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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Advanced Mathematics – II

Time: 3 hrs.

Note: Answer any FIVE full questions.

- 1 a. Find the direction cosines of the line which is perpendicular to the lines with direction cosines $(3, -1, 1)$ and $(-3, 2, 4)$. (06 Marks)
- b. If $\cos \alpha, \cos \beta, \cos \gamma$ are the direction cosines of a line, then prove the following:
 - i) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$
 - ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ (07 Marks)
- c. Find the projection of the line AB on the line CD where $A = (1, 2, 3)$, $B = (1, 1, 1)$, $C = (0, 0, 1)$, $D = (2, 3, 0)$. (07 Marks)
- 2 a. Find the equation of the plane through $(1, -2, 2)$, $(-3, 1, -2)$ and perpendicular to the plane $2x - y - z + 6 = 0$. (06 Marks)
- b. Find the image of the point $(1, -2, 3)$ in the plane $2x + y - z = 5$. (07 Marks)
- c. Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. (07 Marks)
- 3 a. Find the constant 'a' so that the vectors $2i - j + k$, $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar. (06 Marks)
- b. Prove that $\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a} \right] = 2 \left[\vec{a}, \vec{b}, \vec{c} \right]$. (07 Marks)
- c. Find the unit normal vector to both the vectors $4i - j + 3k$ and $-2i + j - 2k$. Find also the sine of the angle between them. (07 Marks)
- 4 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction of $2i + 3j + 6k$. (06 Marks)
- b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at the point $(2, -1, 2)$. (07 Marks)
- c. Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(1, -2, -1)$ in the direction of the normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$. (07 Marks)
- 5 a. Prove that $\text{div}(\text{curl } \vec{A}) = 0$. (06 Marks)
- b. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)
- c. Show that the vector $\vec{F} = (3x^2 - 2yz)i + (3y^2 - 2zx)j + (3z^2 - 2xy)k$ is irrotational and find ϕ such that $\vec{F} = \text{grad } \phi$. (07 Marks)

- 6 a. Find: $L\{\cos t \cos 2t \cos 3t\}$. (06 Marks)
- b. Find: i) $L\{e^{-t} \cos^2 t\}$, ii) $L\{te^{-t} \sin 3t\}$. (07 Marks)
- c. Find: $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. (07 Marks)
- 7 a. Find: $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$. (06 Marks)
- b. Find: i) $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$, ii) $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$. (07 Marks)
- c. Find: $L^{-1}\left\{\frac{1}{s^2(s+1)}\right\}$. (07 Marks)
- 8 a. Using Laplace transforms, solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2t}$ with $y(0) = 0$, $y'(0) = 1$. (10 Marks)
- b. Using Laplace transformation method solve the differential equation $y'' + 2y' - 3y = \sin t$, $y(0) = y'(0) = 0$. (10 Marks)

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