(1) A rectangular pen is built with one side against a barn. Two hundred meters of fencing are used for the other three sides of the pen. What dimensions maximize the area of the pen?

(2) Suppose an airline policy states that all baggage must be box-shaped with a sum of length, width, and height not exceeding 108in. What are the dimensions and volume of a square-based box with the greatest volume under these conditions?

(3) A typical cylindrical soda can has a volume of 354 cm$^3$. The top and bottom of the soda can are twice as thick as the side of the can. What dimensions will minimize the material needed to manufacture the can? (Hint: minimize the surface area, but use twice the surface area for the top and bottom) Compare your answer to an actual soda can, which typically has a radius of 3.1 cm and a height of 12.0 cm.