① ⓐ $\frac{d}{dx}\int_1^{\sqrt{x}} \sin(t^2)\,dt$

BUT FIRST:

Recall the FTC ("Fundamental Theorem of Calculus")

$\int_a^b f(x)\,dx = F(b) - F(a)$ so thus

ⓐ $\frac{d}{dx}\int_1^{\sqrt{x}} \sin(t^2)\,dt = \frac{d}{dx}\left(F(\sqrt{x}) - F(1)\right)$ if $f(x) = \sin(x^2)$

by the chain rule,

$$\frac{d}{dx}\left(F(\sqrt{x}) - F(1)\right) = f(\sqrt{x})\cdot\frac{d}{dx}\sqrt{x}$$

$$= \sin((\sqrt{x})^2)\cdot\frac{1}{2}x^{-1/2}$$

so $\frac{d}{dx}\int_1^{\sqrt{x}} \sin(t^2)\,dt = \boxed{\sin x\cdot\frac{1}{2\sqrt{x}}}$

ⓑ $\frac{d}{dx}\int_x^0 \frac{ds}{\sqrt{s^2+1}} = \frac{d}{dx}\left(F(0) - F(x)\right)$

$$= \boxed{-\frac{1}{\sqrt{x^2+1}}}$$

ⓒ $\frac{d}{dx}\int_{-x}^{x^2} e^{r^2-r}\,dr = \frac{d}{dx}\left(F(x^2) - F(-x)\right)$

$$= f(x^2)\cdot 2x - f(-x)\cdot -1$$

$$= e^{(x^2)^2 - x^2}\cdot 2x + e^{(-x)^2 + x}$$

$$= \boxed{2xe^{x^4-x^2} + e^{x^2+x}}$$

2 ⓐ $\int_1^4 \frac{1-\sqrt{t}}{t}\,dt = \int_1^4 \frac{1}{t} - \int_1^4 \frac{\sqrt{t}}{t} = \ln t\Big|_1^4 - \int_1^4 \frac{1}{\sqrt{t}}$

$= [\ln 4 - \ln 1] - \int_1^4 t^{-1/2}$

$= [\ln 4 - \ln 1] - 2t^{1/2}\Big|_1^4$

$= [\ln 4 - \ln 1] - 2[\underbrace{\sqrt{4}}_{=2} - \underbrace{\sqrt{1}}_{=1}]$

$= \ln(4/1) - 2 = \boxed{\ln(4) - 2}$

ⓑ $\int_0^1 \underbrace{(1-\sqrt{s})(s+2)}_{*FOIL*}\,ds = \int_0^1 s + 2 - \sqrt{s}\cdot s - \sqrt{s}\cdot 2$

$= \int_0^1 s + \int_0^1 2 - \int_0^1 s^{3/2} - \int_0^1 2s^{1/2}$

$= \frac{s^2}{2}\Big|_0^1 + 2s\Big|_0^1 - \frac{2}{5}s^{5/2}\Big|_0^1 - \frac{4}{3}s^{3/2}\Big|_0^1$

$= \boxed{\frac{1}{2} + 2 - \frac{2}{5} - \frac{4}{3}}$

ⓒ $\int_0^1 \frac{1}{e^x}\,dx = \int_0^1 e^{-x}\,dx = -e^{-x}\Big|_0^1 = -(\frac{1}{e} - e^0)$

$= -(\frac{1}{e} - 1) = \boxed{1 - \frac{1}{e}}$