EXTRA PRACTICE PROBLEMS FOR EXAM 1

1. Evaluate the following limits analytically
   (a) $\lim_{x\to 3^-} \frac{x+3}{9-x^2}$.
   (b) $\lim_{x\to\infty} \frac{x^2-7x+100}{x^3+1}$.
   (c) $\lim_{x\to 0} x^4\cos(1/x)$.
   (d) $\lim_{h\to 0} \frac{3(x+h)-3}{h}$.
   (e) $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{(x-1)(x+3)}$
   (f) $\lim_{x\to -1^+} \frac{x^2+3x+2}{x^2-2x-3}$.
   (g) $\lim_{x\to\infty} \frac{x^5-3x^2}{2x+8x^5}$.

2. Find all horizontal and vertical asymptotes of the following
   (a) $\frac{(x-1)(x^2-x-6)}{x^2+3x+2}$.
   (b) $\frac{x^3+5x^2+6x}{x^3-3x^2+2x}$.
   (c) $\frac{1}{x^2+1}$.

3. Find the average velocity of the following position function $s(t)$ over the corresponding interval I.
   (a) $s(t) = -t^2 + 10t + 2, I = [0,3]$,
   (b) $s(t) = -6t^2 - 7t + 12, I = [0,4]$,
   (c) $s(t) = -3t^2 + 11t + 5, I = [1,3]$,
   (d) $s(t) = -t^2 + t - 7, I = [1,5]$,

4. Suppose we define a piecewise function $h(x) = \begin{cases} x^2+2x-1 & x \le 3 \\ c-x & x > 3 \end{cases}$ where c is a constant. Determine a value for c which makes $\lim_{x\to 3} h(x)$ exist, and find the value of the limit.

5. Suppose we define a piecewise function $h(x) = \begin{cases} -5x^2+x+1 & x \le 4 \\ c-x & x > 4 \end{cases}$ where c is a constant. Determine a value for c which makes $\lim_{x\to 4} h(x)$ exist, and find the value of the limit.

6. Suppose f, g, and h are *continuous* functions for all real numbers. Suppose $f(0) = 1$ and $h(0) = 2$. Determine $g(0)$ given that $\lim_{x\to 0} \frac{f(x)^2+g(x)h(x)}{x^2+3} = 1$.

7. Compute $f'(x)$ by *its definition*, for the following functions
   (a) $f(x) = 6x^2 + 2x + 1$
   (b) $f(x) = 1/x$
   (c) $f(x) = x^3 + x$

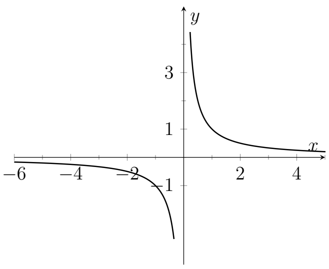
8. Compute the equation of the line tangent to $f(x) = 3x^2 - 5$ at the point $(1, -2)$.

9. Use the intermediate value theorem to prove that $f(x) = x^2 - x - 1$ has a root in $(0, 2)$.

10. Use the intermediate value theorem to prove that $f(x) = -x^2 + x + 1$ has a root in $(-5, 0)$.

11. For the following, match the given function to its graph and give a written **explanation** of why the choice is correct. (Hint: Consider asymptotes.)

    (i) $f(x) = \dfrac{1}{x}$, (ii) $f(x) = \dfrac{x}{x^2 - 1}$, (iii) $f(x) = \dfrac{2x}{x + 1}$, (iv) $f(x) = \dfrac{-1}{x^2}$.



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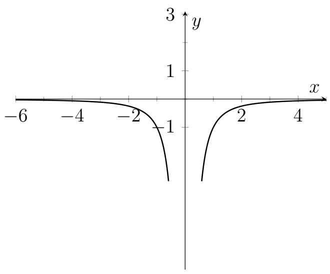

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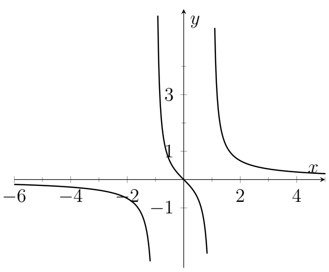

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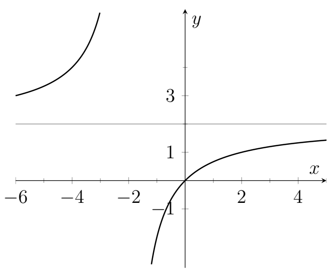

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