2.4 – Infinite Limits

MATH 2554 – Calculus I

Fall 2019

Warm-up Problem: Compute $\lim_{x \to 0} \frac{1}{x}$ graphically and numerically.

In the next two sections, we will be studying the limit scenarios involving infinity. They are:

- Infinite Limits: As the independent variable x approaches a finite number, the dependent variable y becomes arbitrarily large in magnitude.
- Limits at Infinity: As the independent variable x approaches an arbitrarily large number (either positive or negative), the dependent variable y approaches a finite number.

Definition (Infinite Limits)

Suppose f is defined for all x near a. If $f(x)$ grows arbitrarily large for all x sufficiently close (but not equal) to a, we write

$$\lim_{x \to a} f(x) = \infty$$

and say that the limit of $f(x)$ as x approaches a is infinity.

If $f(x)$ is negative and grows arbitrarily large in magnitude for all x sufficiently close (but not equal) to a, we write

$$\lim_{x \to a} f(x) = -\infty$$

and say that the limit of $f(x)$ as x approaches a is infinity. In both cases, the limit does not exist.

There are analogous definitions for one-sided limits.

Summary Statement

Question: What is a sufficient amount of work to obtain full credit on infinite limit problems?

Answer: Use the following framework:

"Since the numerator approaches _____ and the denominator approaches 0 and is (positive/negative), and since (analyze signs here), then (insert limit problem) $= (\pm)\infty$."

Example: Let $f(x) = \dfrac{1}{x^2(x-1)}$.

Analyze the limits graphically and analytically:

1. $\lim\limits_{x \to 0} f(x)$
2. $\lim\limits_{x \to 1^+} f(x)$
3. $\lim\limits_{x \to 1^-} f(x)$
4. $\lim\limits_{x \to 1} f(x)$.

Definition (Vertical Asymptote)

If $\lim_{x \to a} f(x) = \pm\infty$ OR $\lim_{x \to a^+} f(x) = \pm\infty$ OR $\lim_{x \to a^-} f(x) = \pm\infty$, then the line $x = a$ is called a vertical asymptote of f.

What are the vertical asymptotes of the previous example $f(x) = \dfrac{1}{x^2(x-1)}$?

Exercise: Find $\lim_{x \to -1^+} \dfrac{3x+4}{x+1}$ and $\lim_{x \to -1^-} \dfrac{3x+4}{x+1}$ analytically.

Exercise: Find the vertical asymptotes of $f(x) = \dfrac{3x^2 - 48}{(x-4)(x-1)}$.

Also compute $\lim\limits_{x \to 1} f(x)$ and $\lim\limits_{x \to 4} f(x)$.

Homework Problems: Section 2.4 (pp.88-90):#6-8, 16,17, 21-27, 47-50, 54,57.