Exam Review

Overarching Idea

Throughout this section we try to apply Calculus to real world problems.

3.11 (Related Rates) shows us how to use the chain rule (implicit differentiation) to determine how a situation changes over time

4.1-4.4 Teaches us about extrema (maxima and minimas) in order to prepare us for **4.5** which deals with optimization. We can use this idea of maximas or minimas to find a way to use the least (minimum) about of material in order to get the most (maximize) volume or area.

In **4.6** we learn how to approximate computationally difficult functions and in **4.7** we learn about L'Hopital's which simply adds another tool to our toolbox.

Related Rates (3.11)

Related Rates

You’re generally interested in the rate of change with respect to time. You must determine the relationship between variables and potentially substitute known numerical values or variable relationships in order to solve the problem.

Related Rates

Related Rates heavily rely on the concepts from the chain rule, so let's reconsider this rule real fast in a few scenarios:

The changing area of a circle:

$$A(t) = \pi r(t)^2 = A(r(t))$$

$$\frac{d}{dt}A(t) = A'(r(t)) \cdot r'(t) = 2\pi \cdot r(t) \cdot r'(t)$$

The changing volume of a cube:

$$V(t) = x(t)^3 = V(x(t))$$

$$\frac{d}{dt}V(t) = V'(x(t)) \cdot x'(t) = 3x(t)^2 \cdot x'(t)$$

Related Rates

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Problems using pythagorean theorem:

$$y(t) = x(t)^2$$

$$D^2(t) = x(t)^2 + y(x(t))^2 = x(t)^2 + x(t)^4$$

$$\frac{d}{dt}2D = 2D'(x(t)) = 2 \cdot x(t)\frac{dx}{dt} + 4 \cdot x(t)^3\frac{dx}{dt}$$

Extrema and Graphing Functions (4.1-4.4)

Extrema and Graphing Functions

Finding absolute extrema over an interval:

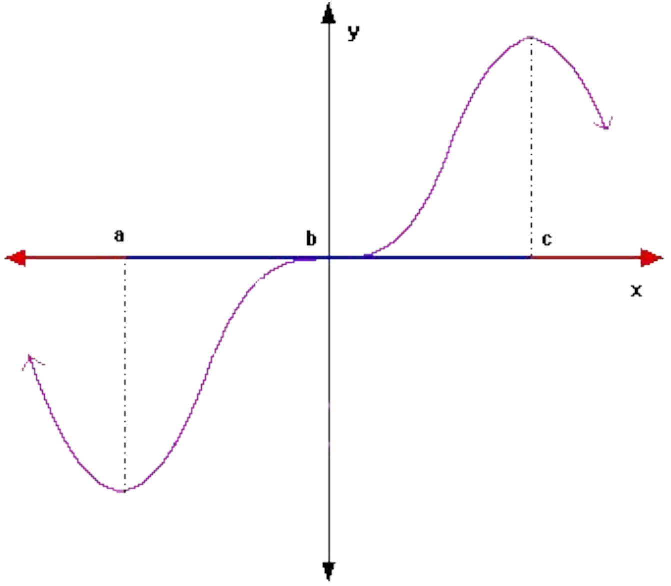
- Find all critical points (when f’ = 0 of f’ is undefined) x=a
- Compare values of function at critical points (f(a)) and values at endpoints e (f(e))

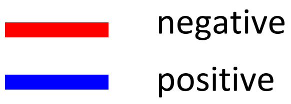
Finding local extrema:

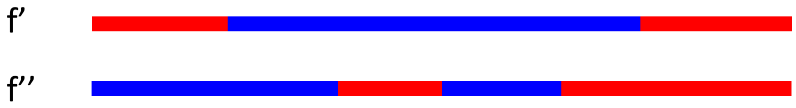
- Find all critical points (when f’ = 0 of f’ is undefined) x=a
- Use 1st or 2nd derivative test to determine if each critical point is a maxima or minima or neither.

Extrema and Graphing Functions

Thinking about the 1st and 2nd derivative test intuitively:





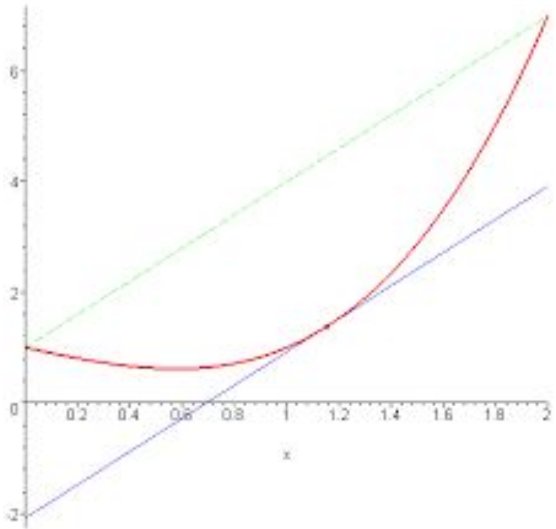


Extrema and Graphing Functions

Mean Value Theorem

If f is continuous on a closed interval $[a, b]$ and differentiable on (a, b), then there is at least one point c in (a, b) so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$



Extrema and Graphing Functions

Mean Value Theorem

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Rolle's Theorem

Let f be a continuous function on a closed interval $[a, b]$ and be differentiable on (a, b) with $f(a) = f(b)$. There is at least one point c in (a, b) with $f'(c) = 0$.

Optimization Problems (4.5)

Optimization Problems Guideline

1. Read the problem, identify the the variables, draw a picture if possible
2. Identify objective function
3. Identify constraints
4. Use constraints to remove all but one variable
5. Find the interval of interest for the one remaining variable (and use this as your endpoints to **prove** you've maximized or minimized the function)
6. Find the absolute maximum or minimum

Linear Approximation (4.6)

Linear Approximation

Let $L(x)$ be the linear approximation to $f(x)$ at the point $(a, f(a))$. Then

$$L(x) = f(a) + f'(a)(x - a).$$

Example:

- **Question: Approximate ln(1.01)** Answer: f(x) = ln(x) and a=1, so L(1.01) = ln(1) + 1/(1) (x-1)
- **Question: Approximate sqrt(64.2)** Answer: f(x) = sqrt(x) and a=64...

L'Hopital's (4.7)

L'Hopital's

If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$, then $\lim_{x \to a} \frac{f'(x)}{g'(x)}$

"0/0" and "infinity/infinity" are called "indeterminate forms"

Make sure you **mention** and **justify** your use of L'Hopital's EVERY time you use it.