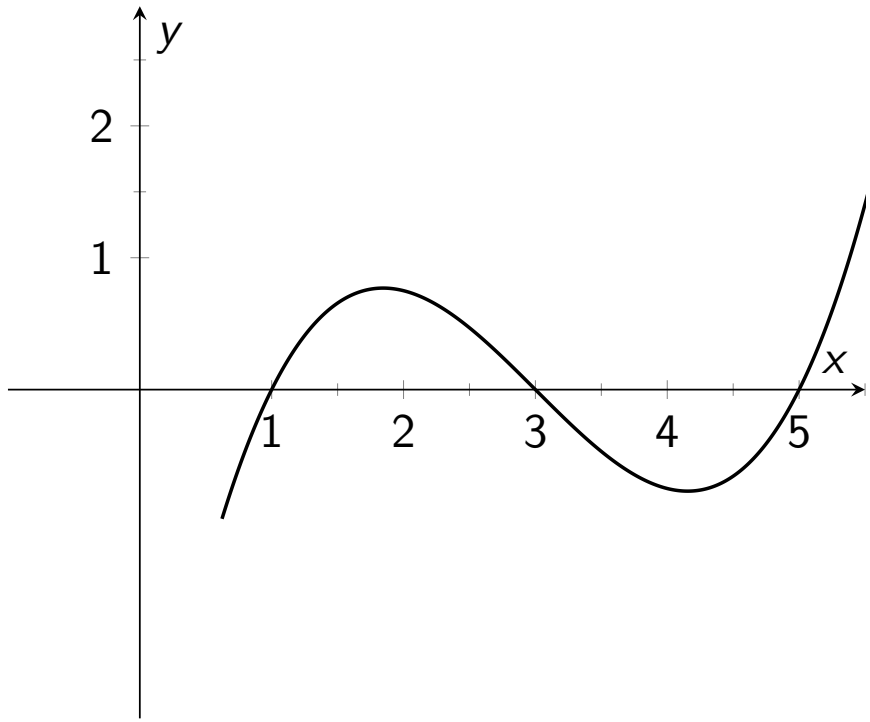
3.2 – Working with Derivatives

MATH 2554 – Calculus I

Fall 2019

Graphs of derivatives:



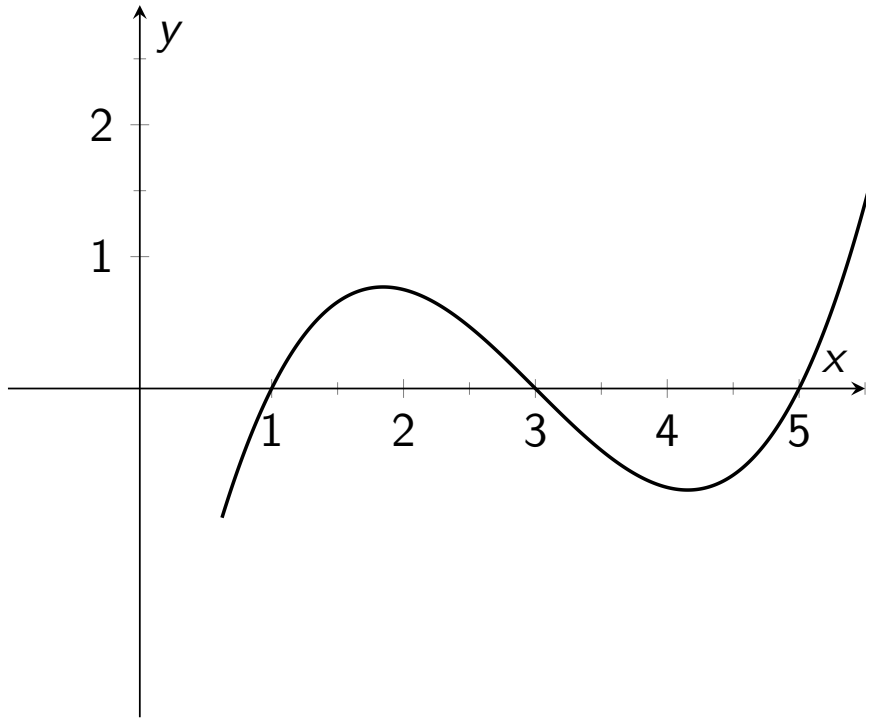
Question: How can we understand the derivative from the graph of $f(x)$?

The graph of the derivative is essentially the graph of the collection of slopes of the tangent lines of a graph. If you just have a graph (without an equation for the graph), the best you can do is approximate the graph of the derivative.

Simple checklist:

1. Note where $f'(x) = 0$.
2. Note where $f'(x) > 0$ (what does this look like?)
3. Note where $f'(x) < 0$ (what does this look like?)

Problem: Sketch the graph of $f'(x)$.



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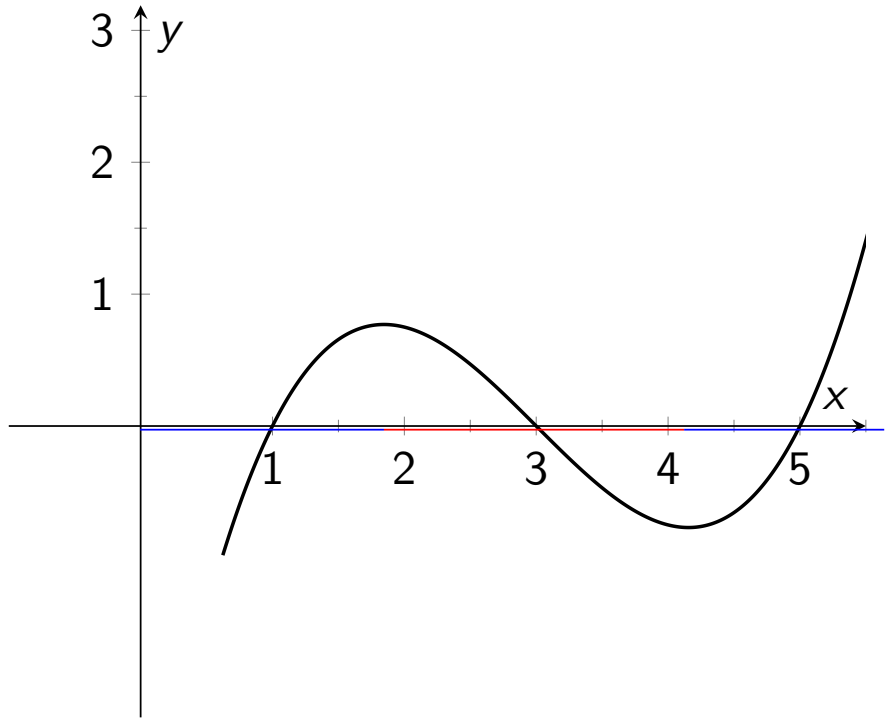

Solution: Highlight Regions where f is increasing and where Regions where f is decreasing.

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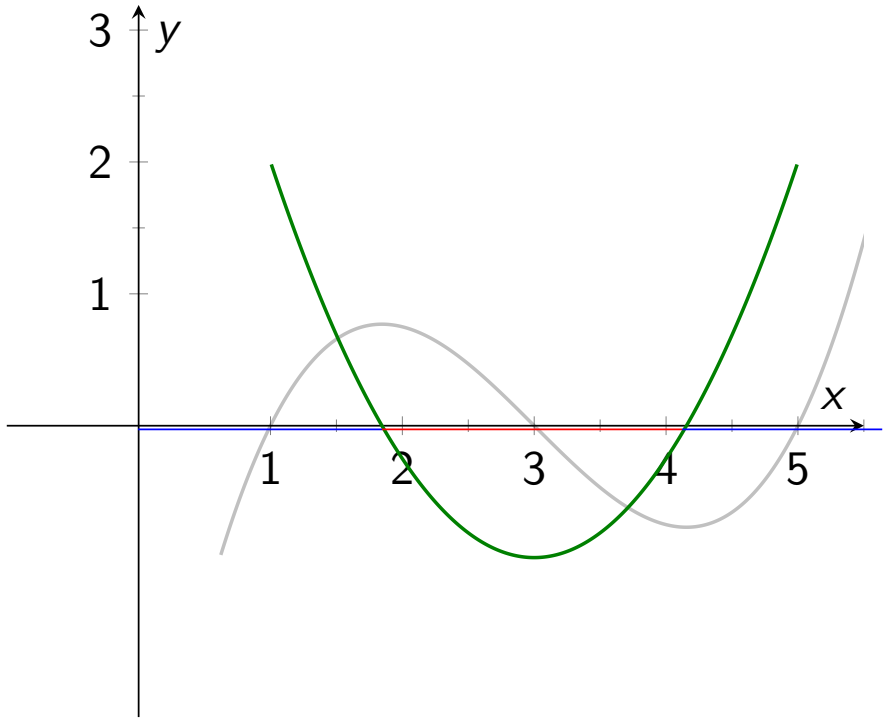
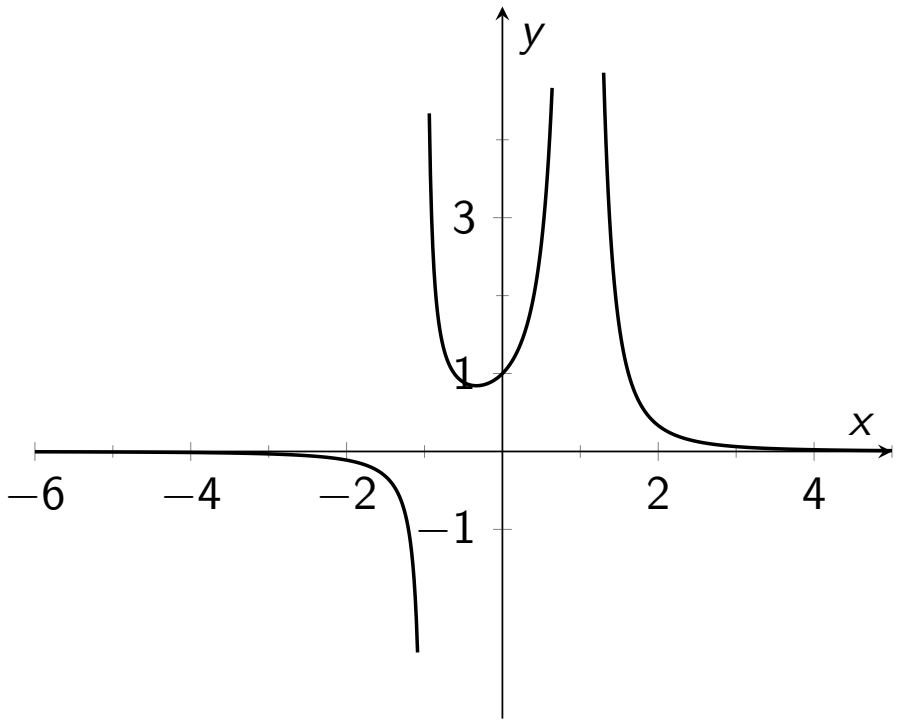


Solution: The graph of $f'(x)$ is positive where f is increasing and negative where f is decreasing.

An example with asymptotes. Consider $f(x) = \frac{1}{(x+1)(x-1)^2}$.



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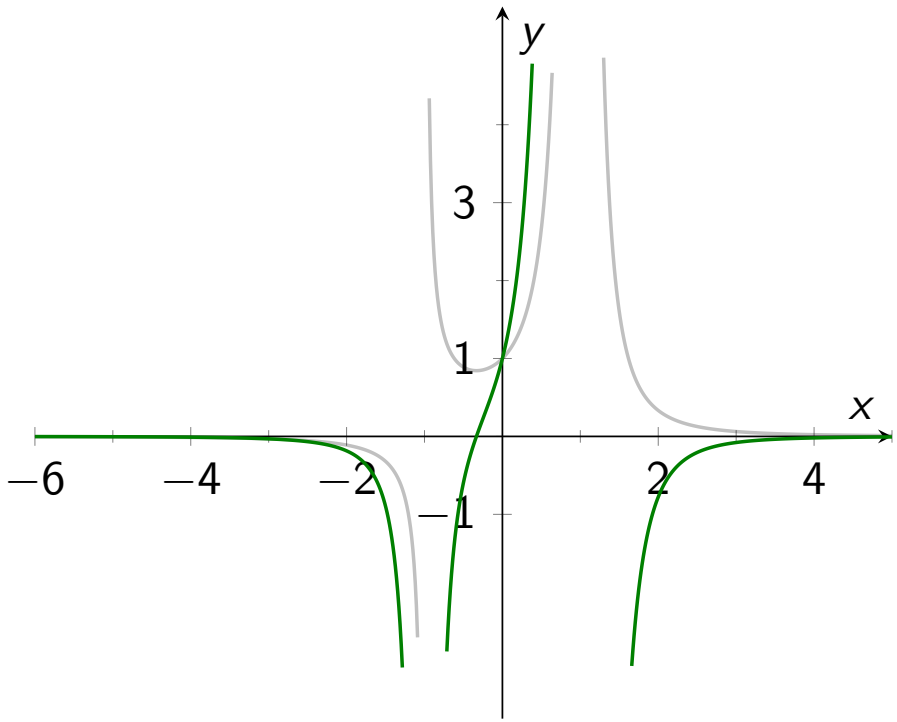

Solution: The graph of $f'(x)$.

Continuity and Differentiability

Theorem (Differentiability implies Continuity)

If f is differentiable at x, then f is continuous at x.

Note: The converse is false!!

Example: $f(x) = |x|$.

Find $f'(x)$.

Obstructions to Differentiability

A function f is NOT differentiable at a if any of the following conditions hold:

1. f is not continuous at a.
2. f has a corner at a.
3. f has a vertical tangent at a

Example: $f(x) = \sqrt{|x|}$.

Homework Problems: Section 3.2 (pp. 148-149): # 16-30, 35,37,47-50, 64