MATH 2554 : Cheat Sheet (No, you can't bring me to the exam)

Nifty rules

Derivation

1. $\frac{d}{dx}c = 0$
2. $\frac{d}{dx}f(x) + g(x) = f'(x) + g'(x)$
3. $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
4. $\frac{d}{dx}x^n = xn^{n-1}$
5. $\frac{d}{dx}cf(x) = cf'(x)$
6. $\frac{d}{dx}f(x) - g(x) = f'(x) - g'(x)$
7. $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$
8. $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$

The above show the following rules : constant rule (1), constant multiple rule (5), sum rule (2 & 6), product rule (3), quotient rule (7), power rule (4), chain rule (8)

Integration

1. $\int \frac{1}{x} = \ln |x| + C$
2. $\int f(g(x))g'(x)dx = \int f(u)du$
3. $\int x^n dx = \frac{x^{n-1}}{n-1} + C$
4. $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

The above show the following rules : power rule (3), an exception to the power rule (1), substitution rule (2 & 4)

Intermediate Value Theorem : Suppose f is continuous on the interval $[a, b]$ and L is a number strictly between $f(a)$ and $f(b)$. Then there exists at least one number c in (a, b) satisfying $f(c) = L$

Definition of the Derivative :

$$f'(x) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

Mean Value Theorem : If f is continuous on the closed interval $[a, b]$, then there is at least one point c in a, b such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Note that **Rolle's Theorm** is a special case of MVT where $f(a) = f(b)$

Linear Approximation to f at a : Suppose f is differentiable on an interval I containing the point a. The linear approximation to f at a is the linear function

$$L(x) = f(a) + f'(a)(x - a)$$

L'Hopital's Rule : Suppose f and g are differentiable on an open interval I containing a with $g'(x) \neq 0$ when $x \neq a$. If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Fundamental Theorem of Calculus : If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$(1) \quad F(x) = \int_a^x f(t)dt$$

$$(2) \quad \int_a^b f(x)dx = F(b) - F(a)$$

Basic derivative forms

Trig derivatives :

1. $\dfrac{d}{dx}\sin x = \cos x$
2. $\dfrac{d}{dx}\cos x = -\sin x$
3. $\dfrac{d}{dx}\tan x = \sec^2 x$
4. $\dfrac{d}{dx}\cot x = -\csc^2 x$
5. $\dfrac{d}{dx}\sec x = \sec x \tan x$
6. $\dfrac{d}{dx}\csc x = -\csc x \cot x$

Inverse Trig derivatives :

1. $\dfrac{d}{dx}\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$
2. $\dfrac{d}{dx}\arccos x = -\dfrac{1}{\sqrt{1-x^2}}$
3. $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$
4. $\dfrac{d}{dx}\operatorname{arccot} x = -\dfrac{1}{1+x^2}$
5. $\dfrac{d}{dx}\operatorname{arcsec} x = \dfrac{1}{|x|\sqrt{x^2-1}}$
6. $\dfrac{d}{dx}\operatorname{arccsc} x = -\dfrac{1}{|x|\sqrt{x^2-1}}$

Exponential/Log derivatives :

1. $\dfrac{d}{dx}e^x = e^x$
2. $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$
3. $\dfrac{d}{dx}b^x = b^x \ln b$
4. $\dfrac{d}{dx}\log_b|x| = \dfrac{1}{x\ln b}$

...And for integrals, all these backwards !

Other

Limits

1. $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$
2. $\lim\limits_{x\to 0}\dfrac{\cos x - 1}{x} = 0$

Deriving Antiderivatives :

1. $\dfrac{d}{dx}\displaystyle\int_a^x f(t)dt = f(x)$
2. $\dfrac{d}{dx}\displaystyle\int_a^{g(x)} f(t)dt = f(g(x))g'(x)$
3. $\dfrac{d}{dx}\displaystyle\int_x^b f(t)dt = -f(x)$
4. $\dfrac{d}{dx}\displaystyle\int_{h(x)}^{g(x)} f(t)dt = f(g(x))g'(x) - f(h(x))h'(x)$

Common integrals utilizing substitution

1. $\displaystyle\int \cos ax dx = \frac{1}{a}\sin ax + C$
2. $\displaystyle\int \sec^2 ax dx = \frac{1}{a}\tan ax + C$
3. $\displaystyle\int \sec ax \tan ax dx = \frac{1}{a}\sec ax + C$
4. $\displaystyle\int e^{ax}dx = \frac{1}{a}e^{ax} + C$
5. $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
6. $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\operatorname{arcsec}|\frac{x}{a}| + C$
7. $\displaystyle\int \sin ax dx = -\frac{1}{a}\cos ax + C$
8. $\displaystyle\int \csc^2 ax dx = -\frac{1}{a}\cot ax + C$
9. $\displaystyle\int \csc ax \cot ax dx = -\frac{1}{a}\csc ax + C$
10. $\displaystyle\int b^x dx = \frac{1}{\ln b}b^x + C$
11. $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$