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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2017/Jan.2018

Advanced Mathematics – II

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions selecting atleast TWO questions from each part.

PART – A

- 1 a. Find the direction cosines l, m, n of the line :
 $x + y + z + 1 = 0$
 $4x + y - 2z + 2 = 0.$ (06 Marks)
- b. Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. (07 Marks)
- c. Find the angle between the line $\frac{x+4}{4} + \frac{y-3}{-3} = \frac{z+2}{1}$ and the plane $2x + 2y - z + 15 = 0.$ (07 Marks)
- 2 a. Find the equation of the plane which passes through the points A(0, 1, 1), B(1, 1, 2), C(-1, 2, -2). (06 Marks)
- b. Find the equation of the plane which passes through the point(3, -3, 1) and normal to the line joining the points (3, 2, -1) and (2, -1, 5). (07 Marks)
- c. Find the equations to the two planes which bisects the angle between the planes :
 $3x - 4y + 5z = 3$
 $5x + 3y - 4z = 9.$ (07 Marks)
- 3 a. Find the sides and the angle A of the triangle whose vertices are $\overrightarrow{OA} = I - 2J + 2K$, $\overrightarrow{OB} = 2I + J - K$, $\overrightarrow{OC} = 3I - J + 2K$. (06 Marks)
- b. Show that the points $-6I + 3J + 2K$, $3I - 2J + 4K$, $5I + 7J + 3K$ and $-13I + 17J - k$ are coplanar. (07 Marks)
- c. Prove that : $[\overrightarrow{B} \times \overrightarrow{C}, \overrightarrow{C} \times \overrightarrow{A}, \overrightarrow{A} \times \overrightarrow{B}] = [\overrightarrow{A} \overrightarrow{B} \overrightarrow{C}]^2.$ (07 Marks)
- 4 a. A particle moves along the curve $x = t^2 + 1$, $y = t^2$, $z = 2t + 3 + \sin(\pi t)$ where t is the time. Find the velocity and acceleration at $t = 1$. (06 Marks)
- b. If $\overrightarrow{A} = (\cos t)I + (\sin t)J + (4t)K$ and $\overrightarrow{B} = (t^3 + 1)I + J + (8t^2 - 3t^3)K$ then find :
 i) $\frac{d}{dt}(\overrightarrow{A} + \overrightarrow{B})$ ii) $\frac{d}{dt}(\overrightarrow{A} \cdot \overrightarrow{B}).$ (07 Marks)
- c. If $\phi = 3x^2y - y^3z^2$, find grad ϕ at (1, -2, 1). Also find a unit normal vector to the surface $3x^2y - y^3z^2 = 6$ at (1, -2, 1). (07 Marks)

PART – B

- 5 a. If $\overrightarrow{A} = xyz I + 3x^2y J + (xz^2 - y^2z)K$ then find curl \overrightarrow{A} at (1, 2, 3). (06 Marks)
- b. Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point A(1, -1, -1) in the direction of $2\hat{i} + 3\hat{j} + 2\hat{k}$. (07 Marks)
- c. If $u = x^2 + y^2 + z^2$ and $\vec{r} = xI + yJ + zK$ then find $\text{div} (u\vec{r})$ in terms of u . if $\vec{f} = \nabla(x^3 + y^3 + 2^3 - 3xyz)$ find $\nabla \cdot \vec{f}$ and $\nabla \times \vec{f}$. (07 Marks)

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- 6 a. Find the Laplace transform of $f(t)$ defined as :
- $$f(t) = \begin{cases} \frac{t}{6}, & \text{when } 0 < t < 6 \\ 1, & \text{when } t < 6 \end{cases}.$$
- (05 Marks)
- b. Find : i) $L(\cos t^2)$ ii) $L(t \sin h at)$ iii) $L\left(\frac{1}{t} \sin 2t\right)$.
- (15 Marks)
- 7 a. Find : $L(e^{2t} \cos 3t)$.
- (06 Marks)
- b. Find : $L^{-1}\left(\frac{2h-5}{9s^2-25}\right)$.
- (07 Marks)
- c. Find : $L^{-1}\left(\frac{s^2+4}{x^2+9}\right)$.
- (07 Marks)
- 8 a. Using Laplace transforms, find the solution of the initial value problem $y''-4y'+4y=64 \sin 2t$, $y(0) = 0$, $y'(0) = 1$.
- (10 Marks)
- b. Using Laplace transforms, solve $y'' + 9y = \cos 2t$, $y(0) = 1$, $y'(0) = \frac{12}{5}$.
- (10 Marks)

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