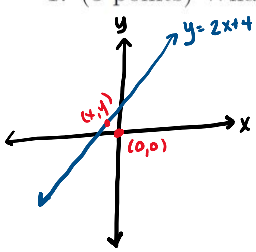
Quiz 10 (4/2/20) Name: Drill Time:

TA name:

Directions: This is a take-home quiz. It should be turned in online through blackboard using GradeScope by 11:59pm on **Tuesday April 7**.

Write your solutions on another sheet of paper. The only resources you may use are notes, books, other students *in* the class, the TAs and your instructor. Any other resources (e.g., a friend on your floor, the Internet in general, etc.) are *prohibited* and constitute cheating. When caught you will be referred to the Academic Integrity Office. **You will be graded for completeness and correctness. Include all supporting work. Because you have a long time to complete this, late work will NOT be accepted.**

1. (3 points) What point on the line $y = 2x + 4$ is closest to the origin?



$d = \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2+y^2}$

interval of interest: $(-\infty, \infty)$

$y = 2x+4$

$d(x) = \sqrt{x^2+(2x+4)^2}$

$= \sqrt{x^2+4x^2+16x+16}$

$= \sqrt{5x^2+16x+16}$

$d'(x) = \frac{1}{2}(5x^2+16x+16)^{-1/2}(10x+16)$

$\frac{10x+16}{2(5x^2+16x+16)^{1/2}} = 0$

$10x+16 = 0$

$10x = -16$

$x = -8/5$

$2(5x^2+16x+16)^{1/2} = 0$

$5x^2+16x+16 = 0$

$x = \frac{-16 \pm \sqrt{16^2-4(5)(16)}}{2(5)}$

no real solution

$y = 2x+4$

$y(-4/5) = 2(-8/5)+4$

$= -\frac{16}{5} + \frac{20}{4}$

$= \frac{4}{5}$

$-$ $+$

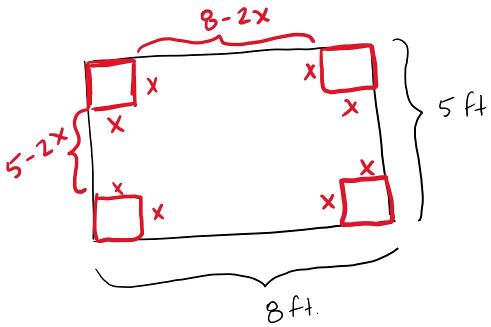
$-8/5$

$d'(-2) = \frac{-4}{4} = -1 < 0$

$d'(0) = \frac{16}{8} = 2 > 0$

Since d' changes from negative to positive at $x=-8/5$, then by the first derivative test, there is a relative minimum at $x=-8/5$. Since this is the only critical value, there is an absolute minimum at $x=-8/5$. Hence, the point on the line $y=2x+4$ closest to the origin is $(-8/5, 4/5)$

2. (4 points) Squares with sides of length x are cut out of each corner of a rectangular piece of cardboard measuring 5 ft by 8 ft. The resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way.



$$V = x(8-2x)(5-2x)$$
$$= x(40 - 10x - 16x + 4x^2)$$
$$= 40x - 26x^2 + 4x^3$$

$$V'(x) = 40 - 52x + 12x^2$$
$$12x^2 - 52x + 40 = 0$$
$$3x^2 - 13x + 10 = 0$$
$$(3x-10)(x-1) = 0$$
$$x = -\frac{10}{3} \qquad x = 1$$

not in interval

$3x^2 - \cancel{13}\, 3x - 10x + 10$

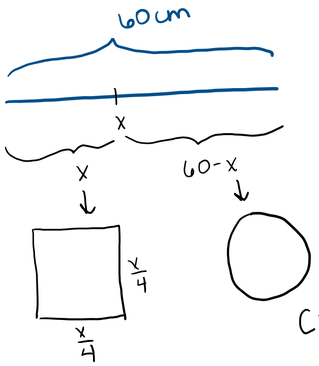
$3x(x-1) - 10(x-1$

Interval of Interest:
$[0, 5/2]$

$$V(0) = 0$$
$$V(5/2) = \frac{5}{2}(8-5)(0) = 0$$
$$V(1) = (8-2)(5-2) = 18$$

Since $V(x)$ is a continuous function on the closed interval $[0, 5/2]$, then by the Extreme Value theorem, the absolute maximum is guaranteed to exist. Indeed, this maximum occurs when $x = 1$, and the volume of the largest such box would be $18\ ft^3$.

3. (3 points) A piece of wire of length 60 cm is cut and the resulting two pieces are formed to make a circle and a square. Where should the wire be cut to minimize the combined area of the circle and the square? Hint: Theorem 4.9 on page 262 of textbook allows you to avoid complicated comparisons.



$C = 60 - x = 2\pi r$

$r = \frac{60-x}{2\pi}$

$A_s = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$

$A_c = \pi\left(\frac{60-x}{2\pi}\right)^2 = \pi\left(\frac{3600 - 120x + x^2}{4\pi^2}\right) = \frac{900}{\pi} - \frac{30}{\pi}x + \frac{1}{4\pi}x^2$

Interval of interest: $[0, 60]$

$A(x) = \frac{1}{16}x^2 + \frac{900}{\pi} - \frac{30}{\pi}x + \frac{1}{4\pi}x^2$

$A'(x) = \frac{1}{8}x - \frac{30}{\pi} + \frac{1}{2\pi}x$

$\left(\frac{1}{8} + \frac{1}{2\pi}\right)x - \frac{30}{\pi} = 0$

$\left(\frac{\pi+4}{8\pi}\right)x = \frac{30}{\pi}$

$x = \frac{30}{\pi}\left(\frac{8\pi}{\pi+4}\right) = \frac{240}{\pi+4} \approx 33.61$

$A''(x) = \frac{1}{8} + \frac{1}{2\pi}$

$A''\left(\frac{240}{\pi+4}\right) = \frac{1}{8} + \frac{1}{2\pi} > 0$

$60 - x = 60 - \frac{240}{\pi+4} \approx 26.39$

Since $A''\left(\frac{240}{\pi+4}\right) > 0$, then there is a relative minimum at $x = \frac{240}{\pi+4}$. Since this is the only critical value on our interval, then there is an absolute minimum at $x = \frac{240}{\pi+4}$. Hence, to maximize the combined area of the circle and square, the wire should be cut so that 33.61 cm are used for the square and 26.39 cm for the circle.