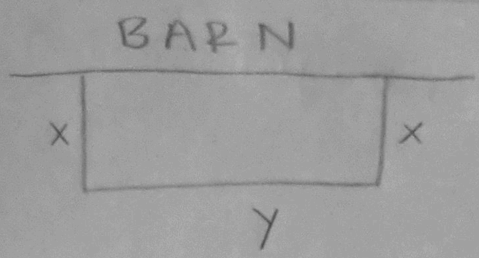
①



"200 m of fencing are used for other three sides" OR

$P = 200$

$200 = 2x + y$

" What dimensions maximize $\underline{\text{area}}$ "

$$\underbrace{A = xy}_{\text{objective function}} \qquad \underbrace{200 = 2x + y}_{\text{constraints}}$$

Reduce objective function to $\underline{\text{one}}$ variable using the constraint.

$200 = 2x + y \qquad y = 200 - 2x \qquad \underline{\underline{\text{so}}}$

$A = x(200 - 2x) = -2x^2 + 200x$

Domain of A: $(0, 100)$ $\quad A(0) = 0 \quad A(100) = 0$

Take derivative to find maximum:

$A' = -4x + 200$

$200 = 4x \quad$ if $A' = 0$

so $x = 50$

$y = 200 - 2x \quad$ so $\quad y(50) = 200 - 2 \cdot 50$
$= 100$

So dimensions of $\boxed{50\text{m} \times 100\text{m}}$ maximize area

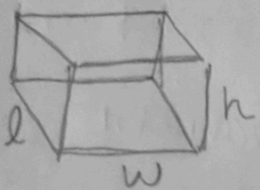
since endpoints of $x=0$ & $x=100$ give 0, this value is the maximum.

(2) "Sum of length, width, height cannot exceed 108in"

$108 = l + w + h$

"What are the dimensions and volume of a square-based box w/ greatest volume under these conditions"

square-based $\equiv$ $l = w$ so



$108 = 2w + h$ (constraint)

$V = lwh = w^2 h$ (objective)

* derive V to find maximum *

first reduce V to one variable using constraint...

$108 = 2w + h$ $\quad h = 108 - 2w$

$V = w^2(108 - 2w) = -2w^3 + 108w^2$

$V' = -6w^2 + 216w$

$= -6w(w - 36)$ so $w = 0, w = 36$

However, Domain of V is $(0, 54)$

and $V(0) = 0$ $\quad V(54) = 0$ so $w = 36$ will maximize V.

$h = h(36) = 108 - 2 \cdot 36 = 36$ so

$\boxed{36m \times 36m \times 36m \text{ maximizes } V}$

(3)

" Top and bottom of can twice as thick as sides " so count these <u>twice</u> ...

" what dimensions <u>minimize</u> material needed to manufacture can"

$$SA = 2(2\pi r^2) + h(2\pi r) \quad \text{(objective funct)}$$

$\uparrow$ Top/Bottom

$$V = 354\,cm^3 = \pi r^2 h \quad \text{(constraint)}$$

$$h = \frac{354}{\pi r^2} \quad \text{so}$$

$$SA = 4\pi r^2 + \left(\frac{354}{\cancel{\pi} r^{\cancel{2}}}\right)(2\cancel{\pi} \cancel{r})$$

$$SA = 4\pi r^2 + \frac{708}{r}$$

Not endpoints: $(0, \infty)$ $\lim_{r \to 0} SA = \infty$ $\lim_{r \to \infty} SA = \infty$ so our crit point is a <u>max</u>

* Now we can derive to find minimum *

$$SA' = 8\pi r - \frac{708}{r^2} \quad \text{now set to } SA' = 0$$

$$\frac{708}{r^2} = 8\pi r$$

$$708 = 8\pi r^3$$

$$\frac{88.5}{\pi} = r^3 \qquad r = \sqrt[3]{\frac{88.5}{\pi}} \approx \boxed{3.04273\ cm}$$

$$h(3.04273) = \frac{354}{\pi(3.04273)^2} \approx \boxed{12.17095}$$