Linear Approximation

if $f(x)$ near $x=a$,

$f(x) \approx L(x) = f(a) + f'(a)(x-a)$

$\Delta y \approx f'(a)\Delta x$

① ⓐ $f(x) = \sin x$, $a=0$ $f(0.2)$

$L(x) = f(0) + f'(0)(x-0)$

$f(0) = \sin 0 = 0$; $f'(x) = \cos x$

$f'(0) = \cos 0 = 1$

$L(x) = 0 + (x-0) = x$ so

$\boxed{f(0.2) \approx L(0.2) = 0.2}$

(to you physics/ mech e kids, this is the small angle approximation!)

② $f(x) = \cos x$, $a=0$, $f(0.05)$

$f(0) = \cos 0 = 1$; $f'(x) = -\sin x$

$f'(0) = 0$

$L(x) = 1 + 0(x-0) = 1$

$\boxed{f(0.05) \approx L(0.05) = 1}$

③ $f(x) = e^x$, $a=0$, $f(-0.1)$

$f(0) = e^0 = 1$; $f'(x) = e^x$

$f'(0) = e^0 = 1$

$L(x) = 1 + (x-0) = 1+x$ $\boxed{f(-0.1) \approx L(-0.1) = 0.9}$

* L'Hôpital's *

Remember : you can only use it if $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ for $\lim_{x \to a} \frac{f(x)}{g(x)}$ ... (or limits are +/- ∞ ...) as $x \to +/- \infty$

(2) (a) $\lim_{x \to 1} \frac{\ln x}{x-1} \underset{=}{LHR} \lim_{x \to 1} \frac{1/x}{1} = 1$

(more specifically, $\lim_{x \to 1} \ln x = 0$ + $\lim_{x \to 1} x-1 = 0$ so by LHR...)

(b) $\lim_{x \to \infty} \frac{e^{2x} - 4}{e^{3x} + 5} \underset{=}{LHR} \lim_{x \to \infty} \frac{2e^{2x}}{3e^{3x}} = \lim_{x \to \infty} \frac{2}{3} e^{-x}$
$= 0$

(more specifically, $\lim_{x \to \infty} e^{2x} - 4 = \infty$ + $\lim_{x \to \infty} e^{3x} + 5 = \infty$ so by LHR...)

(c) $\lim_{x \to 0} \frac{e^x - x - 1}{x^2} \underset{=}{LHR} \lim_{x \to 0} \frac{e^x - 1}{2x}$

$e^0 - 0 - 1 = 0$

$\underset{=}{LHR} \lim_{x \to 0} \frac{e^x}{2} = \boxed{\frac{1}{2}}$

(more specifically $\lim_{x \to 0} e^x - x - 1 = e^0 - 0 - 1 = 0$ and $\lim_{x \to 0} x^2 = 0$ so by LHR ... then similarly by LHR again...)

NOTE: You must justify your use of L'Hôpitals rule and mention your use of LHR every time. (using $\underset{=}{LHR}$ counts as mentioning)