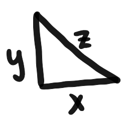
Quiz 8 (3/12/20) Name: Drill Time:

TA name:

Directions: This is a take-home quiz. It should be turned in at the beginning of drill on **Tuesday March 17**.

Your solutions must be written on this paper. No other pages can be included, so you should first work the solution on another page and then **neatly** copy your answer onto this quiz. Failure to meet any of these requirements will result in a zero score. The only resources you may use are notes, books, other students *in* the class, the TAs, the MRTC, and your instructor. Any other resources (e.g., a friend on your floor, the Internet in general, etc.) is *prohibited* and constitutes cheating. When caught you will be referred to the Academic Integrity Office. **You will be graded for completeness and correctness. Include all supporting work. Because you have almost a week to complete this, late work will NOT be accepted.**

1. (3 points) Two boats leave a port at the same time. One travels north at 15 miles per hour and the other travels east at 18 miles per hour. How fast is the distance between the boats changing after two hours of travel?



$\frac{dy}{dt} = 15$ mi/hr $\quad \frac{dx}{dt} = 18$ mi/hr

After 2 hours:

$x = 2(18) = 36$ mi

$y = 2(15) = 30$ mi

$x^2 + y^2 = z^2$

$z = \sqrt{x^2+y^2}$

$z = \sqrt{36^2 + 30^2}$

$z = \sqrt{2196} = 6\sqrt{61}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(36)(18) + 2(30)(15) = 2(6\sqrt{61})\frac{dz}{dt}$

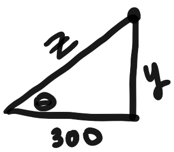
$2196 = (12\sqrt{61})\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{183}{\sqrt{61}} \approx 23.43$ mi/hr

After 2 hours of travel, the distance between the two boats is increasing at a rate of 23.43 mi/hr.

More on back!

2. (3 points) An observer stands 300 ft from the launch site of a hot-air balloon at an elevation equal to the elevation of the launch site. The balloon is launched vertically and maintains a constant upward velocity of 30 ft/s. What is the rate of change of the angle of elevation of the balloon when it is 400ft from the ground? (Note the angle of elevation is the angle θ between the observer's line of sight to the balloon and the ground.)



$\frac{dy}{dt} = 30$ ft/s

When $y = 400$:

$z = \sqrt{400^2 + 300^2} = 500$

$\cos\theta = \frac{300}{500} = \frac{3}{5}$

$\tan\theta = \frac{y}{300}$

$\sec^2(\theta)\,\frac{d\theta}{dt} = \frac{1}{300} \cdot \frac{dy}{dt}$

$\frac{d\theta}{dt} = \left(\frac{1}{300}\right)\cos^2\theta\,\frac{dy}{dt}$

$\frac{d\theta}{dt} = \left(\frac{1}{300}\right)\left(\frac{3}{5}\right)^2(30) = .036$ radians/s

When the balloon is 400 ft from the ground, the angle of elevation is increasing at a rate of .036 radians/s.

3. (4 points) Let $f(x) = x^{1/2}(3 - x)$. Find the absolute maximum and absolute minimum values of the $f(x)$ on the interval $[1, 3]$.

$f(x) = 3x^{1/2} - x^{3/2}$

$f'(x) = \frac{3}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$

$\frac{3}{2}x^{-1/2} - \frac{3}{2}x^{1/2} = 0$

$\frac{3}{2}x^{-1/2}(1 - x) = 0$

$\frac{3}{2x^{1/2}} = 0$ undef. @ $x = 0$

$1 - x = 0$

$x = 1$

Critical values: $x = 0, 1$

not in interval.

Since $f(x)$ is a continuous function on the closed interval $[1,3]$, then by the Extreme Value theorem, the absolute maximum and minimum values are guaranteed to exist.

$f(1) = 1^{1/2}(3-1) = 1(2) = 2$

$f(3) = 3^{1/2}(3-3) = 0$

On the interval $[1,3]$, there is an absolute maximum of 2 at $x = 1$ and an absolute minimum of 0 at $x = 3$.