

Working for questions

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MPAGS Stats

1 Question 3

I have run the MotorcycleMicromorts.ipynb. Looking at the code and thinking about the question, this seems to follow a binomial distribution for each step, which can be described by the equation:

$$P = \binom{n}{x} p^x q^{n-x} \quad (1)$$

With n being the total number of steps, x being the number of successful trials, p the probability of surviving the step and q being the probability of not surviving the step.

I have completed this at the bottom of the jupyter notebook. For a number of iterations the probability of the death will be the sum of all the probabilities over the number of iterations

2 Question 4

$$P(\text{Infected}|\text{Identified}) = \frac{P(\text{Infected}) \times P(\text{Identified}|\text{Infected})}{P(\text{Identified}|\text{NotInfected})} \quad (2)$$

$$P(\text{Infected}|\text{Identified}) = \frac{0.2 \times 0.99}{0.015} = 13.2\% \quad (3)$$

3 Question 5: Which distribution? Which distribution best describes the following:

3.1 a) The number of ashes of lightening within one hour of a thunder-storm.

This would be best described by the **Poisson** distribution

3.2 b) The number of Higgs events at the LHC in a year of running.

This would be best described by the **Poisson** distribution

3.3 c) The number of students per hundred carrying the H1F1 virus.

This would be best described by the **Binomial** distribution because it is almost a hit or miss, infected or not infected.

3.4 d) Weight of individual A4 pieces of paper in a notebook

As this is continuous this would be best described by the **Gaussian** distribution

3.5 e) The number of sand grains in 1kg of sand.

This would be best described by the **Poisson** distribution as there would be an expected outcome due to the known weight of the sand

4 Question 6

- 4.1 a) Estimate the significance of this observation, by calculating the probability of observing such an increased number of events as a result of a statistical fluctuation, taking into account that the physicists are looking simultaneously in 84 bins in the mass range from 160 GeV=c2 to 1000 GeV=c2 (for clarity, only some of that range is shown in the plot above).

Using the equation used to calculate significance values shown below we can answer this question

$$\int_{-n}^n \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \quad (4)$$

Where n is the number of sigmas away from the expected value and u is $x - \mu$, where x is the measured value and μ is the expected value. From the plot found in the problem sheet we can calculate these values.

- $\mu = 28$
- $x = 48$
- $\sigma = \sqrt{\mu} = \sqrt{28}$
- $u = 20$

We can now calculate the number of sigmas the measured value is from expected $n = 3.78$. This produces a significance value of 0.9998429

- 4.2 b) Your answer above should be a fairly small number - less than a percent. So, this is quite an interesting result. Why do you think this has still not been published as a discovery of a new particle (that would have been all over the media)?

The significance value of this result is 3.78σ . Usually this doesn't equate to a discovery as this would be 5σ . It could be published as evidence but more tests would have to be done before calling this a discovery.

5 Question 7: Error bars

Looking at the graphs, you would first say that plot d fits quite well with the line but this only really because the errors on the values are massive. In order to say if this data fitted, i would propose to repeat the experiment and try to reduce the errors

Plot a does somewhat fit the theory, however as you would expect due to the very small errors in the y dimension (all most no error in large values of x) this result would not stand up to scrutiny. Plot c doesn't fit well in the center of the range and doesn't really resemble a straight line.

I would argue that Plot b is the best fit, with reasonable errors, with points distributed around the line.

6 Question 8: Probability questions with a twist

- 6.1 i) When tossing a coin which has an equal chance of giving you heads or tails, which of the following (ordered) sequences is the most likely?

I would say that they are already in order, due to the fact that the probability is related to the number of flips: 2^n

- HHHHH

- TTHHHHH
- TTHHHHHTT

6.2 ii) In a terrible road accident on a usually save and accident-free stretch of road in South Yorkshire, 5 people get severely injured. Rank the following statements by the probability that they are true.

I would say that B) (The accident was caused by an experienced driver who was drunk) is most likely, as even though that there are more non drunk drivers, there is not usually accidents on this road, so there is a higher probability that it was caused by something influencing the driver or car.

7 Question 9: The probability density for an unstable particle that exists at time $t_0 = 0$ to decay at a later time t is given by

7.1 a)

First need to normalise the equation:

$$P(t) = Ke^{-\frac{t}{\tau}} \quad (5)$$

$$\int_0^{\infty} Ke^{-\frac{t}{\tau}} dt = 1 \quad (6)$$

$$K\tau = 1 \quad (7)$$

$$K = \frac{1}{\tau} \quad (8)$$

7.2 b) What is the expectation value and the standard deviation of $P(t)$?

$$\langle t \rangle = \int_0^{\infty} tP(t) dt = \int_0^{\infty} tKe^{-tK} dt = \frac{1}{K} \quad (9)$$

$$\langle t^2 \rangle = \int_0^{\infty} t^2P(t) dt = \int_0^{\infty} t^2Ke^{-tK} dt = \frac{2}{K^2} \quad (10)$$

$$\langle \text{Variance} \rangle = \langle V \rangle = \langle t^2 \rangle - \langle t \rangle^2 \quad (11)$$

$$\langle V \rangle = \frac{2}{K^2} - \frac{1}{K^2} = \frac{1}{K^2} \quad (12)$$

$$\langle \sigma \rangle = \sqrt{\langle V \rangle} = \frac{1}{K} \quad (13)$$

7.3 c) There are 3 particles at $t = 0$. What is the probability that at least one of these particles still exists (i.e. has not decayed) at $t = 3\tau$

First we need to work out the probability of a single particle existing at time t :

$$P(t) = \int_0^{3\tau} Ke^{-tK} dt \quad (14)$$

$$P(t) = e^{-tK} \quad (15)$$

$$P(t) = 0.9502 \quad (16)$$

Which is the probability of decaying so for the particle to exist $P(t)_{\text{exist}} = 1 - P(t) = 0.0497$. As this is only for one particle, we times this by 3 $P(t)_{3\text{exist}} = 0.149$. This is also the same if you calculate it through the Poisson distribution:

$$P(\lambda, x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (17)$$

7.4