MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2019
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the ratio in which the point P(5, 4, –6) divides the line joining the points Q(3, 2, –4) and R(9, 8, –10). **(06 Marks)**

b. Find the angles between any two diagonals of a cube. **(07 Marks)**

c. Find the projection of AB on the line CD, where A = (1, 2, 3), B = (1, 1, 1), C = (0, 0, 1) and D = (2, 3, 0). **(07 Marks)**

2 a. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, –1, –1) are coplanar. Find the equation of the plane containing them. **(06 Marks)**

b. Find the equation of the plane through the intersection of the planes $2x + 3y - z = 5$ and $x - 2y - 3z + 8 = 0$ and perpendicular to the plane $x + y - z = 2$. **(07 Marks)**

c. Find the shortest distance between the lines :

$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ from a right – angled triangle. **(06 Marks)**

(07 Marks)

b. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ then find the angle between $(2\vec{a} + \vec{b})$ and $(\vec{a} + 2\vec{b})$.

c. Find the sine of the angle between $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

4 a. Find the constant 'a' so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar. **(06 Marks)**

b. Find the angle between the tangents to the curve $x = t^2$, $y = t^3$, $z = t^4$ at $t = 2$ and $t = 3$. **(07 Marks)**

c. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(07 Marks)**

5 a. Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(06 Marks)**

b. Find the constants a, b, c such that the vector :

$\vec{F} = (x + y + az)\hat{i} + (x + cy + 2z)\hat{k} + (bx + 2y - z)\hat{j}$ is irrotational. **(07 Marks)**

c. Prove that $\nabla^2(\log r) = \frac{1}{r^2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. **(07 Marks)**

6 a. Find the laplace transform of $5\sin 2t + 3\cos 4t$. **(05 Marks)**

b. Find Lapace transform of $e^{-3t}\cos 4t$. **(05 Marks)**

c. Find $\alpha\left\{\frac{1-\cos t}{t}\right\}$. **(05 Marks)**

d. Find $\alpha\{f(t)\}$ where $f(t) = \begin{cases} 3t, & 0 < t < 2 \\ 6, & 2 < t < 4 \end{cases}$

Given that f(t) is the paiodic function with the period 4. **(05 Marks)**

7 a. Find the inverse Laplace transform of $\frac{5S+1}{S^2+16}$. **(06 Marks)**

b. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)(s+3)}$. **(07 Marks)**

c. Find the inverse Laplace transform of log(1-a/s). **(07 Marks)**

8 a. Solve $y'' - 3y' + 2y = 12e^{-t}, y(0) = 2, y'(0) = 6$ using Laplace transform method. **(10 Marks)**

b. Solve the simultaneous equation using Laplace transforms $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$, given that x = 1, y = 0 when t = 0. **(10 Marks)**