MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Advanced Mathematics – II

Time: 3 hrs.   Max. Marks: 100

Note: *Answer any FIVE full questions.*

1 a. Prove that the angle between two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) is $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$ **(07 Marks)**

    b. Find the value of K if the angle between the lines with direction ratios $-2, 1, -1$ and $1, -K, -1$ is $\dfrac{2\pi}{3}$. **(07 Marks)**

    c. Find the projection of the line segment AB on CD where $A = (3, 4, 5)$, $B = (4, 6, 3)$, $C = (-1, 2, 4)$, $D = (1, 0, 5)$ **(06 Marks)**

2 a. Derive the equation of the plane in the intercept form $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. **(07 Marks)**

    b. Find the image of the point $(2, -1, 3)$ in the plane $2x + 4y + z - 24 = 0$. **(07 Marks)**

    c. Find the equation of the plane containing the line $\dfrac{x+1}{2} = \dfrac{y+2}{3} = \dfrac{z+3}{4}$ and is perpendicular to the line $x - 2y + 3z = 4$. **(06 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $2i - j + k$, $i - 3j - 5k$ and $3i - 4j - 4k$ form a right angled triangle. **(07 Marks)**

    b. Find the cosine and sine of the angle between the vectors $2i - j + 3k$ and $i - 2j + 2k$. **(07 Marks)**

    c. Find the value of λ such that the vectors $\vec{a} = \lambda i - 5j - 2k$, $\vec{b} = -7i + 14j - 3k$ and $\vec{c} = 11i + 4j + k$ are coplanar. **(06 Marks)**

4 a. A particle moves along a curve $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$. Determine its velocity and acceleration and also the magnitude of velocity and acceleration at $t = 2$. **(07 Marks)**

    b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. **(07 Marks)**

    c. Find the directional derivative of the function $\phi = xyz$ along the direction of the normal to the surface $xy^2 + yz^2 + zx^2 = 3$ at the point $(1, 1, 1)$ **(06 Marks)**

5 a. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$. **(07 Marks)**

    b. Show that $\operatorname{curl}(\operatorname{grad}\phi) = 0$. **(06 Marks)**

    c. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

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6 a. Find the Laplace transform of t^n, where n is a positive integer. **(05 Marks)**

   b. Find $L(\sin 5t \cos 2t)$. **(05 Marks)**

   c. Find $L(t \cos at)$. **(05 Marks)**

   d. Find $L\left(\dfrac{\cos at - \cos bt}{t}\right)$. **(05 Marks)**

7 a. Find $L^{-1}\left[\dfrac{s+5}{s^2 - 6s + 13}\right]$. **(07 Marks)**

   b. Find $L^{-1}\left[\dfrac{1}{s(s+1)(s+2)(s+3)}\right]$. **(07 Marks)**

   c. Find $L^{-1}\left[\log\left(\dfrac{s+a}{s+b}\right)\right]$. **(06 Marks)**

8 a. Using Laplace transform solve $\dfrac{d^2 y}{dt^2} + 4\dfrac{dy}{dt} + 4y = e^{-t}$, $y(0) = 0 = y'(0)$ **(10 Marks)**

   b. Using Laplace transform solve $\dfrac{dx}{dt} + y = \sin t$, $\dfrac{dy}{dt} + x = \cos t$ given $x(0) = 1, y(0) = 0$ **(10 Marks)**