For problems 1.-4., let $f(x) = xe^x$.

1. (2 points) Identify the critical point(s) of f. Show supporting work.

$f(x) = xe^x$

$f'(x) = xe^x + e^x(1)$

$xe^x + e^x = 0$

$e^x(x+1) = 0$

$e^x = 0 \quad x+1 = 0$

no sol $\quad x = -1$

$f(-1) = (-1)e^{-1} = -\frac{1}{e}$

critical point: $(-1, -\frac{1}{e})$

2. (2 points) Find the intervals where $f(x)$ is increasing and decreasing.

$-$ $+$

-1

$f'(-2) = e^{-2}(-2+1) = -e^{-2} = -\frac{1}{e^2} < 0$

$f'(0) = e^{0}(0+1) = 1 > 0$

Since $f'(x) < 0$ on $(-\infty, -1)$, then $f(x)$ is decreasing on $(-\infty, -1)$. Since $f'(x)$ is increasing on $(-1, \infty)$, then $f(x)$ is increasing on $(-1, \infty)$.

3. (2 points) Find the intervals where f is concave up and concave down.

$f'(x) = e^x(x+1)$

$f''(x) = e^x(1) + (x+1)e^x$

$= e^x(1+x+1)$

$= e^x(x+2)$

$e^x(x+2) = 0$

$e^x = 0 \quad x+2 = 0$

$\downarrow \quad x = -2$

no sol

$-$ $+$

-2

$f''(-3) = e^{-3}(-3+2) = \frac{-1}{e^3} < 0$

$f''(0) = e^{0}(0+2) = 2 > 0$

Since f'' is negative on the interval $(-\infty, -2)$, then f is concave down on $(-\infty, -2)$. Since $f'' > 0$ on the interval $(-2, \infty)$, then f is concave up on the interval $(-2, \infty)$.

4. (2 points) At each critical point, use BOTH the first and second derivative test to determine if f has a local extremum. If it does, classify the local extremum. Include summary statements explaining your application of the tests.

Since f' changes from negative to positive at $x=-1$, then by the First Derivative Test, there is a relative minimum at $x=-1$.

$f''(-1) = e^{-1}(-1+2) = \frac{1}{e} > 0$

Since $f''(-1) > 0$, then by the Second Derivative Test, there is a relative minimum at $x=-1$.

5. (2 points) Given the following information about the function g, sketch the graph of g. Be sure to label any interesting points (e.g. local extrema and inflection points).

(a) g is continuous everywhere and differentiable everywhere except $x = -1$.

(b) g is increasing on $(-\infty, -1)$ and $(0, 1)$; decreasing on $(-1, 0)$ and $(1, \infty)$

(c) g is concave up on $(-\infty, -1)$ and $(-1, 1/2)$; concave down on $(1/2, \infty)$

(d) $g(-1) = g(1) = 1$ are local and global maximum values; $g(0) = 0$ is a local minimum.

(e) g has an inflection point at $(1/2, 1/2)$.