2.2 – Definitions of Limits

MATH 2554 – Calculus I

Fall 2019

Definition (Limit of a Function)

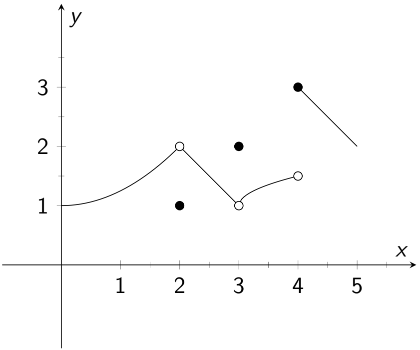
Suppose the function f is defined for all x near a except possibly at a. If $f(x)$ is arbitrarily close to L (that is, as close to L as we like) for all x sufficiently close (but not equal) to a, we write

$$\lim_{x \to a} f(x) = L$$

and say the limit of $f(x)$ as x approaches a equals L.

Question: is it always true that $\lim_{x \to a} f(x) = f(a)$?

Finding limits from a graph



Find $f(1)$ $\qquad$ $\lim_{x \to 1} f(x)$

$f(2)$ $\qquad$ $\lim_{x \to 2} f(x)$

$f(3)$ $\qquad$ $\lim_{x \to 3} f(x)$

$f(4)$ $\qquad$ $\lim_{x \to 4} f(x)$

Definition (One-Sided Limits)

1. Right-sided limit. Suppose that f is defined for all x near a with $x > a$. If $f(x)$ is arbitrariliy close to L for all x sufficiently close to a with $x > a$, we write

$$\lim_{x \to a^+} f(x) = L.$$

2. Left-sided limit. Suppose that f is defined for all x near a with $x < a$. If $f(x)$ is arbitrariliy close to L for all x sufficiently close to a with $x < a$, we write

$$\lim_{x \to a^-} f(x) = L.$$

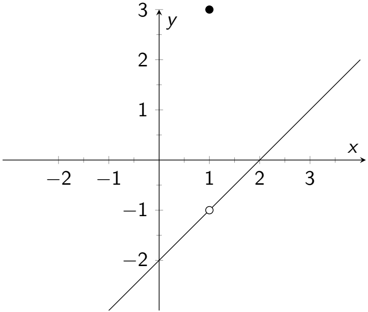
Examining limits graphically and numerically

Find $\lim_{x \to -3} \dfrac{x^2 - 9}{x + 3}$ and $\lim_{x \to 1} \dfrac{x^2 - 9}{x + 3}$.

Theorem (Relationship between One-Sided and Two-Sided Limits)

Assume that f is defined for all x near a except possibly at a. Then $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^+} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$.

A function with a jump



$$g(x) = \begin{cases} x-2 & x \neq 1 \\ 3 & x = 1 \end{cases}$$

Compute $g(1)$ and $\lim\limits_{x\to 1} g(x)$.

Homework Problems: Section 2.2 (pp.67-69):
#1-6,17,18,27,33,35,46,48,