Quiz 11 (4/16/20) Name: Drill Time:

TA name:

Directions: This is a take-home quiz. It should be turned in online through blackboard using GradeScope by 11:59pm on **Tuesday April 21**.

Write your solutions on another sheet of paper. The only resources you may use are notes, books, other students *in* the class, the TAs and your instructor. Any other resources (e.g., a friend on your floor, the Internet in general, etc.) are *prohibited* and constitute cheating. When caught you will be referred to the Academic Integrity Office. **You will be graded for completeness and correctness. Include all supporting work. Because you have a long time to complete this, late work will NOT be accepted.**

1. (1 points) Evaluate the following indefinite integral: $\int x^{-1/3}\,dx$.

$$\int x^{-1/3}dx = \frac{x^{2/3}}{\frac{2}{3}} + c$$

$$= \frac{3}{2}x^{2/3} + c$$

2. (1 points) Give an expression representing all antiderivatives of $f(x) = \frac{1}{1+x^2}$.

$$\int \frac{1}{1+x^2}dx = \arctan(x) + c$$

3. (2 points) Evaluate the following indefinite integral. $\int \frac{3}{x} + \csc^2 x\,dx$. You should not use a computer/calculator for this. Instead, you should only use the rules in Ch. 4 Section 9 of the text book. **Include all steps.**

$$\int \left(\frac{3}{x} + \csc^2 x\right)dx$$

$$= 3\int \frac{1}{x}dx + \int \csc^2 x\,dx$$

$$= 3\ln|x| + C_1 - \cot x + C_2$$

$$= 3\ln|x| - \cot x + C$$

By table 4.10 in Section 4.9, $\int \frac{1}{x}dx = \ln|x| + c$ and by table 4.9 in Section 4.9, $\int \csc^2 x\,dx = -\cot x + c$.

4. (2 points) Solve the initial value problem: $h'(t) = e^t - t^2 + 1$ subject to $h(0) = 3$.

$$h(t) = \int (e^t + t^2 + 1)\,dt = e^t + \tfrac{1}{3}t^3 + t + C$$

$$h(0) = 3$$

$$e^0 + \tfrac{1}{3}(0)^3 + (0) + C = 3$$

$$1 + C = 3$$

$$C = 2$$

$$\boxed{h(t) = e^t + \tfrac{1}{3}t^3 + t + 2}$$

5. (2 points) Let $f(x) = x^2 + 1$. Compute a left Riemann sum over the interval $[0, 2]$ with $n = 4$. Is this an over or under approximation of the area beneath the graph of $x^2 + 1$, above the $x$-axis and between $y = 0$ and $y = 2$?

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

Left endpoints: $0, \tfrac{1}{2}, 1, \tfrac{3}{2}$

| $x$ | $f(x)$ |
|---|---|
| $0$ | $0^2 + 1 = 1$ |
| $\frac{1}{2}$ | $\left(\frac{1}{2}\right)^2 + 1 = \frac{5}{4}$ |
| $1$ | $(1)^2 + 1 = 2$ |
| $\frac{3}{2}$ | $\left(\frac{3}{2}\right)^2 + 1 = \frac{13}{4}$ |

Approximate Area:

$$1\left(\tfrac{1}{2}\right) + \tfrac{5}{4}\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{2}\right) + \tfrac{13}{4}\left(\tfrac{1}{2}\right) = \frac{15}{4}$$

This is an under approximation.

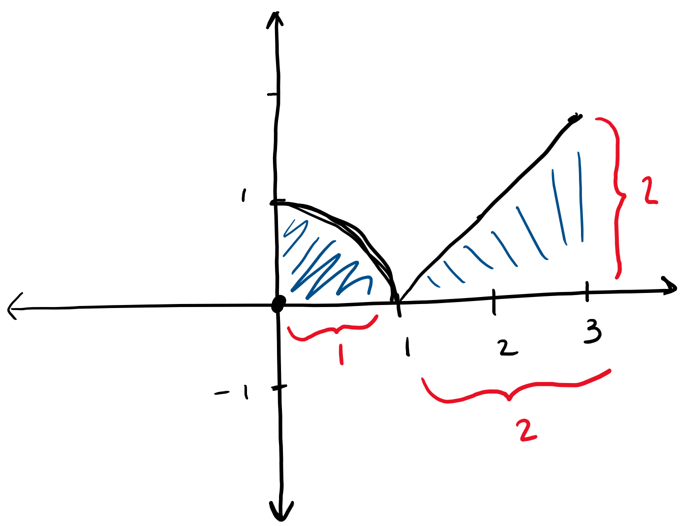
6. (2 points) Using geometry, evaluate $\int_0^3 f(x)\,dx$ given that

$$f(x) = \begin{cases} \sqrt{1-x^2} & 0 \le x \le 1 \\ x-1 & 1 < x \le 3 \end{cases}.$$

Hint: Over $[0, 1]$ the graph of $f(x)$ looks like part of a circle.

$A_C = \pi r^2$

$A_T = \frac{1}{2} bh$




$$A = \frac{1}{4}\pi(1)^2 + \frac{1}{2}(2)(2)$$

$$= \frac{1}{4}\pi + 2 \approx 2.79$$