3.11 – Related Rates

MATH 2554 – Calculus I

Related Rates refer to a class of problems where we are asked to find a rate of change of a quantity but we are not given direct information about the desired quantity.

Instead, we are given information about a quantity that is functionally related to the rate about which we care.

These are some of the most fun problems in Calculus I.

Example A clown inflates a spherical balloon so that the radius increases by 2 cm/sec.

How fast is the volume changing when the radius 24 cm?

- Variables:
- Relationship between the variables:
- Rates Known:
- Rate(s) we seek:
- Use Implicit Differentiation and Solve The Problem.

Steps for Solving Related Rate Problems

1. Read the problem carefully, making a sketch to organize the given information. Identify the rates that are given and the rate that is to be determined.
2. Write one or more equations that express the basic relationships among the variables.
3. Introduce rates of change by differentiating the appropriate equation(s) with respect to time t.
4. Substitute known values and solve for the desired quantity.
5. Check that the units are consistent and the answer is reasonable.

Example: Baseball runners stand at first and second base in a baseball game. At the moment a ball is hit, the runner at first base runs to second base at 18 ft/s; simultaneously, the runner on second runs to third base at 20 ft/s. How fast is the distance between the runners changing 1 second after the ball is hit. Recall that the distance between consecutive bases is 90 ft.

- Variables:
- Relationship between the variables:
- Rates Known:
- Rate(s) we seek:
- Solve!

Exercise: A 13 foot ladder is leaning against a vertical wall when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.5 ft/sec. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?

Exercise: At what rate is iced tea being sipped out of a cylindrical glass that is 6 in tall and has a radius of 3 in? The depth of the tea decreases at a .1 in/min.

A bug on a parabola. A big is moving along the parabola $y = x^2$. At what point on the parabola are the x- and y-coordinates changing at the same rate?

Homework Problems: Section 3.11 (pp.231-233) #11-19 odd, 20-26, 28, 31, 35, 36, 42, 43