

## ■ Big O Notation (complexity theory) $O(n)$

- it's called Landau notation
- it describes the limiting behaviour of a function when the argument tends towards a particular value or infinity.
- used to classify algorithms by how they respond (time and space requirements) to changes in the  $N$  input size
- usually we are interested in large input sizes (Asymptotic Analysis)
- we will drop all the terms that grow slowly & only keep the ones that grow fast as  $N$  (so the input size) becomes larger
- mathematical definition:

$$f(n) = O(g(n))$$

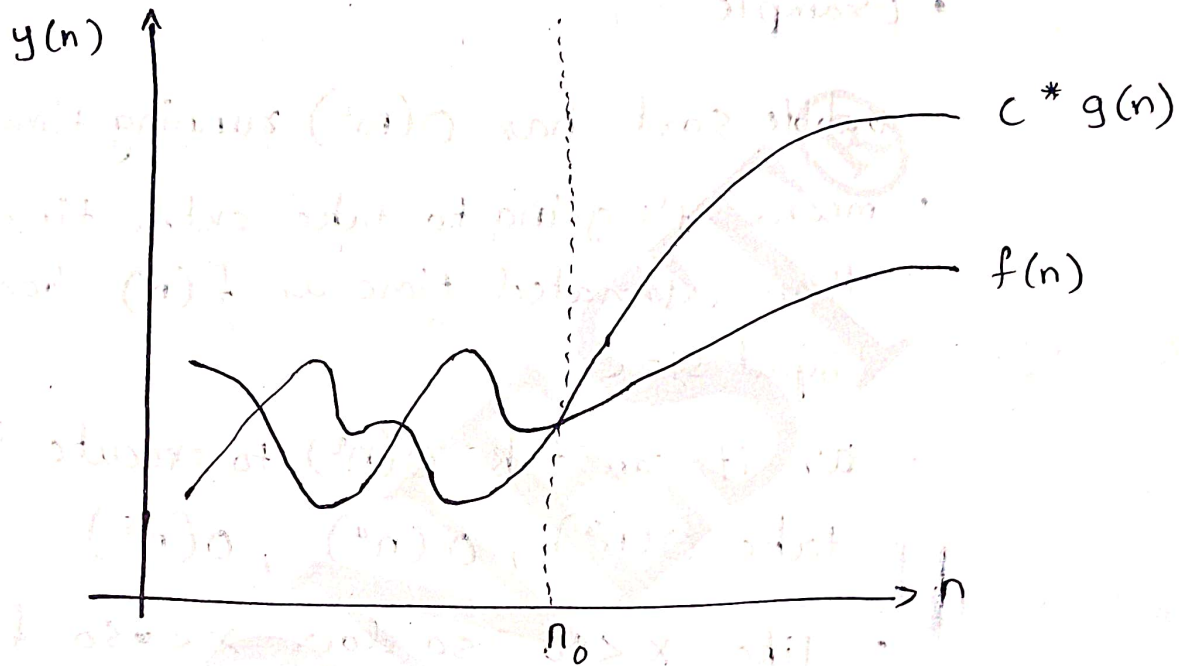
→ where

- $f$  = measures the running time of <sup>algorithm</sup> function
  - $n$  = input size
  - $g$  = also a function.
- this means that there is some  $c > 0$  value & some  $n_0 > 0$  threshold value such that for

• when  $n > n_0$  the absolute (|) value  $|f(n)| \leq c * |g(n)|$

$$|f(n)| \leq c * |g(n)|$$

• graph:



- when  $n$  (input size) is small we don't really care which one is greater  $f(n)$  or  $g(n)$
- in this above image as you can see sometimes  $f(n)$  is greater and sometimes  $c * g(n)$
- what's more crucial is that if  $n$  is larger than  $n_0$ , as we can see in right side of illustration  $c * g(n)$  tends to go larger than  $f(n)$
- if we find  $c = \text{constant}$ ,  $n_0$  threshold such that the  $f(n)$  function is smaller or equals than  $c * g(n)$  ~~that~~ that's where we say

$$f(n) = O(g(n))$$

- essentially  $O(g(n))$  defines the upper bound for the  $f(n)$  function.

### • Example

- bubble sort has  $O(n^2)$  running time complexity
- means it's going to take extra time to execute than estimated time as  $f(n)$  based on the input size
- as it can take  $O(n^2)$  to execute it also may take  $O(n^3)$ ,  $O(n^4)$ ,  $O(n^5)$
- like  $x < 10$  so does  $x < 50$  &  $x < 100$
- we only consider larger elements based tests because that's where you exactly understand how it's efficient otherwise with less elements every algorithm is going to be fast.

there are 7 types of notation.

$O(1)$  = constant

$O(\log n)$  = logarithmic

$O(n)$  = linear

$O(n \log n)$  = linearithmic

$O(n^2)$  = quadratic

$O(2^n)$  = exponential

$O(n!)$  = factorial.



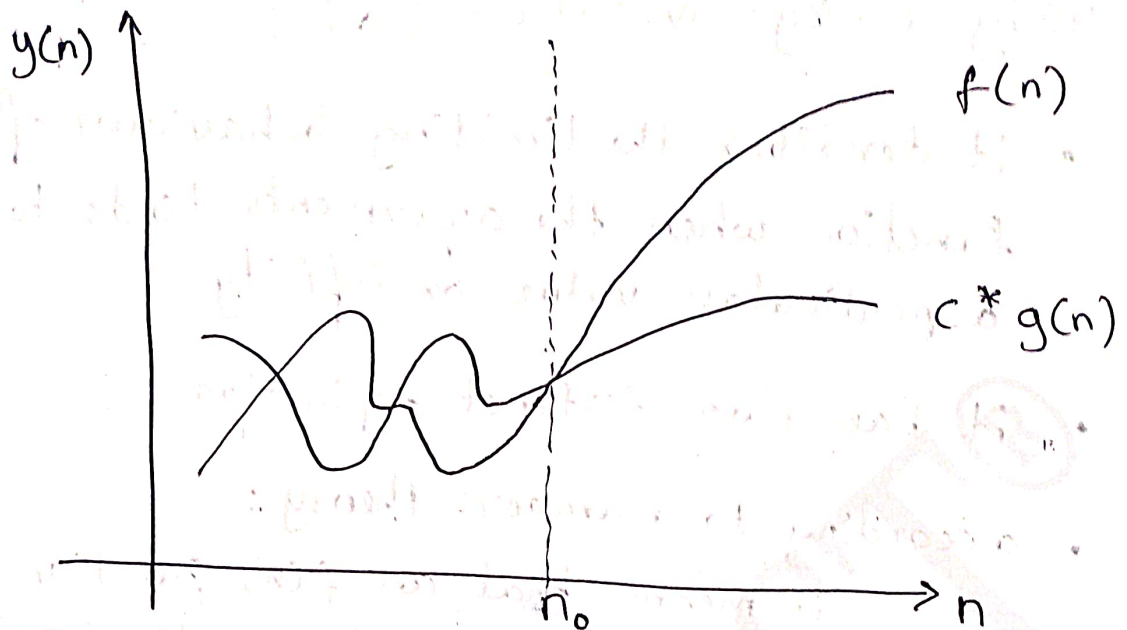
## Big Omega Notation ( $\Omega$ )

- it describes the limiting behaviour of an  $f(n)$  function when the arguments tends towards a particular value or infinity
- it has two distinct definition
- according to numbers theory :  
it means that an  $f(n)$  function is not dominated by  $g(n)$  function asymptotically
- according to complexity theory :  
it means that an  $f(n)$  function is bounded by an  $g(n)$  function asymptotically
- the  ~~$f(n)$~~  mathematical definition

$$f(n) = \Omega(g(n))$$

- this means, that there is some  $c > 0$  value & some  $n_0 > 0$  threshold value such that for  $n > n_0$  absolute value would be

$$|f(n)| \geq c * |g(n)|$$



- when  $n$  is smaller than threshold  $(n_0)$  sometimes  $f(n)$  is greater & sometimes the  $c * g(n)$
- but when  $n$  is larger than threshold  $(n_0)$  the  $f(n)$  function tends to be larger than the  $c * g(n)$
- essentially  $\Omega(g(n))$  defines the lower bound for the  $f(n)$  function.
- Example: let's assume that algorithm has the  $\Omega(N^2)$  time complexity
- so, it's also can also  $\Omega(N)$ ,  $\Omega(\log N)$ ,  $\Omega(1)$
- like, if  $x > 10$  then  $\rightarrow x > 1$  &  $x > -5$

## Big theta Notation ( $\Theta$ )

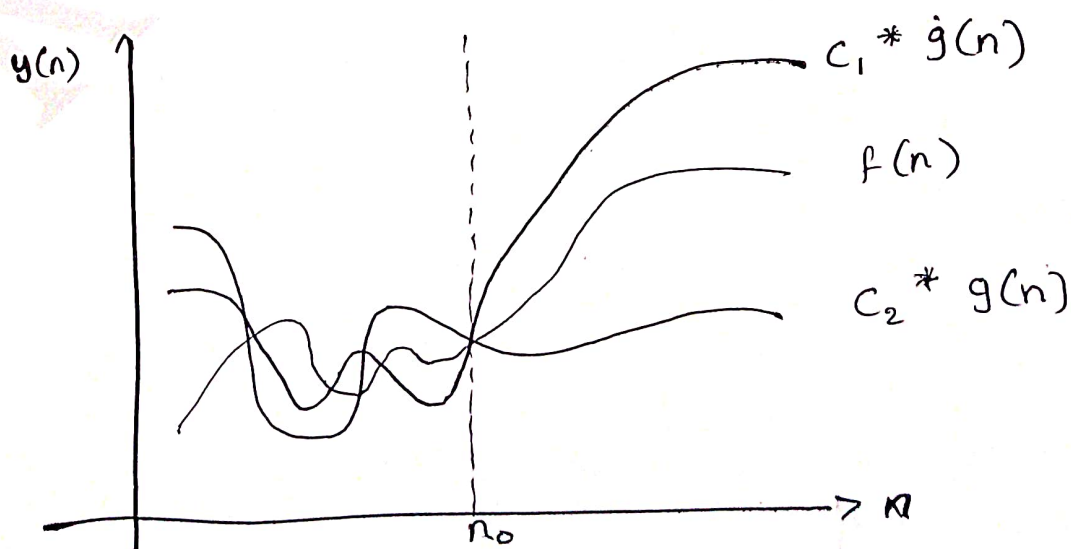
- it describes the limiting behaviour of  $f(n)$  function when the argument tends towards a particular value or infinity
- the  $f(n)$  function is bounded both below & above by a  $g(n)$  function asymptotically such that  $f(n) = \Omega(g(n))$  &  $f(n) = O(g(n))$
- mathematical definition

$c = \text{constant}$ .

$$f(n) = \Theta(g(n))$$

- means that, there is some  $c_1, c_2 > 0$  value and some  $n_0 > 0$  threshold value such that for  $n > n_0$  the

$$|c_1 * |g(n)|| \geq |f(n)| \geq c_2 * |g(n)|$$



- the upper bound of the  $f(n)$  function will be  $C_1 * g(n)$
- and the lower bound of the  $f(n)$  function will be  $C_2 * g(n)$
- essentially  $\Theta(g(n))$  defines the lower & upper bounds for the  $f(n)$  function.