Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If $[l_1, m_1, n_1]$ and $[l_2, m_2, n_2]$ be the direction cosines of two lines subtending an angle θ between them then prove that $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. **(06 Marks)**

b. Find the angle between two lines whose direction cosines satisfy the relations $l + m + n = 0$ and $2lm + 2nl - mn = 0$ **(07 Marks)**

c. Find the co-ordinates of the foot of the perpendicular from $A(1,1,1)$ to the line joining $B(1,4,6)$ and $C(5,4,4)$. **(07 Marks)**

2 a. Find the equation of the plane which bisects the line joining (3, 0, 5) and (1, 2, −1) at right angles. **(06 Marks)**

b. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, −1, −1) are coplanar. Find the equation of the plane containing them. **(07 Marks)**

c. Find the shortest distance and the equations of the line of shortest distance between the lines: $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$. **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $\vec{a} = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{b} = 5\hat{i} + 6\hat{j} + 4\hat{k}$ and $\vec{c} = 6\hat{i} + 4\hat{j} + 5\hat{k}$ form an isosceles triangle. **(06 Marks)**

b. Prove that the points with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $\hat{j} + \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar. **(07 Marks)**

c. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$ where t is the time t. Find the components of velocity and acceleration in the direction of the vector $\hat{i} - 3\hat{j} + 2\hat{k}$ at $t = 1$. **(07 Marks)**

4 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 - z^2 = 3$ at $(2, -1, 2)$. **(06 Marks)**

b. Find the directional derivatives of the function $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $2\hat{i} - \hat{j} - 2\hat{k}$ **(07 Marks)**

c. Find div $\vec{F}$ and curl $\vec{F}$ at the point $(1, -1, 1)$ where $\vec{F} = \nabla(xy^3z^2)$. **(07 Marks)**

5 a. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ then prove that,

(i) $\nabla(r^n) = nr^{n-2}\vec{r}$ (ii) $\nabla.\left(r^n.\vec{r}\right) = (n+3)r^n$ **(06 Marks)**

b. Show that $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Find the value of the constant 'a' such that $\vec{A} = y(ax^2 + z)\hat{i} + x(y^2 - z^2)\hat{j} + 2xy(z - xy)\hat{k}$ is Solenoidal. For this value of 'a' show that curl $\vec{A}$ is also solenoidal. **(07 Marks)**

6 a. Find the Laplace transform of, (i) $\sin 5t \cos 2t$ (ii) $(3t+2)^2$ **(06 Marks)**

b. Find the Laplace transform of $\dfrac{\cos at - \cos bt}{t}$. **(07 Marks)**

c. Find the Laplace transform of $t^2 \sin at$. **(07 Marks)**

7 a. Find the inverse Laplace transform of $\dfrac{s+5}{s^2 - 6s + 13}$. **(06 Marks)**

b. Find $L^{-1}\left\{\log\left(\dfrac{s+a}{s+b}\right)\right\}$. **(07 Marks)**

c. Find $L^{-1}\left\{\dfrac{s}{\left(s^2+a^2\right)^2}\right\}$. **(07 Marks)**

8 a. Using convolution theorem find the Laplace transform of $\dfrac{1}{s\left(s^2+a^2\right)}$. **(10 Marks)**

b. Solve the differential equation, $y'' + 5y' + 6y = 5e^{2x}$ under the condition $y(0) = 2$, $y'(0) = 1$ using Laplace transform. **(10 Marks)**