

Orthogonality and Least Squares

4.1 Inner Product

Let $u = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ u_n \end{bmatrix}$ and $v = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_n \end{bmatrix}$ be any two vectors in \mathbb{R}^n . Then the number $u^T v$ is called the **inner product** of u and v . This inner product is also commonly known as **dot product** and denoted by $\mathbf{u.v}$.

4.1.1 Properties of Inner Product

1. $u.v = v.u$
2. $(u + v).w = u.w + v.w$
3. $(\alpha u).v = \alpha(u.v) = u.(cv)$
4. $u.u \geq 0$ and $u.u = 0 \iff u = 0$

4.1.2 The Length of a Vector

The length of a vector v is called the **norm** of v .

It is denoted by $\|v\|$ and defined by $\|v\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$ so that, $\|v\|^2 = v.v$. There are several kinds of norms actually, this particular norm is called **Euclidean norm**.