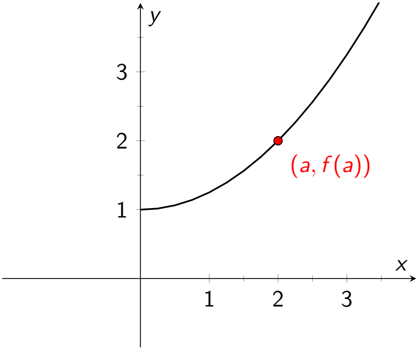
3.1 – Introducing the Derivative

MATH 2554 – Calculus I

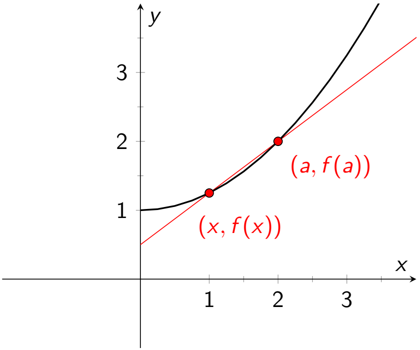
Fall 2019

Consider the following graph:



We want to find the slope of the tangent line to $f(x)$ at a.

Consider the following graph:

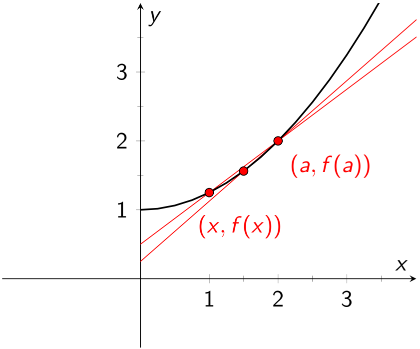


We see the slope of the secant line is

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}.$$

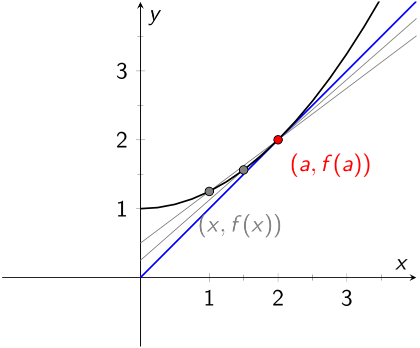
The number m_{sec} is the average rate of change of f in the interval $[a, x]$.

Consider the following graph:



As $x \to a$, the secant lines limit to a tangent line. The slope of the tangent line is

$$m_{\tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$



The slope of the tangent line is

$$m_{\tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

The slope of the tangent line $m_{\tan}$ is also

1. The instantaneous rate of change of f at a;
2. The slope of the tangent line at $(a, f(a))$, provided the limit exists.

The tangent line through $(a, f(a))$ has slope $m_{\tan}$, and consequently is given by the line

$$y - f(a) = m_{\tan}(x - a).$$

Problem: Find the tangent line to $f(x) = x^2 + 2x + 2$ at the point $(1, 5)$.

An alternative definition. By thinking of $x = a + h$, then the average rate of change of f over $[a, a + h]$ is the slope

$$m_{\text{sec}} = \frac{f(a+h) - f(a)}{h}$$

and therefore the instantaneous rate of change of f at a is

$$m_{\text{tan}} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Problem: Find the slope of the tangent line of $f(x) = \frac{1}{x}$ at the point $(2, \frac{1}{2})$.

Definition (The Derivative Function)

The derivative of f is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists and x is in the domain of f. If $f'(x)$ exists, we say that f is differentiable at x. If f is differentiable at every point of an open interval I, we see that f is differentiable on I.

Alternative Notation: The symbol Δ often means "change". So Δx means "change in x", etc.

Then

$$m_{\text{sec}} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\Delta y}{\Delta x}$$

and

$$m_{\tan} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{dy}{dx}.$$

The notation $\frac{dy}{dx}$ is sometimes called Leibniz notation.

Homework Problems: Section 3.1 (pp. 137-139): #9-45 odd, 54, 55.

NOTE: We do not know any rules for differentiation yet (e.g., power rules, chain rules, etc.). In this section, you are strictly using the definition of the derivatives and definition of slope of tangent lines we have derived!!!