① "use symmetry"

$$\int_{-2}^{2} (x^4 - xe^{-x^2})\,dx = \underbrace{\int_{-2}^{2} x^4} - \int_{-2}^{2} xe^{-x^2}$$

* notice $f(-a) = f(a)$ for $\underline{x^4}$
thus it has <u>even</u> symmetry

* now for xe^{-x^2}: $f(a) = ae^{-a^2}$ $\quad f(-a) = -ae^{-(-a)^2}$
$= -ae^{-a^2}$

thus it has <u>odd</u> symmetry so

$$\int_{-2}^{2} xe^{-x^2} = 0$$

and

$$\int_{-2}^{2} x^4 = 2\int_{0}^{2} x^4 = 2\,\frac{x^5}{5}\Big|_0^2 = \frac{64}{5}$$

$$\therefore \boxed{\int_{-2}^{2} (x^4 - xe^{-x^2})\,dx = \frac{64}{5}}$$

② "average value" Recall: $\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$

"avg val of f"

so thus for $f(x) = \sec^2 x$ on $[0, \pi/4]$

$$\bar{f} = 4/\pi \int_0^{\pi/4} \sec^2 x = 4/\pi \tan x \Big|_0^{\pi/4}$$

$$= 4/\pi \left[\tan(\pi/4) - \tan(0)\right]$$

$$= \boxed{4/\pi}$$

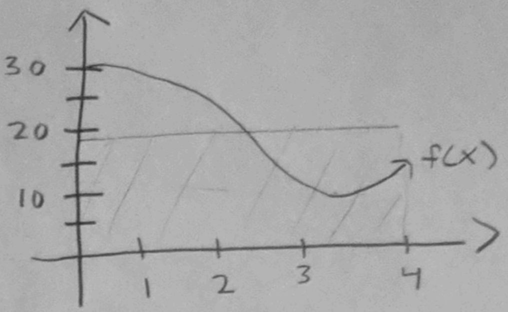
(3) $f(x) = x^3 - 5x^2 + 30$ " avg valve of elev " $0 \le x \le 4$

$$\bar{f} = \frac{1}{4}\int_0^4 x^3 - 5x^2 + 30$$

$$= \frac{1}{4}\left(\frac{x^4}{4}\Big|_0^4 - \frac{5x^3}{3}\Big|_0^4 + 30x\Big|_0^4 \right)$$

$$= \frac{1}{4}\left(64 - \frac{320}{3} + 120\right) = \frac{232}{12}$$

$$= \boxed{19.\bar{3}}$$



(4) " Check avg val of deriv of f over [a, b] is same as avg rate of change of f over same interval

Define our function as $F(x)$
and our derivative as $f(x)$

Thus the avg rate of change of $F(x)$ = $\frac{F(b) - F(a)}{b-a}$ (A)

and the avg val of the deriv of $F(x)$, $f(x)$ = $\frac{1}{b-a}\int f(x)\,dx$

$$= \frac{1}{b-a}\left(F(b) - F(a)\right) = \frac{F(b) - F(a)}{b-a} \quad \text{(B)}$$

Since (A) ≡ (B), they are the same ☑