4.1 – Maxima and Minima

MATH 2554 – Calculus I

Applications of the Dervative: Chapter 4 studies applications of the derivative. Our first application is to maxima and minima of functions

Definition (Absolute Maximum and Minimum)

Let f be defined on a set D containing a real number c. If $f(c) \geq f(x)$ for every x in D, then $f(c)$ is an absolute maximum value of f on D. If $f(c) \leq f(x)$ for every $x \in D$, then $f(c)$ is an absolute minimum value of f on D. An absolute extreme value is either an absolute maximum value or an absolute minimum value.

The existence and location of absolute extreme values depend on both the function and interval of interest.

Examples:

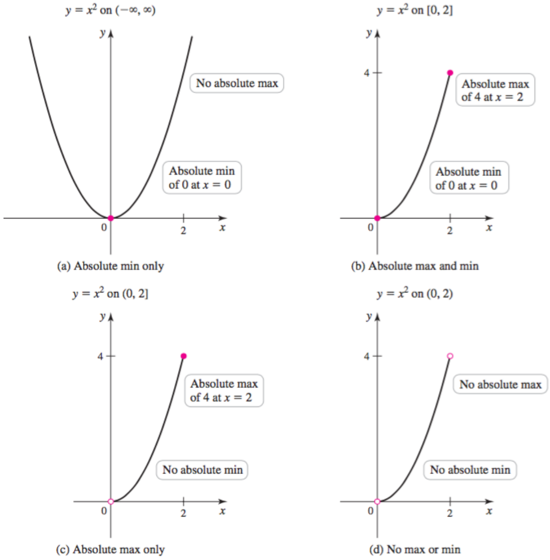


Figure: Examples of Minima and Maxima

Examples:

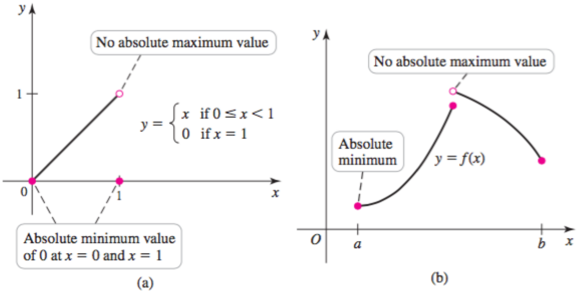


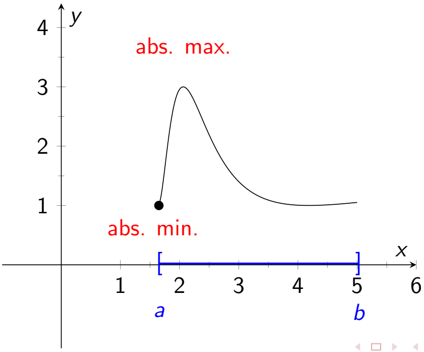
Figure: More examples of Minima and Maxima

Problem: Sometimes absolute extrema exist and sometimes they don't.

Theorem (Extreme Value Theorem)

A function that is continuous on the closed interval $[a, b]$ has an absolute maximum value and absolute minimum value on that interval.

Example:



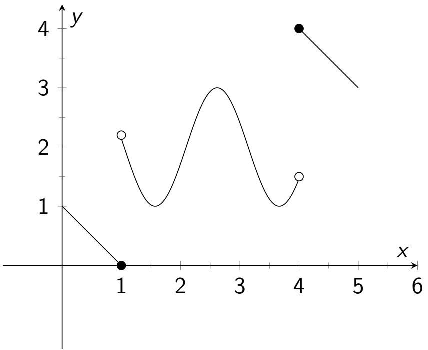
Local Maxima and Minima.

Definition (Local Maximum and Minimum Values)

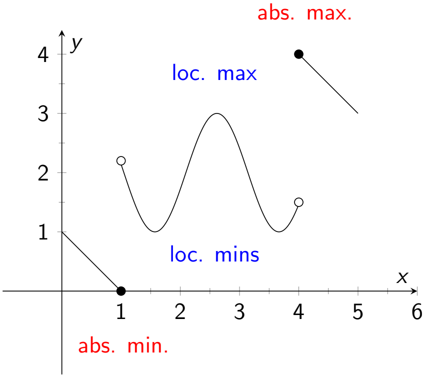
Suppose c is an interior point of some interval I on which f is defined. If $f(c) \geq f(x)$ for all x in I, then $f(c)$ is a local maximum value of f.

If $f(c) \leq f(x)$ for all x in I, then $f(c)$ is a local minimum of f.

Find Local Maxima and Minima.



Local Maxima and Minima.



Local Maxima and Minima. As the pictures suggest,

Theorem (Local Extreme Value Theorem)

If f has a local maximum or minimum value at c and $f'(c)$ exists, then $f'(c) = 0$.

Exercise: Let $f(x) = x^3$. Compute $f'(0)$. What does this mean?

Procedure for Locating Absolute Extreme Values of a Continuous Function on a Closed Interval

Definition (Critical Point)

An interior point c of the domain of f at which $f'(c) = 0$ or $f'(c)$ fails to exist is called a critical point of f.

Procedure to find absolute extrema of f on $[a, b]$ when f is continuous on $[a, b]$:

1. Find all critical points of f in (a, b) (that is, find points c where $f'(c) = 0$ or $f(c)$ DNE.
2. Evaluate f at the critical points and the endpoints of $[a, b]$.
3. Choose the largest and smallest values of f from Step 2 for the absolute maximum and absolute minimum values.

Find the absolute extrema using The Procedure for:

1. $f(x) = x^4$ on $[-2, 2]$
2. $g(x) = (x+1)^{\frac{4}{3}}$ on $[-8, 8]$
3. $h(x) = |2x - x^2|$ on $[-2, 3]$.

Homework Problems: Section 4.1 (pp.247-249) #13-35 odd, 41-65 odd, 72-74, 76