$$\mathrm{Tr}\left[H_{\text{free}} \cdot \mathcal{L}_h \varrho\right]$$

$$= \mathrm{Tr}\Big[\Big(\sum_{i=0}^{3} \hbar\omega_i |i\rangle\langle i| + \hbar\omega_c a^\dagger a\Big) \cdot \Big(\frac{\gamma_h}{2}\left[n(\omega_i T_h)+1\right] \cdot \left(2\sigma_{13}\varrho\sigma_{13}^\dagger - \sigma_{13}^\dagger\sigma_{13}\varrho - \varrho\sigma_{13}^\dagger\sigma_{13}\right) +$$

$$+ \frac{\gamma_h}{2}\left[\bar{n}(\omega_i T_h)\left(2\sigma_{31}\cdot\varrho\sigma_{31}^\dagger - \sigma_{31}^\dagger\sigma_{31}\varrho - \varrho\sigma_{31}^\dagger\sigma_{31}\right)\right)$$

$$= \mathrm{Tr}\left[\hbar\omega_h \frac{\gamma_h}{2}\left(n(\omega_h, T_h)+1\right)\left(|3\rangle\langle 3|\, 2\sigma_{13}\varrho\sigma_{13}^\dagger - |3\rangle\langle 3|\,\sigma_{13}^\dagger\sigma_{13}\varrho - |3\rangle\langle 3|\varrho\sigma_{13}^\dagger\sigma_{13}\right)\right.$$

$$+ \mathrm{Tr}\left[\hbar\omega_h \frac{\gamma_h}{2}\, n(\omega_h, T_h)\quad\left(|3\rangle\langle 3|\, 2\sigma_{31}\varrho\sigma_{31}^\dagger - |3\rangle\langle 3|\,\sigma_{31}^\dagger\sigma_{31}\varrho - |3\rangle\langle 3|\varrho\sigma_{31}^\dagger\sigma_{31}\right)\right.$$

$$+ \mathrm{Tr}\left[\hbar\omega_c \frac{\gamma_h}{2}\left(n(\omega_h, T_h)+1\right)\left(|1\rangle\langle 1|\, 2\sigma_{13}\varrho\sigma_{13}^\dagger - |1\rangle\langle 1|\,\sigma_{13}^\dagger\sigma_{13}\varrho - |1\rangle\langle 1|\varrho\sigma_{13}^\dagger\sigma_{13}\right)\right.$$

$$+ \mathrm{Tr}\left[\hbar\omega_c \frac{\gamma_h}{2}\, n(\omega_h, T_h)\quad\left(|1\rangle\langle 1|\, 2\sigma_{31}\varrho\sigma_{31}^\dagger - |1\rangle\langle 1|\,\sigma_{31}^\dagger\sigma_{31}\varrho - |1\rangle\langle 1|\varrho\sigma_{31}^\dagger\sigma_{31}\right)\right.$$

$$+ \underline{\mathrm{Tr}\left[\hbar\omega_2 |2\rangle\langle 2| \cdot \mathcal{L}_h\varrho\right] = 0}$$

$$+ \underline{\mathrm{Tr}\left[\hbar\omega_1 a^\dagger a \cdot \mathcal{L}_h\varrho\right] = 0} \quad /\!/ \text{ kommutiert}$$

$$= \mathrm{Tr}\Big[\hbar\omega_h \frac{\gamma_h}{2}\left(n(\omega_h, T_h)+1\right)\Big(2\cdot|3\rangle\langle 3|\sigma_{13}\varrho\sigma_{13}^\dagger - |3\rangle\langle 3|\sigma_{13}^\dagger\sigma_{13}\varrho - |3\rangle\langle 3|\varrho\sigma_{13}^\dagger\sigma_{13}\Big)$$

$|3\rangle\langle 3|\varrho = -P_3 \qquad -P_3$

$$+ \mathrm{Tr}\Big[\hbar\omega_h \frac{\gamma_h}{2}\, n(\omega_h, T_h)\cdot\Big(2|3\rangle\langle 3|3\rangle\langle 1|\varrho\sigma_{13}^\dagger\Big) = /\!/ \left(2\cdot\sigma_{31}\varrho\sigma_{13}^\dagger\right) = +P_1$$

$$+ \mathrm{Tr}\Big[\hbar\omega_c \frac{\gamma_h}{2}\, n(\omega_h, T_h)+1)\cdot|1\rangle\langle 1|\left(2\cdot\sigma_{13}\varrho\sigma_{13}^\dagger\right) = 2|1\rangle\langle 1|1\rangle\langle 3|\varrho|3\rangle\langle 1| = +P_3$$

$$+ \mathrm{Tr}\Big[\hbar\omega_c \frac{\gamma_h}{2}\, n(\omega_h, T_h)\cdot|-1\rangle\langle -1|\,\sigma_{31}^\dagger\sigma_{31}\varrho - |1\rangle\langle 1|\varrho\sigma_{31}^\dagger\sigma_{31} = -2P_1$$

$$-|3\rangle\langle 3|\varrho\sigma_{13}^\dagger\sigma_{13} = \varrho|3\rangle\langle 1|1\rangle\langle 3|3\rangle\langle 3| = P_3$$

$$= -\hbar\omega_h\gamma_h\cdot n(\omega_h, T_h)+1)\quad P_3$$
$$+\hbar\omega_h\gamma_h\cdot n(\omega_h, T_h)\quad P_3 \Big\} = -\hbar\omega_h\gamma_h P_3$$
$$+\hbar\omega_c\gamma_h\cdot(n(\omega_h, T_h)+1)\quad P_1$$
$$-\hbar\omega_c\gamma_h\cdot(n(\omega_h, T_h)\quad P_2 \Big\} = +\hbar\omega_c\gamma_h P_1$$

$$= \hbar\gamma_h\cdot(\omega_c P_1 - \omega_h P_3)$$

$$\mathrm{Tr}\left[H_{free}\cdot \mathcal{L}_{cav}\,\varrho\right]$$

$$= \Big(\sum_{i=1}^{3}\hbar\omega_i |i\rangle\langle i| + \hbar\omega\, a^\dagger a\Big)\cdot\left[k(\bar{n}+1)\cdot\left(2a\varrho a^\dagger - a^\dagger a\varrho - \varrho a^\dagger a\right)\right]$$

$$+\Big(\sum_{i=1}^{3}\hbar\omega_i |i\rangle\langle i| + \hbar\omega\, a^\dagger a\Big)\cdot\left[k\,\bar{n}\cdot\left(2a^\dagger\varrho a - a a^\dagger\varrho - \varrho a a^\dagger\right)\right]$$

$$= \mathrm{Tr}\left[\hbar\omega k(n+1)\left[2a^\dagger a a\varrho a^\dagger - a^\dagger a a^\dagger a\varrho - a^\dagger a\varrho a^\dagger a\right]\right.$$

$$\left.+\hbar\omega k n\left[2a^\dagger a a^\dagger\varrho a - a^\dagger a a a^\dagger\varrho - a^\dagger a\varrho a a^\dagger\right]\right.$$

$$= \hbar\omega k(n+1)\left[2\mathrm{Tr}\,a^\dagger a^\dagger a a\varrho\right] - 2\mathrm{Tr}\left[a^\dagger a a^\dagger a\varrho\right] \qquad 2\mathrm{Tr}[a^\dagger a\varrho] - \mathrm{Tr}[a^\dagger a^\dagger a a]$$

$$+\hbar\omega\,\mathcal{K}(\bar{n})\cdot\left[2\mathrm{Tr}\left[aa^\dagger a a^\dagger\varrho\right] - \mathrm{Tr}\left[a^\dagger a a^\dagger a\varrho\right] - \mathrm{Tr}\left[a a^\dagger a^\dagger\varrho\right]\right.$$

$$= -\hbar\omega\,\mathcal{K}(n+1)\cdot\mathrm{Tr}\left[a^\dagger a\varrho\right]$$

$$+\hbar\omega\,\mathcal{K}n\cdot\mathrm{Tr}\left[aa^\dagger(aa^\dagger - a^\dagger a)\varrho\right] + \mathrm{Tr}\left[aa^\dagger aa^\dagger\varrho\right] - \mathrm{Tr}\left[a^\dagger a\,a a^\dagger\varrho\right]$$

$$= -\hbar\omega\,\mathcal{K}(n+1)\,2\mathrm{Tr}\left[a^\dagger a\varrho\right] +$$

$$+\hbar\omega k(n)\,2\mathrm{Tr}\left[aa^\dagger\varrho\right] + \mathrm{Tr}\left[[aa^\dagger - a^\dagger a]\,aa^\dagger\varrho\right]$$

$$= -\hbar\omega k(n+1)\,2\mathrm{Tr}\left[a^\dagger a\varrho\right] + \hbar\omega k n\;2\mathrm{Tr}\left[aa^\dagger\varrho\right]$$

$$= 2\hbar\omega k\left(\bar{n} - \langle a^\dagger a\rangle\right)$$

$\mathcal{D}a \quad aa^\dagger - a^\dagger a = 1$