Unit-1 Mechanics

Chapter 1 Physical quantities

Date 01-15-2080
Page 02

1.2 Physical quantity

:- The quantity which is measurable by some physical means is known as physical quantity. e.g:- mass:- Kilogram, second

1.2.1 Fundamental quantity

:- These physical quantity which is taken as standard to measure other physical quantities is known as fundamental quantity. e.g:- length, mass, time etc.

1.2.2 Derived quantity

:- A quantity obtained from fundamental quantities. is known as derived quantity. e.g:- Area, volume, velocity etc.

1.6. Precision and Accuracy

:- Precision is the degree to which the observed values are least scattered.

1.10 Dimensions

:- Dimensions is defined as the powers raised to the fundamental quantities which are involved in derived physical quantities.

:- An expression which shows how and which basic quantities are involved in the derived quantity is called dimensional formula of that quantity.

(1) Force $= M \cdot a$

$= [M] [L, T^{-2}]$

$= [M, L, T^{-2}]$

2) Frequency $= 1/t$

$= [T^{-1}]$

3) power $= \frac{W}{t}$

$= \frac{F \times d}{t}$

$= \frac{[M, L, T^{-2}] [L]}{[T^{1}]}$

$= [M, L^{2}, T^{-3}]$

4) Torque $=$ Force $\times \perp$ distance

$= [M, L, T^{-2}] [L]$

$= [M, L^{2}, T^{-2}]$

5) strain is dimensionless constant.

strain $= \frac{\text{change in length}}{\text{original length}}$

6) plane Angle is dimensionless Constant.

* classification of physical quantity

:- physical quantity can be divided into 4 type. on the basis of dimension analysis.

(1) Dimensional variables

:- Quantities which have dimension but not fixed value is known as dimensional variable. e.g:- velocity, force etc.

(2) Dimensional constant

:- Quantities having dimension and constant value is known as dimensional constant. e.g:- gravitional constant

(3) Dimensionless variables

:- Quantities having no dimension and do not have fixed value is called dimensionless variables. e.g:- angle, strain etc.

4) Dimensionalless constant

:- Quantities which have no dimension but constant value is called dimensionless constant. e.g:- pie etc.

precision	Accuracy
(i) It means how close the result are with each other.	(i) It is how the result close to the actual value.
(ii) It has multiple factor.	(ii) It has single factor.
(iii) It concerned with random errors.	(iii) It deals with systematic error.

(1)* To check the correctionness of dimension **

(i) $v = u + at$

$[M^0, L^1, T^{-1}] = [M^0, L^1, T^{-1}] + [M^0, L^1, T^{-2}] [M^0, L^0, T^1]$

or, $[M^0, L^1, T^{-1}] = [M^0, L^1, T^{-1}] + [M^0, L^1, T^{-1}]$

or, $[M^0, L^1, T^{-1}] = 2 [M^0, L^1, T^{-1}]$

$\therefore$ L.H.S = R.H.S, thus it is correct dimension.

(ii) $F = m \cdot a$

or, $[M^1, L^1, T^{-2}] = [M^1, L^0, T^0] [M^0, L^1, T^{-2}]$

or, $[M^1, L^1, T^{-2}] = [M^1, L^1, T^{-2}]$

$\therefore$ L.H.S = R.H.S, thus it is correct dimension.

(iii) $v^2 = u^2 + 2as$

$[M^0, L^1, T^{-1}]^2 = [M^0, L^1, T^{-1}]^2 + 2 [M^0, L^1, T^{-2}] [M^0, L^1, T^0]$

$[M^0, L^2, T^{-2}] = [M^0, L^2, T^{-2}] + 2 [M^0, L^2, T^{-2}]$

$[M^0, L^2, T^{-2}] = 3 [M^0, L^2, T^{-2}]$

$\therefore$ L.H.S = R.H.S proved

(2) to convert of unit into other unit

$$n_2 = n_1 \left[\frac{m_1}{m_2}\right]^x \left[\frac{L_1}{L_2}\right]^y \left[\frac{T_1}{T_2}\right]^z$$

(i) Convert 100N into dyne

$$= 100 \left[\frac{1\,kg}{1\,gm}\right]^1 \left[\frac{1\,m}{1\,cm}\right]^1 \times \left[\frac{T'}{T_2}\right]^z$$

$$= 100 \left[\frac{1000}{1}\right] \times \left[\frac{100}{1}\right]^1 \times \left[\frac{0}{0}\right]^{-2}$$

$= 100 \times 1000 \times 1000$

$= 10^7$ dyne. Ans

(ii) Convert 1 dyne into Newton.

$$n_2 = n_1 \left[\frac{m_1}{m_2}\right]^x \left[\frac{L_1}{L_2}\right]^y \left[\frac{T'}{T_2}\right]^z$$

$$= 1 \left[\frac{1\,gm}{1\,kg}\right]^1 \times \left[\frac{1\,cm}{1\,m}\right]^2 \left[\frac{1\,sec}{1\,sec}\right]^{-2}$$

$$= \frac{1}{1000} \times \frac{1}{10000}$$

$$= \frac{1}{10^7}$$

$= 10^{-7}$ Ans

(3) To derive the relation between physical quantities

(i) Time period

$T \propto g^a$

$T \propto l^b$

$T \propto m^c$

$T = K g^a . l^b . m^c$ — (i)

$[M^0, L^0, T] = [L, T^{-2}]^a \, [L]^b \, [M]^c$

$[M^0, L^0, T^1] = [L^{a+b}, T^{-2a}, M^c]$

equating power

$a+b = 0$

$-2a = 1$ or, $a = -\frac{1}{2}$

$c = 0$

Now $T = K g^{-1/2} . l^{1/2} . m^0$

$T = K \frac{l^{1/2}}{g^{1/2}}$

$T = K \sqrt{\frac{l}{g}}$

$T = 2\pi \left(\frac{l}{g}\right)^{1/2}$

(ii) derive formula for pressure equal force and Area.

here:-

$P = \frac{F}{A}$

or, $P \propto F$

or, $P \propto 1/A$

Again on combing

$P \propto F^a . A^b$

$P = K . F^a . A^b$ (i) eq^n

writing dimensions

$[M, L^{-1}, T^{-2}] = [M, L, T^{-2}]^a \, [L^2]^b$

or, $[M, L^{-1}, T^{-2}] = [M^a, L^{a+2b}, T^{-2a}]$

equating power

$a = 1$

$a + 2b = -1$

$2b = -1-1$

$2b = -2$

$b = -1$

$P = K . F^a . A^b$

$= K . F^1 . A^{-1}$

$= \frac{F}{A}$

* Uses of dimensional equation

(i) to check the correctness of formula.

(ii) to convert one unit to another unit.

(iii) to derive the relation betn physical quantities

* limitation of dimensional analysis

(i) Dimensional constant can't be determined.

(ii) It don't give the relation between other physical quantity except mass and time, length

(iii) It doesn't give the information about scalar and vectors.

Unit - 2

Scalar and vector

(i) scalar :-

The physical quantities which have only magnitude but no directions is called scalar quantity. eg:- Distance, speed, volume etc.

(ii) Vector

:- The physical quantities which have both magnitude and directions is called vector quantity. eg:- Velocity, displacement.

Types of vector

(i) Null vector

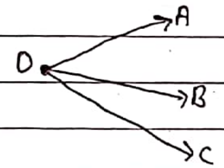
:- Vector which do not have magnitude.

(ii) unit vector

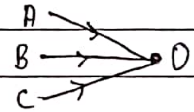
:- Vector which have ~~unique~~ unit magnitude.

(iii) co-initial vector

:-



(iv) co-terminal vector



(v) equal vector

:- Two or more vector that have equal magnitude and same ~~doctor~~ direction.

(vi) Negative vector / unequal vector

:- vector have same magnitude but opposite direction.

(vii) Coplanar vector

:- All the vector lies on the same plan is known coplanar vector.

(viii) Collinear vector

:- Vector that have same vector, parallel vector or anti parallel

(ix) like and unlike vector

:- same direction

opposite direction

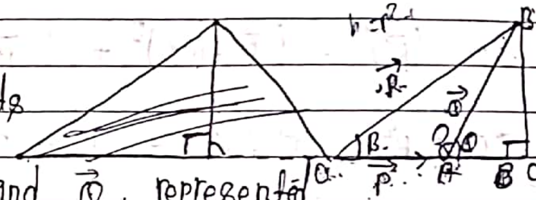
* Triangle law of vector Addition

(i) statement

:- If two vectors are represented by in magnitude and direction by two sides of triangle taken in same order the resultant vector represent in magnitude and direction by the 3rd side of triangle taken in reverse order.

(ii) Derivation

:- suppose $\vec{P}$ and $\vec{Q}$ acts simultaneously on a body such that $\vec{P}$ and $\vec{Q}$, represented by OA and AB side respectively. In same order according to triangle's law resultant vector $\vec{R} = \vec{P} + \vec{Q}$



(i) magnitude of $\vec{R}$.

:- To find the magnitude and direction of resultant vector $\vec{R}$ a $\perp$ line BC is drawn on line OA produ

let 'θ' be the angle betⁿ $\vec{P}$ and $\vec{Q}$ and β be the angle betⁿ $\vec{P}$ and $\vec{R}$.

In $\triangle ACB$ $\sin\theta = \frac{BC}{AB}$

or, $\sin\theta = \frac{BC}{\vec{Q}}$

or, $\therefore$ $\boxed{BC = \vec{Q}\sin\theta}$

In $\triangle ACB$ $\cos\theta = \frac{AC}{AB}$

$\cos\theta = \frac{AC}{\vec{Q}}$

$\therefore$ $\boxed{AC = \vec{Q}\cos\theta}$

From Δ OCB

$h^2 = p^2 + b^2$

or, $(OB)^2 = (BC)^2 + (OC)^2$

or, $(OB)^2 = (BC)^2 + (OA + AC)^2$

or, $(\vec{R})^2 = (\vec{Q}\sin\theta)^2 + (\vec{P} + \vec{Q}\cos\theta)^2$

or, $R^2 = Q^2\sin^2\theta + P^2 + 2\cdot\vec{P}\cdot\vec{Q}\cos\theta + Q^2\cos^2\theta$

or, $R^2 = P^2 + 2\vec{P}\vec{Q}\cos\theta + Q^2(\sin^2\theta + \cos^2\theta)$

or, $R^2 = P^2 + 2\vec{P}\vec{Q}\cos\theta + Q^2$

or, $R = \sqrt{P^2 + 2\vec{P}\vec{Q}\cos\theta + Q^2}$

$\therefore$ This gives the magnitude of resultant vector

(ii) Direction of $\vec{R}$

:- here In Δ OCB

$\tan\beta = \frac{BC}{OC}$

$= \frac{\vec{Q}\sin\theta}{OA + AC}$

$= \frac{\vec{Q}\sin\theta}{OA + \vec{Q}\cos\theta}$

$= \frac{\vec{Q}\sin\theta}{\vec{P} + \vec{Q}\cos\theta}$ $\quad \therefore \beta' = \tan^{-1}\left(\frac{\vec{Q}\sin\theta}{\vec{P} + \vec{Q}\cos\theta}\right)$

$= \tan^{-1}$ ...

* Special Cases

(1) when two vector are same direction $(\theta = 0^\circ)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ $\qquad \beta = \tan^{-1}\left(\frac{\vec{Q}\sin\theta}{P + Q\cos\theta}\right)$

or, $= \sqrt{P^2 + Q^2 + 2PQ\cdot 1}$ $\qquad = \tan^{-1}\frac{\vec{Q}\cdot 0}{\vec{P} + \vec{Q}\cdot 1}$

or, $= \sqrt{P^2 + Q^2 + 2PQ}$

$\beta = 0$

(2) when two vector are perpendicular

$R = \sqrt{P^2 + Q^2 + 2PQ\cdot\cos 90^\circ}$

$= \sqrt{P^2 + Q^2 + 2PQ\cdot 0}$

$= \sqrt{P^2 + Q^2}$

* what is the magnitude and direction of two vector are 4 and 6, which are at 90° to each other.

$\Rightarrow$ Soln:- $P = 4$

$Q = 6$

$R = \sqrt{P^2 + Q^2 + 2PQ\cdot\cos\theta}$

$= \sqrt{4^2 + 6^2 + 2\times 4\times 6\times 0}$

$= \sqrt{52}$ unit

$\beta = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

$= \tan^{-1}\left(\frac{6\times 1}{\ldots}\right)$

$\tan^{-1}\left(\frac{6}{4}\right)$

$= \tan^{-1}(1.5)$

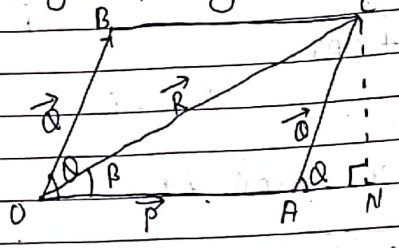
= Ans

parallelogram Law of vector

Date
Page

statement

:- If two vector are represented in magnitude and direction by two adjacent side of a parallelogram drawn from point there resulting is represented in magnitude and direction by the diagonal passing through same point.



If $\vec{P}$ and $\vec{Q}$ are two adjacent vector, then there resultant is given by $\vec{R} = \vec{P} + \vec{Q}$

here OA represents $\vec{P}$ and AC represents $\vec{Q}$ vector where OC represents $\vec{R}$.

let 'θ' be the angle between $\vec{P}$ and $\vec{Q}$, 'β' be the angle between $\vec{R}$ and $\vec{P}$

In Δ CNA

$\sin\theta = \frac{CN}{AC}$

$\sin\theta = \frac{CN}{Q}$

or, $CN = Q\sin\theta$ (i)

and $\cos\theta = \frac{AN}{AC}$

or, $\cos\theta = \frac{AN}{Q}$

or, $AN = Q\cos\theta$ (ii)

Now

In Δ ONC,

$(OC)^2 = (ON)^2 + (CN)^2$

or, $R^2 = (OA + AN)^2 + (CN)^2$

or, $R = \sqrt{(OA+AN)^2 + (CN)^2}$ (iii)

using (i) (ii) and (iii)

$R = \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2}$

$= \sqrt{P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta}$

$= \sqrt{P^2 + 2PQ\cos\theta + Q^2(\cos^2\theta + \sin^2\theta)}$

$= \sqrt{P^2 + 2PQ\cos\theta + Q^2}$ (iv)

This gives the magnitude of resultant vector.

Direction of $\vec{R}$

$\tan\beta = \frac{P}{b} = \frac{CN}{ON}$

or, $\tan\beta = \frac{CN}{OA + AN}$

or, $\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$

$\beta = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

Special Case

(i) when two vector are parallel to each other
here,

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{P^2 + Q^2 + 2PQ\cos 0}$

$= \sqrt{P^2 + Q^2 + 2PQ}$

$\beta = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

$= \tan^{-1}\left(\frac{Q\sin 0}{P + Q \cdot 1}\right)$

$= \tan^{-1}\left(\frac{0}{P + Q \cdot 1}\right)$

$\beta = 0$

Product of Vector

(1) Scalar (dot) product

$\therefore \vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$

(2) Vector (cross) product

$\vec{A} \times \vec{B} = |\vec{A}| \cdot |\vec{B}| \sin\theta\, \hat{n}$

Difference between vector and scalar products.

	Scalar product	Vector product
(i)	It gives scalar quantity.	(i) It gives vector quantity.
(ii)	The magnitude of scalar product of two vector $\vec{A}$ & $\vec{B}$ is $AB\cos\theta$	(ii) The magnitude of $\vec{A}$ and $\vec{B}$ vector is $AB\sin\theta$.
(iii)	It obey commutative rule. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$	(iii) It does not obey commutative rule. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

Resolution of vector

:- The process of spliting of vector into two or more vecto

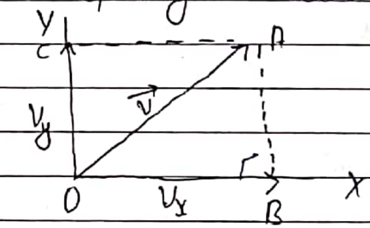


Figure shows the resolution of vector $\vec{V}$ and X and Y-axis. $\vec{V}$ is splitted into component OB and OC along X and Y-axis In Δ OBA, we have

$\cos\theta = \frac{OB}{OA} = \frac{V_x}{V}$ Also, $\sin\theta = \frac{AB}{OA} = \frac{OC}{OA} = \frac{V_y}{V}$

$\therefore V_x = V\cos\theta$ $\therefore V_y = V\sin\theta$

Special Case:

If $\theta = 0$ then,
$v_x = v\cos 0^\circ = v$ (max)
$v_y = v\sin 0 = 0$ (min)

If $\theta = 90^\circ$, then,
$v_x = v\cos 90^\circ = 0$ (min)
$v_y = v\sin 90^\circ = v$ (max)

* If two vector are $\vec{A}$ and $\vec{B}$. Find it's magnitude.

⇒ soln:- here:-

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

* Find $\vec{A}\cdot\vec{B} = 0$, find the angle between them

⇒ soln:

~~$\cos\theta = \vec{A}$.~~

$\vec{A}\cdot\vec{B} = 0$

$\vec{A}\times\vec{B} = AB\cos\theta$

or, $0 = AB\cos\theta$

or, $\frac{0}{AB} = \cos\theta$

or, $\cos\theta = 0$

or, $\cos\theta = \cos 90^\circ$

or, $\theta = 90^\circ$

* If $\vec{A}\cdot\vec{B} = \vec{A}\times\vec{B}$. Find angle between them

⇒ Soln:-

$\vec{A}\cdot\vec{B} = \vec{A}\times\vec{B}$

or, $\cancel{AB}\cos\theta = \cancel{AB}\sin\theta$

or, $\tan\theta = 1$

or, $\tan\theta = \tan 45^\circ$

$\theta = 45^\circ$

	0°	30°	45°	60°	90°
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
Tan	0	$\sqrt{3}/2$	1	$1/\sqrt{3}$	∞

chapter 3

Kinematics

(1) Define speed, velocity, uniform velocity, non-uniform velocity, distance, displacement.

(2) prove that

$S = ut + \frac{1}{2} at^2$

$v = u + at$

$v^2 = u^2 + 2gs$

* Define kinematics

:- The study of motion without its cause is known as kinematics.

* projectile motion

:- If the body is thrown in the air and ~~moves~~ moves under the action of gravity then it is said to be in projectile motion. And it's motion is called projectile motion.

The path followed by the ~~called~~ projectile ~~motion~~ in the space is called trajectory.

(a) projectile fired from a height

:- Suppose a projectile is fired with velocity 'v' of height 'h'.

Equation of trajectory:-

Suppose x and y are horizontal and vertical distances travelled by projectile in time 't' respectively.

For the horizontal motion, $u_x = u$ and $u_y = 0$, so;

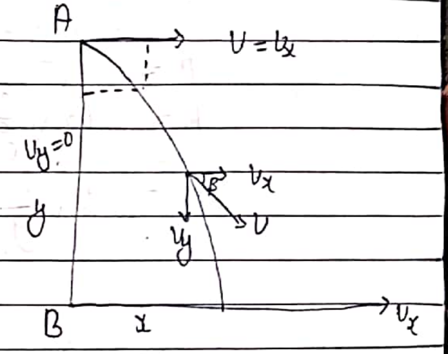
$x = u_x t = ut \quad (\because s = ut)$

or, $t = x/u$ — (i)

For vertical motion

$y = u_y t + \frac{1}{2} gt^2 = 0 \times t + \frac{1}{2} gt^2$

$\therefore y = \frac{1}{2} gt^2$ — (ii)



From (i) and (ii)

$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2$

or, $y = \frac{1}{2} g \frac{x^2}{u^2}$

or, $y = kx^2$

where, $k = \frac{g}{2u^2}$ is constant.

This is the equation of parabola, hence the path in parabolic.

CS CamScanner

a) Time of flight

:- Time taken by the projectile to come down in the ground is known as time occupied.

For this,

$y = 'h'$ and $t = 'T'$

So, from eqn (ii)

$$h = \frac{1}{2} g t^2$$

or, $$\frac{2h}{g} = T^2$$

or, $$T = \sqrt{\frac{2h}{g}}$$

b) Horizontal Range

:- The horizontal distance travelled by the the projectile during time of flight is called horizontal Range.

It is denoted by 'R'.

$$R = v_x T$$

$$R = v_x \sqrt{\frac{2h}{g}}$$

$$R = v \sqrt{\frac{2h}{g}}$$

(c) velocity of projectile at any instance

:- let 'v_x' and 'v_y' be the horizontal and vertical component of velocity after time (T) respectively.

$v_x = v$, $v_y = gt$

then the resultant velocity $v = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{v^2 + (gt)^2}$$

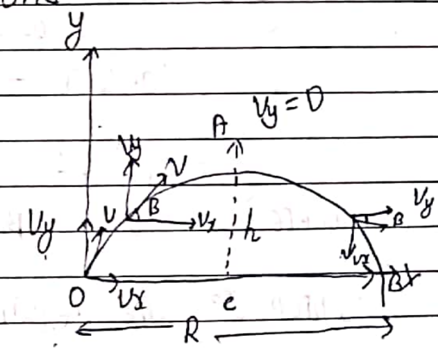
If 'β' be the angle made by resultant and ~~param~~ horizontal then,

$$\tan\beta = \frac{v_y}{v_x}$$

$$\tan\beta = \frac{gt}{v}$$

$$\beta = \tan^{-1}\left(\frac{gt}{v}\right)$$

2) projectile fired from ground



Suppose a projectile is fired from ground with initial velocity (u) as shown in the figure 'H' is the height attain by the projectile seems It's velocity is splited into :-

$$u_x = u\cos\theta \quad , \quad u_y = u\sin\theta$$

The horizontal distance travelled by the projectile is $x = u_x t$ [$s = ut$]

$$x = u\cos\theta\, t$$

$$\therefore t = \frac{x}{u\cos\theta}$$

And the vertical distance travelled by projectile is :

$$y = u_y t - \tfrac{1}{2} g t^2$$

$$y = u\sin\theta\, t - \frac{1}{2} g t^2 \quad \text{--- (ii)}$$

$$y = u\sin\theta \frac{x}{u\cos\theta} - \tfrac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$$

$$y = \tan\theta\, x - \left(\frac{g}{2u^2\cos^2\theta}\right) x^2$$

$$\therefore y = Ax - Bx^2$$

where, $A = \tan\theta$, $B = \dfrac{g}{2\cos^2\theta\, u^2}$ is constant.

which is the required eqn of parabolla

(9) Time of Flight

:- Time taken by the projectile to come down in the ground is known as time of flight.

here, $t \Rightarrow T$, $y = 0$

From (ii) eqn

$$y = u\sin\theta t - \frac{1}{2} g t^2$$

$$0 = u\sin\theta T - \frac{1}{2} g T^2$$

$$\frac{1}{2} g T^2 = u\sin\theta T$$

$$\frac{1}{2} g T = u\sin\theta$$

or, $T = \dfrac{2u\sin\theta}{g}$ Ans

Horizontal range:-

(10) The ~~This is~~ ~~required value~~ horizontal distance travelled by projectile is known as horizontal Range.

For horizontal range (R) $= u_x T$

$$= u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$= \frac{2u^2 \cos\theta \cdot \sin\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

Date ____
Page ____

(iii) maximum Height

:- The maximum height attained by the projectile during it's motion is known as maximum height.

here $v_y = 0$

$$v^2y = u^2y - 2gh$$

or, $0 = (u\sin\theta)^2 - 2gh_{max}$

or, $2gh_{max} = u^2\sin^2\theta$

$\therefore h_{max} = \dfrac{u^2\sin^2\theta}{2g}$

(iv) Two angle of projection for the same range

since, horizontal Range is

$$R = \frac{u^2\sin2\theta}{g}$$

If $(90-\theta)$ be the another angle of projection

$$R = \frac{u^2\sin2(90-\theta)}{g}$$

$$= \frac{u^2\sin(180-2\theta)}{g}$$

$$R = \frac{u^2\sin2\theta}{g}$$

hence, it is required solution.

Date ____
Page ____

* If the stone is thrown from the height 6m then calculate it's time flight or time period, horizontal Range with velocity 2m/s.

=> soln:-

Time of flight $= \sqrt{\dfrac{2h}{g}}$

$= \sqrt{\dfrac{2\times6}{10}}$

$= \sqrt{\dfrac{6}{5}}$

Again

Horizontal Range $= u\sqrt{\dfrac{2h}{g}}$

$= 2\sqrt{6/5}$ ~~s~~ s

$=$

* A stone is throw from the ground with angle 30° having horizontal Range 20m. calculate it's max height

=> soln:-

$$R = \frac{u^2\sin2\theta}{g}$$

$$R = \frac{u^2\sin2(30)}{g}$$

$$R = \frac{u^2\sin60}{9.8}$$

$$20\times9.8 = \frac{u^2\sqrt{3}}{2}$$

$$\frac{20\times9.8\times2}{\sqrt{3}} = u^2$$

$u^2 =$ ~~196~~ 226.58

$u =$ ~~#~~

$$h_{max} = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{196 \sin^2 30}{2\times 9.8} \qquad = \frac{226.58 \sin^2 30}{2\times9.8}$$

$$= \frac{196\ (\frac{1}{2})^2}{2\times 9.8} \qquad = \frac{226.58\ (1/2)^2}{2\times9.8}$$

$$= 196 \times \frac{2}{4} \qquad = \frac{226.58 \times 1/4}{2\times 9.8}$$

$$= 196 \times 2/4$$

$$= \frac{56.64}{19.6}$$

$$= 2.88 \text{ Ans}$$

Unit 4

Dynamics

1) Define Force, Newton 2nd law of motion

2) prove $F = ma$, Define mass, Inertia with Inertia of rest and motion

* Linear Momentum *

:- The product of mass and velocity of a body is known as linear momentum. It is denoted by $(\vec{p})$

$$\vec{p} = m \times \vec{v}$$

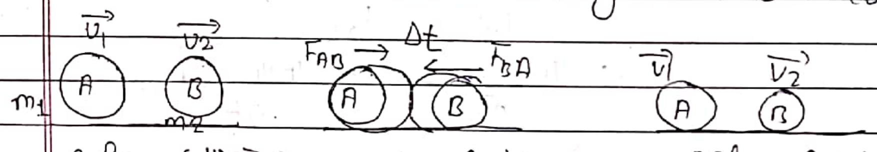
* Impulse

:- The product of force and time for which it acts is called Impulse. It is denoted by 'I'

$$I = F \times t$$

* principle of Conservation of Linear momentum

:- "If no external force acts on the system then the total linear momentum before collision is equal to the total linear momentum of the system after collision".



Before Collision　　During Collision　　After Collision

Suppose, two sphere 'A' and 'B' of masses 'm_1' and 'm_2' moving in a same direction with velocity 'u_1' and 'u_2'. After Sometime sphere 'A' collides with sphere 'B' and finally velocity becomes v_1 and v_2.

* If the force exerted by 'A' on 'B' is action. Then the force exerted by sphere 'B' on 'A' will be reactioned. From Newton 3rd law of motion.

Force exerted by ball A on B = $\frac{\text{change in linear momentum of B}}{\text{time of collision}}$

$$F_{AB} = \frac{m_2 v_2 - m_2 u_2}{\Delta t} = \text{action} \quad \text{--- (i)}$$

Also, force exerted by B on A = $\frac{\text{change in linear momentum of A}}{\text{time of collision}}$

$$F_{BA} = \frac{m_1 v_1 - m_1 u_1}{\Delta t} = \text{reaction} \quad \text{--- (ii)}$$

From Newton's 3rd law :-

Action = − reaction

$F_{AB} = -F_{BA}$

$\therefore \frac{m_2 v_2 - m_2 u_2}{\Delta t} = -\frac{(m_1 v_1 - m_1 u_1)}{\Delta t}$

or, $m_2 v_2 + m_1 v_1 = m_1 u_1 + m_2 u_2$

or, ~~$m_2 v_2$~~

hence, the linear momentum before collision is equal to the linear momentum after collision.

* Collision

:- The interaction between the particle in which they exchange their momentum and energy in short interval of time is known as collision.

There are two types of collision

(i) Elastic collision

:- Collision in which both kinetic energy and momentum conserved is called elastic collision.

→ Mechanical energy is also conserved.

(ii) Inelastic collision

:- The collision in which linear momentum is conserved but not kinetic energy is called inelastic collision.

→ Mechanical energy may or may not be conserved.

* Equilibrium

→ Types

1) Stable equilibrium

:- In this equilibrium, a body is in stable equilibrium if it returns to its equilibrium position after it has been displaced slightly.

2) Unstable equilibrium

:- A body is in unstable equilibrium if it doesnot returns to its equilibrium position after

It has been displaced slightly.

3) Neutral equilibrium

:- A body is in neutral equilibrium if it always stays in the displaced position after it has been displaced slightly.

	Center of mass (CM)	Center of gravity (G)
(i)	It refers to the mass of the body.	(i) It refers to the weight acting on all particles of the body.
(ii)	In case of small and regular body CM and G coincide.	(ii) In case of very large body CM and G will not coincide.

Numerical

* A ball of mass 0.1 kg moving with velocity 6 m/s collides with ball B of mass 3 kg. If A had rebounded with a velocity of 2 m/s in the opposite direction, what is the new velocity of ball 'B'.

=> Soln:-

mass of ball 'A' = 0.1 kg (m_1)
mass of ball 'B' = 3 kg (m_2)
velocity of ball 'A' Initial = 6 m/s (u_1)
re-boun velocity of ball 'A' Final = −2 m/s (v_1)

by Formula

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or $0.1\times6 + 3\times0 = 0.1\times(-2) + 3v_2$

or, $0.6 = -0.2 + 3v_2$

or, $0.8/3 = v_2$

or, v_2 0.26 m/s

∴ new velocity of 'B' is 0.26 m/s.

* A 1 kg object moving with velocity of 8 m/s collide with 2 kg object moving " " 6 m/s. Calculate their common velocity.

=> Soln:- $m_1 = 1$ kg
$m_2 = 2$ kg
$u_1 = 8$ m/s
$u_2 = 6$ m/s

Now

$m_1u_1 + m_2u_2 = v(m_1 + m_2)$

or, $1\times8 + 2\times6 = v(\cancel{3})$

or, $\frac{20}{3} = v$

or, $v = 6.66$ m/s Ans

* If A 6 kg mass having speed 4 m/s collides with a bus of 100 kg with speed ~~at~~ rest. Calculate the velocity of bus if final velocity of A mass is 12 m/s.

⇒ Soln:-

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$$

or, $6\times4 + 100\times\cancel{0} = 6\times12 + 100\times v_2$

or, $24 = 72 + 100v_2$

or, $24-72 = 100v_2$

or, $\frac{-48}{100} = v_2$

or,

72
24

48
72

Chapter 5 Work, Energy and power

Define work, power, energy.

Types of work done

(i) work done against friction

(ii) ~~work~~ when a body moves against the friction with this type of work is said to be work done against friction.

$$W = F\times s \ (F.s)$$

(ii) work done against gravity

:- If a body moves against the gravity then this is work done against gravity.

$$W = mg$$

Types of Energy

(i) kinetic energy

:- The energy possessed by a body by virtue of its motion is called kinetic energy.

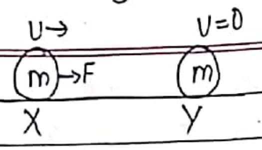
If 'm' and 'v' is the mass and velocity of a body then,

$$K.E = \tfrac{1}{2}mv^2$$

proof, let a considered a body of mass 'm' moving with initial velocity (u) then. K.E possess the body at 'Y' is equal to the work done by Force 'F' as distance 's'

$W = F\times d$

'S'



$W = F \times S = mas$ -- (i) eqn

using

$v^2 - u^2 = 2as$; we get

$0 - v^2 = -2as$

or, $as = \frac{1}{2}v^2$ (ii)

From (i) and (ii)

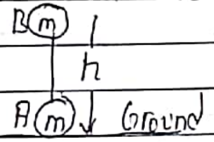
$W = \frac{1}{2}mv^2$

$\therefore W = \frac{1}{2}$ mass $\times$ (velocity)^2

(ii) potential Energy

:- It is the energy possess by a body by virtue of its position or state of condition.

Let us considered a body of mass 'm' is at height 'h' from the ground.



then, from figure work done is given by

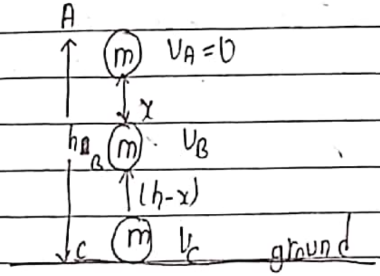
$W = F.h$

$= m.g h$

$P.E = mgh$

* principle of conservation of Energy

It states" that energy can neither be created nor destroyed but can be transformed from one from to another."



Let us considered a body of mass 'm' is at height 'h' from the ground. i.e at A. After some time it is on the position B at distance x from A and finally on the ground.

At point A ;

K.E of body $= \frac{1}{2}mv^2_A = 0$

P.E of body $= mgh$

total energy $(E_A) =$ K.E + P.E $= mgh$ -- (i) eqn

At point B, here body falls from point A to B, through distance 'x' with height (h-x) from ground.

If V_B is the velocity of it ;

$V_B^2 = V_A^2 + 2gx = 0 + 2gx = 2gx$

K.E $= \frac{1}{2}mv^2_B = \frac{1}{2}m.2gx = mgx$

~~K.E~~ again P.E $= mg(h-x) = mgh - mgx$

total energy $= K.E + P.E = \cancel{mgx} + mgh - \cancel{mgx}$

total energy $(E_B) = mgh$ — (ii) eqn

At a point C, since, it velocity become v_C

So, $v_C^2 = v_A^2 + 2gh$

$v_C^2 = 0 + 2gh$

$= 2gh$

$K.E = \frac{1}{2} m v_C^2$

$= \frac{1}{2} \times m \times 2gh = mgh$ ∴ (iii) eqn

$P.E = mgh = mg \times 0$ [∵ $h = 0$]

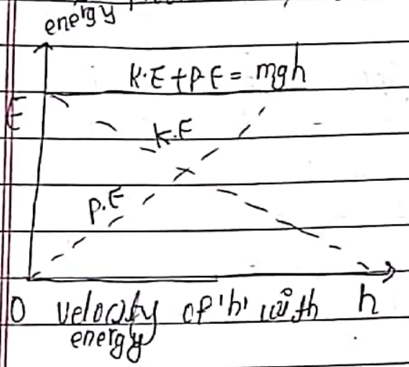
$= 0$

Total energy $(E_C) = mgh + 0$

Since from (i) & (ii) & (iii) eqn

Energy $= E_A = E_B = E_C = mgh =$ Constant

This proof conservation of energy



Conservative Force	Non-Conservative Force
→ Force is said to be conservative if work done by it on moving particle around a closed path is zero	→ Force is said to be non-conservative if work done by it on moving a particle around a closed path is not zero.
→ For e.g:- gravitational force, magnetic force etc.	→ For e.g:- Frictional force. etc.

$F = mg\sin\theta + \mu R$ (Resultant Force)

$F = mg\sin\theta + \mu \times mg\cos\theta$

Least force (miu)

$\mu =$ Coefficient of Frictional

* A 2 kg ball moving with velocity 10 m/s collides with another ball of mass 12 kg moving from opposite direction with velocity 7 m/s. Calculate common velocity

⇒ Soln:- According to formula

$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{2 \times 10 - 7 \times 12}{2 + 12}$$

$$\Rightarrow \frac{20 - 84}{14}$$

$$= \frac{-64}{14} = \frac{-32}{7}$$

$$= -4.6 \text{ m/s}$$

*

Unit 6

circular motion

→ A motion of a body in a circular path with constant speed but variable acceleration is known as circular motion.

Some terms :

(1) Angular velocity

:- It is defined as the angle travelled by the body in unit time. It is denoted by 'ω' (omega)

$$\omega = \frac{\text{angular displacement}}{\text{time taken}} = \frac{\theta}{t}$$

$\therefore \theta = \omega t$ It's unit is rad/sec.

(2) Time period

:- Time taken by the body to complete one revolution. It is denoted by 'T'.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\therefore \omega = \frac{2\pi}{T}$$

3) Frequency

:- Number of revolutions completes, per second by an object in circular motion. It is denoted by 'f' (small 'f').

$$f = \frac{1}{T}$$

$$\therefore \omega = 2\pi f$$

unit of Frequency in Hertz (Hz).

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$$f = \frac{1}{T}$$

$$\therefore \omega = 2\pi f$$

unit of Frequency in Hertz (Hz).

d) Angular acceleration :

:- The rate of change of angular velocity of rotating body with time.

It is denoted by 'α'

$$\alpha = \frac{\text{change in angular velocity}}{\text{time taken}}$$

$$\alpha = \frac{d\omega}{dt}$$

If ω_0 and ω be the initial and final angular velocity with time 't'.

$$\alpha = \frac{\omega - \omega_0}{t}$$

or, $\alpha t = \omega - \omega_0$

or, $\omega = \omega_0 + \alpha t$

* Relation between linear velocity and angular velocity.

→ Suppose a body moving on circular path of radius 'r' from A to B, in time 't' so, that

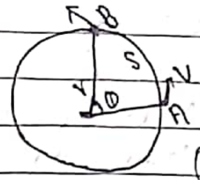


Fig (i)

arc length = AB = S

$\therefore \theta = \frac{\text{arc length}}{\text{radius}} = \frac{S}{r}$

$\therefore S = \theta r$

dividing both side by 't'.

$\frac{S}{t} = \frac{\theta r}{t} = \frac{\theta}{t} \cdot r$

$\therefore V = \omega r$

* Expression for centripetal force.

→ Suppose a body of mass 'm' is moving in circular path of radius 'r'.

Let $\vec{V_P}$ and $\vec{V_Q}$ be the velocities of body at point 'P' and 'Q'.

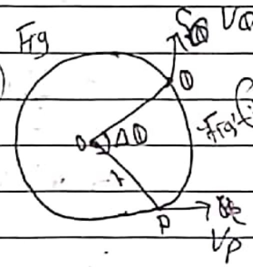


Fig (i)

So, change in velocity of internal of time Δt is;

$\Delta V = \vec{V_Q} - \vec{V_P}$

From fig (ii) OA and OB represent $\vec{V_P}$ and $\vec{V_Q}$ and AB represent direction of resultant. So, change in velocity.

$\Delta V = \vec{V_Q} - \vec{V_P} = \vec{AB}$ —— (1)

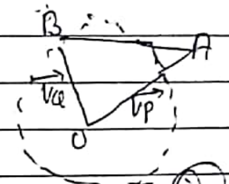


Fig (ii)

Since, acceleration is given by;

$$\vec{a} = \frac{\vec{AB}}{\Delta t} = \frac{\Delta V}{\Delta t} \quad \text{—— (i)}$$

If $\Delta\theta$ be angle by $\vec{v_P}$ and $\vec{v_Q}$ in small interval of time

$$|v_P| = |v_Q| = v$$

then, $$\Delta\theta = \frac{AB}{OA} = \frac{\Delta v}{v}$$

$$\therefore \Delta v = v\Delta\theta \quad \text{—— (ii)}$$

from (i) and (ii)

$$a = \frac{\Delta v}{\Delta t} = \frac{v\Delta\theta}{\Delta t} \quad \text{—— (iii)}$$

~~in lim~~ in limiting case

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$$

So from (iii)

$$a = v\omega = v \cdot \frac{v}{r} = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

* Centripetal Force

:- Force that acts on a body moving in circular towards it's center is called centripetal force.

we know,

$$F = m \cdot a = m \cdot \frac{v^2}{r}$$

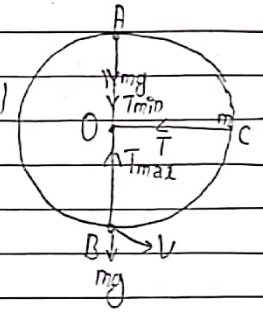
$$\therefore F = \frac{mv^2}{r}$$

* Centrifugal Force

:- The outward force acting on a body when they move in a circular path is called centrifugal force.

$$F = \frac{mv^2}{r}$$

* motion in vertical circle:



At point 'A' (min)

$$F = T_{min} + mg$$

$$\frac{mv^2}{r} = T_{min} + mg$$

$$T_{min} = \frac{mv^2}{r} - mg$$

At point 'B' (max)

$$F = T_{max} - mg$$

$$T_{max} = \frac{mv^2}{r} + mg$$

At point 'C'

$$F = \frac{mv^2}{r}$$

* A stone mass 2 kg tied with string in vertical circle of radius 1m with constand speed 10m/s. Find the max and min tension of strings. ($g = 9.8 m/s^2$)

⇒ Solution:- $m = 2\,kg$

$v = 10\,m/s$

$r = 1\,m$

Now

$$T_{min} = \frac{mv^2}{r} - mg$$
$$= \frac{2\times(10)^2}{1} - 2\times9.8$$
$$= 2\times100 - 2\times9.8$$
$$= 200 - 2\times9.8$$
$$= 180.4\ N$$

$$T_{max} = \frac{mv^2}{r} + mg$$
$$= \frac{2\times(10)^2}{1} + 2\times9.8$$
$$= 200 + 2\times9.8$$
$$= 219.6\ N$$

* A mass of 0.2 kg is rotate by string at constant speed in vertical circle of radius 1m. If the minimum tension in the string is 3N. Calculate the speed and maximum tension of the string.

=> Soln:- mass = 0.2 kg

$r = 1m$

$T_{min} = 3N$ $\quad g = 9.8 m/s^2$

$v = ?$

Now $T_{min} = \frac{mv^2}{r} - mg$

or, $3 = 0.2v^2 - 0.2\times9.8$

or, $\frac{3+0.2\times9.8}{0.2} = v^2$

or, $v^2 = 24.8$

or, $v = 4.97\ m/$

$$T_{max} = \frac{mv^2}{r} + mg$$
$$= 0.2\times24.8 + 9.8\times0.2$$
$$= 4.96 + 1.96$$
$$= 6.92N$$

* When a body moves in a uniform circular motion net work done in it is
a) zero b) positive c) infinity d) Negative
Ans: a)

* particle moving in circle with uniform speed. It has constant
a) Velocity b) acceleration c) K.E d) P.E

* work done by frictional force is
a) zero b) Negative c) positive d) infinity
Ans: b)

* Two bodies of heaving equal mass and velocity 4 m/s and 6 m/s. Their ratio of K.E will be
=> 2:3 Ans

$\frac{8}{4}\ \frac{3}{2}$ 3:2

* Two bodies moving with the mass 6 and 12 kg and velocity is half of respective one. Find the ratio of second to first.
=> 8:1 Ans

$m_1 = 6$
$m_2 = 12\ kg$

$\frac{K.E_2}{K.E_1} : \frac{\frac{1}{2}m_2v_2^2}{\frac{1}{2}m_1v_1^2}$

$\frac{12\times36}{6\times9}$ $\quad \frac{12\times36}{6\times6}$

$m \cdot \overset{F}{a}$ $F \times T$

$[M, L, T^{-2}]$ [

Date ____ Page ____

* The dimension of impulse
$[M, L, T^{-1}]$

* If $h = 6m$, the, Find it's Time period

$$T = \sqrt{\frac{2h}{g}}$$
$$= \sqrt{\frac{2 \times 6}{9.8}}$$
$$= \sqrt{\frac{12}{9.8}}$$
$$= \cancel{0.35}\ 1.10$$

* what is resultant of 6 and 7 vector which ~~are equal~~ move in equal direction

$$R = \sqrt{p^2 + \omega^2 + 2p\omega \cos\omega}$$
$$= \sqrt{6^2 + 7^2 + 2 \times 6 \times 7 \times 1}$$
$$= \sqrt{169}$$
$$= 13 \text{ Ans}$$

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* what is the common velocity of having two mass equal and velocity also equal.

Suppose common mass = 2 and velocity = 2

$$V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$
$$= \frac{2 \times 2 + \cancel{2 \times 2}}{\cancel{2 + 2}}$$
$$= \frac{mv + mv}{m + m}$$
$$= \frac{2mv}{2m}$$
$$= \frac{V}{1} \text{ Ans}$$

* If 0.2 kg and time 6 sec travelled a distance then calculate its impulse

⇒ Soln:-

$$\text{impulse} = F \times T$$
$$= m \cdot g \cdot T$$
$$= 0.2 \times 9.8 \times 6$$
$$= 11.76 \text{ Ns}$$

\+ Difference betn

(1) Define inertia also discuss it's type. And Reduce the
(2) expression for the conservation of linear momentum.

Inertia is the property of object in which it's want to continue it's original state when no external force is applied.

Types

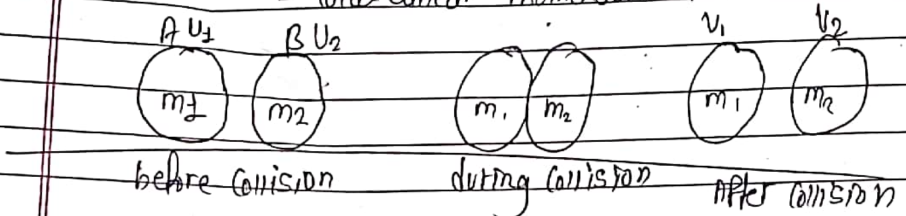
(i) Inertia of Rest

:- It is the property of a object in which it's remain in rest phase unless external force is applied.

(ii) Inertia of motion

(iii) Inertia of direction.

statement :- Total linear momentum before collision = Total linear momentum after collision



A and B

Suppose two bodies of mass m_1 and m_2 are moving with initial velocity u_1 and u_2. After some time A collides with B and their final velocity becomes v_1 and v_2.

The force exerted by A on B bodies is action and force exerted by B on A is action.

From Newton 3rd law

Force exerted by A on B = $\frac{\text{linear momentum of B}}{\Delta t}$

F_{AB}

$$= \frac{m_2v_2 - m_2u_2}{\Delta t} = \text{action (i)}$$

Force exerted by B on A = $\frac{\text{linear momentum of A}}{\Delta t}$

F_{BA}

$$= \frac{m_1v_1 - m_1u_1}{\Delta t}$$

Now $F_{AB} = -F_{BA}$

$$\frac{m_2v_2 - m_2u_2}{\cancel{\Delta t}} = -\frac{(m_1v_1 - m_1u_1)}{\cancel{\Delta t}}$$

$w = F \times d$
$= m \cdot a \cdot d$

or, $m_2v_2 + m_1v_1 = m_2v_2 + m_1v_1$

∴ This shows the eqn of linear momentum.

$\frac{m \cdot a}{M, L, T^{-2}}$

* Convert 2J of work in erg. CGS

by Formula Dimension $W = [M, L^2, T^{-2}]$
$x = 1 \quad y = 2 \quad z = -2$

by Formula

$n_2 = n_1 \left[\frac{m_1}{m_2}\right] \left[\frac{L_1}{L_2}\right] \left[\frac{T_1}{T_2}\right]$

$= 2 \left[\frac{1kg}{1gm}\right]^1 \left[\frac{1m}{1cm}\right]^2 \left[\frac{1s}{1s}\right]^{-2}$

$= 2 \left(\frac{1000}{1}\right) \left(\frac{100}{1}\right)^2$

$= 2 \times 1000 \times 10000$

$= 20000000$

$= 2 \times 10^7$ erg

Chapter 7

Gravitation

The mutual force of attraction between any two bodies in the universe is called gravitation and the acted force is called Gravitational force.

The Force by which an object are attract towards earth centre is called gravity.

The field around the material where gravitational pull can be experienced by other bodies is called gravitational fields.

* Newton's law of gravitation
$$F = \frac{GM_1m_2}{d^2}$$

* Acceleration Due to gravity (g)
∴ The acceleration produced on a body due to the action of gravity is known as acceleration due to gravity. It is denoted by 'g'

Suppose mass and Radius of earth are 'M' and 'R'. Suppose a body of mass 'm' lying on the surface of earth. From Newton law of Gravitation, force attraction between body and earth is
$$F = \frac{GMm}{R^2} \quad (i)$$

Also, weight of the body is

$$F = m \cdot g \quad \text{(ii)}$$

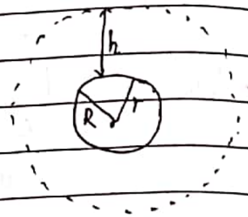
From (i) and (ii)

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

$$\therefore \boxed{g = \frac{GM}{R^2}}$$

* Variation of 'g'

a) variation of 'g' with altitude



let us considered 'R' and 'M' are the Radius and mass of earth.
'g' and 'g'' be the acceleration due to gravity on surface of earth and at height 'h'. from the surface of earth.

The acceleration due to gravity 'g' on the surface of earth is equal to $g = \frac{GM}{R^2}$ (i)

g' on height 'h' from surface of earth

$$g' = \frac{GM}{(R+h)^2} \quad \text{(ii)}$$

dividing (ii) ~~and (ii)~~ by (i)

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\frac{g'}{g} = \frac{\cancel{GM} \cdot R^2}{\cancel{GM} \cdot (R+h)^2}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{\cancel{R^2}}{\cancel{R^2}\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g'}{g} = 1\Big/\left(1+\frac{h}{R}\right)^2$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{R^2}{\left[R\left[1+\frac{h}{R}\right]\right]^2}$$

$$\frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$= \frac{R^2}{(R+h)^2}$$

$$\frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

$$g' = g\left(1-\frac{2h}{R}\right)$$

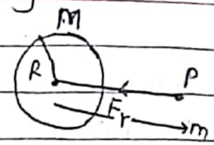
g' →

* Gravitational Field Intensity / strength

:- At a point, In the gravitational force experienced by a unit mass placed at that point. It is denoted by E.

i.e $E = \frac{\text{Force } (F)}{\text{mass } (m)}$

** Expression of Gravitational Field intensity :-



Considered a test mass 'm' is at point 'P' at distance 'r' away from centre.
From Figure 'M' and 'R' are the mass and Radius of earth respectively.

Then From Newton law of Gravitation

$$F = \frac{GMm}{r^2} \quad \text{--(i)}$$

again, For Gravitational field intensity.

$$E = F/m \quad \text{--(ii)}$$

From (i) and (ii)

$$E = \frac{\frac{GMm}{r^2}}{m} = \frac{GM\cancel{m}}{\cancel{m}\cdot r^2} = \boxed{\frac{GM}{r^2}}$$

It is seen that E decreases as distance increases and becomes zero at infinity.

If the test mass is at surface of earth (r=R)

so, $E = \frac{GM}{R^2} = g$

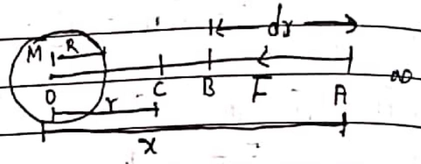
hence this is the required eqn of E.

* Gravitational potential ~~intensity~~ Energy.

:- At a point in the gravitational Field is defined as the amount of work done in bringing the body from infinity to that point at constant velocity.

→ Considered the earth of Radius 'R' and mass 'M'; Gravitational force on mass 'm' at point A due to earth is $F = \frac{GMm}{x^2}$

Let, mass moves from dx distance from A to B as shown in Fig:-



$\therefore \; dw = F dx$

$dw = \frac{GMm}{x^2} dx$

Then total work done in bringing mass 'm' from infinity to point 'c' is

$$w = \int_{\infty}^{r} dw = \int_{\infty}^{r} \frac{GMm}{x^2} dx = GMm \int_{\infty}^{r} x^{-2} dx$$

$$= GMm \left[-\frac{1}{x} \right]_{\infty}^{r} = GMm \left[-\frac{1}{r} + \frac{1}{\infty} \right] = -\frac{GMm}{r}$$

$$V = w = -\frac{GMm}{r}$$

* Escape Velocity

: The minimum velocity with which a body must be thrown upward so that it may escape the gravitational pull of the planet is called escape velocity.

Suppose 'M' and 'R' be mass and Radius of earth respectivly. Suppose 'm' be the mass of body thrown vertically upward with escape velocity (V_e) from earth surface.

then the Gravitational potential Energy

$$V = -\frac{GMm}{r}$$

amount of work required to move mass 'm' from surface of earth to infinity is

$$W = V = \frac{GMm}{R}$$

Then, K.E. by which body is projected with velocity V_e is

$$K.E = \frac{1}{2} m v_e^2$$

The escape the body gravitational potential energy should be equal to K.E.

So, $\frac{1}{2} m v_e^2 = \frac{GMm}{R}$

or, $\frac{V_e^2}{2} = \frac{Gm}{R}$

or, $V_e = \sqrt{\frac{2gR^2}{R}}$

$\therefore \; V_e = \sqrt{2gR}$

* satellite

:- The heavenly body that revolves around the planet in its orbit is called satellite.

* orbital velocity

:- The minimum velocity at which the body revolves around the earth is known as orbital velocity.

let us consider 'M' and 'R' be the mass and Radius of earth. m be the mass of satellite moving around the earth with radius $r = r+h$

In order to moving satellite around the earth
Gravitational Force = centripetal force

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r}$$

or, $\frac{GM}{r} = v^2$

or, $v^2 = \frac{GM}{r}$

or, $v = \sqrt{\frac{GM}{r}}$

or, $v = \sqrt{\frac{g \times R^2}{R+h}}$

or, $v = R\sqrt{\frac{g}{R+h}}$

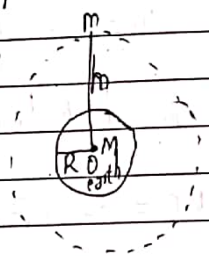


Fig :- motion of satellite

For time period :-

Time period $(T) = \frac{2\pi}{\omega}$

$= \frac{2\pi}{v/r}$

$= \frac{2\pi r}{v}$

$= \frac{2\pi r}{R\sqrt{\frac{g}{R+h}}}$ or, $\frac{2\pi r}{\sqrt{\frac{Gm}{r}}}$

∴ This is required eqn for time.

chapter 8 Elasticity

Date ____
Page ____

* Elastic Body

The property of body under which it tends to regain in original shape and size after the removal of deforming force is called Elastic body. and this property is called elasticity. Deformation is reversible in elastic.

* plasticity

:- The proprety of a body under which it doesnot regain it's original shape and size after the removal of deforming force. is called plastic body and this property is plasticity. Deformation is ireversible in elastic.

* stress

:- The force which when applied to a body changes its configuration is known as deforming force.

Thus, stress is defined as the ration deforming force per unit area.

$$S = \frac{F}{A}$$

Types of stress

1) Normal stress

:- here, normal stress = $\frac{\text{Normal Force}}{\text{Area}}$

Normal stress are two types :-

(i) Tensile stress

:- If there is increase in length of body at stress developed is known Tensile stress.

Date ____
Page ____

(ii) compression stress

:- If there is decrease in length of body an stress developed which is called compression stress.

2) Tangential (or shearing) stress

:- If the deforming force is applied in the direction parallel to the surface of the body called tangential stress.

$$\text{Tangential stress} = \frac{\text{tangential force}}{\text{area}}$$

3) valumetric (or pressure) stress

:- The valumetric stress is defined as the normal force per unit of total surface area that produce valume change.

* strain

:- strain is defined as the ratio of change in dimension to the original dimension. It has no unit and dimension less quantity.

Types

(i) longitudinal / tensile strain

:- It is defined as ratio change in length to original length.

$$\text{longitudinal strain} = \frac{\text{change in length}}{\text{orginal length}}$$

$$\therefore l.s = \frac{\Delta L}{L} = \frac{e}{L}$$

(ii) valumetric strain

:- It is defined as ratio of change in volume to original volume.

$$V.S = -\frac{\Delta V}{V}$$

(iii) shearing strain

:- The angular displacement produced by the tangential stress is called shearing strain.

It is denoted by ϕ.

$$\text{shearing strain} = \phi = \frac{x}{h}$$

8.6. Hooke's law

:- It states that, within elastic limit the applied stress is proportional to strain produced.

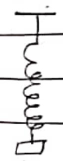
i.e. stress $\propto$ strain

$$\frac{\text{stress}}{\text{strain}} = E$$

where E is constant called coefficient of elasticity

Experimental verification

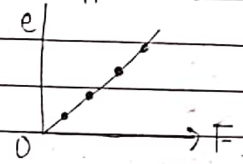
Suppose a string of length 'L' suspended from rigid support. as shown in figure.



Such that it carries a pan in it's lower end (pan is loaded 0.5 kg), corresponding pointers reading are noted.

The difference betn each reading and the first reading gives the extension e produced in the spring by different weights.

Graphs betn applied force 'F' and extension produced e is drawn.



Graph betn e vs F.

This indicates that F is directly proportional

$$F \propto e$$

$$F = k.e \quad (i)$$

and dividing (i) by A $\quad \frac{F}{A} = \frac{k.e}{A} = \left(\frac{k.L}{A}\right) \cdot \frac{e}{L}$

since k and L are constant

so, $\frac{F}{A} \propto \frac{e}{L}$ [within elastic limit].

$\therefore$ $\boxed{\text{strain} \propto \text{stress}}$

stress $\propto$ strain

This verify Hook's law

Types of coefficient / modules of elasticity :-

There are 3 types of coefficient of elasticity :-

1) Young's modules of Elasticity (Y)

:- It is defined as the ratio of normal stress to the longitudinal strain.

It is the property of solid materials only.

$$Y = \frac{\text{normal stress}}{\text{longitudinal strain}} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$= \frac{F/A}{e/L} = \frac{F.L}{A.e}$$

It is an indication of stiffness material.

2) Bulk's modules of Elasticity (k) (B)

:- It is defined as the ratio of normal stress to the volume strain.

$[\because P = F/A]$

$$k = \frac{\text{normal stress}}{\text{volume strain}} = \frac{F/A}{-\Delta V/V} = -\frac{F.V}{A.\Delta V} = \frac{-PV}{\Delta V}$$

3) Shear's modules of Elasticity (β)(η)

:- It is defined as the ratio of tangential stress to the shear strain.

(neta) $\eta = \frac{\text{tangential stress}}{\text{shear strain}} = \frac{F/A}{\phi}$

Since $\phi = x/h$ $= \frac{F/A}{x/h} = \frac{F.h}{A.x}$ $[\because P = \frac{F}{A}]$

* If 250N acts on the $6m^2$. Find its tension.

$\Rightarrow T = F/A = 250/6 = 41.67$ N. Ans

* Find the Dimension of Bulk's moduls of Elasticity

$$k = \frac{\text{normal stress}}{\text{volume strain}}$$

$$= -\frac{F\,V}{A.\Delta V}$$

$$= \boxed{-\frac{m.\cancel{A}.V}{\cancel{A}\,\Delta V}} \qquad = \frac{F}{A}$$

$$= [M\,L^{-2}] \qquad = \frac{m.A}{A}$$

$$= \frac{[M][\cancel{M}L^{2}T^{-2}]}{\cancel{M}\,L^{2}}$$

$$= [M\,L^{-1}\,T^{-2}]$$

$$P = \frac{F}{A} \qquad \text{Force} = M.a = [M\,L\,T^{-2}]$$

$$= \frac{M.a}{A}$$

$$= \frac{[M][L\,T^{-2}]}{[L^2]}$$

$$= [M\,L^{-1}\,T^{-2}] \text{ Ans}$$

$Y = \frac{\text{stress}}{\text{strain}}$ [where strain is dimensionless]

$= \text{stress}$

$= F/A$

$= [M\,L^{-1}\,T^{-2}]$ Ans

* Elastic limit

:- The maximum value of deforming force which a body can experienced and still regain original shape and size owce the force is removed is known as Elastic limit.

* Elastic after ~~deffect~~ effect

:- The temporary delay in regaining the original configuration by a body after the deforming force is removed is called Elastic after effect.

* poisson's Ratio :

:- The increase in dimension per unit original dimension is called longitudinal strain (α).

The decrease in dimension per unit original dimension perpendicular to ~~appli~~ direction of applied force is called lateral strain (β).

Within elastic limit $\beta \propto \alpha$

$$\beta = \sigma\alpha$$

$$\frac{\beta}{\alpha} = \sigma$$

where σ is constant called poisson's ratio. Thus, poisson's Ratio is the ration of lateral strain to the longitudinal strain.

* Elastic Fatigue

:- The phenomenon of decrease of elasticity of body due to repeatdly applyied alternating deforming forces is called Elastic Fatigue

* Elastic Hysteresis

:- The dictionary meaning of hysteresis is delayed or coming late. Thus process of lagging of strain behind the applied stress is known elastic hysteresis.

* Elastic potential Energy stored in a stretched wire,

:- Considered, a wire of original length (l) and stretched by length (e) when force 'F' is applied. This force increased from 0 to 'F'. So, average force $= \frac{0+F}{2} = \frac{F}{2}$

Thus, work done by average force = Force x distance

$$= \text{Average Force} \times \text{distance}$$

$$= F/2 \times e$$

$$W = \frac{1}{2} Fe$$

Thus, it can be written as :-

$$U = \frac{1}{2} Fe$$

$$\therefore U = \frac{1}{2} \text{Force} \times \text{extension}$$

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Unit 9 Head and Temperature

Heat :- The total sum of kinetic energy of the molecules contained on a body is called heat.

1 calorie = 4.2 joule

→ The science of temperature and its measurement is known as thermometry.

* Thermal Equilibrium and Zeroth law of Thermodynamics

:- When two or more than two bodies are said to be in thermal equilibrium if there is no transfer of heat among them when kept in contact.

According to zeroth's law of thermodynamics, if two systems A and B are in thermal equilibrium with another system C separately, then A and B are also in thermal equilibrium and have the same temperature.

* Absolute zero

:- The temperature at which the K.E of the molecule of a substance becomes zero is called absolute zero temperature.

* Calibration of thermometer

:- The process of fixing lower fixed point and upper fixed point of a thermometer is called calibration of thermometer.

(i) Lower Fixed point
:- The temperature at which the pure ice melts under standard atmospheric pressure is taken as the lower fixed point of the thermometric scale.

(ii) upper Fixed point
:- The temperature at which the pure water boils under standard atmospheric pressure is called upper fixed point.

* Types of temperature scale
i) Centrigrade scale or Celsius (°C)
LFP :- 0°C
UFP :- 100°C

ii) Fahrenheit scale
LFP — 32°F
UFP — 212°F

iii) Kelvin scale
LFP — 273 K
UFP — 373K

iv) Reaumer scale
LFP — 0°R
UFP — 80°R

Relation between °C, °F, °R, K

$$\frac{C-0}{100} = \frac{F-32}{180} = \frac{R-0}{80} = \frac{K-273}{\cancel{180}\ 100}$$

* In what value celsius and Fahrenheit scales show same read.
Soln:- $C = F = x$ (suppose)
by relation

$$\frac{C}{100} = \frac{F-32}{180}$$

or, $\frac{x}{100} = \frac{x-32}{180}$

or, $180 \times x = 100(x-32)$
or, $180x = 100x - 3200$
or, $180x - 100x = -3200$
or, $80x = -3200$
or, $x = \frac{-3200}{80} = -40$
$\therefore -40°C$ or $-40°F$.

* In what value celsius and Reaumer scales show same reading.
Soln:- $C = R = x$ (suppose)
By relation

$$\frac{C}{100} = \frac{R}{80}$$

or, $x \times 80 = x \times 180$
or, $0 = 180x - 80x$
or, $0 = 80x$
or, $\frac{0}{80} = x$
or, $x = 0$ $\therefore$ (0°C or 0°R)

* At what values F and R give common reading.

Soln:- $F = R = x$ (x suppose)

then,

By relation

$$\frac{F-32}{180} = \frac{R}{80}$$

or, $(x-32)\,80 = x \times 180$

or, $-2560 = 180x - 80x$

or, $-2560 = 80x$

or, $\frac{-2560}{100} = x$

or, $x = -25.6$

$$\frac{F-32}{9} = \frac{R}{4}$$

or, $(x-32)\,4 = 9x$

or, $4x - 128 = 9x$

or, $\frac{-128}{5} = x$

or, $x = -25.6$

($\therefore$ $-25.6°R$ or $-25.6°F$)

* Celsius scale and kelvin scale never gives common readings.

Checked

Chapter 10

Thermal Expansion

The phenomenon in which the dimension of a body is increased on heating is called thermal expansion.

* Expansion of solid

1) Linear Expansion

a:- The increase in the length of a solid body on heating is called linear Expansion. It is also called one dimensional (1D).

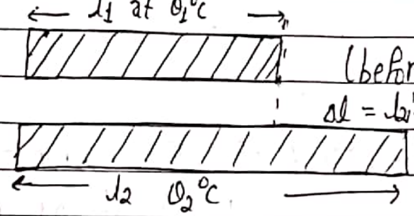


Fig:- Rod Expansion

For Mathmatical Expression

Let us suppose a metal rod of initial length l_1 at temperature θ_1°C, when it is heated to temperature θ_2°C, then it's length becomes l_2 as shown in above figures.

Experimentally, it is found that the change in length $(\Delta l = l_2 - l_1)$ is directly proportional to

(i) it's original length i.e. $\Delta l \propto l_1$ ... (i) eqn

(ii) the change in temperature $\Delta l \propto \Delta\theta$ ... (ii) eqn where $\Delta\theta = \theta_2 - \theta_1$

Combing (i) and (ii) eqⁿ

$\Delta l \propto l_1 \Delta \theta$

$\Delta l = \alpha l_1 \Delta \theta$ (where α linear expansivity) (3) eqⁿ

From (3) eqⁿ $\alpha = \frac{\Delta l}{l_1 \Delta \theta}$

If $l_1 = 1$ unit, $\Delta \theta = 1^oC$ or $1K$ then,

$\alpha = \frac{\Delta l}{1 \times 1}$ $\therefore \alpha = \Delta l$

Therefore, the coefficient of linear expansion is defined as the change in length per unit original length per unit change in temperature.

Again from (3) eqⁿ

$\Delta l = \alpha l_1 \Delta \theta$

or, $l_2 - l_1 = \alpha l_1 \Delta \theta$

or, $l_2 = l_1 + \alpha l_1 \Delta \theta$

or, $l_2 = l_1 (1 + \alpha l_1 \Delta \theta)$ which is the final expansion of length of given metal rod when $\theta_1 = 0^oC$, l_1: $\theta_2 = \theta^oC$, l_2:

then, $l_\theta = l_0 (1 + \alpha \theta)$

unit of linear expansivity α

we have $\alpha = \frac{\Delta l}{l_1 \Delta \theta} = \frac{m}{m \times {}^oC \text{ or } K} = {}^oC^{-1}$ or K^{-1}

2) Superficial Expansion

:- The increase in the area of solid body on heating is called superficial expansion. It is also known as 2D.

mathmatically,

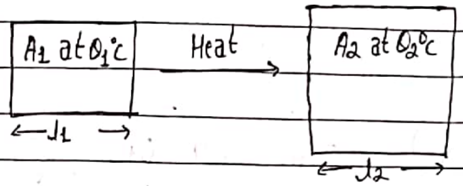


Fig:- Expansion of a metal sheet

Suppose metal sheet of area A_1 at a temperature θ_1^oC. When it is heated at temperature θ_2^oC then it's area becomes A_2 as shown in above fig.

Experimentally it was found that change in area ($\Delta A = A_2 - A_1$) directly proportional to

i) it's original area $\Delta A \propto A_1$ -- (i) eqⁿ

ii) change in temperature ~~$\Delta A \propto \theta_2 - \theta_1$~~ $\Delta A \propto \Delta \theta$ (ii) eqⁿ

by combing (i) and (ii) eqⁿ

$\Delta A \propto A_1 \Delta \theta$

$\Delta A = \beta A_1 \Delta \theta$ (where β is superficial expansivity) (iii) eqⁿ

Now $\beta = \frac{\Delta A}{A_1 \Delta \theta}$

If $A_1 = 1$ unit, $\Delta \theta = 1^oC$ or, $1K$

then, $\beta = \Delta A$

There fore coefficient of superficial expansion is defined as the change in area per unit area per unit change in temperature.

From (iii) eqn

$\Delta A = \beta A_1 \cdot \Delta\theta$

or, $A_2 - A_1 = \beta A_1 \cdot \Delta\theta$

$\therefore A_2 = A_1(1 + \beta\Delta\theta)$ also when $\theta_1°C = 0°C$ $A_1 = A_0$, $\theta_2°C = \theta°C$ $A_2 = A_\theta$

This is the expression for the final area of metal sheet.

3) cubical / volumetric expansion

:- The increase in volume of a solid body on a heating is known as cubical / volumetric expansion. It is also known as 3D.

Mathematical Expression

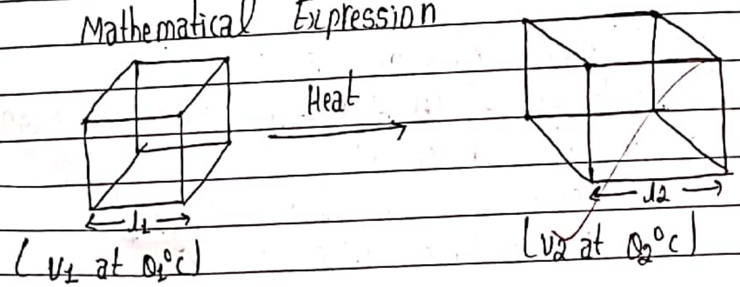


Fig:- Expansion of metallic box

let us considered a cubical metal box of volume V_1 at temperature $\theta_1°C$ when it is heated to the temperature $\theta_2°C$, it's volume become V_2 as shown in fig.

Experimentally, it is found that change in volume (ΔV) is directly proportional to,

i) original volume $\Delta V \propto V_1$ (i) eqn

ii) change in temperature $\Delta V \propto \Delta\theta$ (ii) eqn

by combining (i) and (ii) eqn

$\Delta V \propto V_1 \Delta\theta$

$\Delta V = \gamma V_1 \Delta\theta$ (where γ is cubical expansivity) (iii) eqn

again

$\gamma = \frac{\Delta V}{V_1 \Delta\theta}$

If $V_1 = 1$ unit, and $\Delta\theta = 1°C$ or $1K$ then,

$\gamma = \Delta V$

Therefore the coefficient of cubical expansion is defined as the change in volume per unit original volume per unit change in temperature.

Now,

From (iii) eqn $\Delta V = \gamma V_1 \Delta\theta$

or, $V_2 - V_1 = \gamma V_1 \Delta\theta$

or, $V_2 = V_1(1 + \gamma\Delta\theta)$

when, $\theta_1°C = 0°C$ $V_1 = V_0$

$\theta_2°C = \theta°C$ $V_2 = V_\theta$

$\therefore$ This is the final expansion of cubical metal.

* Relation between α and β

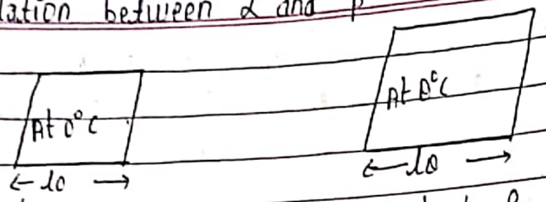


Fig:- Expansion of thin metal sheet from l_0 to l_θ.

Consider a square metallic sheet having length of l_0 and area A_0 at temperatures 0°C when it is heated up to θ°C. it's length and also area becomes l_θ and A_θ respectively. let α and β be the linear and superficial expansivity of material or square sheet.

Then, we can write

$$l_\theta = l_0(1+\alpha\theta)$$

squaring on both sides,

$$l_\theta^2 = l_0^2(1+2\alpha\theta+\alpha^2\theta^2) \text{ -- (i) eqn}$$

Also,

$$A_\theta = A_0(1+\beta\theta)$$

since, $A_0 = l_0^2$ and $A_\theta = l_\theta^2$

$$\therefore l_\theta^2 = l_0^2(1+\beta\theta) \text{ -- (ii) eqn}$$

From (i) and (ii) eqn

$$\cancel{l_0^2}(\cancel{1}+2\alpha\theta+\alpha^2\theta^2) = \cancel{l_0^2}(\cancel{1}+\beta\theta)$$

$$2\alpha\theta+\alpha^2\theta^2 = \beta\theta$$

since, the value of α is very small so the terms containing higher power of α can be neglected.

$$2\alpha\theta = \beta\theta$$

$$\text{or, } \alpha = \beta/2 \qquad \therefore \beta = 2\alpha$$

Thus, the coefficient of superficial expansion of a sheet is twice the coefficient of linear expansion.

* Relation betn α and γ

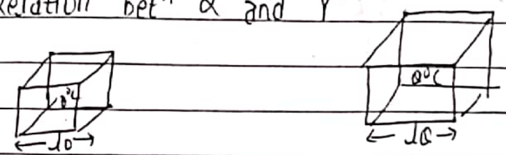


Fig:- volume expansion of metallic cube.

Consider a metal cube having length, l_0 and volume V_0 at temp of 0°C. when it is heated up to θ°C. it's length and volume becomes l_θ and V_θ respectively.

Let α and γ be the linear and cubical expansivity of material of cube, we can write

$$l_\theta = l_0(1+\alpha\theta)$$

cubing on both side

$$l_\theta^3 = l_0^3(1+3\alpha\theta+3\alpha^2\theta^2+\alpha^3\theta^3) \text{ -- (i) eqn}$$

Also,

$$V_\theta = V_0(1+\gamma\theta)$$

since, $V_\theta = l_\theta^3$ and $V_0 = l_0^3$

∴ $l_\theta^3 = l_0^3 (1+\gamma\theta)$ -- (ii)

From (i) and (ii) eqn

$$l_0^3 (1+\gamma\theta) = l_0^3 (1 + 3\alpha\theta + 3\alpha^2\theta^2 + \alpha^3\theta^3)$$

or, $\gamma\theta = 3\alpha\theta + 3\alpha^2\theta^2 + \alpha^3\theta^3$

since, the value of α is very small so the containing higher power can be neglected.

$$3\alpha\theta = \gamma\theta$$

or, $3\alpha = \gamma$

∴ $\alpha = \frac{\gamma}{3}$

Thus, the coefficient of cubical expansion of a solid is three times it's coefficient of linear expansion.

* Relation between α, β and γ we have,

$\alpha = \frac{\beta}{2}$ -- (i)

and, $\alpha = \frac{\gamma}{3}$ -- (ii)

combining both (i) and (ii) eqn we get,

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$$

∴ $\boxed{\alpha : \beta : \gamma = 1:2:3}$. which is the required relation between α, β and γ.

* Force (Tension) set up is a bar due to expansion or ~~concentration~~ contraction.

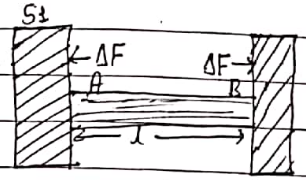


Fig:- Thermal stress

When a metal rod fixed between two rigid support is heated, it tries to expand, but rigid support do not set for it's expansion. Hence, metal rod feels compressive strain. The corresponding stress produced in metal when it is heated betn two rigid supports is known as thermal stress.

mathematically

let us consider a metal rod, AB of original length, l. As it is heated, the change in length of rod is given by $\Delta l = l\alpha\Delta\theta$

where α = linear expansivity of rod.

or, $\frac{\Delta l}{l} = \alpha\Delta\theta$

Compressive strain $= \frac{\Delta l}{l} = \alpha\Delta\theta$ -- (i)

From definition of young's modulus of elasticity,

$$Y = \frac{\text{stress}}{\text{compressive strain}}$$

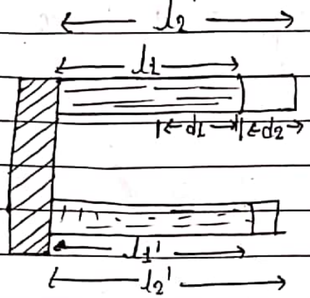
or, $Y = \frac{\Delta F / A}{\Delta l / l}$ where, A is class section area of rod

or, $Y = \frac{\Delta F/A}{\alpha \Delta \theta}$ (from eqn i)

or, $Y = \frac{\Delta F}{A\alpha\Delta\theta}$

$\therefore \Delta F = Y A \alpha \Delta\theta$

* Differential expansion.



$d_1 = l_1 - l_1'$

$d_2 = l_2 - l_2'$

Fig:- Differential expansion of two metal strips.

We know that different materials have different value of linear expansivities, so they expand differently at same change in temperature. The difference in expansion of two dissimilar materials when they are heated at same temperature is known as differential expansion.

chapter 11

Quantity of Heat

The branch of physics which deals with the measurement of the quantity of heat when two bodies share heat is known as calorimetry.

* principle of calorimeter
(Heat lost by hot body = Heat gain by cold body) at physical contact between them.

* Specific heat capacity
: when The amount of heat required to change the temperature of unit mass of substance through one degree.

* Determination of specific heat capacity of a solid by method of mixture

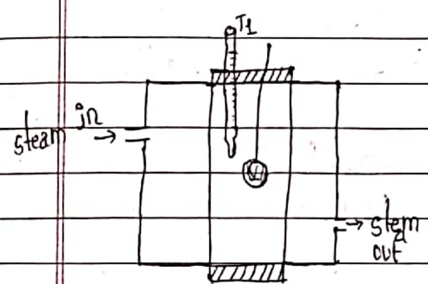


Fig:- Regnault's appartus

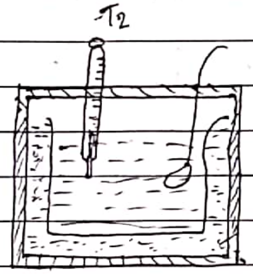


Fig:- Calorimeter with stirrer

Derivation

Let, mass of calorimeter with stirrer = m_c

mass of water = m_w

mass of solid sphere = M

Initial temp. of calorimeter and water = θ_1°C
steady temperature of heated solid sphere = θ_2°C
Final steady temperature = θ°C

specific heat capacity of calorimeter = S_c
" " " " water = S_w
" " " " solid sphere = S

Now Heat lost by hot solid sphere = $ms(\theta_2-\theta_1)$
Heat gain by calorimeter and water = $m_cS_c(\theta-\theta_1) + m_w S_w(\theta-\theta_1)$

By using calorimetry principle
heat lost = heat gain

$$ms(\theta_2-\theta_1) = m_cS_c(\theta-\theta_1) + m_wS_w(\theta-\theta_1)$$

$$\text{or, } ms(\theta_2-\theta_1) = (m_cS_c + m_wS_w)(\theta-\theta_1)$$

$$\text{or, } S = \frac{(m_cS_c + m_wS_w)(\theta-\theta_1)}{m(\theta_2-\theta_1)}$$

∴ By knowing all R.H.S values we can find out specific heat capacity of solid.

Newton's law of cooling

:- It states that the rate of loss of heat by the liquid to it's surrounding is directly proportional to temperature difference between liquid and the surrounding.

If θ and θ_s be the temperature of liquid and surrounding and $\frac{dQ}{dt}$ be the rate of loss of heat by liquid to the surrounding, then, by newton's law of cooling,

$$\frac{dQ}{dt} \propto (\theta-\theta_s)$$

$$\text{or, } -\frac{dQ}{dt} = K(\theta-\theta_s) \quad \text{.. (i)}$$

where, k is a proportionality constant whose value depends on nature of liquid and surface area exposed to surrounding. The negative sign indicates that liquid loses more heat as time increases.

let 'm' be the mass of liquid and 'S' be specific heat capacity of the liquid then the amount of heat lost by the liquid when it's temperature changes by θ°C, then, $Q = ms\theta$

differenciating eqn above : i.e. $Q = ms\theta$ with respect to time (t), we get,

$$\frac{dQ}{dt} = ms\frac{d\theta}{dt} \quad \text{-- (ii) eqn}$$

From (i) and (ii) eqn

$$-ms\frac{d\theta}{dt} = k(\theta-\theta_s)$$

or, $$\frac{d\theta}{\theta-\theta_s} = \frac{-k}{ms}\,dt$$

integrating both side, we get

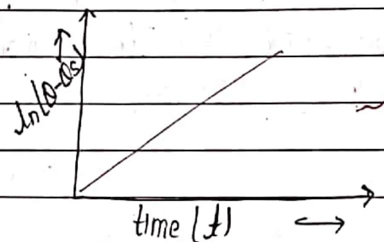
$$\int \frac{d\theta}{\theta-\theta_s} = \int -\frac{k}{ms}\,dt$$

or, $$\int \frac{d\theta}{\theta-\theta_s} = \frac{-k}{ms}\int dt$$

or, $$\ln(\theta-\theta_s) = \frac{-k}{ms}t + c \quad \text{-- (iii)}$$

This is the eqn of straight line in the form $y = mx + c$.

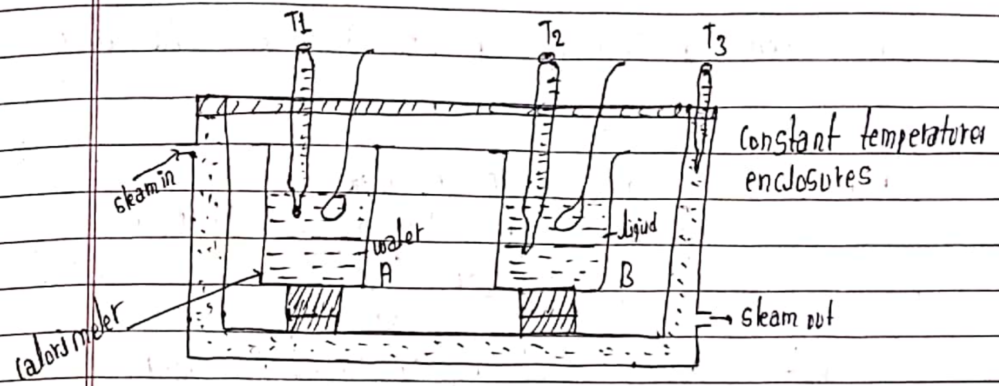
If a graph is plotted ~~where~~ between $\ln(\theta-\theta_s)$ and t, a straight line is obtained as shown in fig: below which velocity Newton's law of cooling.



Graph Fig:→ to show $\ln(\theta-\theta_s)$ vs time (t).

+ Determination of specific heat capacity of a liquid by method of cooling

principle :- when two liquid are cooled under identical condition their rate of cooling are equal.



working :- we take water in a calorimeter 'A' and the liquid whose specific heat capacity is to be determined in the calorimeter 'B' such that volume of both water and liquids are equal. Now both calorimeter A & B are cooled from θ_1°C to θ_2°C.

let, mass of calorimeter A with stirrer = m_1

mass " " B " " = m_2

" " water = m_w

" " liquid = M

S.p heat capacity of calorimeter = S_c

" " " " water = S_w

" " " " liquid = S

Time taken by calorimeter A and water to cool down from $\theta_1°C$ to $\theta_2°C = t_1$

Time taken by calorimeter B and liquid to cool down from $\theta_1°C$ to $\theta_2°C = t_2$

Heat lost by calorimeter A and water

$$= m_1 s_c (\theta_1 - \theta_2) + m_w s_w (\theta_1 - \theta_2)$$
$$= (m_1 s_c + m_w s_w)(\theta_1 - \theta_2)$$

$\therefore$ Rate of loss of heat by calorimeter A & water

$$= \frac{(m_1 s_c + m_w s_w)(\theta_1 - \theta_2)}{t_1}$$

Similarly, Rate of loss of heat by calorimeter B & liquid

$$= \frac{(m_2 s_c + m s)(\theta_1 - \theta_2)}{t_2}$$

Since, they are cooled under identical condition so,

$$\frac{(m_1 s_c + m_w s_w)\cancel{(\theta_1 - \theta_2)}}{t_1} = \frac{(m_2 s_c + m s)\cancel{(\theta_1 - \theta_2)}}{t_2}$$

or, $$\frac{(m_1 s_c + m_w s_w)}{t_1} \times t_2 - m_2 s_c = m s$$

or, $$\therefore s = \frac{(m_1 s_c + m_w s_w)}{m}\frac{t_2}{t_1} - \frac{m_2 s_c}{m}$$

If all values of R.H.S is known ...

* change of state :-

change in phase :-
The transition of substance from one state (or phase) to another state/phase at constant temperature is known as change in phase or phase transition.

→ Different process of change of phase
:- There are different processes during transition of substance from one state to another which are given below.

(i) melting or Fusion
:- The change in phase of matter from it's solid state to liquid state at constant temperature is known as melting or fusion.

ii) Vapourization or Evapouration
:- The change in phase of matter from it's liquid state to gaseous state at constant temperature is known as vapourization.

Latent Heat

Date
Page

The amount of heat required to change the phase of the substance at constant temperature is known as latent heat.

→ purpose for using latent heat

(i) To decrease external atmospheric pressure to create space between molecule of that substance (i.e. external latent heat)

(ii) To decrease the intermolecular force of attraction betn molecules (i.e. internal latent heat)

Thus, latent heat energy changes the P.E of substance not

Specific latent heat

:- It is defined as the amount of heat required to change the phase of substance of unit mass at constant temperature. It is denoted by 'L'.

amount of heat (Q) required to change the phase of substance is directly proportional to it's mass 'm'

$Q \propto m$

or, $Q = Lm$ (where L is a proportionality constant of called specific latent heat. It's value depends on the pressure exerted on it.

Date
Page

Unit of L:

$Q = \frac{L}{m}$

or, $L = \frac{Q}{m}$

In S.I unit, $L = \frac{joule}{kg} = J\,kg^{-1}$

In cgs unit, $L = cal\,gm^{-1}$

→ Types of latent heat

(i) latent heat of Fusion

:- It is defined as the amount of heat required to change the phase of unit mass of substance from it's solid state to liquid state at it's melting point.

The latent heat of fusion of ice is, $80\ cal\,gm^{-1}$

$= \frac{80 \times 4.2}{\frac{1}{100}}$

$= 80 \times 4.2 \times 10^3$

$= 3.36 \times 10^5\ J/kg$ (In S.I unit)

(ii) latent heat of vapourization

:- It is defined as the amount of heat required to change the phase of unit mass of substance from it's liquid state to gaseous state at it's boiling point.

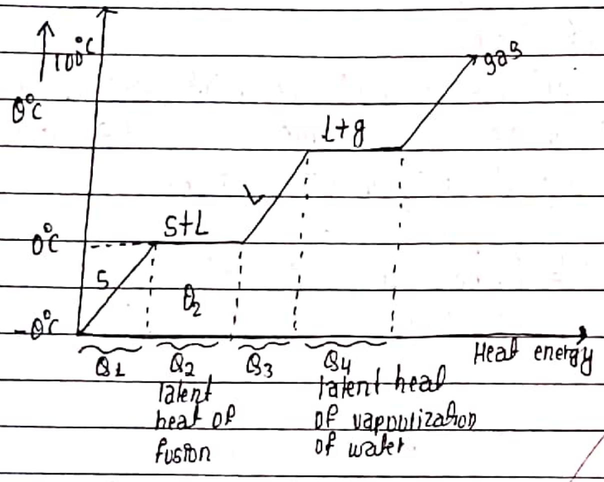
The latent heat of vapourization of water is 540 cal gm^{-1}

$= 540 \times 4.2 \times 1000$

$= 2.26 \times 10^6$ J/kg (In S.I unit)

* Total heat required to change solid (i.e. Ice) at -θ°C to gas (i.e. steam) at 100°C.

The total amount of heat required to change ice at -θ°C to stream at 100°C is obtained from diagram below



∴ Total amount of heat

$Q_T = Q_1 + Q_2 + Q_3 + Q_4$

$= m_i s_i \Delta\theta + m_i l_i + m_w s_w \Delta\theta + m_w l_v$

-θ°C		0°C		0°C		Ice at 100°C		100°C
m_i	→	m_i	→	m_w	→	m_w	→	m_s
Ice		Ice		water		water		steam

* Calculate the amount of heat required to convert 10 kg of ice at -10°C into the steam at 100°C. (Give s.p heat capacity of ice = 2100 J/kg°C, latent heat fusion of ice = 3.36×10^5 J/kg, latent heat of steam = 2.26×10^6 J/kg.

⇒ Soln:-

mass of Ice (m) = 10 kg

initial Temper. of ice (θ_1) = -10°C

Final " " steam (θ_2) = 100°C

S.p heat capacity of ice = 2100 J/kg°C

~~S.p~~ latent heat of fusion of ice (l_i) = 3.36×10^5 J/kg

" " " vapourization of water (l_s) = 2.26×10^6 J/kg

Total amount of heat required to convert 10 kg of -10°C ice into 100°C steam is

$Q = Q_1 + Q_2 + Q_3 + Q_4$

$= m_i s_i (0-(-10)) + m_i l_i + m_w s_w (100-0) + m_w l_s$

$= 10 \times 2100 \times 10 + 10 \times 3.36 \times 10^5 + 10 \times 4200 \times 100 + 10 \times 2.26 \times 10^6$

$= 210000 + 3.36 \times 10^6 + 4200000 + 2.26 \times 10^7$

$= 210000 + 3360000 + 4200000 + 22600000$

$= 30370000$

$= 3.03 \times 10^7$ ~~J/kg°~~ J Ans

4. A substance takes 3 min in cooling from 50°C to 45°C & takes 5 min in cooling from 45°C to 40°C. what is the temp. of the surrounding?

Ans: Soln:- Given,

$t_1 = 3 \times 60 = 180$ sec

$t_2 = 5 \times 60 = 300$ sec

First temp. of substance $(\theta_1) = 50°C$

2nd " " " $(\theta_2) = 45°C$

3rd " " " $(\theta_3) = 40°C$

let,

temp. of surrounding $= \theta_s$

1st case, From Newton's law of cooling,

$$\frac{d\theta}{dt} \propto (\theta - \theta_s)$$

or, $$\frac{d\theta}{dt} = k(\theta - \theta_s)$$

Also, from heat eqn, $Q = ms d\theta$

or, $$\frac{d(msd\theta)}{dt} = k(\theta - \theta_s)$$

or, $$ms\frac{d\theta}{dt_1} = k(\theta - \theta_s)$$

or, $$ms\frac{(50-45)}{180} = k\left(\frac{50+45}{2} - \theta_s\right)$$

or, $$ms\frac{5}{180} = k\left(\frac{95}{2} - \theta_s\right) \quad \text{--- (i) eqn}$$

similarly 2nd eqn

$$ms\frac{d\theta}{dt_2} = k(\theta-\theta_s)$$

or, $ms\frac{45-40}{300} = k\left[\frac{45+40}{2}-\theta_s\right]$

or, $ms\frac{5}{300} = k\left(\frac{85}{2}-\theta_s\right)$ (ii)

dividing (i) by (ii) we get

$$\frac{\cancel{ms}\,\frac{5}{180}}{\cancel{ms}\,\frac{5}{300}} = \frac{\cancel{k}\left(\frac{95}{2}-\theta_s\right)}{\cancel{k}\left(\frac{85}{2}-\theta_s\right)}$$

$$\frac{\cancel{5}\times 30\cancel{0}}{\cancel{5}\times 18\cancel{0}} = \frac{\left(\frac{95}{2}-\theta_s\right)}{\left(\frac{85}{2}-\theta_s\right)}$$

or, $30\left(\frac{85}{2}-\theta_s\right) = 18\left(\frac{95}{2}-\theta_s\right)$

or, $\frac{85\times \cancel{30}^{15}}{\cancel{2}} - 30\theta_s = \frac{\cancel{18}^{9}\times 95}{\cancel{2}} - 18\theta_s$

or, $1275 - 30\theta_s = 855 - 18\theta_s$

or, $1275 - 855 = 30\theta_s - 18\theta_s$

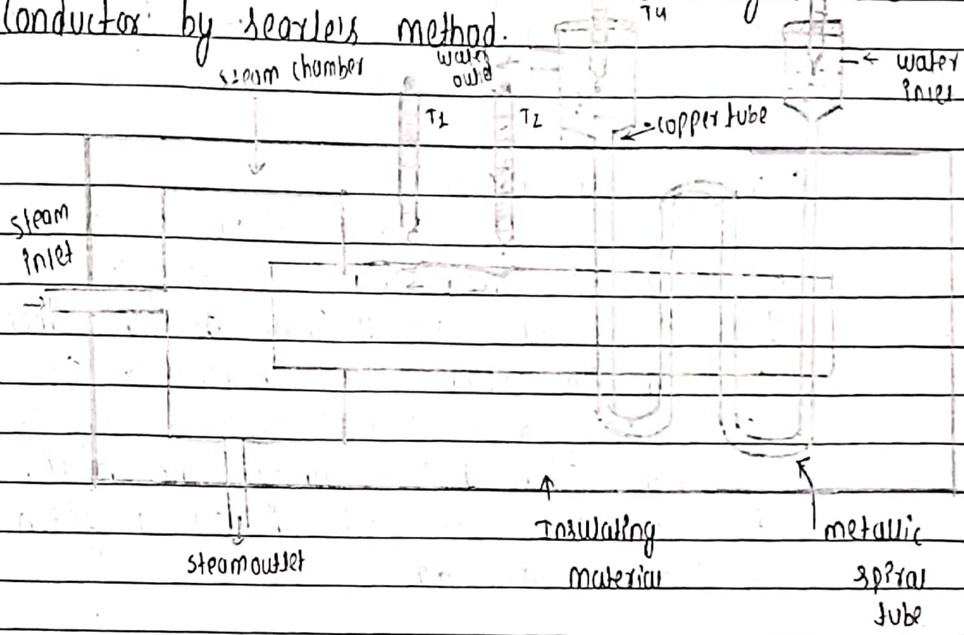
or, $\frac{420}{12} = \theta_s$

or, $\theta_s = 35^\circ C$ Ans

$40 \to 20$
$(40^\circ \to 20)$

* A body cools in 10 m from 60°C to 40°C. Calculate what will be it's temp. after the next 10 m. The tempr of the surrounding is 10°C. Assume that newton's law of cooling holds good.

Measurement of method Thermal Conductivity of a Conductor by Searle's method.



Thermal conductivity of a conductor can be determined in the laboratory by searle's method. In this method, conductor should be taken the form of solid rod.

Experimental arrangement :- It consist of uniform metallic rod whose thermal conductivity is to be measured one end of the rod is heated by passing steam while cold water is circulated through metallic spiral tube at the other end. Two thermometer, T_3 and T_4 are provided in the tube to measure the temperature of incoming water and outgoing water respectively. Also, two thermometer, T_1 & T_2 are inserted in a conductor at a given distance to measure temperature (change in temperature per length) Finally, whole apparatus is perfectly covered by insulating material.

Working and theory:-

When steam is passed through one end of the conductor & cold water is circulated through metallic spiral tube at the other end of rod. The reading of thermometer. $T_1, T_2, T_3 \& T_4$ goes on increasing and becomes steady after some time. Then their reading are noted. Also the mass of water collected in a given interval of time is measured.

Let, steady reading of thermometer $(T_1) = \theta_1°C$
steady reading of thermometer $(T_2) = \theta_2°C$
steady reading of thermometer $(T_3) = \theta_3°C$
mass of water collected per second $= \left(\frac{M}{t}\right) = m$

cross sectional area of conductor $= A$
sp. heat capacity of water $= S_w$.
Thermal conductivity of cold water $= K$

Now, Rate of flow of heat through the conductor is given by

$$\left(\frac{Q}{t}\right)_1 = \frac{KA(\theta_1 - \theta_2)}{l} \quad -(1)$$

And the rate at which heat is absorbed by the water is given by,

$$\left(\frac{Q}{t}\right)_2 = \frac{M}{t} S_w (\theta_4 - \theta_3)$$

$$\therefore \left(\frac{Q}{t}\right)_2 = m S_w (\theta_4 - \theta_3) - (2)$$

[where, $m = M/t$]

At steady state condition
Rate of flow of heat = Rate at which heat is absorbed.

$$\left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2$$

$$\text{Or, } \frac{KA(\theta_1 - \theta_2)}{l} = m S_w (\theta_4 - \theta_3)$$

$$\boxed{\therefore K = \frac{m S_w (\theta_4 - \theta_3) l}{A(\theta_1 - \theta_2)}}$$

By knowing the value of the quantities are right hand side thermal conductivity of a conductor can be determined.

* Convection:-

Convection is the transfer of heat by actual motion of molecules. The hot air furnance, the hot water heating system and flow of blood in the body are its some examples.

It is process in which heat is transform through a substance from one point to another with actual motion of heated particles. such process takes place in liquid & gas. when water is heated in a beakers, first of all water molecules at bottom becomes hot and rise up.

It is two types:-
i. Natural convection of free convection
ii. Forced convection

Natural convection of free convection :- If the heated molecules move due to different in density then such type of convection.

Force convection :- If the heated molecules are forced to move by blower or pump then such type of convection.

* Convection coefficient
:- Experimentaly, it is observed that rate of transfer of heat by convection is directly proportional to the temperature difference between two parts of convective fluids and to the surface area of the fluid exposed i.e.

$$\frac{Q}{t} \propto A\Delta\theta$$

$$\therefore \boxed{\frac{Q}{t} = hA\Delta\theta}$$

OR

$H = Q/t$ = Heat current

or, $H = hA\Delta\theta$

$\therefore h = \frac{H}{A\Delta\theta}$

where, h is a constant called convection coefficient where value depends on nature of material of fluids. This equation is applicable to natural convection only.

* Application of convection
i. ventil-ation
ii. chimney
iii. monsoon
vi. Trade winds

* chimney :- A chimney is provided above the fire in a room or kitchen

* monsoon :- It is based on the principle of convection.

* Trade winds :- winds are due to the convection current set up in the atmosphere by unequal heating earth surface gets heated more at the equator than at the poles, so that warm air at equator rises up and cold air from the poles moves

towards the equator.

* Radiation :- It is the process by which heat is transmitted from one place to another place without any medium. Large amount of heat energy is approaching us from the sun by this process Heat energy by this process travels in a straight line in the form of electromagenetic wave.

The term radiation refers to continuous emission of energy from the surface of all body above 0 k and the emitted energy is called radiant energy.

* Heat radiation or thermal radiation
:- It is the infrared radiation which have longer wave length than that of light.

* Black body
:- The body which absorbs most of the radiation falling on it is called black body.

* Perfectly black body :- perfectly black body is one which absorbs all the radiation falling on it. It neither reflects nor transmits the radiation falling on it.

Ferry's Black Body

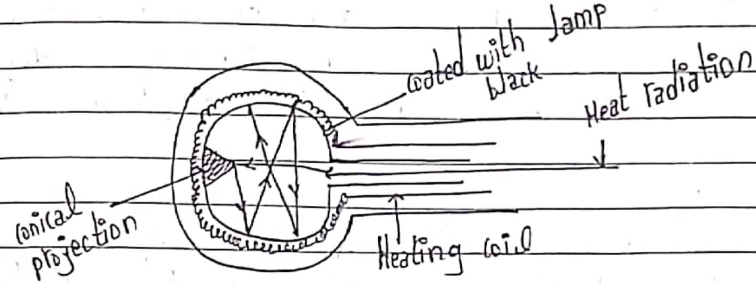


Fig:- Ferry's black body

The simplest and most commonly used blackbody was designed by Ferry known as Ferry's black body. It consists of hollow double wall metallic sphere having small opening on one side and a conical projection just opposite to it, the inner side of sphere is coated with lamp black.

A heating coil is placed between two wall to heat the body. All the radiation entering into it gets trapped due to multiple reflection. It absorbs about 99% of incident radiation.

When the black body is heated by passing elasticity to the heating coil, all possible radiation are emitted known as Black body radiation.

Date ____
Page ____

* Emissive power

:- Emissive power of a body at a particular temperature is defined as amount of heat energy radiated per second per unit area by the body at that temperature. (Q/At).

* Emissivity

:- It is defined as the ratio of emissive power of a body of a given temperature to a emissive power of a perfectly black body at the same temperature.

i.e. Emissivity (e) = $\dfrac{\text{emissive power of a body at given temperature}}{\text{emissive power of perfectly black body at same temperature}}$

It's values lies between 0 & 1. For a perfectly black body, it's value is 1.

* Stefan - Boltzmann's Law

:- It states that "total amount of heat energy radiated per second per unit area of a perfectly black body is directly proportional to the fourth power of it's absolute temperature. i.e. $E \propto T^4$

$\therefore E = \sigma T^4$ (where σ is proportionality constant called stefan constant whose value is $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$) ---- (i)

If the black body of temperature 'T' is surrounded by a encloses of temperature (T_0) (for absolute temperature, T_0).

Date ____
Page ____

$E = \sigma (T^4 - T_0^4)$ - (ii)

If the black body is not perfectly black body and having emissivity, e, then above eqn becomes

$$E = e\sigma (T^4 - T_0^4) \quad \text{-- (iii)}$$

If the black body has a surface area, A then amount of heat energy radiated per second by the body is given by.

$$E = Q/At$$

$$E = \frac{P}{A} \quad (\because P = Q/t)$$

or, $P = EA$

$\therefore P = e\sigma(T^4 - T_0^4)A$

* A spherical black body of radius 5cm has its temp. 127°C and it's emissivity is 0.6. Calculate its radiant power.

=) Soln:- Radius (r) = 5cm = 0.05m

temp (t) = 127°C = 127 + 273 = 400K

emissivity (e) = 0.6

Now, $A = 4\pi r^2$

$= 4 \times \frac{22}{7} \times 0.05 \times 0.05 = 0.031$

Stefan's constant $(\sigma) = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$

Radiant power (P) =

$P = e\sigma A T^4$

$= 0.6 \times 5.67 \times 10^{-8} \times 0.031 \times (400)^4$

$= 0.6 \times 0.031 \times 9.56 \times 10^{10} \times 5.67 \times 10^{-8}$

$= 0.6 \times 0.031 \times 5.67 \times 9.56 \times 10^{2}$

$= 0.0186 \times 5.67 \times 2.56 \times 100$

$= 26.99$ W Ans

* Estimate the power loss through unit area from a perfectly black body at 327°C to surrounding at 27°C.

⇒ Soln:- area $= 1\ m^2$

$T_0 = 27°C = 27 + 273 = 300$ K

$T = 327°C = 327 + 273 = 600$ K

$\sigma = 5.67 \times 10^{-8}\ Wm^{-1}K^{-4}$

power loss $(P) = \sigma A (T^4 - T_0^4)$

$= 5.67 \times 10^{-8} ((600)^4 - (300)^4)$

$= 5.67 \times 10^{-8} (1.29 \times 10^{11} - 8.1 \times 10^{9})$

$= 5.67 \times 10^{-8} \times 1.209 \times 10^{11}$

$= 6855.03$ W Ans

* Estimate the radiant power loss from a human body at temperature 38°C to the environment at 0°C. If the surface area of body is 1.5 m² and it's emissivity is 0.6.

⇒ Soln. $T = 38°C = 38 + 273 = 311$ K

$T_0 = 0°C = 0 + 273 = 273$ K

$A = 1.5\ m^2$

$e = 0.6$

Now: $P = e\sigma A (T^4 - T_0^4)$

$= 0.6 \times 5.67 \times 10^{-8} \times 1.5 ((311)^4 - (273)^4)$

$= 193.99$ W Ans

lesson 16

wave and optics

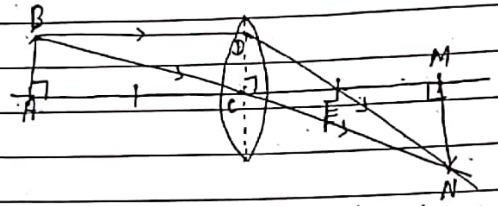
Lenses

i) Define lense.
ii) convex lens. define
iii) concave lense define

} self study

Lens Formula $\left(\frac{1}{f} = \frac{1}{u} + \frac{1}{v}\right)$

i) For convex lens



Suppose an object AB is placed vertically on the principal axis of a convex lens having optical centre C and principal focus, F, so that it's real and inverted image, MN is formed.

Now

In ΔABC and ΔMNC

$\angle BAC = \angle NMC$ (being 90°)

$\angle BCA = \angle MCN$ (being vertically opposite angle)

$\angle ABC = \angle MNC$ (Remaining angle)

$\therefore \Delta ABC \sim \Delta MNC$ (similar triangle)

$\frac{MN}{AB} = \frac{MC}{AC}$ — (i) eqn

similarly ΔDCF and ΔMNF are similar

$\frac{MN}{DC} = \frac{MF}{CF}$

or, $\frac{MN}{AB} = \frac{MF}{CF}$ — (ii) $\because [AB = CD]$

Equating equation (i) and (ii) we get,

$\frac{MC}{AC} = \frac{MF}{CF}$

or, $\frac{MC}{AC} = \frac{MC - CF}{CF}$ $\therefore (MF = MC - CF)$

MC = image distance $= +v$
AC = object distance $= +u$
CF = focal length $= +f$

$\therefore \frac{v}{u} = \frac{v-f}{f}$

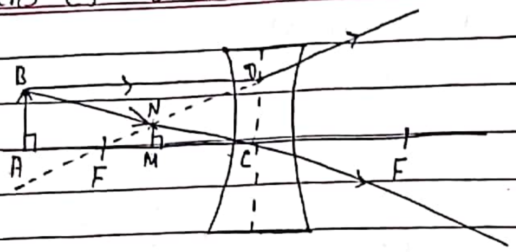
or, $vf = uv - uf$

dividing both sides by uvf

or, $\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$

or, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (hence proved)

ii) Concave lens $\left(\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \right)$



Suppose an object AB is placed vertically on the principal axis of a concave lense having optical centre C and principal focus F, so that virtual and erect image MN is formed.

From Δ ABC and ΔMNC

$\angle BAC = \angle NMC$ (being right angle)

$\angle ACB = \angle MCN$ (being common angle)

$\angle ABC = \angle MNC$ (remaining angle)

$\therefore \Delta ABC \sim \Delta MNC$

$\frac{MN}{AB} = \frac{MC}{AC}$

Similarly Δ MNF and Δ CDF are similar

$\frac{MN}{CD} = \frac{MF}{CF}$

Or $\frac{MN}{AB} = \frac{MF}{CF}$ (CD = AB) (ii) eqn

From (i) and (ii)

$\frac{MC}{AC} = \frac{MF}{CF}$

Or, $\frac{MC}{AC} = \frac{CF - MC}{CF}$

where, MC = Image distance = $-v$

AC = object distance = $+u$

CF = focal length = $-f$

Now $\frac{MC}{AC} = \frac{CF - MC}{CF}$

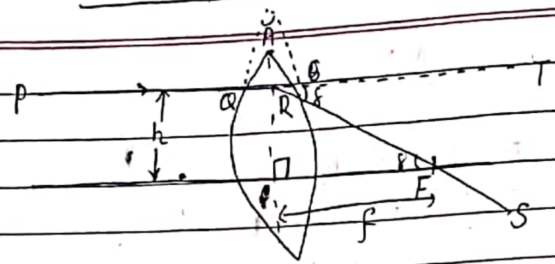
$\frac{-v}{u} = \frac{f - v}{f}$

or, $-vf = uf - uv$ (dividing both side by uvf)

or, $\frac{-1}{u} = \frac{1}{v} - \frac{1}{f}$

or, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ (hence proved)

len's maker's Formula



consider a ray of light PQ is parallel to the principal axis of convex lens having optical centre C and principal focus F is incident on the lens at a height h from optical centre and refracted through F along RS

If δ be angle of deviation produced by lens

$$\delta \simeq \tan\delta = \frac{h}{f} \quad \text{(i) eq}^n$$

As the lens is regarded as the combination of small angle prisms, the deviation produced by lens should be in the form of deviation produced by a small angle prisms

$\therefore \delta = (\mu-1)A$ -- (ii) where A is angle of prism and μ is refractive index of material

equating (i) and (ii) eq^n

$$\frac{h}{f} = (\mu-1)A \quad \text{-- } \cancel{\text{(iii) eq}^n}$$

or, $\frac{1}{f} = \frac{(\mu-1)A}{h}$ (iii) eq^n

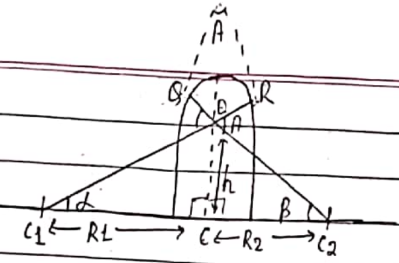


Fig:- lens as the small angle prisms

let C_1 and C_2 are centre curvature of two spherical surfaces of the lenses. Now, join C_1 and R and join C_2 and Q, so that C_1R and C_2Q being radius. Let O be the point of intersection of these two radii.

let, $\angle RC_1C_2 = \alpha$ and $\angle QC_2C_1 = \beta$

For small angle

$$\left.\begin{aligned}\alpha &\simeq \tan\alpha = \frac{h}{R_1}\\ \beta &\simeq \tan\beta = \frac{h}{R_2}\end{aligned}\right\} \text{(iv) eq}^n$$

It is found that the angle between two radii is equal to the angle between their tangent.

$\therefore \angle ROC_2 = \angle QOC_1 = A$

Now

In $\triangle OC_1C_2$

$\alpha + \beta = A$ -- (v) eq^n (being exterior angle)

From eq^n (iv) and (v)

or, $\frac{h}{R_1} + \frac{h}{R_2} = A$

or, $h\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = A$

or, $\frac{A}{h} = \frac{1}{R_1} + \frac{1}{R_2}$ (vi) eq^n

By substituting this value of A/h in eqn (iii) we get

$$\therefore \frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \therefore \text{Proved}$$

unit 13

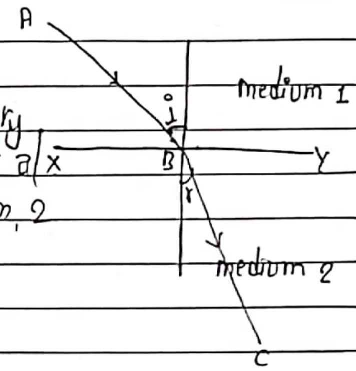
Refraction at ~~curved Mirror~~ plane surface

Define refractive Index
Define Absolute refractive Index
laws of refraction
causes of refraction
} self study

Refractive Index (μ) is depend upon following medium

i) Nature of medium
ii) wavelength or colour of light

prove that a) $_1\mu_2 = \frac{1}{_2\mu_1}$



consider a ray of light AB travelling in medium 1 is incident on a boundary between medium 1 and medium 2 at a angle i and is refracted into medium 2 along BC with the angle of r

From snell's law

$$_1\mu_2 = \frac{\sin i}{\sin r} \text{ --- (i) eq}^n$$

From the reversibility of light, if the light is allowed to be incident along CB, it will be refracted along BA

From snell's law

$$_2\mu_1 = \frac{\sin r}{\sin i} \text{ -- (ii)}$$

multiplying both (i) and (ii)

$$_1\mu_2 \times {}_2\mu_1 = \frac{\sin i}{\cancel{\sin r}} \times \frac{\cancel{\sin r}}{\sin i}$$

or, $_1\mu_2 \times {}_2\mu_1 = 1$.

or, $_1\mu_2 = \dfrac{1}{_2\mu_1}$ proved

This relation shows the reversibility of light. Therefore, the principal of reversibility suffering from a number of reflection or refraction has it's final path reversed, it will travel along the same paths but in a opposite direction.

prove that: $_a\mu_b \times {}_b\mu_c \times {}_c\mu_a = 1$

or, $_a\mu_b \times {}_b\mu_c = {}_a\mu_c$

or, $_b\mu_c = \dfrac{_a\mu_c}{_a\mu_b}$

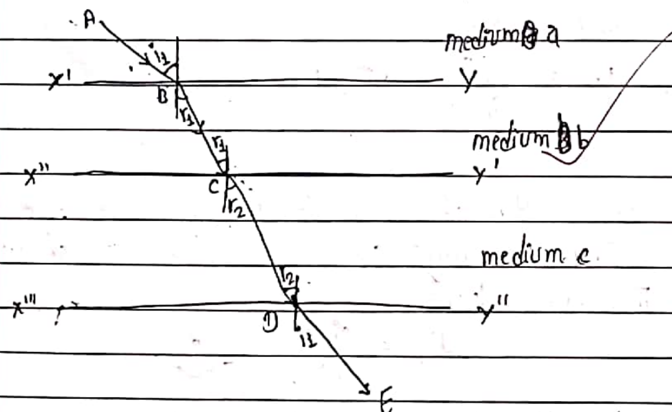


Fig:- refraction of light in different medium.

Consider a ray of light AB travelling in medium A is refracted into medium B along BC is again refracted into medium C along CD and finally emerges into medium A along DE provided their boundaries being parallel.

Let, i_1, r_1 and r_2 are being incidence angle in medium A, B and C respetively.

From Snell's law $_a\mu_b = \dfrac{\sin i_1}{\sin r_1}$ ... (i) eqn

$_b\mu_c = \dfrac{\sin r_1}{\sin r_2}$ ... (ii) eqn

$_c\mu_a = \dfrac{\sin r_2}{\sin i_1}$ ... (iii) eqn

multiplying (i) (ii) and (iii) eqn

$$_a\mu_b \times {}_b\mu_c \times {}_c\mu_a = \frac{\cancel{\sin i_1}}{\cancel{\sin r_1}} \times \frac{\cancel{\sin r_1}}{\cancel{\sin r_2}} \times \frac{\cancel{\sin r_2}}{\cancel{\sin i_1}}$$

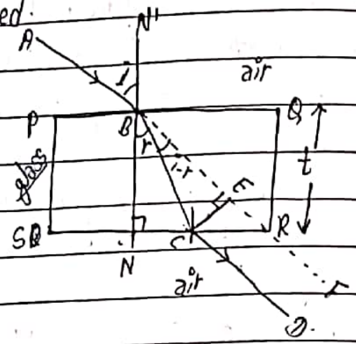
$\therefore$ $_a\mu_b \times {}_b\mu_c \times {}_c\mu_a = 1$

or, $_a\mu_b \times {}_b\mu_c = \dfrac{1}{_c\mu_a}$

or, $_b\mu_c = \dfrac{_a\mu_c}{_a\mu_b}$ proved

Lateral shift

:- It is defined as the perpendicular distance between emergent ray and incident ray produced.



Consider a rectangular glass slab, PQRS of thickness, t. A ray of light, AB travelling in air is refracted into glass along BC and again emerges into air along CD. The emergent ray CD is parallel to the incident ray produce AF the perpendicular distance CE between them, is lateral shift.

let i and r be the angle of incidence in air, and angle of refraction in glass respectively

$\therefore \angle ABN' = i$ and $\angle NBC = r$

so, that $\angle CBE = (i-r)$

Now In ΔCBE

$$\sin(i-r) = \frac{CE}{BC}$$

or, $CE = BC \times \sin(i-r)$ -- (i) eqn

Again In ΔNBC

$$\cos r = \frac{BN}{BC}$$

or, $BC = \frac{BN}{\cos r}$

$\therefore BC = t/\cos r$ (ii) eqn

From (i) and (ii) eqn

$$CE = \frac{t}{\cos r} \times \sin(i-r)$$

$$\therefore l = \frac{t}{\cos r}(\sin(i-r))$$ proved

Special cases

i) when $i = 90^\circ$

$l = \frac{t}{\cos r}\sin(90-r) = \frac{t}{\cancel{\cos r}}\cancel{\cos r}$ t

i.e $l = t$

ii) when $i = 0^\circ$ or, $r = 0^\circ$

$\therefore l = 0$

* ray of light is incident on the upper surface of glass slab of thickness 6cm at an angle of 45°. The refractive index of glass is 1.52. Calculate the lateral shift produced.

⇒ Soln: $t = 6cm$

$\mu = 1.52$

$i = 45$

$r = 45 - r$

$$\mu = \frac{\sin i}{\sin r}$$

or, $1.52 = \frac{0.70}{\sin r}$

or, $\sin r \cdot 1.52 = 0.70$

or, $\sin r = 0.70/1.52 = 0.46$

$r = \sin^{-1} 0.46$

$r = 27.38$

Now
$$L = \frac{t}{\cos(27.3)} \sin(45-27.3)$$
$$= \frac{6 \times 0.30}{\cos(27.3)}$$
$$= 2.05 \text{ Ans}$$

* Calculate the critical angles of
i) glass-water and (ii) water air interface. If object lies in the denser medium.

$_a\mu_g = 1.5$, $_a\mu_w = 1.33$

⇒ Soln:- here:-
Refractive index of glass = 1.5
Refractive index of water = 1.33

Now critical angles of glass water
$$_g\mu_w = \frac{1}{\sin c}$$
or, $$\frac{_a\mu_w}{_a\mu_g} = \frac{1}{\sin c}$$
or, $$\frac{1.33}{1.5} = \frac{1}{\sin c}$$
or $$\frac{0.88}{1} = \frac{1}{\sin c}$$
or, $\sin c =$

$$\sin c = \frac{1.5}{1.33}$$ or, $$\sin c = \frac{1}{0.88}$$
or, $c = \sin^{-1}(1.)$

* A ray of light is incident at 60° in air on air-glass plane surface. Find the angle of refraction in the glass.

⇒ Soln:- Angle of incidence = 60° = i
$\mu = 1.5$
Angle of refraction (r) ?

by snell law $$\mu = \frac{\sin 60}{\sin r}$$
$$1.5 \times \sin r = \frac{\sqrt{3}}{2}$$
$$\sin r = \frac{\sqrt{3}}{2 \times 1.5}$$
$$\sin r = 0.57$$
$$r = \sin^{-1}(0.57)$$
$$r = 34.78 \text{ Ans}$$

Date
Page

Define
→ Concave mirror } self study
→ Convex mirror

prove that: $R = 2f$ or, $f = R/2$

a) For concave mirror
Considered a ray of light AB parallel to the principal axis, is incident on a concave mirror having pole, P and centre of curvature, C and is reflected through the principal focus, F along BD.

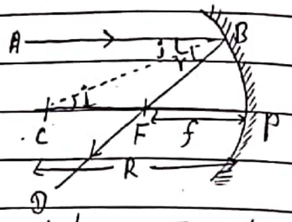


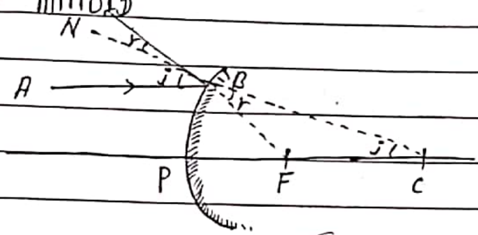
Fig:- Relation between F and R for concave mirror

Now, join ~~CDB~~ C and B, such that CB is normal to B.
$\angle ABC = i$ and $\angle CBD = r$
Also, $\angle ABC = \angle BCF = i$ (being alternate angle)
$\angle BCF = \angle CBF$ ($\angle i = \angle r$)
$\therefore$ $\triangle BCF$ is an isoseles triangle
$CF = FB$ — (i)
If the aperture of the mirror is small the point B lies close to the point P.
$FB \simeq FP$ — (ii)
Now $CP = CF + FP$
or, $CP = FB + FP$ [from (i) eqn]

Date
Page

or, $CP = FP + FP$
or, $CP = 2FP$
[here CP = Radius of curvature ($+R$)
FP = focal length $= +f$
or, $R = 2f$ or, $f = R/2$ proved

b) For convex mirror



Consider a ray of light AB parallel to the principal axis is incident on a convex mirror having pole, P and centre of curvature, C and is reflected along B which appears to pass through the principal focus F.

Now join C and B and produce to N, so that NB is normal to B.
$\angle ABN = i$, $\angle DBN = r$
Also, $\angle DBN = \angle CBF = r$ (vertically opposite angle)
and $\angle ABN = \angle BCF = i$ (corresponding angle)
$\therefore \angle BCF = \angle CBF$ ($\because \angle i = \angle r$)
So, $\triangle BCF$ is an isosceles triangle
$\therefore BF = FC$ — (i)

If the aperture of the mirror is small B will close to point P.

$\therefore PF \approx BF$ — (ii)

Now $CP = PF + FC$

or, $CP = PF + BC$ (from (i) eqn)

or, $CP = PF + PF$ (from (ii) eqn)

$\therefore CP = 2PF$

where CP = Centre of curvature = $-R$

PF = focal length = $-f$

or, $\cancel{-}R = \cancel{-}2f$

or, $R = 2f$

or, $f = \frac{R}{2}$ } proved