Nov 5th Handout: ☆: extra explination for review material, prob not necessary

To graph functions w/ Calc you should (i) find all critical points (ii) identify all VA's and HA's (asymptotes) (iii) determine where function is increasing / decreasing and concave up / down (iv) specifically mark any local min/max's and inflection points (v) find x/y interception pts

① $f(x) = (x-2)^2(x+2) = (x-2)^2 \cdot (x+2)$ = product rule =

(i) $f'(x) = 2(x-2)(x+2) + (x-2)^2 \cdot 1$

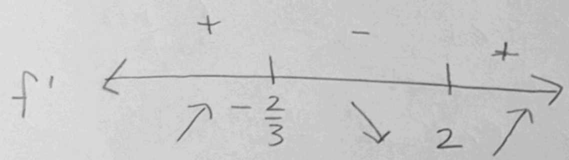
$= (x-2)[2(x+2) + (x-2)]$

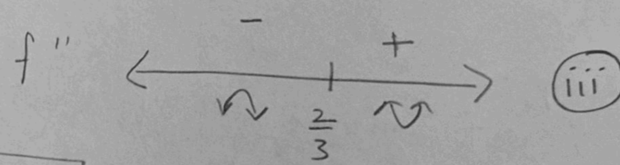
$= (x-2)(3x+2)$ so c.p.'s at $\boxed{x = 2, \; x = -\frac{2}{3}}$

$f''(x) = (3x+2) + (x-2)\cdot 3$

$= 6x - 4$

$x = \frac{2}{3}$ is potential inflect point





$\boxed{f(-\frac{2}{3}) = (-\frac{8}{3})^2(\frac{4}{3}) = 9.\overline{481}}$ is a $\boxed{\text{local max}}$ by 1st or 2nd deriv test (iv)

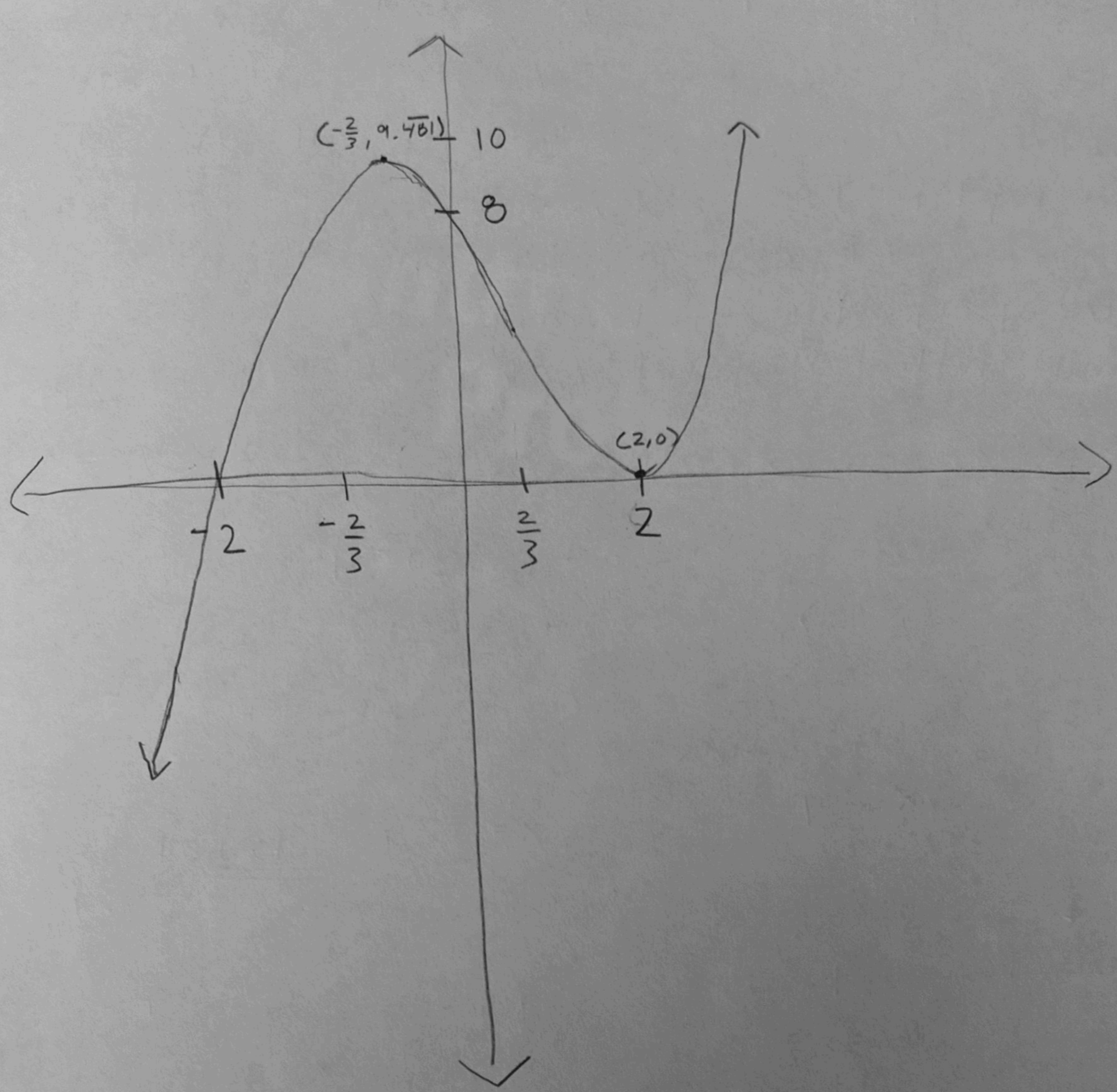
$\boxed{f(2) = 0}$ is $\boxed{\text{local min}}$ by either test

☆ { we could say since f' flips from + to - at $x = -2/3$, $f(-2/3)$ is a local max by 1st deriv test ... just as an example

☆ { we could say since $x = 2$ is a crit point and $f''(2) > 0$, $f(2)$ is a local min by 2nd deriv test ... just as an example

y intercept: $f(0) = (-2)^2(2) = 8$ $\quad f = 0$ at $x = \pm 2$ (v)

HA's: $\lim_{x \to -\infty} f(x) = -\infty$ $\quad \lim_{x \to \infty} f(x) = \infty$ (ii) (whoops out of order...)


$(-\frac{2}{3}, 9.\overline{481})$
10
8
(2,0)
-2
$-\frac{2}{3}$
$\frac{2}{3}$
2

(2) $f(x) = \ln(x^2+1)$ Domain is $(-\infty, \infty)$

$f'(x) = \dfrac{2x}{x^2+1}$

(i) $\boxed{\text{c.p.}: x = 0}$, differentiable everywhere

$$f''(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

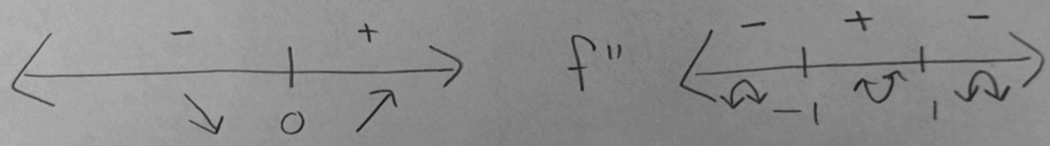
$$= \frac{-2(x^2-1)}{(x^2+1)^2} = \frac{-2(x-1)(x+1)}{(x^2+1)^2}$$ ☆ see below star

$\boxed{\text{potential inflec points}: \quad x = \pm 1}$

(ii) no VA's,

HA's: $\lim_{x \to \infty} \ln(x^2+1) = \infty$

$\lim_{x \to -\infty} \ln(x^2+1) = \infty$

(iii) 

use sign analysis to find these

example:

☆ since $-2(x-1)(x+1) < 0$ if $x < -1$, $f'' < 0$

$\quad - \cdot - \cdot - = -$

since $-2(x-1)(x+1) > 0$ if $-1 < x < 1$, $f'' > 0$

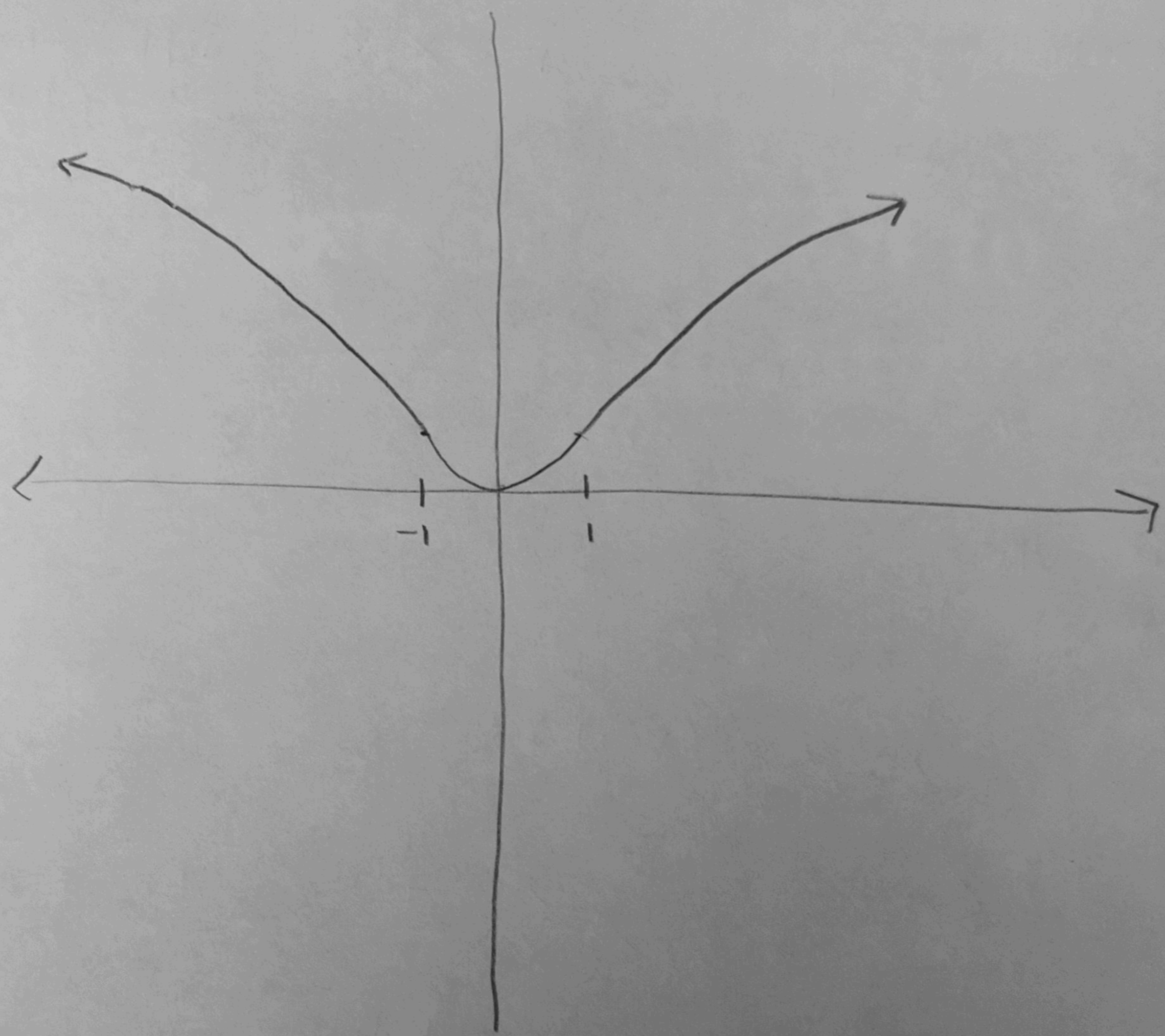
$\quad - \cdot - \cdot + = +$

...

$f(0) = \ln(1) = 0$ is a local min by 1st or 2nd DT (derivative test)

(iv) $x = \pm 1$ is an inflection point $((-1, \ln(2))$ & $(1, \ln(2)))$ since f'' flips from − to + and + to − respectively

(v) $y = 0$ at $x = 0$ only as it is {decreasing before and increasing forever after that



-1
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