10M

Discreat time Signals & Systems

→ Classification of DTS :-

1) Even & odd DTS.
2) periodic & non periodic DTS
3) Deterministic & Random DTS
4) Energy & power DTS.

1) Even & odd DTS :-

→ A Discreat time signals satisfy the Condition $x(n) = x(-n)$.

Hence Its Even Signal & Even DTS.

Ex: $\cos(n) = x(n)$

→ A DTS satisfy the Condition

$$x(+n) = -x(-n)$$

is called odd DTS Ex: $x(n) = \sin(n)$

To calculate odd & Even Component of DTS using following Equations

$$\text{Even} \Rightarrow x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$\text{odd} \Rightarrow x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

Ex: $x(n) = \cos(n)$

$$\left.\begin{aligned} x(1) &= \cos(1) = 0.540 \\ x(-1) &= \cos(-1) = 0.540. \end{aligned}\right\} \text{Even}$$

$y(n) = \sin(n)$
$y(1) = \sin(1) = 0.84$
$y(-1) = \sin(-1) = -0.84$
$y(n) = -y(-n)$
$n=1$ $y(1) = -y(-1)$
$0.84 = -(-0.84)$

2) periodic & non-periodic DTS :-

→ If DT signal is periodic, its ~~satisfy's~~ satesfies $x(n) = x(n+N)$

otherwise Its non periodic DT signals

Here, N should be Integer & its fundamental period of $x(n)$

$$\boxed{\Omega = \frac{2\pi}{N}} \text{ (radians/sec)}$$

3) Deterministic & Random DTS :-

The descreate ~~th~~ time signal is predictable, with respect to time (n domain) is called Deterministic.

Ex: $x(n) = n^2$, $0 < n \leq 10$.

→ Random DTS : The DT signal is not predictable exert value of the signal with respect to particular time (n domain) is called Random signal.

EX: ECG signal & Tossing a Coint
EMG signal

1) ~~Even~~ Energy & power signal :-

A Energy of DTS

$$\boxed{E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2}$$

or

$$\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$

→ power of DTS given by

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

or

$$\boxed{P = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n)}$$

note : 1) The signal is referred to as an energy signal if the total energy 'E' of the signal satisfies the condition is $0 < E < \infty$ [E is finite]

2) where it is refer to as power signal if the average power 'p' of the signal satisfies the condition

$0 < P < \infty$ ['p' is finite]
or must

3) All periodic signals are power signal but all power signals are not periodic signals.

4) Generally signals which are both deterministic & non periodic signals are example for energy signals but not all

* problems :-

1) Find & plot the Even & odd component of following DTS.

(a) $x(n) = u(n)$

(b) $x(n) = 3u(n-2)$

(c) $x(n) = \alpha u(n) \quad 0 < \alpha < 1$

(d) $x(n) = \alpha^n u(n-3), \quad 0 < \alpha < n$

(e) $x(n) = u(n) - u(n-4)$

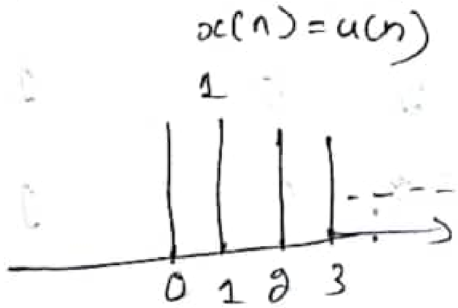
(f) $x(n) = \cos(\pi/2\, n)\, u(n)$

(g) $x(n) = [\underset{\uparrow}{2}, 3, 4, 5, 6]$

Ex: 0 1 2 3 4

(a) $x(n) = u(n)$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



we need to calculate even & odd component

$$\text{Even} = x_e(n) = \tfrac{1}{2}\left[x(n) + x(-n)\right]$$

$$\text{odd} = x_o(n) = \tfrac{1}{2}\left[x(n) - x(-n)\right]$$

$\rightarrow x(-n) = u(-n)$

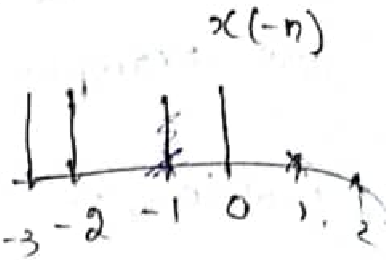
$n=0 ; \quad x(-0) = u(-0) = 1$

$n=1 ; \quad x(-1) = u(-1) = 0$

$n=-1 ; \quad x(-(-1)) = u(1) = 1$

$n=-2 ; \quad x(-(-2)) = u(2) = 1$

$n=-3 ; \quad x(-(-3)) = u(3) = 1$



$\rightarrow x(n) = u(n)$

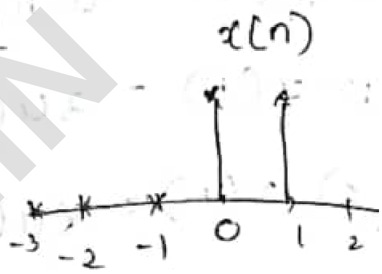
$n=0 ; \quad x(+0) = u(0) = 1$

$n=1 ; \quad x(+1) = u(1) = 1$

$n=-1 ; \quad x(-1) = u(-1) = 0$

$n=-2 ; \quad x(-2) = u(-2) = 0$

$n=-3 ; \quad x(-3) = u(-3) = 0$



-4 -3 -2 -1

n	$x(n)$	$x(-n)$	Even $x(n)+x(-n)$	odd $x(n)-x(-n)$
0	1	1	2	0
1	1	0	1	1
-1	0	1	1	-1
-2	0	1	1	-1
-3	0	1	1	-1

$$\text{Even} = x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x(n) + x(-n) = 2x_e(n)$$

$$= 2 \times \frac{1}{2}(2)$$

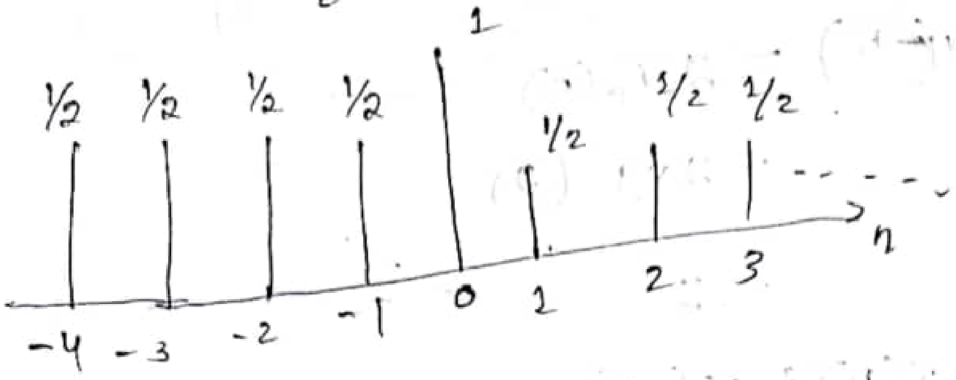
$$= 2$$

$n = 1$; $x(n) + x(-n) = 2 \times \frac{1}{2}(1)$

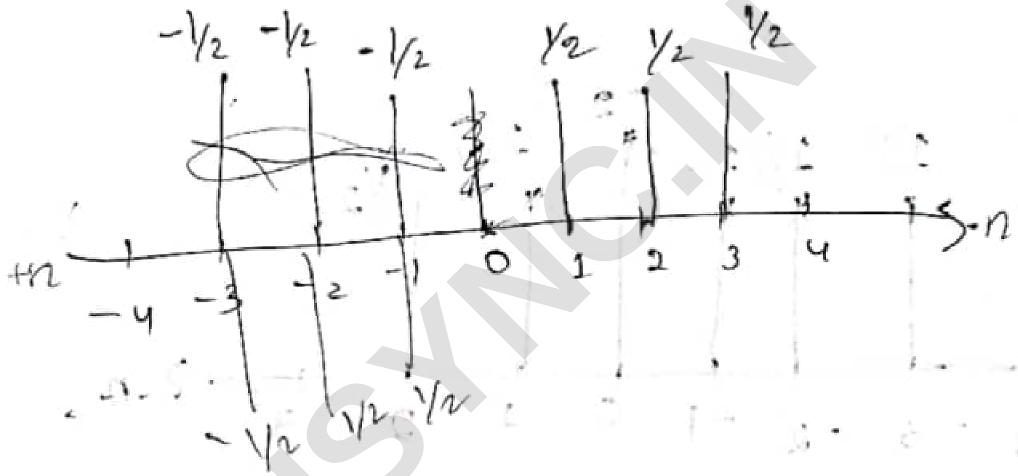
$$= 1$$

→ Even sketch

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$



$$\rightarrow x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$



b) $x(n) = 3u(n-2)$

$n=0$; $x(0) = 3u(0-2)$

$= 3u(-2) = 3(0) = 0$

$n=1$; $x(1) = 3u(1-2)$

$= 3u(-1)$

$= 3(0) = 0$

$n=-1$; $x(-1) = 3u(-1-2)$

$= 3u(-3)$

$= 3(0)$

$= 0$

b) $x(n) = 3u(n-2)$

Note:- Time shifting :

If $y(n) = x(n-n_0)$

where,

case I : $n_0 > 0$

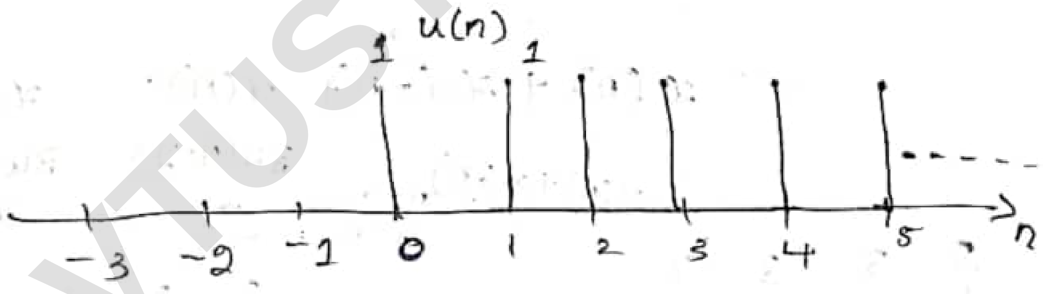
The signal is shifted to the right (n_0 times)

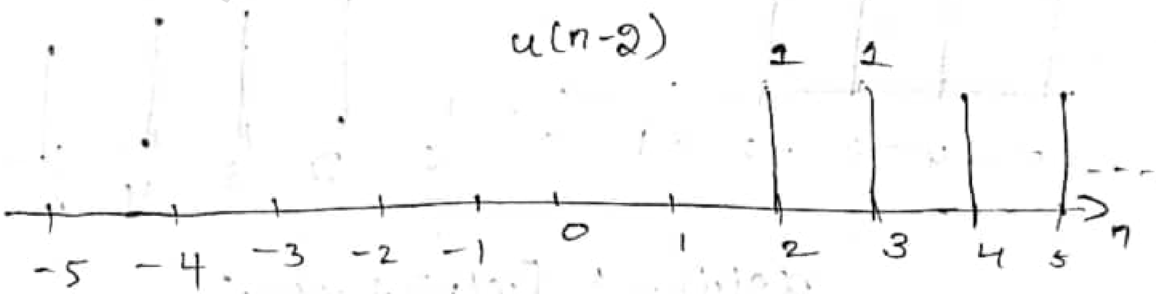
case II : $n_0 < 0$

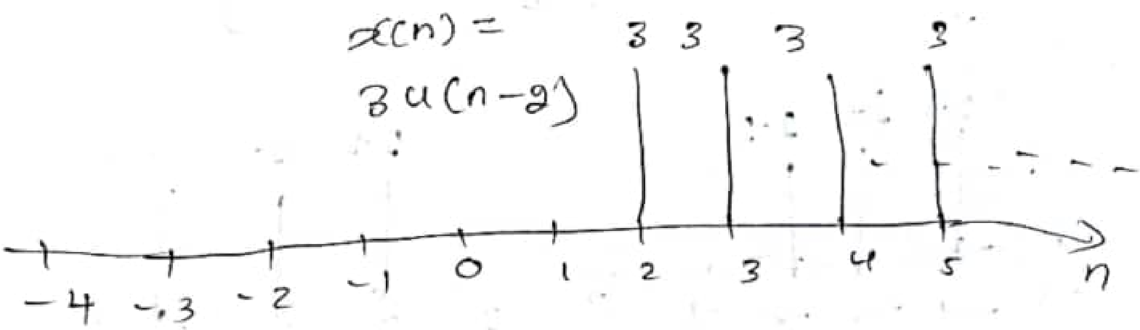
The signal is shifted to the left (n_0 times)

$x(n) = 3u(n-2)$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0. \end{cases}$$







$\therefore \quad x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$

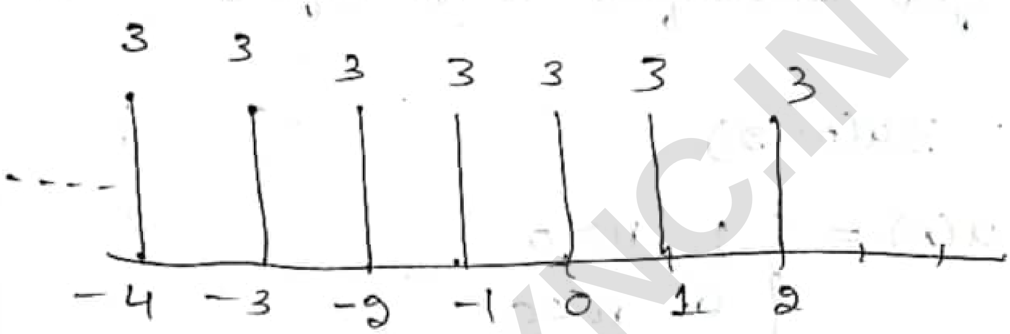
note :- $x(-n)$

Reflection / Time folding

$y(n) = x(-n)$

$y(n)$ is reflected version of $x(n)$

(or) Time folding.

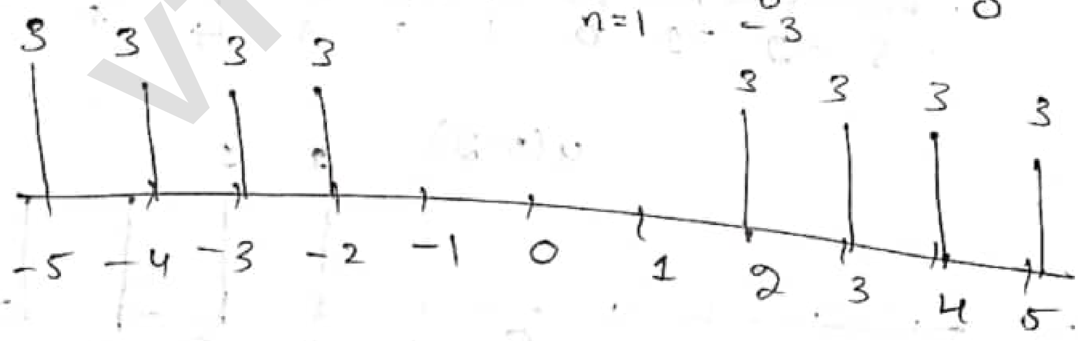


~~$x(n)+x(-n)$~~ $\quad x(n) \quad\quad x(-n)$

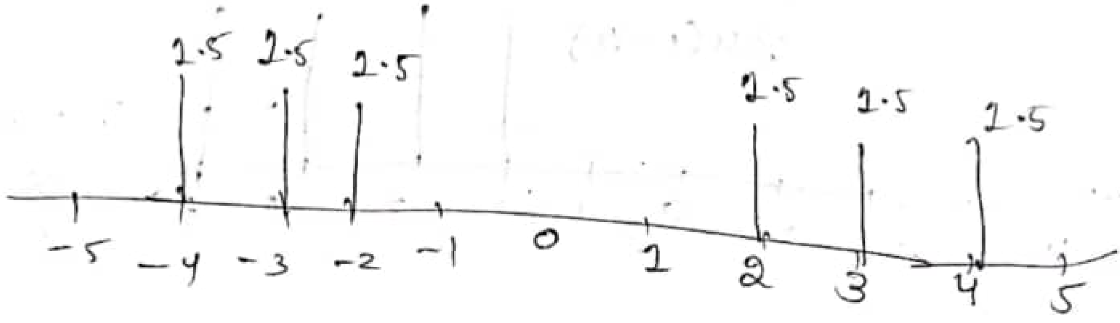
$3u(n-2) \quad\quad 3u(-n-2)$

$x(n)+x(-n)_{n=0} \quad 0 \quad\quad 0$

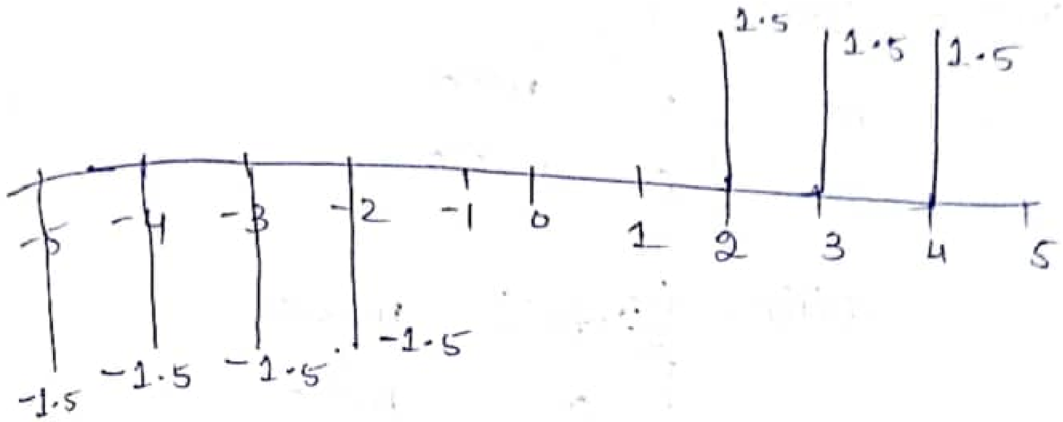
$n=1 \quad -3$



$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$



$\cancel{x(n)+x(-n)}$ $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$



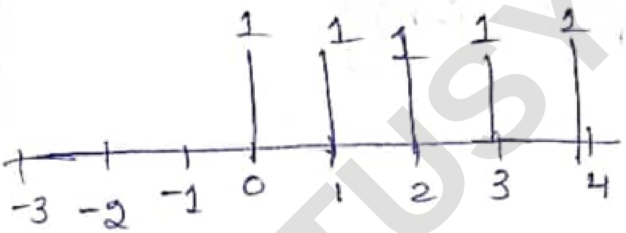
* plot and calculate the even & odd for the following

1) $x(n) = 2u(n-3)$ 2) $x(n) = 4u(n-4)$

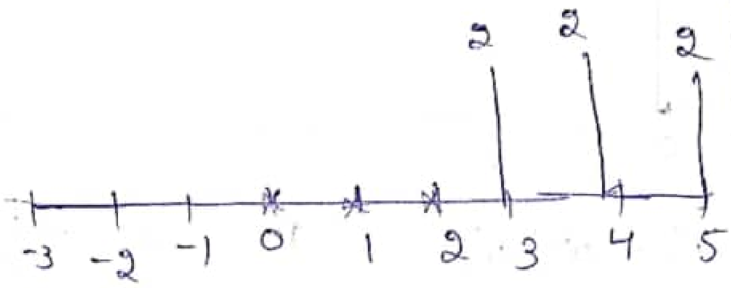
soln:- Given :-

$x(n) = 2u(n-3)$

• $x(n) = u(n)$

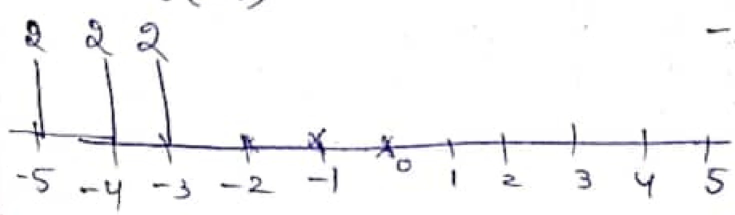


• $x(n) = 2u(n-3)$

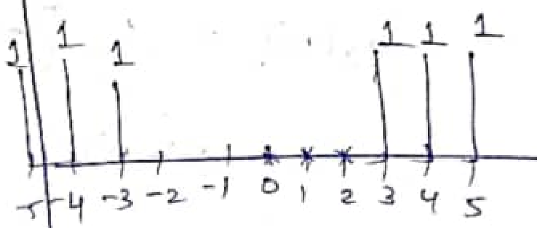


• $x_e(n) = \frac{1}{2}[x(n)$

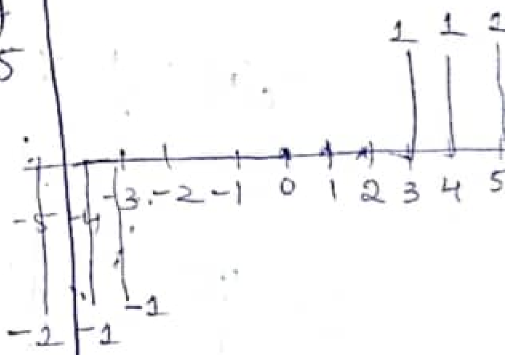
$x(-n)$



• $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$



• $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$



(d) $x(n) = \alpha^n u(n)$, $0 < \alpha < 1$

Consider $\alpha = 0.75$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

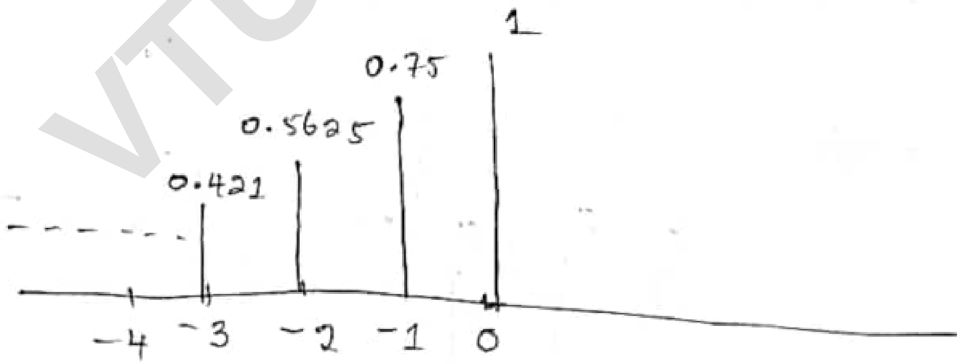
$$x(n) = \begin{cases} (0.75)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$n = 0$ $x(n) = 1$

$n = 0$ $x(n)$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$x(-n)$



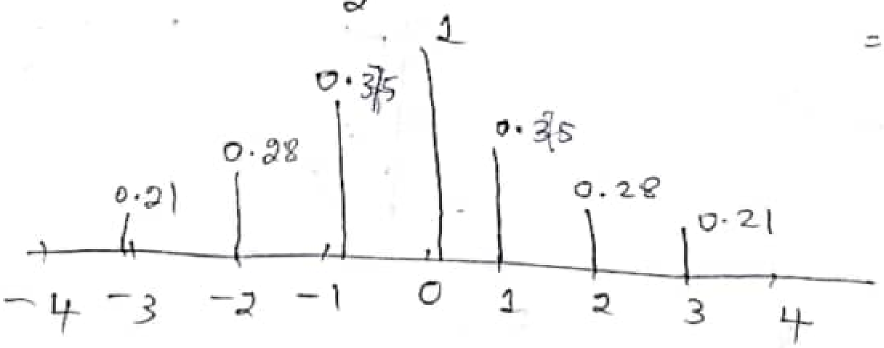
$$x_e(n) = \frac{1}{2}\, x(n) + x(-n)$$

$1 + 1 = 2 \times \frac{1}{2} = 1$

$= \frac{0.75 + \cdot}{2}$

$\frac{0.56}{2}$

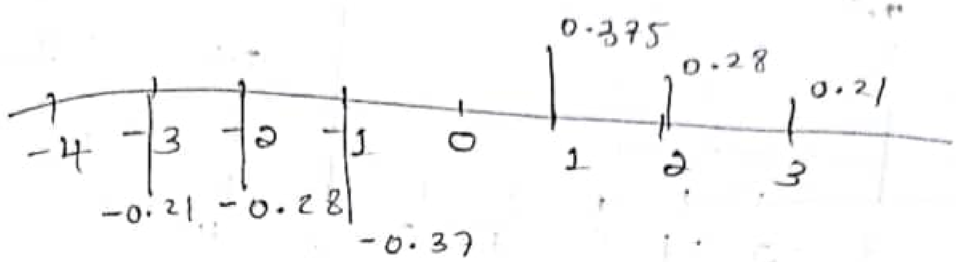
$\frac{0.42}{2} = 0.21$



$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$1 - 1 = 0$

$1 - 1 = 0$



d) $x(n) = \alpha^n u(n-3)$, $0 < \alpha < 1$

$\alpha = 0.75$

$0.75(1) \rightarrow$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 3 \end{cases}$$

$$x(n) = \begin{cases} (0.75)^n, & n \geq 0 \\ 0, & n < 3 \end{cases}$$

3 & 4

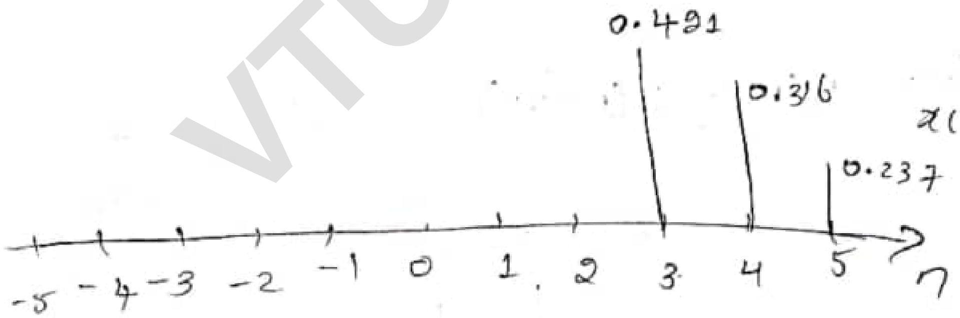
$x(3) = (0.75)^3$

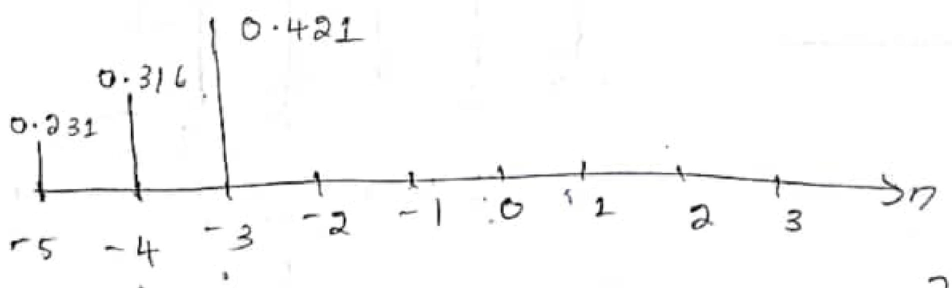
$= 0.421$

$x(4) = (0.75)^4$

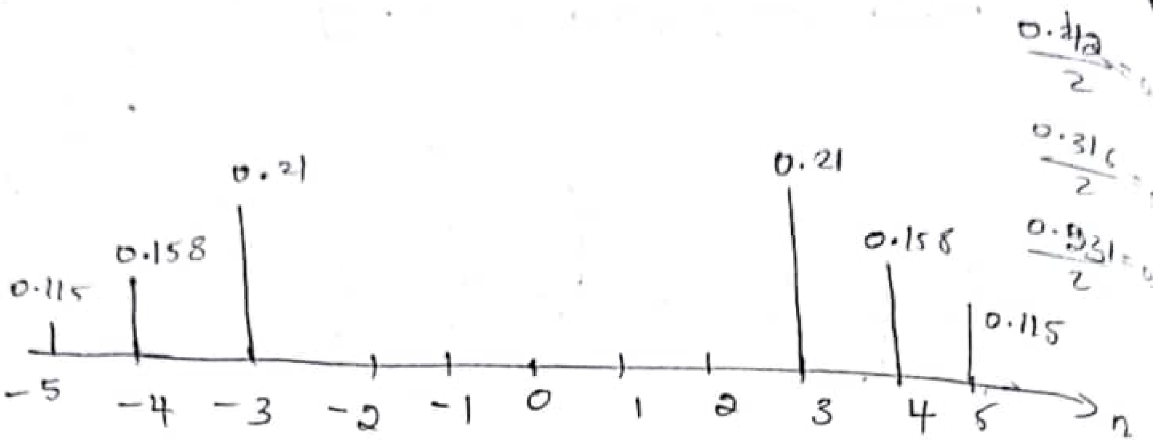
$= 0.316$

$x(5) = 0.237$

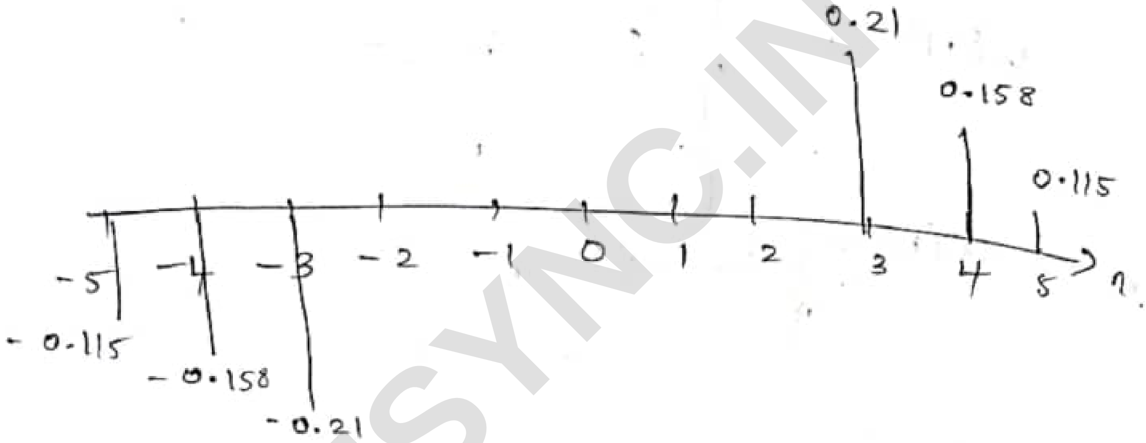




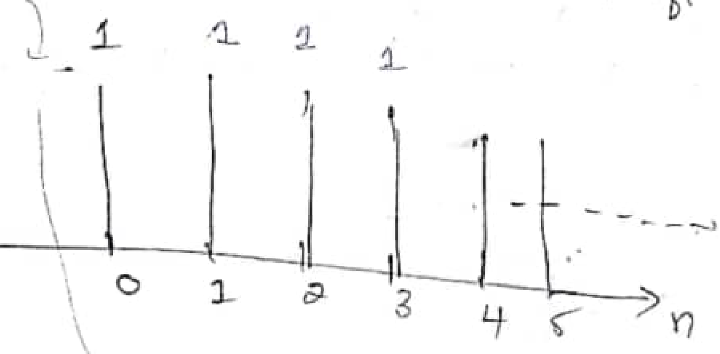
$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

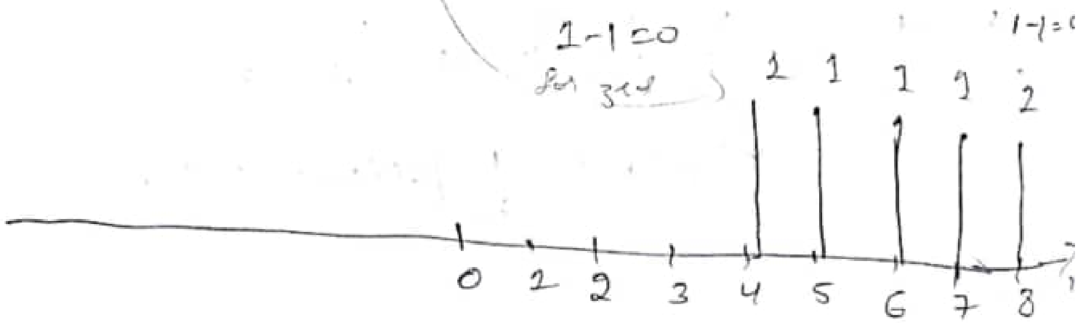


$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

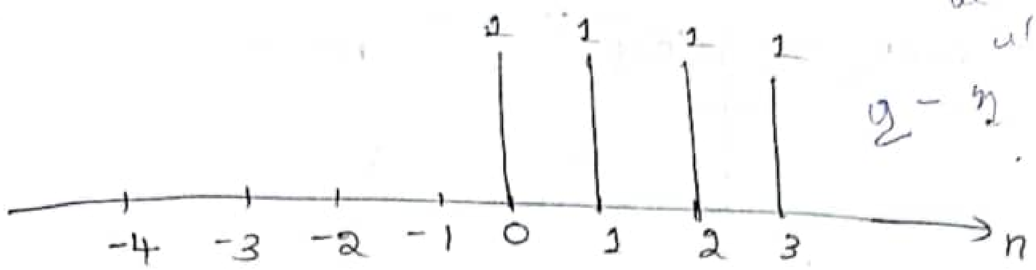


⑦ $x(n) = u(n) - u(n-4)$ → delay is 4
so 4 shift

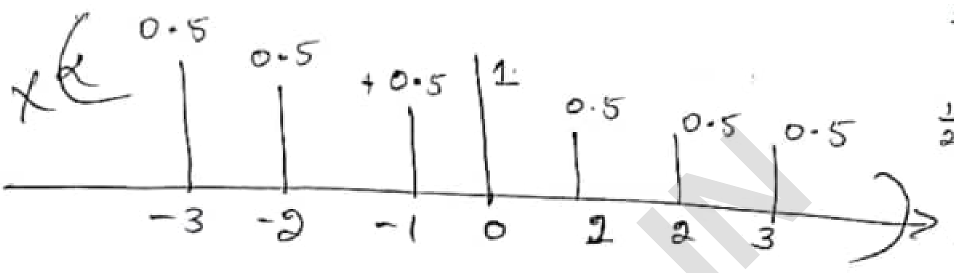




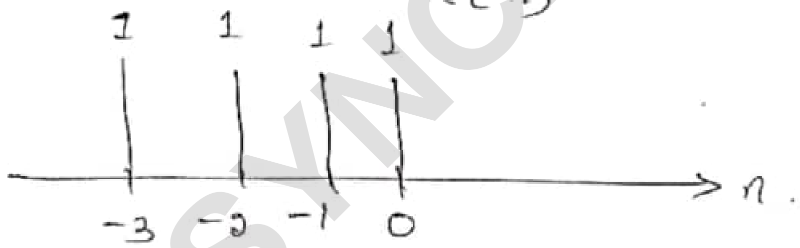
$$x(n) = u(n) - x(n-4)$$



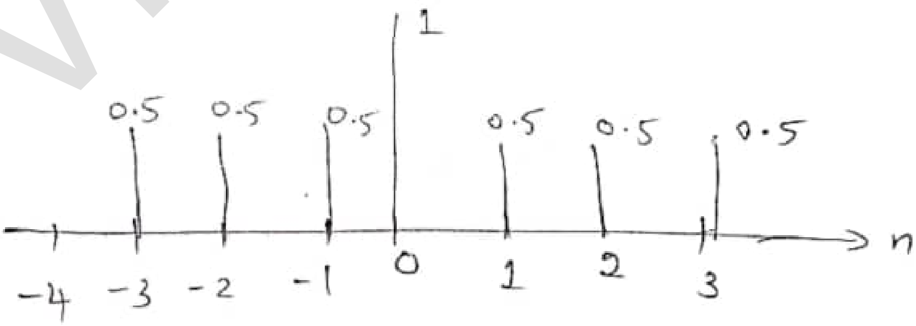
$x(-n)$



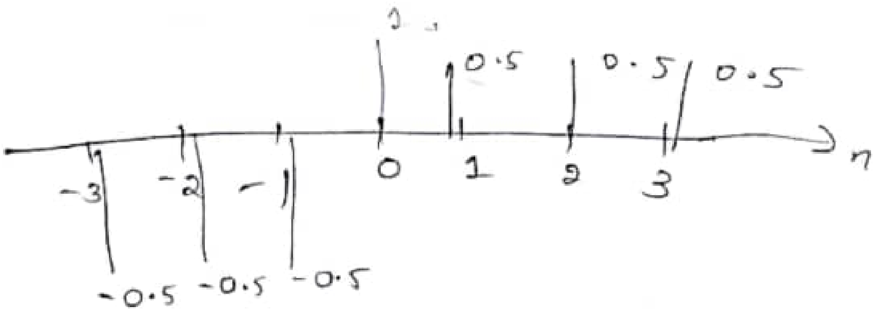
both part
1 only
1+1=2
$\frac{1}{2}[2]$
=1
so
0 taken
1



$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$



$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

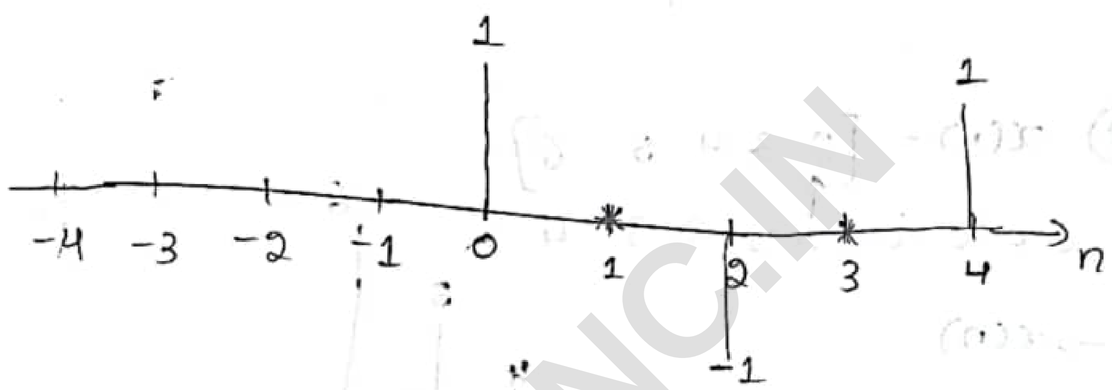


f) $x(n) = \cos\left(\frac{\pi}{2}n\right)u(n)$

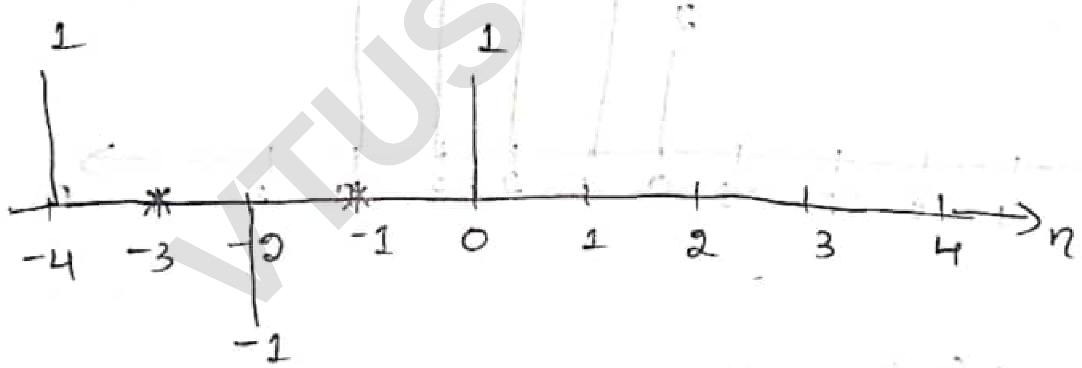
Soln: $x(n) = \cos\left(\frac{\pi}{2}n\right)u(n)$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$ when $n = 0, 1, 2, 3, 4$

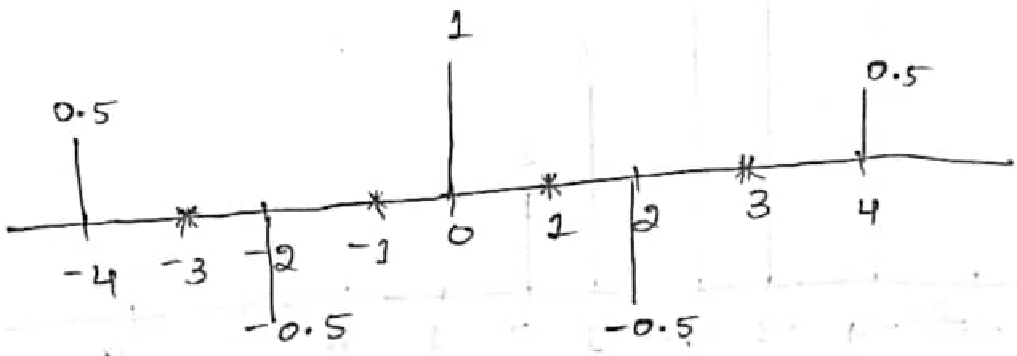
→ $x(n)$



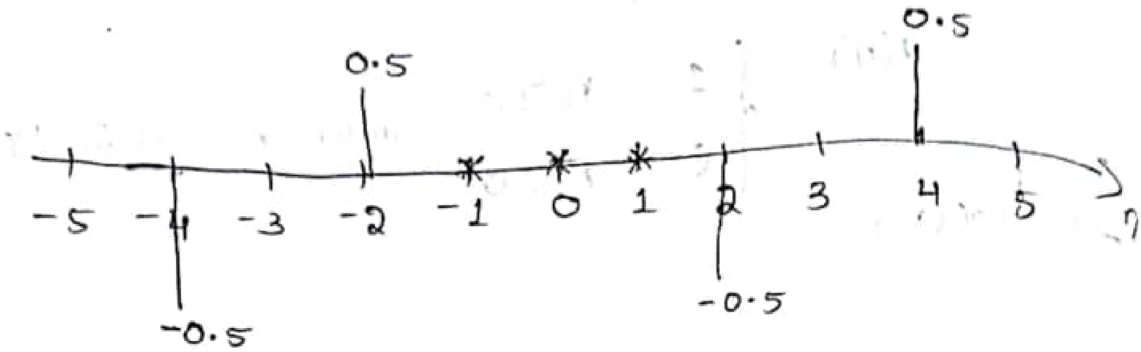
→ $x(-n)$



→ $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$



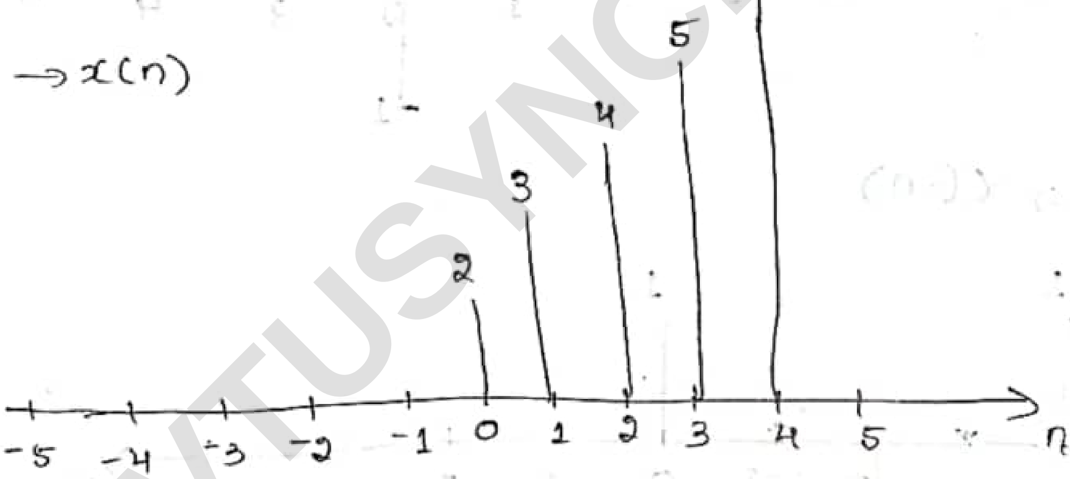
$\rightarrow x_o(n) = \frac{1}{2}[x(n) - x(-n)]$



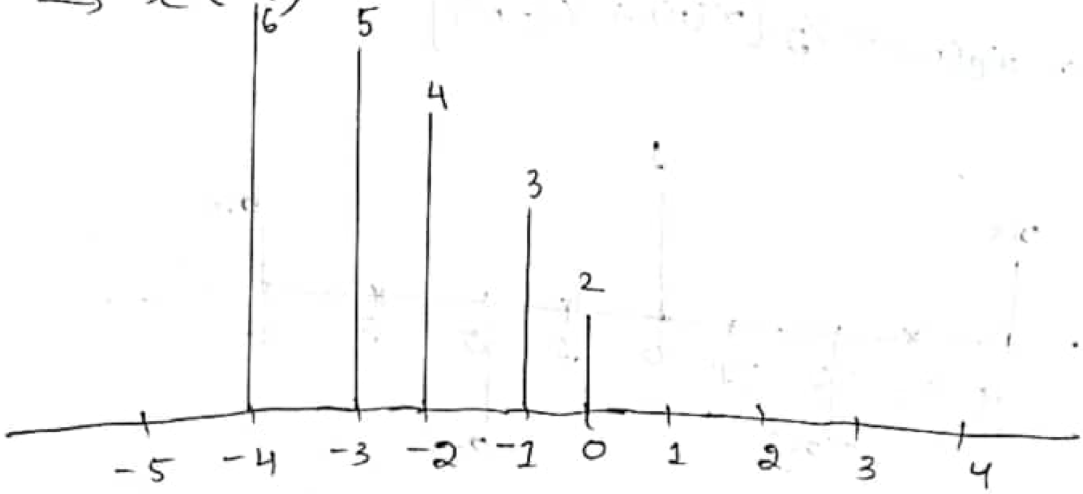
g) $x(n) = [\underset{\uparrow}{2}\ 3\ 4\ 5\ 6]$

0 0 0 0 0 1 2 3 4

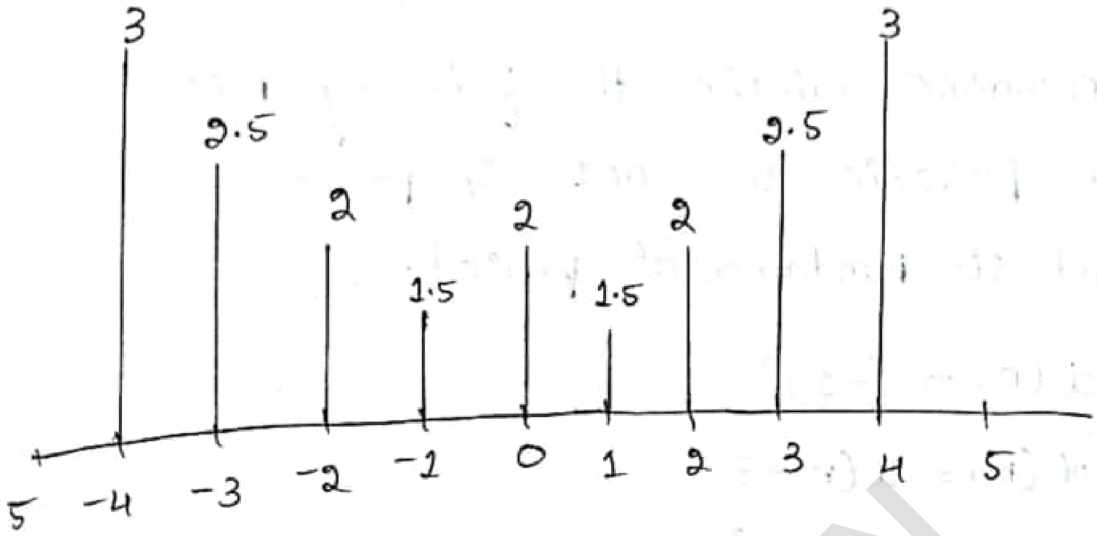
$\rightarrow x(n)$



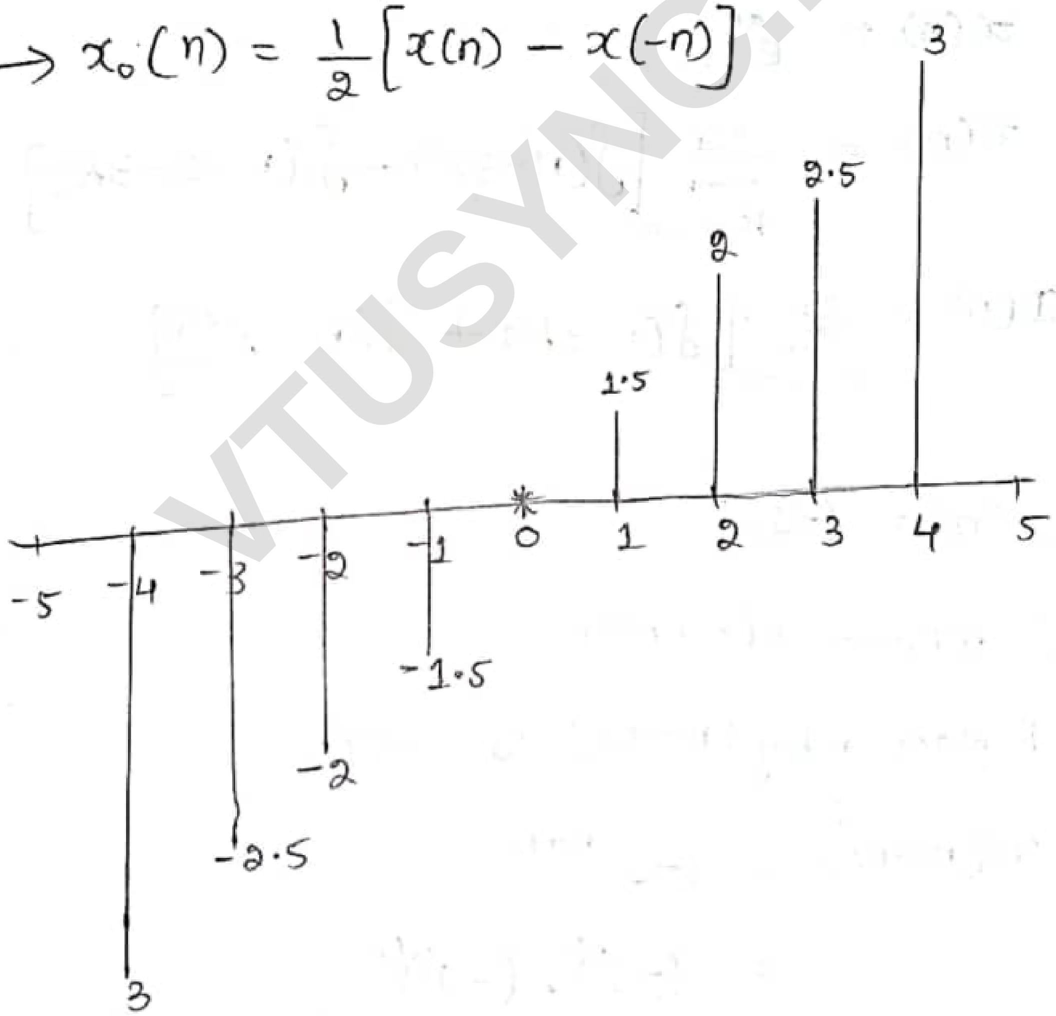
$\rightarrow x(-n)$



$$\rightarrow x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$



$$\rightarrow x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$



* Periodic and non-periodic DTS :

→ problems :

1) Determine whether the following DTS are periodic or not if periodic find its Fundamental period.

a) $x(n) = (-1)^n$

b) $x(n) = u(n-3)$

c) $x(n) = (-1)^{n^2}$

d) $x(n) = e^{an}$

e) $x(n) = \sum_{k=-\infty}^{\infty} [\delta(n-3k) - \delta(n-1-3k)]$

(f) $x(n) = \sum_{k=-\infty}^{\infty} [\delta(n-3k) + \delta(n-k^2)]$

a) $x(n) = (-1)^n$

soln :- $x(n) = x(n+N)$

Replace n by $(n+N)$ in $x(n)$

i,e $x(n+N) = (-1)^{n+N}$

$= (-1)^n \cdot (-1)^N$

$\because$ if $(-1)^N = 1$

$\boxed{x(n+N) = x(n)}$

$\boxed{N = 2m}$

$x(n) = (-1)^n$ is periodic when fundamental period $= 2M$

where, $m = 0, 1, 2, 3 \ldots N$

b) $x(n) = u(n-3)$

soln:- $x(n) = u(n-3)$

$x(n) = x(n+N)$

Replace n by n+N

$x(n+N) = u(n+N-3)$

$= N=0$

Then, $x(n) = u(n-3)$

It satisfies when $N=0$ [fundamental period is zero] so, it is non-periodic DTS.

c) $x(n) = (-1)^{n^2}$

soln:- $x(n) = (-1)^{n^2}$

$x(n) = x(n+N)$

replace n by n+N

$x(n+N) = (-1)^{(n+N)^2} \Rightarrow (-1)^{n^2+2nN+N^2}$

$= (-1)^{n^2} \cdot (-1)^{N^2}$ $\quad (-1)^{2N} = N=1$, $(-1)^{N^2} = 1$ if $\boxed{N = 2m}$

$= (-1)^{n^2} \cdot (1)$

$x(n+N) = (-1)^{n^2}$

$\boxed{x(n+N) = x(n)}$

$x(n) = (-1)^{n^2}$ is a perio

c) $x(n) = (-1)^{n^2}$

Soln: $x(n) = x(n+N)$

Replace n by n+N

$x(n+N) = (-1)^{(n+N)^2}$

$= (-1)^{n^2+2nN+N^2}$

$= (-1)^{n^2} (-1)^{2nN+N^2}$

$= (-1)^{n^2} \cancel{(-1)^{2nN+N^2}}^{1}$ $\because$ if $(-1)^{2Nn+N^2} = 1$

$= x(n)$

$\boxed{2nN + N^2 = 2m}$

$x(n) = (-1)^{n^2}$ is periodic when fundamental period $= 2m$

d) $x(n) = e^{an}$

Soln: $x(n) = x(n+N)$

Replace n by n+N in $x(n)$

$x(n+N) = e^{a(n+N)}$

$x(n+N) = e^{an} \cdot \cancel{e^{aN}}^{1}$ when $N=0$

$x(n) = e^{an}$

It satisfies when $N=0$, so it is non-periodic

$n=3k$

$k=\frac{n}{3}$

$n-1=3k$

$k=\frac{n-1}{3}$

e) $x(n) = \sum_{k=-\infty}^{\infty} \left[\delta(n-3k) - \delta(n-1-3k) \right]$

soln:- $x(n) = x(n+N)$

Replace n by $n+N$.

$$x(n+N) = \sum_{k=-\infty}^{\infty} \left[\delta(n+N-3k) - \delta(n+N-1-3k) \right]$$

if $N = 3$. [Take the coefficient of k], value.

$$= \sum_{k=-\infty}^{\infty} \delta(n+3-3k) - \delta(n+3-1-3k)$$

$$= \sum_{k=-\infty}^{\infty} \delta(n-3(k-1)) - \delta(n-1-3(k-1))$$

put $k-1 = m \Rightarrow k = m+1$

$$= \sum_{k=-\infty}^{\infty} \delta(n-3m) - \delta(n-1-3m)$$

$\delta(n-3(k-1))$

$$= x(n)$$

$\therefore$ $x(n)$ is periodic of period or fundamental period is $N=3$.

f) $x(n) = \sum_{k=-\infty}^{\infty} \left[\delta(n-3k) + \delta(n-k^2)\right]$

$x(n) = x(n+N)$

Sol.

$$x(n+N) = \sum_{k=-\infty}^{\infty} \left[\delta(n+N-3k) + \delta(n+N-k^2)\right]$$

if $N = 0$

$$x(n) = \sum_{k=-\infty}^{\infty} \left[\delta(n-3k) + \delta(n-k^2)\right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} \left[\delta(n-3k) + \delta(n-k^2)\right]$$

It satisfies when $N=0$ [fundamental period is zero]

So, It is non-periodic DTS.

5m or 6m

* Show that a discrete time complex exponential signal

$$x(n) = e^{\pm j\Omega_0 n}$$ is periodic

if only if Ω_0 is Rational multiple of 2π

(ie $\Omega_0 = \frac{m}{N} 2\pi$)

Soln:- A DTS is periodic if

$x(n) = x(n+N)$; for all n

$$\therefore x(n) = e^{\pm j\Omega_0 n}$$

$$x(n+N) = e^{\pm j\Omega_0 (N+n)}$$

$$= \left(e^{\pm j\Omega_0 n}\right)\left(\cancel{e^{\pm j\Omega_0 N}}^{\,1}\right)$$

$\therefore e^{\pm j\Omega_0 N} = [\cos\Omega_0 N \pm j\sin\Omega_0 N]$

[cos function with even multiple with π is $=1$]

consider $\Omega_0 N = m(2\pi)$ where, m is any Integer

$$\boxed{\Omega_0 = \frac{m}{N} 2\pi}$$

*problems :-

*) Determine whether the following DTS are periodic or not if periodic find its fundamental period.

1) $x(n) = e^{j7\pi n}$

2) $x(n) = \cos(3\pi n)$

3) $x(n) = \cos(2n)$

4) $x(n) = 3e^{j3/5(n+1/2)}$

5) $x(n) = 3e^{j3/5\pi(n+1/2)}$

1) $x(n) = e^{j7\pi n}$

soln:- Given: $\Omega_0 = 7\pi$

$\Omega_0 = \frac{7}{2}(2\pi)$

$x(n)$ is periodic. The fundamental period is $N=2$.

2) $x(n) = \cos(3\pi n) \rightarrow (1)$

Soln: Given:-

$$\Omega_0 = 3\pi$$

w.k.T:- sinusoidal signal

$$x(n) = A\cos(\Omega_0 n + \phi) \rightarrow (2)$$

Compare Eq (1) and (2)

$$\Omega_0 = 3\pi$$

$$= \frac{3}{2} 2\pi \Rightarrow \left[\Omega_0 = \frac{m}{N} 2\pi\right]$$

$x(n)$ is periodic with fundamental period;

$$\boxed{N = 2}$$

$\frac{24}{2} 2\pi$

$\frac{4}{2\pi} 2\pi$

3) $x(n) = \cos(2n)$

Sol: $x(n) = \cos(2n)$

w.k.T:-

$$x(n) = A\cos(\Omega_0 n + \phi)$$

{ $x$ly by with π Then it is periodic }

$$\boxed{\Omega_0 = 2}$$

$\frac{2}{2} \cancel{2\pi}$ 2π

$x(n)$ is non-periodic because [Ω_0 is not multiple with π]

4) $x(n) = 3e^{j3/5(n+1/2)}$

soln: Given:-

$$x(n) = 3e^{j3/5(n+1/2)}$$

$$= 3\left[e^{j3/5 n} \cdot e^{j3/5(\frac{1}{2})}\right]$$

$$= 3\left[e^{j\frac{3}{5}n} \quad 3e^{j\frac{3}{10}}\right]$$

$$= 3\left[e^{j\frac{3}{5}n}\left(\cos 0.3 + j\sin 0.3\right)\right]$$

$$= 3\left[0.95 + j0.29\right] e^{j\frac{3}{5}n}$$

$$= \left[2.85 + j0.87\right] e^{j\frac{3}{5}n}$$

Here $\Omega_0 = \frac{3}{5}$ [Ω_0 is not multiple with π]

$\therefore$ $x(n)$ is non periodic.

5) $x(n) = 3e^{j\frac{3}{5}\pi(n+\frac{1}{2})}$

Sol :- $x(n) = 3e^{j\frac{3}{5}\pi(n+\frac{1}{2})}$ $\quad = 3e^{j\frac{3}{5}\pi n + j\frac{3}{5}\frac{1}{2}\pi}$

$$= 3\left[e^{j\frac{3}{5}\pi n} \cdot e^{j\frac{3}{5}\pi\frac{1}{2}}\right] \quad \text{or} \quad = 3e^{j\frac{3}{5}\pi n} \cdot e^{j\frac{3}{5}\frac{1}{2}}$$

$$= 3\left[e^{j\frac{3}{5}\pi n} \cdot e^{j\frac{3}{10}\pi}\right]$$

Here $\Omega_0 = \frac{3}{5}\pi$

$\Omega_0 = \frac{3}{5\times\cancel{2}}\cancel{2}\pi$

$\Omega_0 = \frac{3}{10}2\pi$

$\Omega_0 = \frac{m}{N}2\pi$

$\boxed{N = 10}$

$\therefore$ $x(n)$ is periodic. The fundamental period is

$\boxed{N = 10}$

* Determine whether the following DTS are periodic or not find its fundamental period

1) $x(n) = 5\sin\left(\frac{7\pi}{12}n\right) + 8\cos\left(\frac{14\pi}{8}n\right)$

2) $x(n) = \cos\left(\frac{\pi}{2}n\right) - \sin\left(\frac{\pi}{8}n\right) + 3\cos\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)$

3) $x(n) = 1 + e^{j\frac{4}{7}n} - e^{j\frac{2\pi}{5}n}$

Note: The sum of M periodic signals $x_1(n), x_2(n), x_3(n) \ldots x_M(n)$ is necessarily periodic, the following steps can be used to determine the period

1) Find the LCM of fundamental period of M signals that is $N_1, N_2, N_3 \ldots N_M$

Then, $x_M(n)$

2) Then the period of some signal is given by $N = LCM(N_1, N_2, N_3 \ldots N_M)$

$$\boxed{N = l}$$

1) $x(n) = 5\sin\left(\frac{7\pi}{12}n\right) + 8\cos\left(\frac{14\pi}{8}n\right)$

Sol: Given:

$$x(n) = 5\sin\left(\frac{7}{12}\pi n\right) + 8\cos\left(\frac{14\pi}{8}n\right)$$

$$\Omega_{0_1} = \frac{7}{12}\pi$$

$$= \frac{7}{12\times 2} 2\pi$$

$$= \frac{7}{24} 2\pi \Rightarrow \frac{m_1}{N_1} * 2\pi$$

$$\boxed{N_1 = 24}$$

$$\therefore \Omega_{0_2} = \frac{14}{8}\pi$$

$$= \frac{14}{16} 2\pi \Rightarrow \frac{m_2}{N_2} 2\pi$$

$$\Rightarrow = \frac{7}{8} 2\pi$$

$$\boxed{N_2 = 8}$$

$$LCM(N_1, N_2) = l$$

$$LCM[24, 8] = 24.$$

$\therefore$ Fundamental period of

$$x(n) = N = 24$$

2) $x(n) = \cos\left(\frac{\pi}{2} n\right) - 5\sin\left(\frac{\pi}{8} n\right) + 3\cos\left(\frac{\pi}{4} n + \frac{\pi}{3}\right)$

$$\Omega_0 = \frac{1}{2}\pi$$

$$\Omega_{o_1} = \frac{1}{2\times2}2\pi \Rightarrow \frac{m_1}{N_1}2\pi$$

$$\Omega_{o_1} = \frac{1}{4}2\pi$$

$$\boxed{N_1 = 4}$$

$$\Omega_{o_2} = \frac{\pi}{8}$$

$$\Omega_{o_2} = \frac{1}{8}\pi$$

$$= \frac{1}{8\times2}2\pi$$

$$= \frac{1}{16}2\pi$$

$$\boxed{N_2 = 16}$$

$$\Omega_{o_3} = \frac{1}{4}\pi$$

$$\Omega_{o_3} = \frac{1}{4\times2}2\pi$$

$$= \frac{1}{8}2\pi \Rightarrow \frac{m_3}{N_3}$$

$$\boxed{N_3 = 8}$$

2	4, 16, 8
2	2, 8, 4
2	1 4 2
2	1 2 1
	1 1 1

$$\text{LCM}(N_1, N_2, N_3) = 16$$

$$\text{LCM}(4, 16, 8) = 16 = N.$$

$\therefore$ Fundamental Period of $x(n) = N = 16$

3) $x(n) = 1 + e^{j\frac{4}{7}n} - e^{j\frac{2\pi}{5}n}$

soln:- Given:-

$$x(n) = 1 + e^{j\frac{4}{7}n} - e^{j\frac{2\pi}{5}n}$$

$$\Omega_{01} = \frac{4}{7}$$

$$\Omega_{01} = \frac{4}{7} 2\pi \Rightarrow \frac{m_1}{N_1} 2\pi$$

$$\Omega_{01} = \frac{4}{7} 2\pi$$

$$= \frac{4}{7 \times 2} 2\pi$$

$$= \frac{\cancel{4}\,2}{\cancel{14}\,7} 2\pi$$

$$\Omega_{01} = \frac{2}{7} 2\pi$$

$$\boxed{N_1 = 7}$$

$$\Omega_{02} = \frac{2\pi}{5}$$

$$\Omega_{02} = \frac{2}{5}\pi$$

$$= \frac{2}{5} 2\pi \Rightarrow \frac{m_2}{N_2} 2\pi$$

$$= \frac{2}{5 \times 2} 2\pi$$

$$= \frac{\cancel{2}\,1}{\cancel{10}\,5} 2\pi$$

$$\boxed{N_2 = 5}$$

LCM

7	7	5
5	1	5
	1	1

$7 \times 5 = 35$

$LCM(N_1, N_2) = l$

$LCM[7, 5] = 35$

$\therefore$ Fundamental period of $x(n) = N = 35$

*) Check whether the following discrete time signal are Energy or power signals

a) $x(n) - 1$; $|n| \leq 1$

b) $x(n) = n$; $0 \leq n \leq 5$

$10-n$; $5 < n \leq 10$.

0 ; otherwise.

c) $x(n) = A\delta(n)$.

d) $x(n) = A$

e) $x(n) = u(n)$

f) $x(n) = nu(n)$

g) $x(n) = \cos(\pi n)$; ~~$0 \leq$~~ $-4 \leq n < 4$

0 ; otherwise.

A) a) $x(n) - 1$; $|n| \leq 1$

Soln :- $x(n) = 1$; $|n| \leq 1$

$x(n) = 1$; $-1 \leq n \leq 1$

W.K.T:

$$\text{Energy } E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-1}^{1} |x(n)|^2$$

Substitute n value in above eq

$$= |x(-1)|^2 + |x(0)|^2 + |x(1)|^2$$

$$= 1^2 + 1^2 + 1^2$$

$= 3$ It is finite.

$x(n) = 1 \; ; \; |n| \leq 1$, is an energy signal.

b) $x(n) = n \; ; \; 0 \leq n \leq 5$

$10 - n \; ; \; 5 < n \leq 10.$

$0 \; ; \;$ otherwise.

soln:- ~~$x(n) = n \; , \; 0 \leq n \leq 5$~~

~~$x(n) = n \; ; \; 5$~~

$$x(n) = \begin{cases} n, & 0 \leq n \leq 5 \\ 10-n, & 5 < n \leq 10 \\ 0, & \text{otherwise.} \end{cases}$$

n	$x(n)$
0	0
1	1
2	2
3	3
4	4
5	5
6	4 $(10-n)$
7	3
8	2
9	1

w.k.T:-

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=0}^{10} |x(n)|^2$$

$$E = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2$$

$$E = 0 + 1 + 4 + 9 + 16 + 25 + 16 + 9 + 4 + 1.$$

$$E = 85 < \infty \rightarrow \text{Finite}.$$

Hence $x(n)$ is Energy signal

25
16
16
18
75
5
85

c) $x(n) = A\delta(n)$.

Soln: Given:-

$$x(n) = A\delta(n).$$

$$\delta(n) = \begin{cases} 1, & n=0 \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = \begin{cases} A & n=0 \\ 0 & \text{otherwise} \end{cases}$$

w.k.T:-

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0} |x(n)|^2$$

$$\boxed{E = A^2} \Rightarrow E = A^2 < \infty$$

∴ Finite

Hence $x(n)$ is Energy signal

d) $x(n) = A$

Soln:- Given :-

$$x(n) = A$$

w.k.T :-

$$\cancel{E = \lim_{n\to\infty} \sum_{n=-\infty}} \qquad E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$\cancel{E = \lim_{N\to\infty}}$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} A^2$$

$$E = A^2 \lim_{n\to\infty} \left[\sum_{n=-N}^{N} 1 \right] \quad \left[\because \sum_{n=-N}^{N} 1 = 2N+1\right]$$

$$E = A^2 \lim_{N\to\infty} (2N+1) \qquad N = \infty$$

$$\boxed{E = \infty} \rightarrow \text{Infinite}$$

$x(n)$ is not Energy signal.

→ power of DTS is given by.

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \qquad P = \lim_{N\to\infty} \frac{1}{2N+1}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} A^2$$

$$= A^2 \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= A^2 \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} \cancel{(2N+1)}$$

$\therefore P = A^2 < \infty$

Finite

Hence $x(n)$ is power signal

$$= \lim_{N\to\infty} 1$$

e) $x(n) = u(n)$

$\sum_{0}^{N} \to N+1$

soln: $x(n) = u(n)$

$\sum_{-N}^{N} \to 2N+1$.

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \text{ or otherwise.} \end{cases}$$

w.k.T:

$$E = \lim_{N\to 0} \cancel{E=} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \lim_{N\to\infty} \cancel{E=} \sum_{n=0}^{N} |x(n)|^2$$

$$E = \lim_{N\to\infty} \cancel{E=} \sum_{n=0}^{N} 1$$

$$\boxed{E = \infty}$$

Hence $x(n)$ is not a Energy signal.

power :-

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} N+1$$

$$\boxed{P = \infty}$$

neither En nor power

?) $x(n) = n\,u(n)$

Soln:- Given :-

$$x(n) = n\,u(n)$$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \quad x(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$x(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

W.k.T:-

$$E = \lim_{N\to\infty} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \lim_{N\to\infty} \sum_{n=0}^{\infty} n^2$$

$$E = \lim_{N\to\infty} \sum_{n=0}^{\infty} n^2$$

$$E = n^2 \lim_{N\to\infty} \sum_{n=0}^{\infty} 1 \quad \because \sum_{K=1}^{N} K^2 = \frac{N(N+1)(2N+1)}{6}$$

$$E = n^2 \lim_{N\to\infty} N(N+1) \longrightarrow 0^2 + \sum_{n=1}^{N} n^2$$

$$= \lim_{N\to\infty} \frac{N(N+1)(2N+1)}{6}$$

$\boxed{E = \infty}$ $\rightarrow$ Infinite

not an energy signal

w.k.T

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$$

$$= n \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0} 1$$

$$= n^2 \lim_{N\to\infty} \frac{1}{2N+1} (N+1)$$

$$\frac{N(N+1)\times(2N+1)}{6}$$

$\boxed{P = \infty}$ $x(n)$ signal neither Energy nor power

g) $x(n) = \cos(\pi n)$; $-4 \le n \le 4$
$\quad 0$; otherwise.

soln: Given:-

$$x(n) = \begin{cases} \cos(\pi n) & ; -4 \le n < 4 \\ 0 & ; \text{otherwise.} \end{cases}$$

w.k.T:-

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=-4}^{4} |x(n)|^2$$

$n = -4, -3, -2, -1, 0, 1, 2, 3$

n	$x(n)$
-4	$\cos(\pi\times -4) = 1$
-3	$\cos(-3\pi) = -1$
-2	$\cos(-2\pi) = 1$
-1	$\cos(-1\pi) = -1$
0	$\cos(0\pi) = 1$
1	$\cos(1\pi) = -1$
2	$\cos(2\pi) = 1$
3	$\cos(3\pi) = -1$
4	$\cos(4\pi) = 1$

even = 1
odd = -1

$$= |x(-4)|^2 + |x(-3)|^2 + |x(-2)|^2 + |x(-1)|^2 + |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2.$$

$$= 1^2 + (-1)^2 + 1^2 + (-1)^2 + 1^2 + (-1)^2 + 1^2 + (-1)^2 + 1^2$$

$$= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$

$$\boxed{E = 9}$$

$$\boxed{E = 9 < \infty} \rightarrow \text{Finite}$$

Hence $x(n)$ is Energy signal. //

h) Find the energy of the sequence $x(n) =$

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n ; & n \geq 0 \end{cases}$$

h) soln:- Given:-

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n ; & n \geq 0 \\ 0 ; & \text{otherwise} \end{cases}$$

w.k.T:-

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n\right|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= \frac{1}{1-\frac{1}{4}}$$

$$E = \frac{1}{\frac{4-1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$$

i) Determine whether the given signal

$$x(n) = \begin{cases} 3(-1)^n ; & n \geq 0 \\ 0 ; & \text{otherwise.} \end{cases}$$ power (or) Energy

Soln:- Given:-

$$x(n) = \begin{cases} 3(-1)^n ; & n \geq 0 \\ 0 , & \text{otherwise} \end{cases}$$

$$E = \lim_{N \to \infty} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \lim_{N \to \infty} \sum_{n=-N}^{N} |3(-1)^n|^2$$

$$= \lim_{N \to \infty} \sum_{n=+0}^{N} [3(-1)^n]^2$$

$$= \lim_{N \to \infty} \sum_{n=+0}^{N} 9((-1)^2)^n$$

$$= \lim_{N \to \infty} \sum_{n=0}^{N} 9(1)$$

$$= 9 \lim_{N \to \infty} \sum_{n=0}^{N} 1$$

$$= 9 \lim_{N \to \infty} (N+1)$$

$$= 9(\infty + 1)$$

$$\boxed{E = \infty}$$ Infinite

It is not a Energy signal

* power signal

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} [3(-1)]^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 9(1)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 9$$

$$= 9 \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1$$

$$= 9 \lim_{N\to\infty} \frac{1}{2N+1} (N+1)$$

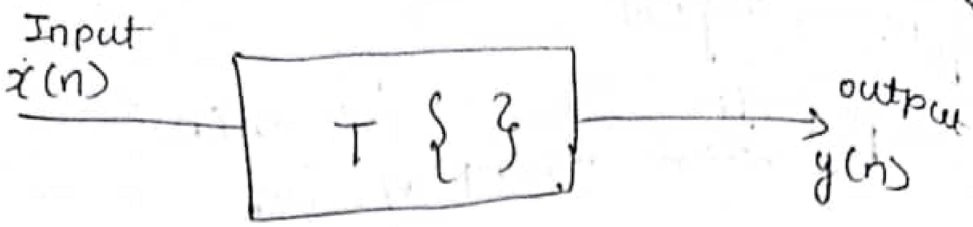
$$P = 9(\infty+1)$$

$$\boxed{P = \infty}$$

$\therefore$ ~~It~~ $x(n)$ is neither Energy nor power.

* Discrete Time Systems :-

One more discrete time signals as a input modified according to a desired manner we will get o/p in the form of Discrete sequence (discrete signal) is called DT System.



$$y(n) = T\{x(n)\}$$

$$y(n) = T\{x_1(n)\} + T\{x_2(n)\}$$

$$y(n) = y_1(n) + y_2(n)$$

Consider a DT system represented by a Operator T. And input signal $x(n)$ applied to Discrete time system results in an output signal $y(n)$ described as

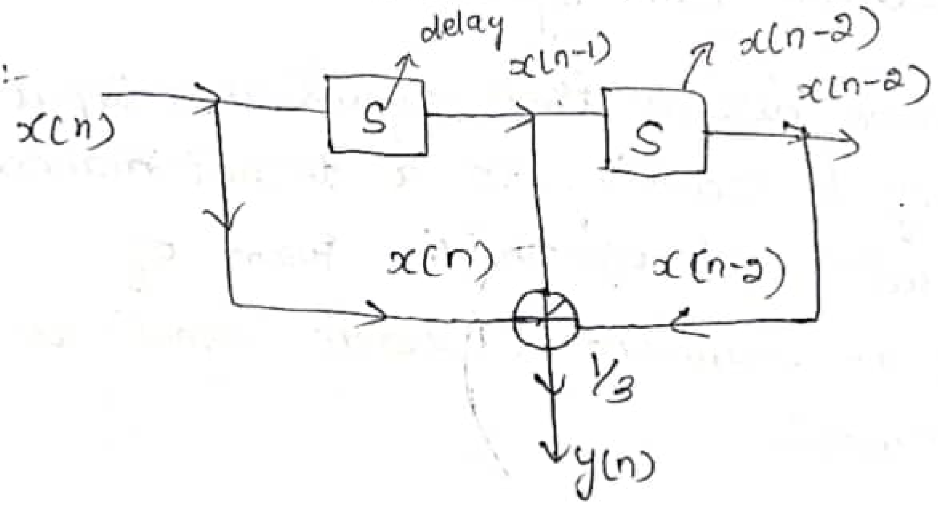
$$\boxed{y(n) = T\{x(n)\}}$$

Example :- 1) Find the overall operator of a system whose o/p signal $y(n)$ is given by

$$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]$$

also draw the block diagram representation.

Soln:-

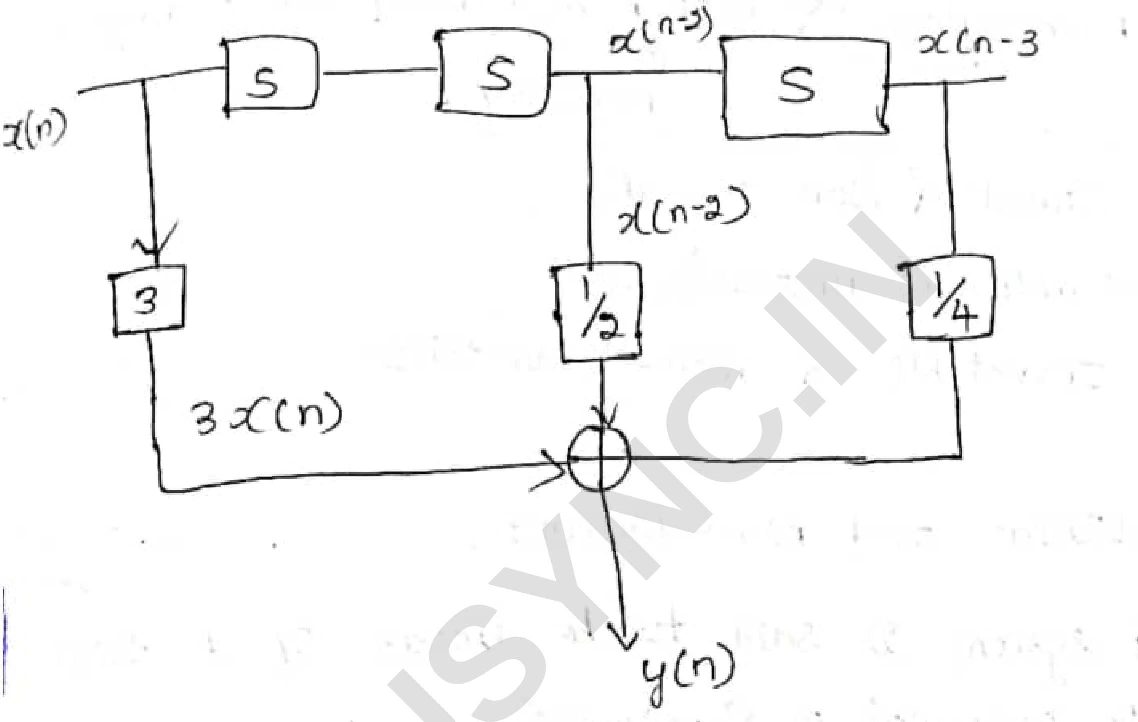


2) Find the overall operator of a system whose o/p signal $y(n)$ is given by

$$y(n) = 3x(n) + \frac{1}{2}x(n-2) + \frac{1}{4}x(n-3)$$

Soln:- Given :-

$$y(n) = 3x(n) + \frac{1}{2}x(n-2) + \frac{1}{4}x(n-3)$$

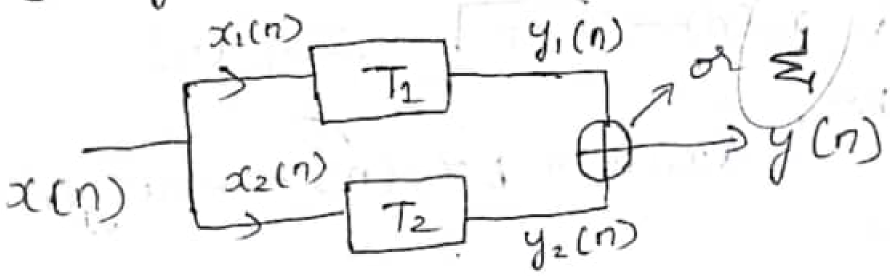


3) A system consist of several sub-system connected as shown in figure below find the operator T relating $x(n)$ to $y(n)$ for the sub system operators is given by

$$T_1 = y_1(n) = x_1(n)\, x_1(n-1)$$

$$T_2 = y_2(n) = |x_2(n)|$$

Soln:-



soln:- $y(n) = x_1(n)\, x_1(n-1)$

* properties/Classification system :-

1) Linear and non-linear
2) Time invariant & variant
3) Memoryless & Not memoryless (or) memory / Static & Dynamic
4) Causal & Non-causal
5) Stable & unstable
6) Invertible & non-Invertible

1) Linear and non-linear :-

A system is said to be linear if it satisfies the principal of Superposition.

i,e If an input consist of the weighted sum of several signals then the o/p is weighted sum of the responses of the system, is called principle of Superposition

If $x_1(n) \rightarrow \boxed{T} \rightarrow y_1(n)$

& $x_2(n) \rightarrow \boxed{T} \rightarrow y_2(n)$

Then,

$$a x_1(n) + b x_2(n) \longrightarrow a y_1(n) + b y_2(n)$$

→ Non linear system: A system is said to be non-linear if it's not satisfies the principle of superposition

2) Time invariant & variant

→ Time invariant :- A time invarient system is one in which a time shift of the input signal causes ~~the to~~ a corresponding time shift in the output signal.

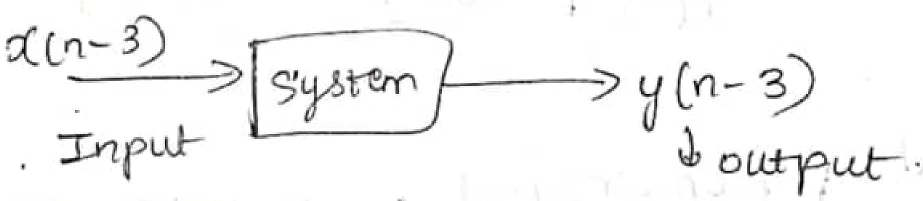
The shift may be advance or Delay.

if $x(n) \longrightarrow y(n)$

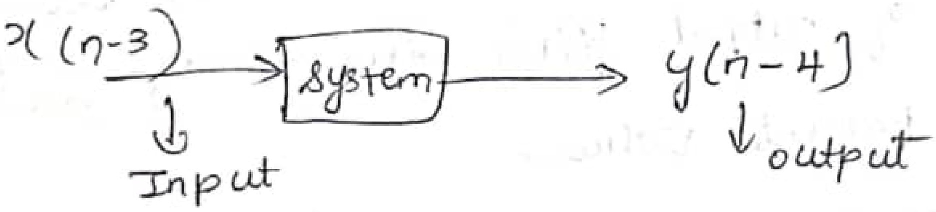
Then, ~~$x(n)$~~ $x(n-n_0) \longrightarrow y(n-n_0)$

→ Variant :- If it is not satisfies the time invariant condition that type of systems are called time variant

Ex:- For time Invarient



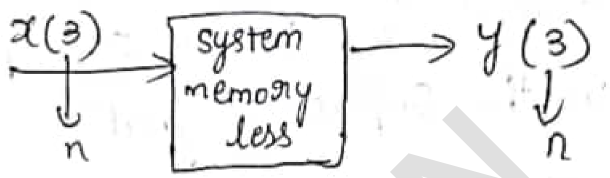
EX: for time Variant.



3) Memory less and ... or memory
Dynamic

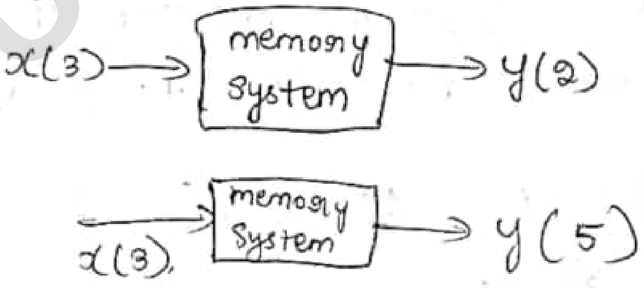
→ Memory less / mem :- A Discreate time signal is referred to as if the output y(n) at every value of 'n' depends on the input x(n) at the same value of n

Ex :-



→ Not memory less :- A system is said to not memory less (memory/Dynamic) memory if its output is depends on past and or future values of input.

Ex:



4) Causal & Non-causal :- A discreate time signal is said to be causal if present value of output y(n) depends ~~of~~ on past or present values.

→ If it not satisfies the causal condition the system is called as Non-causal

A system is said to be non-causal if present value of output $y(n)$ is depends on future value of Input $x(n)$.

Ex: for causal condition

$x(n) \longrightarrow \boxed{\text{System Causal}} \longrightarrow y(n-1) + \cancel{\text{Past}}$
$\downarrow y(n)$

Ex: for non-causal.

$x(n) \longrightarrow \boxed{\text{non-causal system}} \longrightarrow y(n+1)$

5) Stable and unstable:-

→ stable :- A system is said to be stable ~~then~~ bounded input $x(n)$ with respect to if output $y(n)$ is finite quantity (bounded).

$$Ex: -3 < x(n) < 3 \longrightarrow -3 < y(n) < 3$$
$$2.5 \longrightarrow 1$$

→ unstable :- If it is not satisfies the stable condition that is bounded input & bounded output that type of systems are called unstable.

$$Ex:- -3 < x(n) < 3 \longrightarrow -3 < y(n) < 3$$
$$2.5 \longrightarrow 5$$

Amplitude.

6) Inventible & non-inventible

→ Inventible :- A system is said to be inventible if the input of the system can be recovered from the system output.

→ Non-inventible :- A system is said to be non-inventible if the input of the system cannot be recovered from the system output.

Ex:-

for inventible

Ex: for non-inventible :-

* problem : Based on properties or Classification DTS

1) For the following DTS whether the system is i) Linear ii) Time Invarience iii) memory less iv) causal v) Stable.

a) $y(n) = n\,x(n)$

b) $y(n) = x(-n)$

c) $y(n) = x(2n)$

d) $y(n) = x^2(n)$

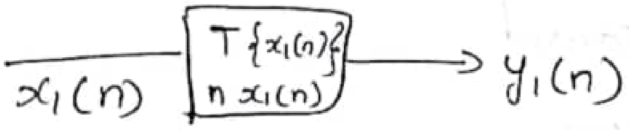
a) $y(n) = nx(n)$

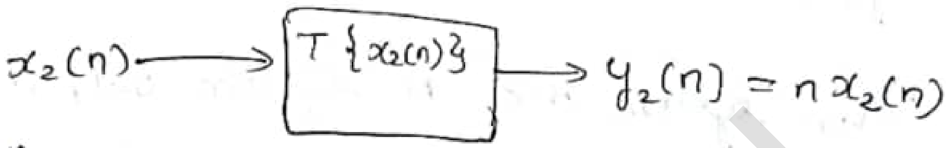
soln:-

i) Linear :

if $x_1(n) \longrightarrow y_1(n)$

& $x_2(n) \longrightarrow y_2(n)$





Then,

$$a x_1(n) + b x_2(n) \longrightarrow a y_1(n) + b y_2(n)$$

$$a x_1(n) + b x_2(n) \longrightarrow T\{a x_1(n) + b x_2(n)\}$$

$$\longrightarrow T\{a x_1(n)\} + T\{b x_2(n)\}$$

$$\longrightarrow a T\{x_1(n)\} + b\, T\{x_2(n)\}$$

$$\longrightarrow a n x_1(n) + b n x_2(n)$$

$$a x_1(n) + b x_2(n) \longrightarrow a y_1(n) + b y_2(n)$$

$\therefore$ Given system is linear.//

ii) Time invariant / variant :-

if $n \longrightarrow n - n_0$.

$$y(n-n_0) = n - n_0\, x(n-n_0),$$

$$y(n-n_0) = \overset{n-n_0}{n}\, x(n-n_0)$$

$\hookrightarrow$ variant

Given system is Time variant

iii) memory less / memory

$$y(n) = \cancel{x}\, n\, x(n)$$

$\therefore$ Given system is memoryless

iv) Causal / Non-Causal

$$y(n) = n\, x(n)$$

$\therefore$ Given system ~~is~~ is A system output depends on present value of Input hence given system is causal.

v) stable / un-stable :-

$$y(n) = n\, x(n)$$

if n is finite quantity

Then given system is stable.

b) $y(n) = x(-n)$

i) Linear :

$$\text{if } y_1(n) = T\{x_1(n)\} = x_1(-n)$$

$$y_2(n) = T\{x_2(+n)\} = x_2(-n)$$

$$y(n) = T\{a x_1(n) + b x_2(n)\}$$

$= T\{a x_1(n) + b x_2(n)\}$

$= T a x_1(n) + T b x_2(n)$

$= a T\{x_1(n)\} + b T\{x_2(n)\}$

$= a x_1(-n) + b x_2(-n).$

$y(n) = a y_1(n) + b y_2(n).$

Hence it is linear.

ii) Time invarient / varient

Given :- $y(n) = x(-n)$

$n \to n - n_0$

$y(n) = x(-(n-n_0))$

$y(n-n_0) = x(-n+n_0)$

$2-3 \quad -2+3$

$-1 \quad +1$

ex: $n = 2$, & $n_0 = 3$.

$\therefore$ Given system is variant

iii) memory less / memory.

$y(n) = x(-n)$

$\therefore$ It is memory system.

iv) Causal / Non - Causal.

$y(-3) = x(3)$

$y(n) = x(-n)$ ex: $n = -3$.

$\therefore$ Non causal.

v) Stable & unstable.

~~if n is~~ $y(n) = x(-n)$

if n is finite quantity

Then given system is stable

c) $y(n) = x(2n)$

i) Linear:

$y_1(n) = \{T x_1(n)\} = x(2n)\}$

$y_2(n) = \{T x_2(n)\} = x_2(2n)]$

$y(n) = T\{a x_1(n) + b x_2(n)\}$

$= T\{a x_1(n)\} + T\{b x_2(n)\}$

$= aT\{x_1(n)\} + bT\{x_2(n)\}$

$= a x_1(2n) + b x_2(2n)$

$y(n) = a y_1(n) + b y_2(n).$

∴ Hence it is linear

ii) Time invariant / variant

Given: $y(n) = x(2n)$

$n - n_0 \Rightarrow y(n-n_0) = x(2(n-n_0))$

$= \cancel{x(}$ If $n = 2$, $n_0 = 3$

$y(2-3) = x(2(2-3))$

... invarient .

iii) memory less / memory.

$y(n) = x(2n)$.

$y(n) = x(2n)$

Ex: -

$\therefore$ It is memory.

iv) causal & non-causal.

$$y(n) = x(2n)$$

$n = -3$ $\quad y(-3) = x(2(-3))$

$$y(-3) = x(-6) \text{ present \& future}$$

$\therefore$ It is non causal.

v) stable / un-stable.

if n is finite quantity
Then given system is stable.

$\therefore$ The given system is Linear, time invarient, memory, noncausal & stable.

d) $y(n) = x^2(n)$

i) Linear / non linear :-

if $\quad y_1(n) = \{Tx_1(n)\} = x_1^2(n)$

$\quad y_2(n) = \{Tx_2(n)\} = x_2^2(n)$

$$y(n) = [a y_1(n) + b y_2(n)]$$

$$= T[a x_1(n) + b x_2(n)] \Rightarrow x^2(n)$$

$$= \cancel{[a T x_1(n) + b T x_2(n)]}$$

$$= \cancel{a T y x} \cdot [a x_1(n) + b x_2(n)]^2$$

$$a y_1(n) + b y_2(n) \neq T\{a x_1(n) + b x_2(n)\}$$

$\therefore$ It is non linear.

ii) Time invarient / variant

$$y(n) = x^2(n) \rightarrow n = n - n_0$$

$$y(3) = x^2(3)$$

$\therefore$ Time invarient

iii) memory less / memory.

if $y(n) = x^2(n)$

Ex: $n = 3$ $y(3) = x^2(n)$

$\therefore$ It is memory less

iv) causal / non-causal

if $y(n) = x^2(n)$

$$y(4) = x^2(4)$$

$\therefore$ It is causal

v) stable

if $y(n) = x^2(n)$

n is finite quantity Then given system is stable.

HW e) $y(n) = m\,x(n^2)$ linear TV Non-causal memory stable

f) $y(n) = n\,x^2(n)$ → non linear ; Time V memless causal stable

g) $y(n) = x(1-n)$ linear TV memory non causal stable

h) $y(n) = p\,x(n) + q$. where p & q are the constant

e) $y(n) = m\,x(n^2)$

Soln:- i) Linear / non linear

$$y(n) = m\,x(n^2)$$

$$y_1(n) = \{T x_1(n)\} = m\,x_1(n^2)$$

4.

* Linear Time invarient system :-

↳DTS

A system satisfies linearity property and time invariant property that type of systems are called linear time invariant system.

The output of an LTI system is given by a weighted superposition of time shifted impulse responses.

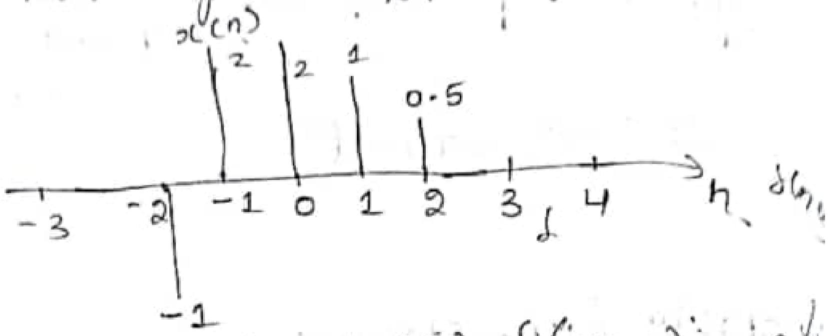
→ This weighted superposition is known as convolution theorem.

→ covulution theolem.

* Impulse Response :- Representation ... time LTI system. or Graphical represent...

(1)

⇒ Consider signal :- representation



$x_1(n) = x(-2)\,\delta(n+2)$

$n=0$; $x_1(0) = (-1)\,\delta(2)$

$n=0$; $x_1(0) = (-1)\,0$

$\boxed{x_1(0) = 0}$

$n = 1$; $x_1(1)$; $x_1(1) = (-1)\,\delta(1)$

$x_1(1) = (-1)\,(0)$

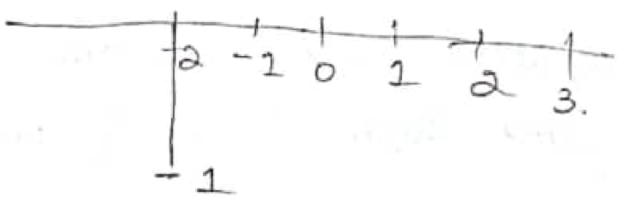
$\boxed{x_1(1) = 0}$

$n = -1$; $x_1(1)$; $x_1(2) = (-1)\,\delta(0)$ 1

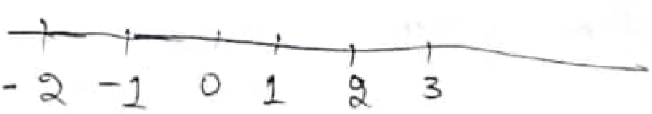
$x_1(2) = -1\,(1)$

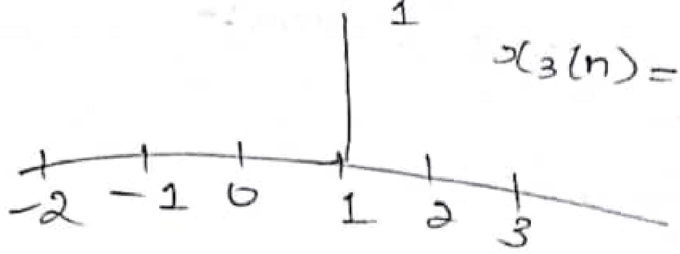
$\boxed{x_1(2) = -1}$

- $x_1(n) = x(-2)\,\delta(n+2) \rightarrow (1)$

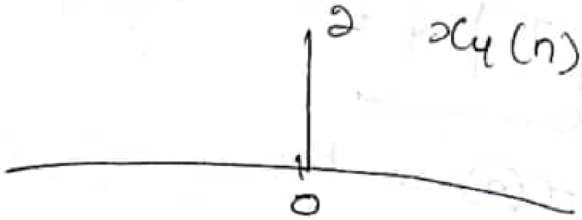


- $x_2(n) = 0.5 = x(2)\,\delta(n-2) \rightarrow (2)$

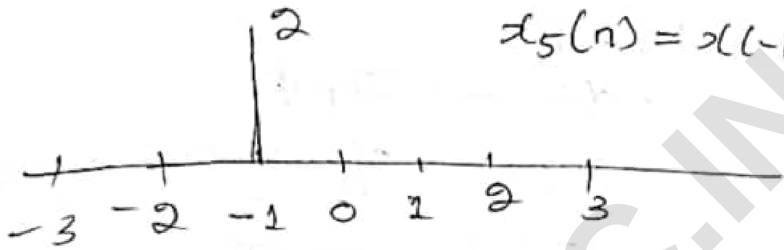




$x_3(n) = x(1)\delta(n-1) \rightarrow (3)$



$x_4(n) = x(0)\delta(n) \rightarrow (4)$



$x_5(n) = x(-1)\delta(n+1) \rightarrow (5)$

- Add Eq (1), (2), (3), (4) & (5) $\rightarrow$ $x(n)$ signal

$\Rightarrow x(-2)\delta(n+2) + x(2)\delta(n-2) + x(1)\delta(n-1) + x_1(0)\delta(n) + x(-1)\delta(n+1)$

$\Rightarrow x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2)$.

$x(n) = x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2)$.

$$\boxed{x(n) = \sum_{k=-2}^{2} x(k)\delta(n-k)}$$

(or) In general

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)}$$

Imp

* Expression for Impulse response for Discrete time LTI System :-

Consider a system :

$$x(n) \longrightarrow \boxed{T\{\ \}} \longrightarrow y(n)$$

* → we express $x(n)$ as the weighted sum of time shifted impulse.

we have :

- $x(n) \rightarrow$ Input
- $y(n) \rightarrow$ o/p of the system

we have $T\{\ \}$ operator.

$$y(n) = T\{x(n)\} \longrightarrow ①$$

$$y(n) = T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right] \rightarrow ②$$

$n \geq 0$

1

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,T\delta(n-k)$$

Consider :

$$T(\delta(n-k)) = h(n-k)$$

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)}$$

(or)

$$\boxed{y(n) = x(n) \circledast h(n)}$$

convolution symbol

1) show that (a) $x(n) * \delta(n) = x(n)$

3M each or total

(b) $x(n) * \delta(n-n_0) = x(n-n_0)$

(c) $x(n) * u(n-n_0) = \sum_{k=-\infty}^{n-n_0} x(k)$

Soln a) $x(n) * \delta(n) = x(n)$

soln: $x(n) * \delta(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

when $k=n$

$= \sum_{k=n} (x k) \quad \because \delta(0)=1$

$y(n) = x(n) * \delta(n) \Big|_{k=n} = x(n)\,\delta(0)$

Hence proved. $= x(n)$

b) $x(n) * \delta(n-n_0) = x(n-n_0)$

soln:- $x(n) * \delta(n-n_0) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-n_0-k)$

when $k = n - n_0$. $\delta(n-n_0-k) = 1$.

$x(n) * \delta(n-n_0)\Big|_{k=(n-n_0)} = \sum_{k=-\infty}^{\infty} x(n-n_0)$

Hence proved.

c) $x(n) * u(n-n_0) = \sum_{k=-\infty}^{n-n_0} x(k)$

soln:- $x ~~(n) *~~ \delta(n) = \sum$

~~u(n)~~

→ by definition of unit step signal

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \qquad \delta(0) = 1$$

$$u(n-n_0) = \begin{cases} 1 & n-n_0 \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(n) * u(n-n_0) = \sum_{k=-\infty}^{\infty} x(k)\, u(n-n_0-k)$$

$u(n-n_0-k)$

$$= \begin{cases} 1 & n-n_0-k \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(n) * u(n-n_0) = \sum_{k=-\infty}^{\infty} x(k)$$

2) Find the convolution sum of 2 sequences $x_1(n)$ & $x_2(n)$

$x_1(n) = (\underset{\uparrow}{1}, 2, 3) = x_1(0) = 1,\ x_1(1) = 2,\ x_1(2) =$

$x_2(n) = (\underset{\uparrow}{2}, 1, 4) = x_2(0) = 2,\ x_2(1) = 1,\ x_2(2) =$

Soln: $x_1(n) * x_2(n) = y(n)$

$$x_1(n) = 1\delta(n) + 2\delta(n-1) + 3\delta(n-2)$$

$$x_2(n) = 2\delta(n) + 1\delta(n-1) + 4\delta(n-2)$$

$$x_1(n) * x_2(n) = [\delta(n) + 2\delta(n-1) + 3\delta(n-2)] * [2\delta(n) + 1\delta(n-1) + 4\delta(n-2)]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$$= \sum_{k=0}^{2} x_1(k)\, x_2(n-k)$$

$$y(n) = x_1(0)\, x_2(n-0) + x_1(1)\, x_2(n-1) + x_1(2)\, x_2(n-2)$$

$$y(n) = x_2(n) + 2x_2(n-1) + 3x_2(n-2)$$

$n=0$ $\quad y(0) = x_2(0) + 2x_2(-1) + 3x_2(-2)$

$$\boxed{y(0) = 2}$$

$n=1$; $\quad y(1) = x_2(1) + 2x_2(0) + 3x_2(-1)$

$$= 1 + 2\times 2 + 0$$

$$\boxed{y(1) = 5}$$

$n=2;\ y(2) = x_2(2) + 2x_2(1) + 3x_2(0)$

$= 4 + 2 \times 1 + 3 \times 2$

$= 4 + 2 + 6$

$\boxed{y(2) = 12}$

$n=3;\ y(3) = x_2(3) + 2x_2(2) + 3x_2(1)$

$= 2 \times 4 + 3 \times 1$

$= 8 + 3$

$\boxed{y(3) = 11}$

$n=4;$

$y(4) = x_2(4) + 2x_2(3) + 3x_2(2)$

$= 3 \times 4$

$\boxed{y(4) = 12}$

$\therefore x_1(n) * x_2(n) = y(n)$

$y(n) = [2, 5, 12, 11, 12]$

3) Determine $y(n) = x(n) * h(n)$.

HW

where, $x(n) = [\underset{\uparrow}{1}\ 3\ 1]$

$h(n) = [\underset{\uparrow}{1}\ 4\ 2]$

HW

4) Calculate convolution sum of two sequences

$x_1(n) = [\underset{\uparrow}{1}, 6, 8, 4]$

$x_2(n) = [1, 3, 4]$

$= 32 + 12$

$\boxed{y(4) = 44}$

$\therefore \; x_1(n) * x_2(n) = y(n)$

$y(n) = [1, 9, 30, 52, 44]$ //

Imp

*4) Consider an input $x(n)$ & an unit impulse response $h(n)$ is given by.

$$x(n) = \alpha^n u(n) \; ; \; 0 < \alpha < 1$$

$$h(n) = u(n)$$

Evaluat & plot the o/p signal $y(n)$

Soln:- Given :-

$$y(n) = x(n) * h(n)$$

$$= \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

$x(k) = \alpha^n u(k)$

$x(k) = \alpha^1 u(k)$

Here, $x(K) = \alpha^n u(K)$ $\quad n \geq 0 =$

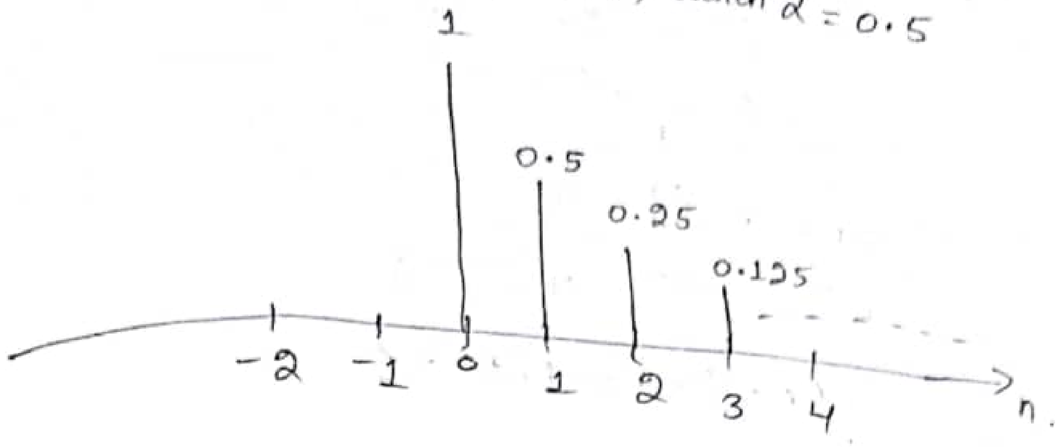
$\& \; h(n) = u(n)$

$\alpha = 0.5$, $\quad n \geq 0 \quad x(n) = \alpha^n$

$\qquad n < 0 \quad x(n) = 0.$

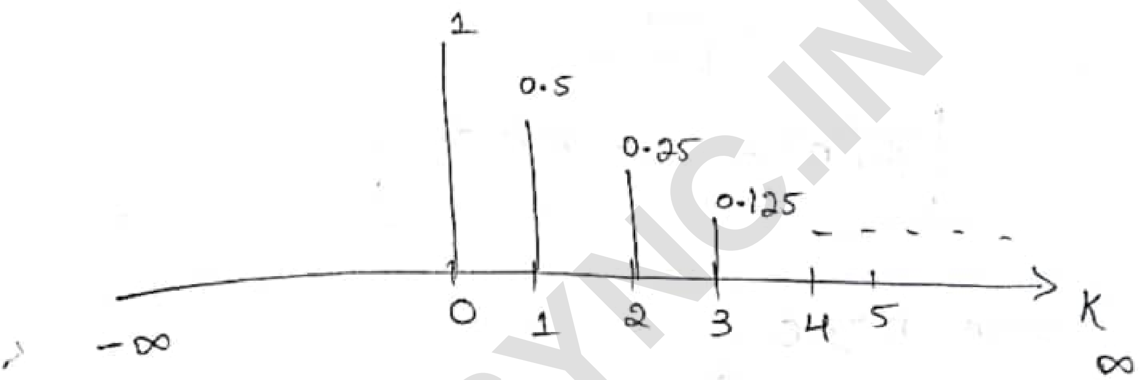
$n = 0, 1, 2, 3,$

$x(n)$, when $\alpha = 0.5$

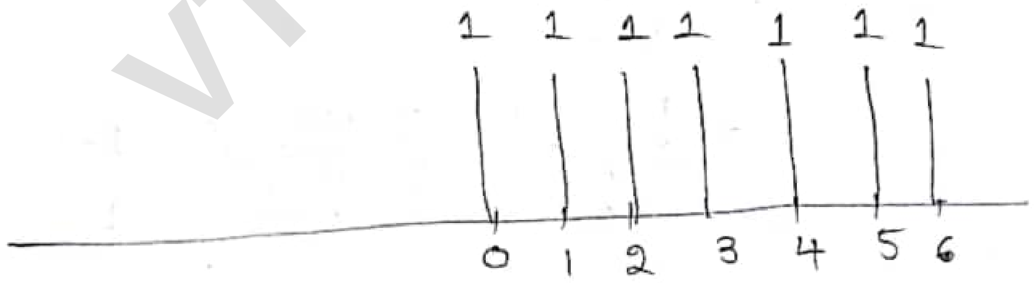


→ when $n = k$

$x(k) = \alpha^k u(k)$

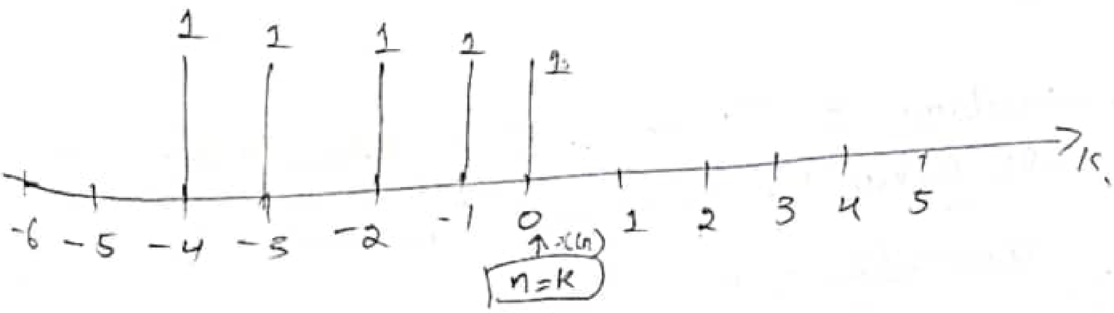


→ $h(n) = u(n)$



Replace n by $(n-k)$

$h(n-k) = u(n-k)$



$$h(n-k) = u(n-k)$$
$$= u(0) = 1$$
$$u(0-1) = u(-1) = 0$$

$k = -1$ $\quad u(0-(-1)) = u(1) = 1$
$$u(0-(-2)) = u(2) = 1$$
$$u(0-(-3)) = u(3) = 1$$

when, $n < 0$
$$y(n) = x(n) * h(n)$$
$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$
$$\boxed{y(n) = 0 \;, \; n < 0}$$

→ when $n \geq 0$
$$y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$
$$= \sum_{k=0}^{n} \alpha^{k} \cdot 1$$
$$= \sum_{k=0}^{n} \alpha^{k} \quad \left[\because \sum_{n=0}^{N-1} \alpha^{n} = \frac{1-\alpha^{N}}{1-\alpha}\right]$$

$$y(n) = \begin{cases} \dfrac{1-\alpha^{n+1}}{1-\alpha} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

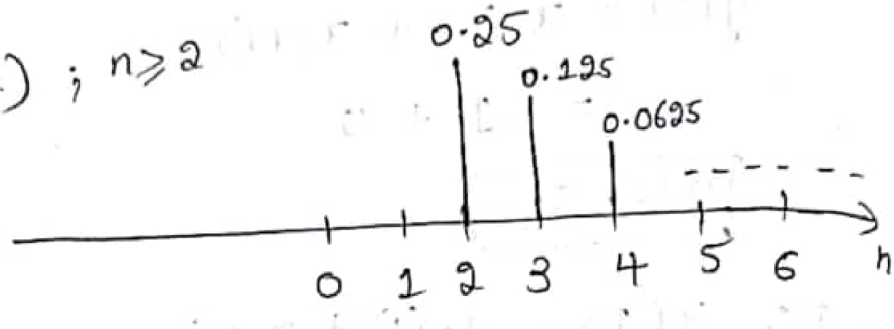
6) Evaluate a discreate time convolution sum is given by $y(n) = \left(\frac{1}{2}\right)^{n} u(n-2) * u(n)$ Convolution with $u(n)$

6).

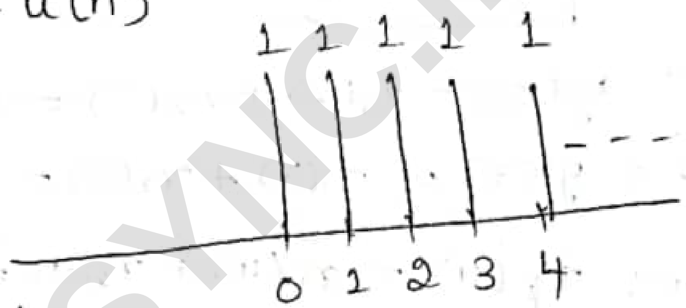
Given:-

$$y(n) = \left(\frac{1}{2}\right)^n u(n-2) * u(n)$$

$\left(\frac{1}{2}\right)^n u(n-2)$; $n \geq 2$

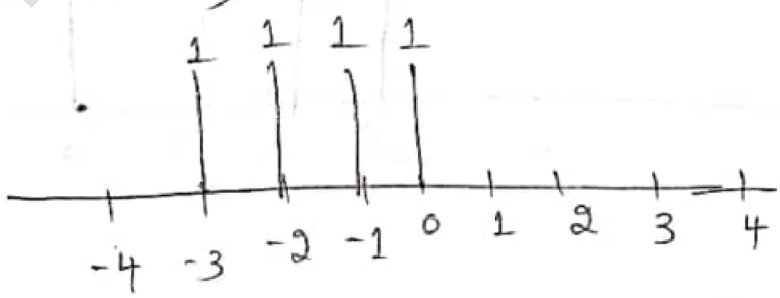


$$h(n) = u(n)$$



Replace n by $n-k$

$h(n-k) = u(n-k)$ for $k \leq n$



when $n < 0$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$x(k) = \left(\frac{1}{2}\right)^k \quad h(n-k)$$

$k=-\infty$

$\boxed{y(n) = 0} \quad n \leq 0$

when $n \geq$

$$y(n) = \sum_{k=2}^{n} x(k)\, h(n-k)$$

$$= \sum_{k=2}^{n} \left(\frac{1}{2}\right)^{k} 1$$

$$= \sum_{k=2}^{n} \left(\frac{1}{2}\right)^{k} 1 \qquad \left[\because \sum_{n=0}^{N} (\alpha)^{n} = \frac{1-\alpha^{N}}{1-\alpha}\right]$$

$$= \sum_{k=2}^{n} \left(\frac{1}{2}\right)^{k} \qquad \left[\because \sum_{n=0}^{N} \alpha^{n} = \begin{cases} \frac{1-\alpha^{N}}{1-\alpha}; & \alpha \neq 1 \\ N; & \alpha = 1 \end{cases}\right.$$

or

$$= \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} \qquad S = a\,\frac{1-\alpha^{N}}{1-\alpha}$$

$$= 2\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

or

$$= \sum_{k=2}^{n} \left(\frac{1}{2}\right)^{k} 1 \qquad \begin{matrix} k=2 \text{ to } n \\ n-2+1 = n-1 \end{matrix}$$

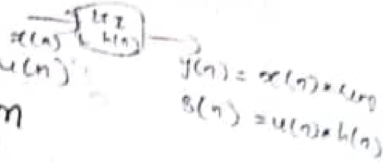
$$= \frac{1}{4}\,\frac{1-\left(\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}} = \frac{1}{4}\,2\left(1-\left(\frac{1}{2}\right)^{n-1}\right)$$

$$= \frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{n-1}\right)$$

$$\therefore y(n) = \begin{cases} \frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{n-1}\right) & ; n \geq 2 \\ 0 & ; n < 2 \end{cases}$$

* The unit step Response [S(n)] of an LTI system

The output of DTLTI system characterised by an impulse response h(n) with x(n)



$$y(n) = x(n) * h(n)$$

If the input is unit step :

$x(n) = u(n)$ then step response is given by

$$S(n) = h(n) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, u(n-k)$$

by defn

$$u(n-k) = \begin{cases} 1 & n-k \geq 0 \text{ or } n \geq k \\ 0 & n-k < 0 \end{cases}$$

$$s(n) = \sum_{k=\infty}^{n} h(k) ,$$

$$\boxed{s(n) = \sum_{k=-\infty}^{n} h(k)}$$

is the required unit step response of an LTI system.

Imp:

1) Determine the step response of the following LTI system.

i) $h(n) = u(n)$

ii) $h(n) = (-1)^n [u(n+2) - u(n-3)]$

iii) $h(n) = \alpha^n u(n)$

iv) $h(n) = (1/2)^n u(n)$

i) $h(n) = u(n)$

soln:- Given:-

$$h(n) = u(n)$$

w.k.T:-

$$s(n) = \sum_{k=-\infty}^{n} h(k)$$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$u(k) = \begin{cases} 1 & k \geq 0 \\ 0 & k < 0 \end{cases}$$

$$s(n) = \sum_{k=-\infty}^{-1} \cancel{h(k)} + \sum_{k=0}^{n} h(k)$$

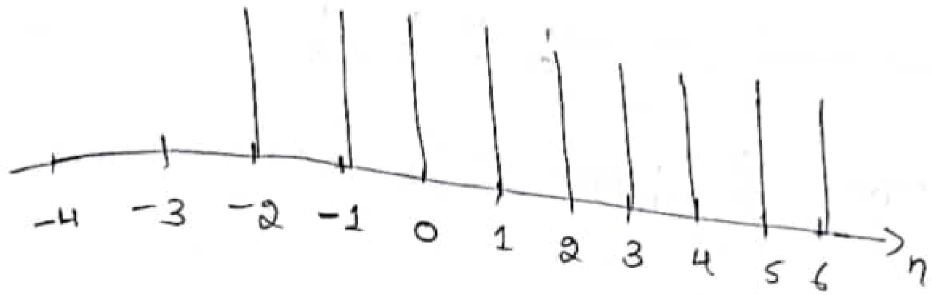
$$= \sum_{k=0}^{n} h(k)$$

$$= \sum_{k=0}^{n} 1 \quad \left[\because \sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \\ N & \alpha = 1 \end{cases}\right]$$

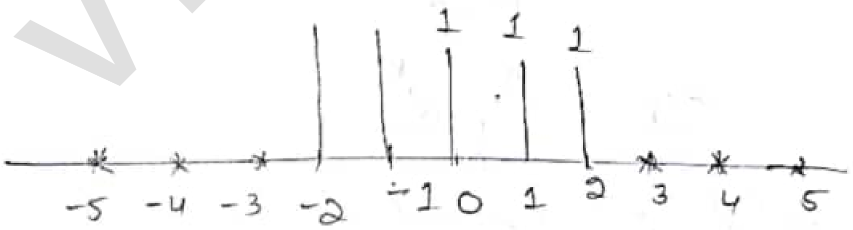
$$\boxed{S(n) = n+1}$$

ii) $h(n) = (-1)^n [u(n+2) - u(n-3)]$

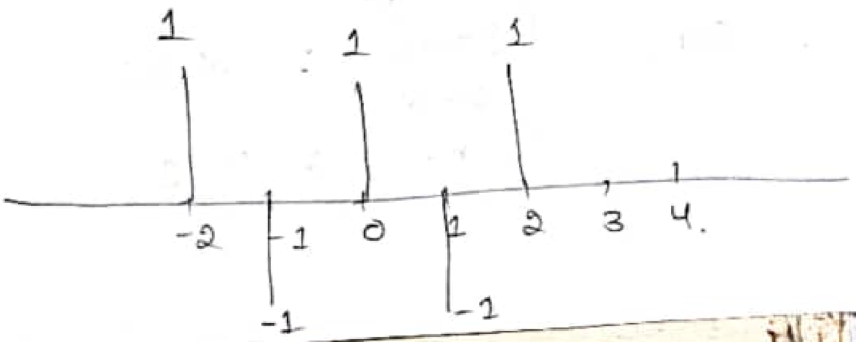
soln:- $u(n+2)$



$u(n+2) - u(n-3)$



$h(n) = (-1)^n [u(n+2) - u(n-3)]$



$$s(n) = \sum_{k=-2}^{2} h(k) = \sum_{k=-2}^{2} (-1)^k .$$

$$\boxed{s(n) = \sum_{k=2}^{2} (-1)^k .}$$ //.

Q iii) $h(n) = \alpha^n u(n)$.

Soln:- Given :-

$$h(n) = \alpha^n u(n) \Rightarrow h(k) = \alpha^k u(k)$$

$$s(n) = \sum_{k=-\infty}^{n} h(k)$$

$$s(n) = \sum_{k=0}^{n} \alpha^k$$

$$\because \sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & ; \alpha \neq 1 \\ N . & \alpha = 1 \end{cases}$$

$$s(n) = \begin{cases} \frac{1-\alpha^{n+1}}{1-\alpha} & ; \alpha \neq 1 \\ n+1 & ; \alpha = 1 \end{cases}$$

iv) $h(n) = \left(\frac{1}{2}\right)^n u(n)$

soln:- Given:-

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$s(n) = \sum_{k=-\infty}^{\infty} h(k)$$

$$s(n) = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$

(or)

start from

$$s(n) = h(n) * u(n)$$

$$s(n) = \sum_{k=-\infty}^{\infty} h(k)\, u(n-k)$$

$$s(n) = \sum_{k=0}^{n} h(k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$

$$s(n) = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} \qquad \left[\because \sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & ; \alpha \neq 1 \\ N & ; \alpha = 1 \end{cases}\right.$$

$$s(n) = 2\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

v) Determine step response for the system

i) $h(n) = (-1)^2 \left[u(n+3) - u(n-6)\right]$

soln:- Given:-

$$h(n) = (-1)^2 \left[u(n+3) - u(n-6)\right]$$

HW

v) Determine step response for the system

i) $h(n) = (-1)^2 [u(n+3) - u(n-6)]$

ii) $h(n) = (-1)^n u(n)$

iii) $h(n) = \delta(n)$