t))) \cdot g'(Q, Q', t)$$

$$= e^{\int_0^t dt_2 \int_0^{t_2} V^x(Q, Q', t_2) \cdot \left( \sum_k c_k^R e^{-iV_k^R(t_2-t_1)} \cdot V^x(Q, Q', t_1) - \right.$$

$$\left. \sum_{ck} c_k^I e^{iV_k^I(t_2-t_1)} \cdot V^0(Q, Q', t_1) \right)$$

$$= i \int_0^t \underbrace{V^x(Q, Q', t) \cdot \left( \sum_k c_k^R e^{iV_k^R(t-t_1)} \cdot V^x(Q, Q', t_1) - \sum_k c_k^I e^{-iV_k^I(t-t_1)} V^0(Q, Q', t_1) \right)}_{\mathcal{S}_-}$$

$\mathcal{S}_+$

$\mathcal{S}_-$

$$\dot{g}_1(Q, Q', t) = \int D[Q] D[Q'] \left[ e^{Q \cdot \dot{Q}(t)} \right. \\ \left. e^{Q'_f(Q-Q') + Q \cdot (Q_f - Q'_f)} \right. \\ \left. - \frac{i}{\hbar} \cdot (H_0(Q(t)) - H_0(Q'(t))) \cdot g_1(Q, Q', t_0) \right]$$

$$\left( e^{\int_0^t dt_2 \int_0^{t_2} V^X(Q, Q', t_2) \cdot \left( \sum_k c_k^R e^{-iV_k^R(t_2-t_1)} \cdot V^X(Q, Q', t_1) - \sum_k c_k^I e^{-iV_k^I(t_2-t_1)} \cdot V^O(Q, Q', t_1) \right)} \right)$$

$$\left( \left( \int_0^t dt_1 \sum_k c_k^R e^{-iV_k^R(t-t_1)} V^X(Q, Q', t_1) - i \sum_k c_k^I e^{-iV_k^I(t-t_1)} V^O(Q, Q', t_1) \right) \right)$$

$$\cdot V^X(Q, Q', t) \cdot \int_0^t \sum_k c_k^R e^{-iV_k^R(t-t_1)} V^X(Q, Q', t_1) - i \sum_k c_k^I e^{-iV_k^I(t-t_1)} V^O(Q, Q', t_1) \\ \cdot g_1(Q, Q', t_0) \Big)$$

$$+ \left( \int_0^t \left( -i \sum_k c_k^R e^{-iV_k^R(t-t_1)} \cdot V_k^R V^X(t_1) - \sum_k c_k^I e^{-iV_k^I(t-t_1)} V_k^I V^O(Q, Q', t_1) \right) \right)$$

$$\cdot g(Q, Q', t_0) \Big)$$

$$+ \left( \left( \sum_k c_k^R V^X(Q, Q', t) - i \sum_k c_k^I V^O(Q, Q', t) \cdot g(Q, Q', t_0) \right) \right) \Big]$$

$$G_s(Q, Q', t) = \int D[Q] D[Q'] \left[ e^{Q_f'(Q-Q') + Q \cdot (Q_f - Q_f')} \right. \\ \left. - \frac{i}{\hbar} \cdot (H_0(Q(t)) - H_0(Q'(t))) \cdot g'(Q, Q', t_0) \right]$$

$$\left( \left( \int_0^t dt_1 \sum_k c_k^R e^{-i v_k^R (t-t_1)} V^X(Q, Q', t_1) - i \sum_k c_k^I e^{-i v_k^I (t-t_1)} V^O(Q, Q', t_1) \right) \right. \\ \cdot V^X(Q, Q', t) \cdot \int_0^t \sum_k c_k^R e^{-i v_k^R (t-t_1)} V^X(Q, Q', t_1) - i \sum_k c_k^I e^{-i v_k^I (t-t_1)} V^O(Q, Q', t_1) \\ + \left( \int_0^t (-i \sum_k c_k^R e^{-i v_k^R (t-t_1)} \cdot v_k^R V^X(t_1) - \sum_k c_k^I e^{-i (t-t_1)} v_k^I V^O(Q, Q', t_1) \right. \\ \left. \left. + \left( \sum_k c_k^R V^X(Q, Q', t) - i \sum_k c_k^I V^O(Q, Q', t) \cdot g(Q, Q', t_0) \right) \right) \right] \\ \cdot \left( e^{\int_0^t dt_2 \int_0^{t_2} V^X(Q, Q', t_2) \cdot \left( \sum_k c_k^R e^{-i v_k^R (t_2-t_1)} \cdot V^X(Q, Q', t_1) - \sum_k c_k^I e^{-i v_k^I (t_2-t_1)} \cdot V^O(Q, Q', t_1) \right)} \right) \\ \cdot G_s(Q, Q', t_0)$$

$$\dot{g}_1(Q, Q', t) = \int D[Q] D[Q'] \left[ e^{Q \cdot \dot{Q}(t)} \right. \\ \left. - \frac{i}{\hbar} \cdot (H_0(Q(t)) - H_0(Q'(t))) \cdot g_1(Q, Q', t_0) \right]$$

$$\left( e^{\int_0^t dt_2 \int_0^{t_2} V^X(Q, Q', t_2) \cdot \left( \sum_k c_k^R e^{-iV_k^R(t_2-t_1)} \cdot V^X(Q, Q', t_1) - \sum_k c_k^I e^{-iV_k^I(t_2-t_1)} \cdot V^0(Q, Q', t_1) \right)} \right)$$

$$\left( \left( \int_0^t d\tau \sum_k c_k^R e^{-iV_k^R(t-\tau)} V^X(Q, Q', \tau) - i \sum_k c_k^I e^{-iV_k^I(t-\tau)} V^0(Q, Q', \tau) \right) \right)$$

$$\cdot \int_0^t \sum_{k'} V^X(Q, Q', t) \cdot c_{k'}^R e^{-iV_{k'}^R(t-\tau)} V^X(Q, Q', \tau) - i \sum_{c_k} c_k^I e^{-iV_k^I(t-\tau)} V^0(Q, Q', \tau)$$

$$\cdot g_0(Q, Q', t_0)$$

$$+ \left( \int_0^t \left( -i \sum_k c_k^R e^{-iV_k^R(t-\tau)} \cdot V_k^R V^X - \sum_k c_k^I e^{-i(t-\tau)} V_k^I V^0(Q, Q', t) \right) \right)$$

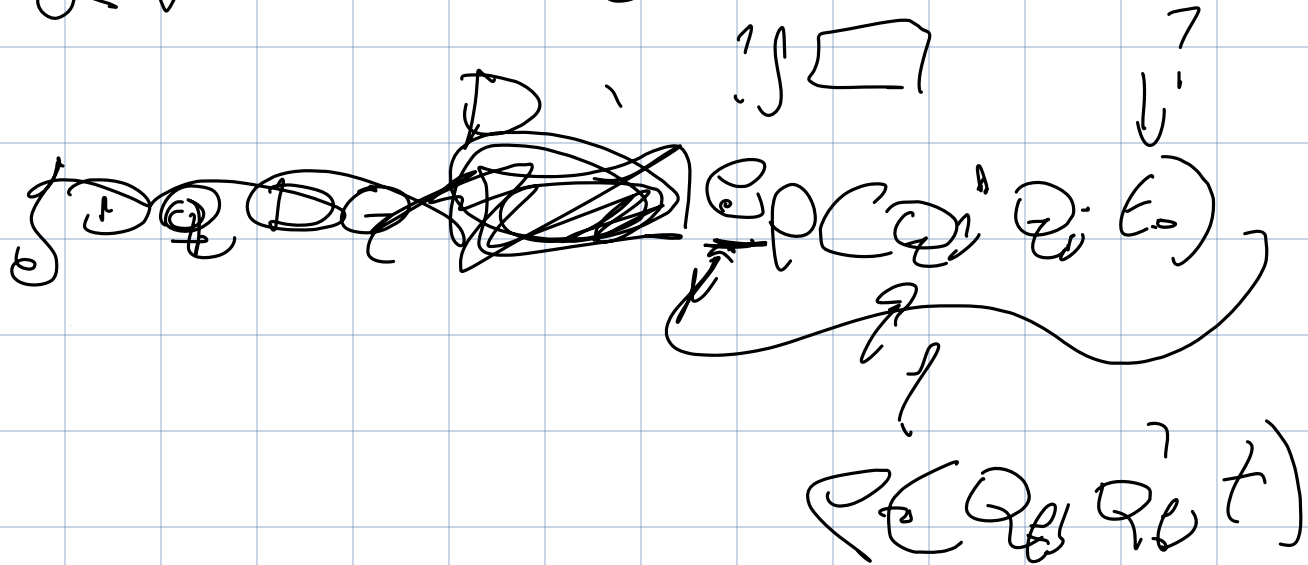
$$\cdot g_0(Q, Q', t_0)$$

$$+ \left( \left( \sum_k c_k^R V^X(Q, Q', t) - i c_k^I V^0(Q, Q', t) \cdot g_0(Q, Q', t_0) \right) \right)$$

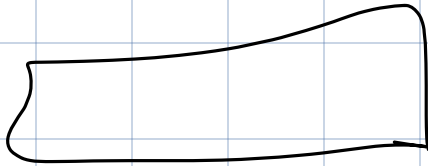
$$\hat{P}_1 = \underbrace{L_2(P_2)}_f + \underbrace{L_0(P_0)}_f + \underbrace{\quad}_f$$

$2V^x$

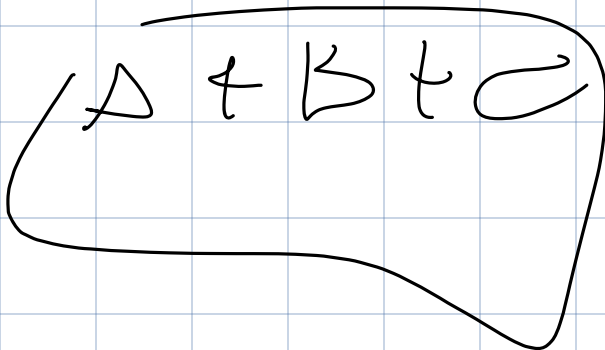
$L_0 \rightarrow H$



$P_0 =$



$P_2 =$



$A =$



$B =$



$C =$



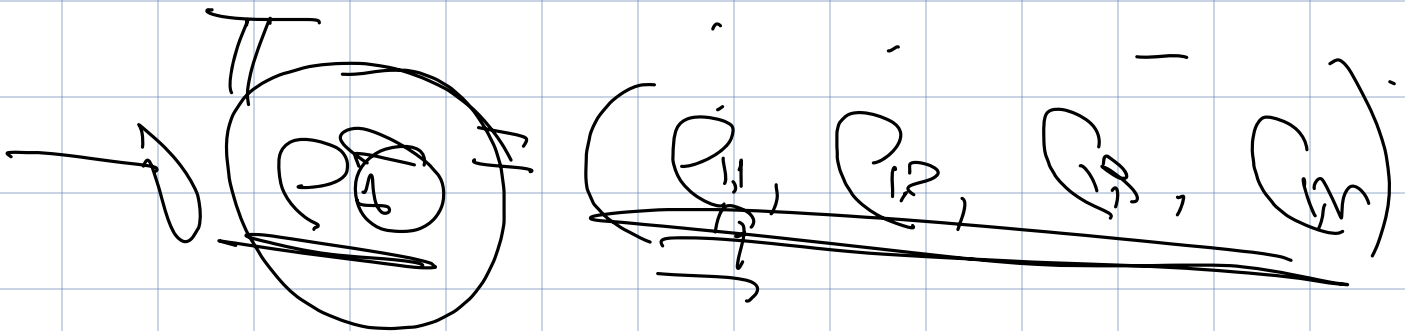


$$\partial \dot{p}_s = \mathcal{L}(p_s) + \dots$$

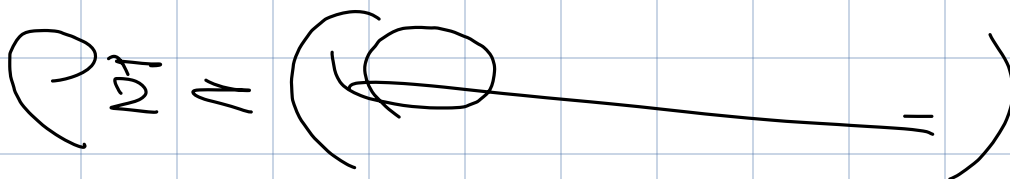
$$\partial_t \sum_k p_{s,k} = \sum \dot{p}_{s,k} = \sum_k \delta_k \mathcal{L}(p_{s,k})$$

$$\sum \partial_t p_{s,k} = \sum_k \delta_k \mathcal{L}(p_{s,k})$$

$$\neq \delta \mathcal{L}(p_s)$$



$$\partial_t p_{11} = \delta \mathcal{L}(p_{2,11})$$



$$Q(t) = (\Phi^*(t), \Phi(t)), \quad Q'(t) = (\Phi^{*'}(t), \Phi'(t)), \quad V^0(Q, Q', t) = V(Q(t)) + V(Q'(t+1))$$

$$V^x(Q, Q', t) = V(Q(t)) - V(Q'(t+1))$$

$$\dot{S}(\Phi_f^*, \Phi_f, t) = \int D[\Phi_f^*, \Phi_f] \int D[\Phi_f^{*'}, \Phi_f'] e^{-\Phi_f^* \Phi_f + \Phi_f^{*'} \Phi_f'} \cdot \left[ \frac{1}{\hbar} (H_S(Q, t) - H_S(Q', t)) \right]$$

$$\cdot S(\Phi^*, \Phi, t_0) = i V^x(Q_f, Q_f', t) \cdot e^{\int_0^t dt_2 \int_0^{t_2} V^x(Q, Q', t_2)} \left( c_{K_1}^R e^{-i(t_2-t_1) V_{K_1}^R} + c_{K_2}^R e^{-i(t_2-t_1) V_{K_2}^R} \right) \cdot V^x(Q, Q', t)$$

$$- c_{K_1}^I e^{-i(t_2-t_1) V_{K_1}^I} + c_{K_2}^I e^{-i(t_2-t_1) V_{K_2}^I} \cdot V^0(Q, Q', t)$$

$$\int_0^t dt \left( c_{K_1}^R e^{-i(t-t_1) V_{K_1}^R} c_{K_2}^R e^{-i(t-t_1) V_{K_2}^R} \right) V^x(Q, Q', t_2) = i (c_{K_1}^I e^{-i(t-t_1) V_{K_1}^I} + c_{K_2}^I e^{-i(t-t_1) V_{K_2}^I}) V^0(Q, Q', t)$$

$$\cdot S(\Phi^*, \Phi, t_0) \int D[\Phi_f^*, \Phi_f] e^{-\Phi_f^* \Phi_f + \Phi_f^{*'} \Phi_f'}$$



$$\mathcal{F}(Q, Q, t) = e^{\int_0^t dt_2 \int_0^{t_2} dt_1} V(Q, Q, t_2) \left( (c_{K_1}^R e^{-i(t_2-t_1) V_{K_1}^R} + c_{K_2}^R e^{-i(t_2-t_1) V_{K_2}^R}) \cdot V^X(Q, Q, t) - c_{K_1}^I e^{-i(t_2-t_1) V_{K_1}^I} + c_{K_2}^I e^{-i(t_2-t_1) V_{K_2}^I} \right) \cdot V^O(Q, Q, t)$$

$$S_1(\Phi_t^*, \Phi_t, t) = \int D[\Phi_{K_1}^* \Phi_{K_2}] D[\Phi_t^*, \Phi_t] \cdot [\mathcal{F}(Q, Q, t)]$$

$$\int_0^t dt \left( c_{K_1}^R e^{-i(t-t_1) V_{K_1}^R} + c_{K_2}^R e^{-i(t-t_1) V_{K_2}^R} \right) V^X(Q, Q, t_2) - i \left( c_{K_1}^I e^{-i(t-t_1) V_{K_1}^I} + c_{K_2}^I e^{-i(t-t_1) V_{K_2}^I} \right) V^O(Q, Q, t)$$

$$\cdot S(\Phi_t^*, \Phi_t, t_0) \Big] e^{-\mathcal{I}_F^* \mathcal{I}_F + \mathcal{I}_F^* \Phi}$$

$$|\varphi(t_f)\rangle = e^{-i\int_{t_0}^{t_f} H(t_f-t_1) dt_1} |\Phi\rangle = \int d[\Phi] e^{-i\int_{t_0}^{t_f} H(t_f-t_1) dt_1} |\Phi\rangle$$

$$S_A(t_f) = |\varphi_f\rangle\langle\varphi_f| = \int d\Phi^* d\Phi |\Phi\rangle\langle\Phi| S_A(\Phi, \Phi^*) |\Phi\rangle\langle\Phi|$$

$$= H(\Phi) - H(\Phi)$$

$$\dot{S}_1(\Phi_t^*, \Phi_t, t) = \int D[\Phi_t^*, \Phi_t] \int D[\Phi_t^*, \Phi_t] e^{-\Phi_t^* \Phi_t + \Phi_t^* \Phi} [g(Q, Q, t) e^{S_1[Q] - S_1[Q]}, F[Q, Q, t]]$$

$$\bullet V^x(Q_t, Q_t, t) \left( \int_0^t dt (C_{K_1}^R e^{-i(t-\tau_1) V_{K_1}^R} + C_{K_2}^R e^{-i(t-\tau_1) V_{K_2}^R}) V^x(Q, Q, t_2) - i(C_{K_1}^I e^{-i(t-\tau_1) V_{K_1}^R} + C_{K_2}^I e^{-i(t-\tau_1) V_{K_2}^R}) V^0(Q, Q, t) \right) V^0(Q, Q, t) \Bigg)^2$$

$$- i \int_0^t d\tau_1 \left( (-i C_{K_1}^R e^{-i(t-\tau_1) V_{K_1}^R} - i C_{K_2}^R e^{-i(t-\tau_1) V_{K_2}^R}) V^x(Q, Q, t_1) \right) V^x(Q, Q, t_1)$$

$$- i \left( (-i C_{K_1}^I e^{-i(t-\tau_1) V_{K_1}^I} - i C_{K_2}^I e^{-i(t-\tau_1) V_{K_2}^I}) V^0(Q, Q, t_1) \right) V^0(Q, Q, t_1)$$

$$+ (C_1^R + C_2^R) V(Q_t^*, Q_t, t) - i(C_1^I + C_2^I) V^0(Q_t, Q_t, t) \Bigg] \Bigg]$$

77

$$+ \left( \int_0^t dt (C_{K_1}^R e^{-i(t-\tau_1) V_{K_1}^R} + C_{K_2}^R e^{-i(t-\tau_1) V_{K_2}^R}) V^x(Q, Q, t_2) - i(C_{K_1}^I e^{-i(t-\tau_1) V_{K_1}^R} + C_{K_2}^I e^{-i(t-\tau_1) V_{K_2}^R}) V^0(Q, Q, t) \right) V^0(Q, Q, t) \Bigg] \underbrace{\partial_t g(Q, Q, t) e^{S_1[Q] - S_1[Q]}}_{\bullet e^{-\Phi_t^* \Phi_t + \Phi_t^* \Phi}}$$

$$\bullet e^{-\Phi_t^* \Phi_t + \Phi_t^* \Phi}$$