MATH 2554 : 5.2-5.4 Review

5.2 Definite Integrals

Reversing Limits and Identical limits of Integration

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
2. $\int_a^a f(x)dx = 0$

And from this follows : $\int_a^b f(x)dx = \int_a^p f(x)dx + \int_p^b f(x)dx$

5.3 The Fundamental Theorem of Calculus

Fundamental Theorem of Calculus : If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Area Function Let f be a continuous function, for $t \geq a$. The area function for f with left endpoint a is

$$A(x) = \int_a^x f(t)dt$$

Deriving Antiderivatives :

1. $\frac{d}{dx}\int_a^x f(t)dt = f(x)$
2. $\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))g'(x)$
3. $\frac{d}{dx}\int_x^b f(t)dt = -f(x)$
4. $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)dt = f(g(x))g'(x) - f(h(x))h'(x)$

5.4 Working with Integrals

DEFINITION Average Value of a Function

The average value of an integrable function f on the interval $[a, b]$ is

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

THEOREM 5.5 Mean Value Theorem for Integrals

Let f be continuous on the interval $[a, b]$. There exists a point c in (a, b) such that

$$f(c) = \bar{f} = \frac{1}{b-a}\int_a^b f(t)\,dt.$$