CHAPTER-8
Circle

Section-A [JEE Advanced/IIT-JEE]

A : Fill in the Blanks

1. If A and B are points in the plane such that $\frac{PA}{PB} = K$(constant) for all P on a given circle, then the value of k cannot be equal to.......(1982)
2. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are..... and.....(1983)
3. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. The radius of this circle is......(1984)
4. Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point (4, 5) with a pair of radii form a quadrilaterall of area ....(1985)
5. From the origin chords are drawm to the circle$(x - 1)^2 + y^2 = 1$. The equation of the locus of the mid-points of these chords is ....(1985)
6. The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$ is ....(1986)
7. From the point A(0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that AM=2AB. The equation of the locus of M is.......(1986)
8. The area of the triangle formed by the tangents from the point (4, 3) to the the circle $x^2 + y^2 = 9$ and the line joining their points of contact is.......(1987)
9. If the circle C:$x^2 + y^2 = 16$ intersects anothere circle C_2 of radius 5 in such manner that common chord is of maximum length and has a slope equal to $\frac{3}{4}$,then the coordinates of the center of C_2 are........(1988)
10. The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is....... (1989)

11. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 1 = 0$, then the value of λ=........(1991)

12. The equation of the locus of the mid-points of the circle $4x^2 + 4y^2 - 2x + 4y + 1 = 0$ that subtend an angle of $\frac{2\pi}{3}$ at its centre is......(1993)

13. The intercept on the line $y = xby$ the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as a diameter is......(1996)

14. For each natural number k, let C_k, denote the circle with radius k centimetres and centre at the origin. On the circle C_k, α-particle moves k centimetres in the counter-clockwise direction. After completing its motion on C_k the particle moves to $C_k + 1$ in the radial direction. The motion of the particle continues in this manner. The particle starts at (1,0). If the particle crosses the positive direction of the x-axis for the first time on the circle C_n. then n=.......(1997)

15. The chords of contact of the pair of tangents drawn from each point on the line $2x+y = 4$ to circle$x^2+y^2 = 1$ pass through the point.....(1997)

B : True/False

1. No tangent can be drawn from the point $(\frac{5}{2}, 1)$ to the circumcircle of the triangle with vertices $(1, \sqrt{3})$,$(1, -\sqrt{3})$, $(3, -\sqrt{3})$.(1985)

2. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$. (1989)

C : MCQ'S with One Correct Answer

1. A square is inscribed in the circle $x^2+y^2-2x+4y+3 = 0$. Its sides are parallel to the coordinate axes. The one vertex of the square is (1980)

   (a) $(1 + \sqrt{2}, -2)$
   (b) $(1 - \sqrt{2}, -2)$
   (c) $(1, -2 + \sqrt{2})$

(d) none of these

2. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1,1) is 1980)

   (a) $x^2 + y^2 - 6x + 4 = 0$

   (b) $x^2 + y^2 - 3x + 1 = 0$

   (c) $x^2 + y^2 - 4y + 2 = 0$

   (d) none of these

3. The centre ofthe circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is (1983)

   (a) $\left[\frac{-16}{5}, \frac{27}{10}\right]$

   (b) $\left[\frac{-16}{7}, \frac{53}{10}\right]$

   (c) $\left[\frac{-16}{5}, \frac{53}{10}\right]$

   (d) none of these

4. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is (1983)

   (a) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$

   (b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

   (c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$

   (d) none of these

5. The locus of the mid-point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is (1984)

   (a) $x + y = 2$

   (b) $x^2 + y^2 = 1$

   (c) $x^2 + y^2 = 2$

   (d) $x + y = 1$

6. Ifa circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k$ orthogonally, then the equation of the locus of ()

   (a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
   (b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
   (c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
   (d) $x^2 + y^2 - 2ax - 3by + (a^2 + b^2 - k^2) = 0$

7. Ifthe two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then (1989)

   (a) $2 < r < 8$
   (b) $r < 2$
   (c) $r = 2$
   (d) $r > 2$

8. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of acircleof area 154 sq. units. Then the equation of this circle is (1989)

   (a) $x^2 + y^2 + 2x - 2y = 62$
   (b) $x^2 + y^2 + 2x - 2y = 47$
   (c) $x^2 + y^2 - 2x + 2y = 47$
   (d) $x^2 + y^2 - 2x + 2y = 62$

9. The centre of a circle passing through the points (0, 0),(1,0) and touching the circle $x2 + y2 = 9$ is (1992)

   (a) $\left[\frac{3}{2}, \frac{1}{2}\right]$
   (b) $\left[\frac{1}{2}, \frac{3}{2}\right]$
   (c) $\left[\frac{1}{2}, \frac{1}{2}\right]$
   (d) $\left[\frac{3}{2}, -2^{\frac{1}{2}}\right]$

10. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation: (1993)

(a) $x^2 - 6x - 10y + 14 = 0$

(b) $x^2 - 10x - 6y + 14 = 0$

(c) $y^2 - 6x - 10y + 14 = 0$

(d) $y^2 - 10x - 6y + 14 = 0$

11. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if (1994)

    (a) $r < 2$

    (b) $r > 8$

    (c) $2 < r < 8$

    (d) $2 \leq r \leq 8$

12. The angle between a pair of tangents drawn from a point p to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is (1996)

    (a) $x^2 + y^2 + 4x - 6y + 4 = 0$

    (b) $x^2 + y^2 + 4x - 6y - 9 = 0$

    (c) $x^2 + y^2 + 4x - 6y + 9 = 0$

    (d) $x^2 + y^2 + 4x - 6y - 4 = 0$

13. If two distinct chords, drawn from the point (p,q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the X-axis, then (1999)

    (a) $p^2 = q^2$

    (b) $p^2 = 8q^2$

    (c) $p^2 < 8q^2$

    (d) $p^2 > 8q^2$

14. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (4,3) respectively, then $\angle PQR$ is equal to (2000)

    (a) $\frac{\pi}{2}$

    (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

15. If the circles $x^2+y^2+2x+2khy+6=0, x^2+y^2+2ky+k=0$ intersect orthogonally, then k is

    (a) 2 or -$\frac{3}{2}$

    (b) -2 or -$\frac{3}{2}$

    (c) 2 or $\frac{3}{2}$

    (d) -2 or $\frac{3}{2}$

16. Let AB be a chord of the circle $x^2+y^2=r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is (2001)

    (a) a parabola

    (b) a circle

    (c) a ellipse

    (d) a pair of straight lines

17. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals (2001)

    (a) $\sqrt{PQRS}$

    (b) $\frac{PQ+RS}{2}$

    (c) $\frac{2PQ \cdot RS}{PQ+RS}$

    (d) $\frac{\sqrt{PQ^2+RS^2}}{2}$

18. Ifthe tangent at the point P on the circle $x^2+y^2+6x+6y=2$ meets a straight line $5x-2y+6=0$ at a point Q on the y- axis,then the length of PQ is (2002)

    (a) 4

    (b) $\sqrt[2]{5}$

    (c) 5

(d) $\sqrt[3]{5}$

19. The centre of circle inscribed in square formed by the lines $x^2-8x+12=0$ and$y^2-14y+45=0$, is (2003)

    (a) (4,7)
    (b) (7,4)
    (c) (9,4)
    (d) (4,9)

20. If one of the diameters of the circle $x^2+y^2-2x-6y+6=0$ is a chord to the circle with centre (2, 1),then the radius of the circle is (2004)

    (a) $\sqrt{3}$
    (b) $\sqrt{2}$
    (c) 3
    (d) 2

21. A circle is given by $x^2+(y-1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is (2005)

    (a) $\{(x,y): x^2=4y\} \cup \{(x,y): y \leq 0\}$
    (b) $\{(x,y): x^2+(y-1)^2=4\} \cup \{(x,y): y \leq 0\}$
    (c) $\{(x,y): x^2=y\} \cup \{(0,y): y \leq 0\}$
    (d) $\{(x,y): x^2=4y\} \cup \{(0,y): y \leq 0\}$

22. Tangents drawn from the point P(1,8) to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is (2009)

    (a) $x^2+y^2+4x-6y+19=0$
    (b) $x^2+y^2-4x-10y+19=0$
    (c) $x^2+y^2-2x+6y-29=0$
    (d) $x^2+y^2-6x-4y+19=0$

23. The circle passing through the point (-1,0) and touchingthe y-axis at (0, 2) also passes through the point (2011)

(a) $\left[-\frac{3}{2}, 0\right]$
(b) $\left[-\frac{5}{2}, 2\right]$
(c) $\left[-\frac{3}{2}, \frac{5}{2}\right]$
(d) $[-4, 0]$

24. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x-5y=20$ to the circle $x^2+y^2=9$ is (2012)

(a) $20(x^2+y^2)-36x+45y=0$
(b) $20(x^2+y^2)+36x-45y=0$
(c) $36(x^2+y^2)-20x+45y=0$
(d) $36(x^2+y^2)+20x-45y=0$

25. A line $y=mx+1$ intersectrs the circle $(x-3)^2+(y+2)^2=25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ? (2019)

(a) $2 \leq m < 4$
(b) $-3 \leq m < -1$
(c) $4 \leq m < 6$
(d) $6 \leq m < 8$

D : MCQ'S with One or More Than One Correct Answer

1. The equations of the tangents drawn from the origin to the circle $x^2+y^2-2rx-2hy+h^2=0$ are (1988)

(a) x=0
(b) y=0
(c) $(h^2-r^2)x-2rhy=0$
(d) $(h^2-r^2)x+2rhy=0$

2. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is (1998)

   (a) 0
   (b) 1
   (c) 3
   (d) 4

3. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1), Q(x_2 Y_2), R(x_3, y_3), S(x_4, y_4)$ then (1998)

   (a) $x_1 + x_2 + x_3 + x_4 = 0$
   (b) $y_1 + y_2 + y_3 + y_4 = 0$
   (c) $x_1 x_2 x_3 x_4 = c^4$
   (d) $y_1 y_2 y_3 y_4 = c^4$

4. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $\sqrt[2]{7}$ on y-axis is (are) (2013)

   (a) $x^2 + y^2 - 6x + 8y + 9 = 0$
   (b) $x^2 + y^2 - 6x + 7y + 9 = 0$
   (c) $x^2 + y^2 - 6x - 8y + 9 = 0$
   (d) $x^2 + y^2 - 6x - 7y + 9 = 0$

5. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then (2014)

   (a) radius of S is 8
   (b) radius of S is 7
   (c) Center of S is (-7,1)
   (d) Center of S is (-8,1)

6. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) (2016)

(a) $\frac{1}{3}, \frac{1}{\sqrt{3}}$

(b) $\frac{1}{4}, \frac{1}{2}$

(c) $\frac{1}{3}, -\frac{1}{\sqrt{3}}$

(d) $\frac{1}{4}, \frac{1}{2}$

7. Let Tbe the line passing through the points P(-2, 7) and Q(2.-5). Let F_1, be the set of allpairs of circles (S_1, S_2) such that T is tangent to S, at P and tangent to S_2, at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1, be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1, and passing through the point R(1, 1) be F_2, Let E_2, be the set of the mid-points of the line segments in the set F_2 Then, which of the following statement(s) is (are) TRUE? (2018)

   (a) The point (2,7) lies in E_1

   (b) The point $(\frac{4}{5}, \frac{7}{5})$ does NOT lies in E_2

   (c) The point $(\frac{1}{2}, 1)$ lies in E_2

   (d) The point $(0, \frac{3}{2})$ does NOT lies in E_1

E : Subjective Problems

1. Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point (5, 5).(1978)

2. Let A be the centre of the circlex $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that B(1,7) and D(4,-2) on the circle meet at the point C.(1981)

3. Find1 the area of the quadrilateral ABCD. Find the equations of the circle passing through 4,3) and touching the lines x+y=2 and x-y =2. (1982)

4. Through a fixed point (h, k) secants are drawn to the circle $x^2+y^2 = P$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$. (1983)

5. The abscissa of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter. (1984)

6. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_2, lies in the first quadrant, find the equation of the circle C_1, which is concentric with C_1 and cuts intercepts of length 8 on these lines the tangents at the points (1986)

7. Let a given line L_1, intersects the x and y axes at P and Q, respectively. Let another line L_2, perpendicular to L_1, cut the x and y axes at R and S,respectively. Show that the locus of the point of intersection of the lines PS and OR is a circle passing through the origin. (1989)

8. The circlec $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{\frac{1}{2}} = 0$. Find k. (1987)

9. If $\left[m_i \frac{1}{m_i}\right]$, $m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle ,then show that $m_1m_2m_3m_4 = 1$ (1989)

10. A circle touches the line $y = x$ at a point P such that $OP = \sqrt[4]{2}$,, where O is the origin. The circle contains the point(-10,2) in its interior and the length of its chord on the line $x + y = 0$ is $\sqrt[6]{2}$. Determine the equation of the circle. (1990)

11. Two circles, each of radius 5 units, touch each other at (1,2).If the equation of their common tangent is $4x + 3y = 10$, find the cquation of the circles. (1991)

12. Let a circle be given by $2x(x - a) + y(2y - b) = 0, (a \neq 0, b \neq 0))$.Find the condition on a and b if two chords,each biscected by the x- axis, can be drawn to the circle from $\left[a, \frac{b}{2}\right]$. (1992)

13. Consider a family of circles passing through two fixed points A(3,7) and B(6,5). Show that the chords in which the circle $x^2+y^2-4x-6y-3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point.

14. Find the coordinates of the point at which the circles $x^2+y^2-4x-2y = -4$ and $x^2+-12x-8y=-36$ touch each other. Also find equations common tangents touching the circles in the distinct points.

15. Find the intervals of values of a for which the line $y+x=0$ bisects two chords drawn from a point $\left[\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right]$ to the circle $2x^2+2y^2-(1+\sqrt{2}a)x+(1-\sqrt{2}a)y=0$. (1996)

16. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance betwcen the point A and the mid-point of the line segment DC is d, prove that the area of the circle is $\frac{\pi d^2 \cos^2\alpha}{\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta\cos(\beta-\alpha)}$ (1996)

17. Let C be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational points can be there on C. (A rational point is a point both of whose coordinates are rational numbers) (1997)

18. C_1 and C_2 are two concentric circles, the radius of C_2 being twice of that C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1.(1998)

19. Let T_1,T_2 be two tangents drawn from (-2,0) on to the circle $C: x^2+y^2=1$. Determine the circle touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time (1999)

20. Let $2x^2+y^2-3xy=0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA.(2001)

21. Let C_1 and C_2 be two circles with C_2 , lying inside C_1 A circle Clying inside C_1, touches C_1 internally and C_2 externally. Identify the locus of the centre of C . (2001)

22. For the circle $x^2+y^2=r^2$, find the value of r for which the area enclosed by the tangents drawn from the point P (6, 8) to the circle and the chord of contact is maximum.(2003)

23. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at (1,-1) and cutting orthogonally the circle having line segment joining (0,3) and (-2,-1) as diameter.(2004)

24. Circles with radii 3,4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact (2005)

F : Match The Following

DIRECTIONS (Q.1): Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled 1, 2, 3 and 4. while the statements in Columa-II are labelled as a,b,c, d and e. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answer to these questions have to be darkened as illustrated in the following example: If the correct matches are 1-a. s and e: 2-b and c: 3-1 and 2: and 4-d

1. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circie $C_3 : (x - h)^2 + (y - ky)^2 = r^2$ satisfies the following conditions:

   (a) Centre of C_3, is collinear with the centres of C_1, and C_2

   (b) C_1 and C_2 both lie inside C_3 and

   (c) C_3 touches C_1 at M and C_2 at N

   (d) Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_2 be a tangent to the parabola $x^2 = 8\alpha y$. There are some expressions given in the List - I whose values are given in List - II below

column-I column-II

1. $(2h+k)$	a) 6
2. $\frac{Length of ZW}{Length of XY}$	b) $\sqrt{6}$
3. $\frac{Area of triangle MZN}{Area of triangle ZMW}$	c) $\frac{5}{4}$
4.	d) $\frac{21}{5}$
5.	e) $\sqrt[2]{6}$
6.	f) $\frac{10}{3}$

Which of the following is the only CORRECT combination?

(a) (i)(f)

(b) (i)(d)

(c) (ii)(e)

(d) (ii)(b)

2. Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circie $C_3 : (x-h)^2 + (y-ky)^2 = r^2$ satisfies the following conditions:

   (a) Centre of C_3, is collinear with the centres of C_1, and C_2

   (b) C_1 and C_2 both lie inside C_3 and

   (c) C_3 touches C_1 at M and C_2 at N

   (d) Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_2 be a tangent to the parabola $x^2 = 8\alpha y$. There are some expressions given in the List - I whose values are given in List - II below

column-I	column-II
1. $(2h+k)$	a) 6
2. $\frac{Length of ZW}{Length of XY}$	b) $\sqrt{6}$
3. $\frac{Area of triangle MZN}{Area of triangle ZMW}$	c) $\frac{5}{4}$
4.	d) $\frac{21}{5}$
5.	e) $\sqrt[2]{6}$
6.	f) $\frac{10}{3}$

Which of the following is the only INCORRECT combination?

(a) (iv)(e)
(b) (i)(a)
(c) (iii)(c)
(d) (iv)(f)

G : Comprehension Based Questions

PASSAGE-1 ABCD is a square of side length 2 units. C_1 is the circle touching all the sides of the square ABCD and C_2, is the circumcircle of square ABCD. L is a fixed line in the same plane and R is a fixed point.

1. P is any point of C_1, and is another point on C_2, then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ is equal to (2006)

   (a) 0.75
   (b) 1.25
   (c) 1
   (d) 0.5

2. Ifa circle is such that it touches the line L and the circle C_1 externally, such that both the circles are on the same side of the line, then the locus of centre of the circleis (2006)

   (a) ellipse
   (b) parabola
   (c) hyperbola
   (d) pair of straight line

3. A line L' through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. 1f locus of S cuts L' at T_2 and T_3 and AC at T_1 then area of $\triangle T_1T_2T_3$ is (2006)

   (a) $\frac{1}{2}$ sq.units
   (b) $\frac{2}{3}$ sq.units
   (c) 1 sq. units

(d) 2 sq. units

PASSAGE-2

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ. QR, RP are D, E, F,respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left[\frac{\sqrt[3]{3}}{2}, \frac{3}{2}\right]$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

4. The equation of circle C is (2008)

   (a) $(x - \sqrt[2]{3})^2 + (y-1)^2 = 1$

   (b) $(x - \sqrt[2]{3})^2 + (y+\frac{1}{2})^2 = 1$

   (c) $(x - \sqrt{3}^2 + (y+1)^2 = 1$

   (d) $(x - \sqrt{3}^2 + (y-1)^2 = 1$

5. Points E and F are given by (2008)

   (a) $\left[\frac{\sqrt{3}}{2}, \frac{3}{2}\right]$, $[\sqrt{3}, 0]$

   (b) $\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$, $[\sqrt{3}, 0]$

   (c) $\left[\frac{\sqrt{3}}{2}, \frac{3}{2}\right]$, $\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$

   (d) $\left[\frac{3}{2}, \frac{\sqrt{3}}{2}\right]$, $\left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$

6. Equations of the sides QR, RP are (2008)

   (a) $y = \frac{2}{\sqrt{3}}x + 1, y = \frac{2}{\sqrt{3}}x - 1$

   (b) $y = \frac{1}{\sqrt{3}}x, y = 0$

   (c) $y = \frac{\sqrt{3}}{2}x + 1, y = \frac{\sqrt{3}}{2}x - 1$

   (d) $y = \sqrt{3}x, y = 0$

PASSAGE-3 A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PTis atangent to the circle $(x-3)^2 + y^2 = 1$.

7. A possible equation of L is (2012)
   (a) $x - \sqrt{3}y = 1$
   (b) $x + \sqrt{3}y = -1$
   (c) $x - \sqrt{3}y = 1$
   (d) $x + \sqrt{3}y = 5$

8. A common tangent of the two circles is
   (a) x=4
   (b) y=2
   (c) $x + \sqrt{3}y = 4$
   (d) $x + \sqrt[2]{2}y = 6$

   PASSAGE-4 Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$

9. Let E_1, E_2 and F_1, F_2, be the chords of S passing through thepoint (P_1, P_2) and parallel to the x-axis and the y-axis respectively. Let G_1, G_2, be the chord of S passing through P_0 and having slope -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3 and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve (2018)
   (a) $x + y = 4$
   (b) $(x-4)^2 - (y-4)^2 = 16$
   (c) $(x-4)(y-4) = 4$
   (d) $xy = 4$

10. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve (2018)
    (a) $(x+y)^2 = 3xy$
    (b) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2^{\frac{4}{3}}$
    (c) $x^2 + y^2 = 2xy$
    (d) $x^2 + y^2 = x^2y^2$

H : Assertion And Reason Type Questions

1. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$ STATEMENT-1: Thetangents are mutually perpendicular because STATEMENT-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$ (2007)

   (a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.

   (b) Statement-1 is True, Statement-2 is True;, Statement-2 is NOT a correct explanation for Statement-1.

   (c) Statement-1 is True, Statement-2 is False.

   (d) Statement-1 is False, Statement-2 is True.

2. Consider $L_1 : 2x + 3y + p - 3 = 0$
   $L_2 : 2x + 3y + p + 3 = 0$
   where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$ STATEMENT-1:If line L_1, is a chord of circle C, then line L_2 is not always a diameter of circle C and STATEMENT-2:If line L_1, is a diameter of circle C, then line L_1, is not a chord of circle C. (2008)

   (a) Statement-1 is True, Statement-2 is True; Statement-2 is acorrect explanation for Statement-1

   (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

   (c) Statement-1 is True, Statement-2 is False

   (d) Statement-1 is False, Statement-2 is True

I : Integer Value Correct Type

1. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segement joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1, and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C (2009)

2. The straight line $2x-3y=1$ divides the circular region $x^2+y^2 \leq 6$ into two parts. If $S = \left[\left[2, \frac{3}{4}\right], \left[\frac{5}{2}, \frac{3}{4}\right], \left[\frac{1}{4}, -\frac{1}{4}\right], \left[\frac{1}{8}, \frac{1}{4}\right]\right]$ then the number of points (s) in S lying inside the smaller part is (2011)

3. For how many values of p, the circle $x^2+y^2+2x+4y-p=0$ and the coordinate axes have exactly three common points? (2017)

4. Let the point B be the reflection of the point A(2,3) with respect to the line $8x-6y-23=0$. Let T_A and T_B be circles of radii 2 and 1 with centers Aand B respectively. Let T be a common tangent to the circles T_A and T_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing throughA and B, then the length of the line segment AC is (2019)

Section-B [JEE Mains /AIEEE]

1. If the chord $y=mx+1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45° at the major segment of the circle then value of m is (2002)

   (a) $2 \pm \sqrt{2}$
   (b) $-2 \pm \sqrt{2}$
   (c) $-1 \pm \sqrt{2}$
   (d) none of these

2. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is (2002)

   (a) $4 \leq x^2+y^2 \leq 64$
   (b) $x^2+y^2 \leq 25$
   (c) $x^2+y^2 \geq 25$
   (d) $3 \leq x^2+y^2 \leq 9$

3. he centre of the circle passing through (0,0) and (1,0) and touching the circle $x^2+y^2=9$ is (2002)

   (a) $\left[\frac{1}{2}, \frac{1}{2}\right]$

(b) $\left[\frac{1}{2}, -\sqrt{2}\right]$

(c) $\left[\frac{3}{2}, \frac{1}{2}\right]$

(d) $\left[\frac{1}{2}, \frac{3}{2}\right]$

4. The equation of a circle with origin as a centre and passing through equilater al triangle whose median is of length 3a is (2002)

   (a) $x^2 + y^2 = 9a^2$

   (b) $x^2 + y^2 = 16a^2$

   (c) $x^2 + y^2 = 4a^2$

   (d) $x^2 + y^2 = a^2$

5. If the two circles $(x-1)^2(y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct point, then (2003)

   (a) $r > 2$

   (b) $2 < r < 8$

   (c) $r < 2$

   (d) $r = 2$

6. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq.units.Then the equation of the circle is (2003)

   (a) $x^2 + y^2 - 2x + 2y = 62$

   (b) $x^2 + y^2 + 2x - 2y = 62$

   (c) $x^2 + y^2 + 2x - 2y = 47$

   (d) $x^2 + y^2 - 2x + 2y = 47$

7. If a circle passes through the point (a,b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is (2004)

   (a) $2ax - 2by - (a^2 + b^2 + 4) = 0$

   (b) $2ax + 2by - (a^2 + b^2 + 4) = 0$

   (c) $2ax - 2by + (a^2 + b^2 + 4) = 0$

   (d) $2ax + 2by + (a^2 + b^2 + 4) = 0$

8. A variable circle passes through the fixed point A(p,q) and touches x-axis. The locus of the other end of the diameter through A is (2004)

   (a) $(y-q)^2 = 4px$
   (b) $(x-q)^2 = 4py$
   (c) $(y-p)^2 = 4qx$
   (d) $(x-p)^2 = 4qy$

9. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameter of a circle of circumference 10π, then the equationofthe circle is (2004)

   (a) $x^2+y^2+2x-2y-23=0$
   (b) $x^2+y^2+2x+2y-23=0$
   (c) $x^2+y^2+-x-2y-23=0$
   (d) $x^2+y^2-2x+2y-23=0$

10. Intercept on the line y=x by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is (2004)

    (a) $x^2+y^2+x-y=0$
    (b) $x^2+y^2+x+y=0$
    (c) $x^2+y^2-x+y=0$
    (d) $x^2+y^2-x-y=0$

11. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line 5x +by-a=0 passes through P and Q for (2005)

    (a) exactly one value ofa
    (b) no value of a
    (c) infinitely many values of a
    (d) exactly two values of a

12. A circle touches the x-axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is (2005)

    (a) an ellipse

(b) a circle

(c) a hyperbola

(d) a parabola

13. If a circle passes through the point (a,b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is (2005)

    (a) $x^2+y^2-3ax-4by+(a^2+b^2-p^2)=0$

    (b) $2ax+2by-(a^2-b^2+p^2)=0$

    (c) $x^2+y^2-2ax-3by+(a^2-b^2-p^2)=0$

    (d) $2ax+2by-(a^2-b^2+p^2)=0$

14. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters ofa circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then (2005)

    (a) $3a^2-10ab+3b^2=0$

    (b) $3a^2+10ab+3b^2=0$

    (c) $3a^2-2ab+3b^2=0$

    (d) $3a^2+2ab+3b^2=0$

15. If the lines 3x-4y -7 = 0 and 2x-3y-5 =0 are two diameters of a circle of area 49π square units, the equation ofthe circle is (2006)

    (a) $x^2+y^2+2x-2y-47=0$

    (b) $x^2+y^2+2x-2y-62=0$

    (c) $x^2+y^2-2x+2y-47=0$

    (d) $x^2+y^2-2x+2y-62=0$

16. Let C be the circle with centre (0,0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is (2006)

    (a) $x^2+y^2=\frac{3}{2}$

    (b) $x^2+y^2=1$

    (c) $x^2+y^2=\frac{27}{4}$

(d) $x^2 + y^2 = \frac{9}{4}$

17. Consider a family of circles which are passing through the point(-1,1) and are tangent to x-axis. If (h,k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval (2006)

    (a) $-\frac{1}{2} \leq k \leq \frac{1}{2}$
    (b) $k \leq \frac{1}{2}$
    (c) $0 \leq k \leq \frac{1}{2}$
    (d) $k \geq \frac{1}{2}$

18. The point diametrically opposite to the point P(1,0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is (2007)

    (a) (3,-4)
    (b) (-3,4)
    (c) (-3,-4)
    (d) (3,4)

19. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is (2008)

    (a) $(x - 2)y'^2 = 25 - (y - 2)^2$
    (b) $(y - 2)y'^2 = 25 - (y - 2)^2$
    (c) $(y - 2)^2 y'^2 = 25 - (y - 2)^2$
    (d) $(x - 2)^2 y'^2 = 25 - (y - 2)^2$

20. If P and Q are the points of intersection of the circle $x^2 + y^2 + 33x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P,Q and (1,1) for: (2009)

    (a) all except one value of p
    (b) all except two values of p
    (c) exactly one value of p
    (d) all values of p

21. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if (2010)

   (a) $-35 < m < 15$
   (b) $15 < m < 65$
   (c) $35 < m < 85$
   (d) $-85 < m < -35$

22. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if (2011)

   (a) $| a |= c$
   (b) $a = 2c$
   (c) $| a |= 2c$
   (d) $2 | a |= c$

23. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is (2012)

   (a) $\frac{10}{3}$
   (b) $\frac{3}{5}$
   (c) $\frac{6}{5}$
   (d) $\frac{5}{3}$

24. The circle passing through (1,-2) and touching the axis ofx at (3,0) also passes through the point (2013)

   (a) (-5,2)
   (b) (5,-2)
   (c) (-5,-2)
   (d) (5,2)

25. Let C be the circle with centre at (1,1) and radius =1. If T is the circle centred at (0,y), passing through origin and touching the circle C externally, then the radius of T is equal (2014)

   (a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{\sqrt{3}}{\sqrt{2}}$

(d) $\frac{\sqrt{3}}{2}$

26. Locus of the image of the point (2,3) in the line $(2x-3y+4)+k(x-2y+3)=0$, $k \in R$, is a: (2015)

    (a) circle of radius $\sqrt{2}$.

    (b) circle of radius $\sqrt{3}$.

    (c) straight line parallel tox-axis.

    (d) straight line parallel to y-axis.

27. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$, is: (2015)

    (a) 3

    (b) 4

    (c) 1

    (d) 2

28. The centres of those circles which touch the circle, $x^2+y^2-8x-8y-4=0$, externally and also touch the x-axis,lie on: (2016)

    (a) a hyerbola

    (b) a parabola

    (c) a circle

    (d) an ellipse, which is not a circle

29. If one of the diameters of the circle, given by the equation, $x^2+y^2-4x+6y-12=0$, is a chord of a circle S, whose centre is at (-3,2),then the radius of S is: (2016)

    (a) 5

    (b) 10

    (c) $\sqrt[5]{2}$

(d) $\sqrt[5]{3}$

30. Ifa tangent to the circle $x^2 + y^2 = l$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is: (2019)

(a) $x^2 + y^2 - 4x^2y^2 = 0$

(b) $x^2 + y^2 - 2xy = 0$

(c) $x^2 + y^2 - 16x^2y^2 = 0$

(d) $x^2 + y^2 - 2x^2y^2 = 0$