Portfolio Optimization with Constrained Optimization Techniques: Balancing Risk and Return

CMSE 831 Final Project

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Introduction

Purpose:

- To construct a portfolio that optimally balances **risk** and **return** using advanced constrained optimization techniques

Real-World Relevance:

- Modern financial markets require **robust and adaptable strategies** to manage diversified asset classes, including stocks, bonds, and cryptocurrencies
- Traditional approaches, such as Markowitz mean-variance optimization, face limitations in handling complex constraints and market dynamics

Research Questions:

1. Can advanced constrained optimization methods achieve **superior risk-adjusted returns** compared to traditional approaches?
2. How does an optimized portfolio dynamically adjust to **changing market conditions**, maintaining stability during volatile periods?

Data Collection

A **diversified portfolio** was constructed, including assets across multiple categories to ensure balanced risk exposure

Data Source:

- Historical price data fetched using the **yfinance** library
- **Daily returns** were calculated for portfolio analysis

Asset Categories:

1. **Stocks**: AAPL (Apple), MSFT (Microsoft), GOOGL (Alphabet)
2. **Bonds**: TLT (iShares 20+ Year Treasury Bond ETF)
3. **Commodities**: GLD (SPDR Gold Shares), DBC (Invesco DB Commodity Index Tracking Fund)
4. **Real Estate**: VNQ (Vanguard Real Estate ETF)
5. **Cryptocurrencies**: BTC-USD (Bitcoin), ETH-USD (Ethereum)

Purpose of Selection:

- To capture the **diverse risk-return characteristics** of traditional and modern asset classes, ensuring portfolio adaptability and robustness

Methodology Overview

Optimization Approach: Sequential Least Squares Programming (SLSQP)	Balances **returns** and **risks** using a constrained optimization framework Ensures portfolio weights sum to 1 and penalizes over-concentration
Risk Metrics	**Sharpe Ratio**: Risk-adjusted return **Conditional Value at Risk (CVaR)**: Average loss in worst-case scenarios **Maximum Drawdown (MDD)**: Largest peak-to-trough loss
Comparative Portfolios	**Optimized Portfolio**: SLSQP-tuned for best risk-return trade-offs **Markowitz Portfolio**: Traditional mean-variance optimization **Baseline Portfolio**: Equal-weighted allocation

Parameter Tuning and Evaluation

Hyperparameter Tuning:

- **Grid Search**: Conducted over λ1 (return weight) and λ2 (risk weight) values to identify optimal combinations
- **5-Fold Cross-Validation**: Ensured robustness and avoided overfitting by validating results across multiple data subsets

Evaluation:

1. **Backtesting**:
    - Assessed **historical performance** of portfolios optimized using the best hyperparameters
2. **Rolling-Window Analysis**:
    - Evaluated the **adaptability** of portfolio metrics (Sharpe Ratio, CVaR, MDD) over time
    - Highlighted the dynamic adjustment of portfolio weights to changing market conditions
3. **Asset-Level Insights**:
    - Investigated **excluded assets** for high correlations or poor diversification benefits
    - Ensured optimal allocations focus on assets with complementary risk-return characteristics

Performance Across Portfolios

- The **Optimized Portfolio (SLSQP)** achieves the **highest Sharpe Ratio (1.362)**, reflecting superior risk-adjusted returns
- It outperforms the **Markowitz Portfolio** and **Baseline Portfolio** across all performance metrics:
  - **Lower CVaR (-0.016)**, indicating reduced exposure to extreme losses
  - **Reduced MDD (0.211)**, showcasing greater resilience to drawdowns
- The **Baseline Portfolio**, with equal weighting, performs the worst:
  - **Lowest Sharpe Ratio (0.867)**, highlighting inefficiency in balancing risk and return
  - **Highest MDD (0.694)**, reflecting poor diversification and vulnerability to market downturns

Portfolio Optimization	Sharpe Ratio	CVaR	MDD
SLSQP	1.362	-0.016	0.211
Markowitz	1.070	-0.014	0.184
Baseline	0.867	-0.033	0.694

Cumulative Returns Comparison

Optimized Portfolio (SLSQP):

- Demonstrates **steady growth** with minimal drawdowns
- Effectively balances **return maximization** and **risk control**, outperforming other approaches

Markowitz Portfolio:

- Shows **reasonable performance**, but lags behind the optimized portfolio in terms of risk-adjusted returns and adaptability

Baseline Portfolio:

- Experiences **high volatility** and **significant drawdowns**, underscoring its lack of diversification and suboptimal allocation

This analysis highlights the superiority of the optimized portfolio in achieving consistent growth while managing risk effectively

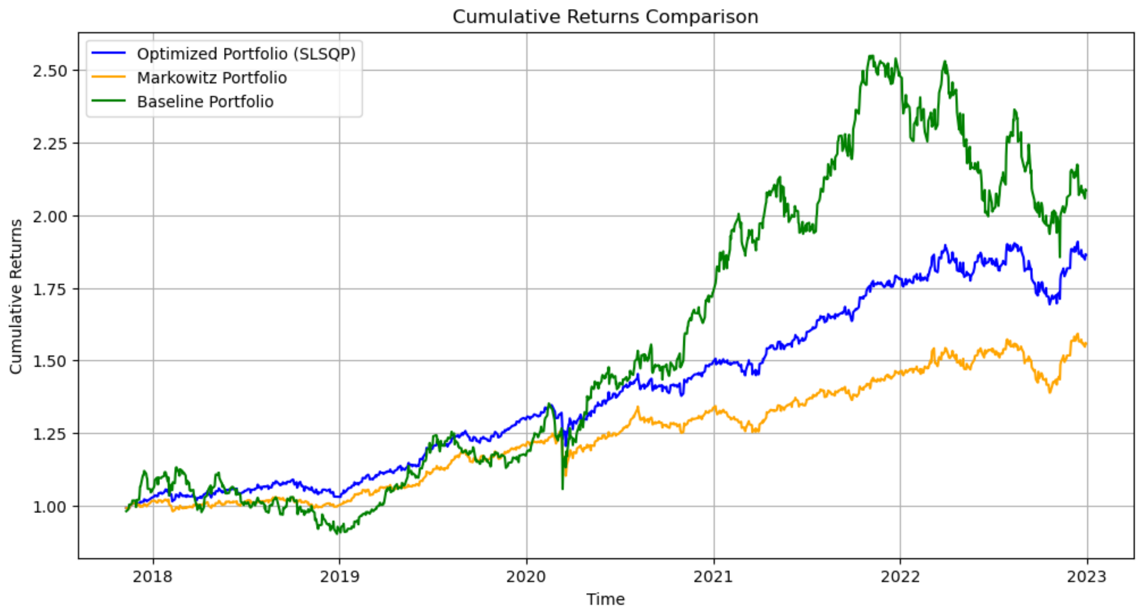


Fig: Portfolio performance over time by comparing cumulative returns across different strategies.

Sensitivity Analysis

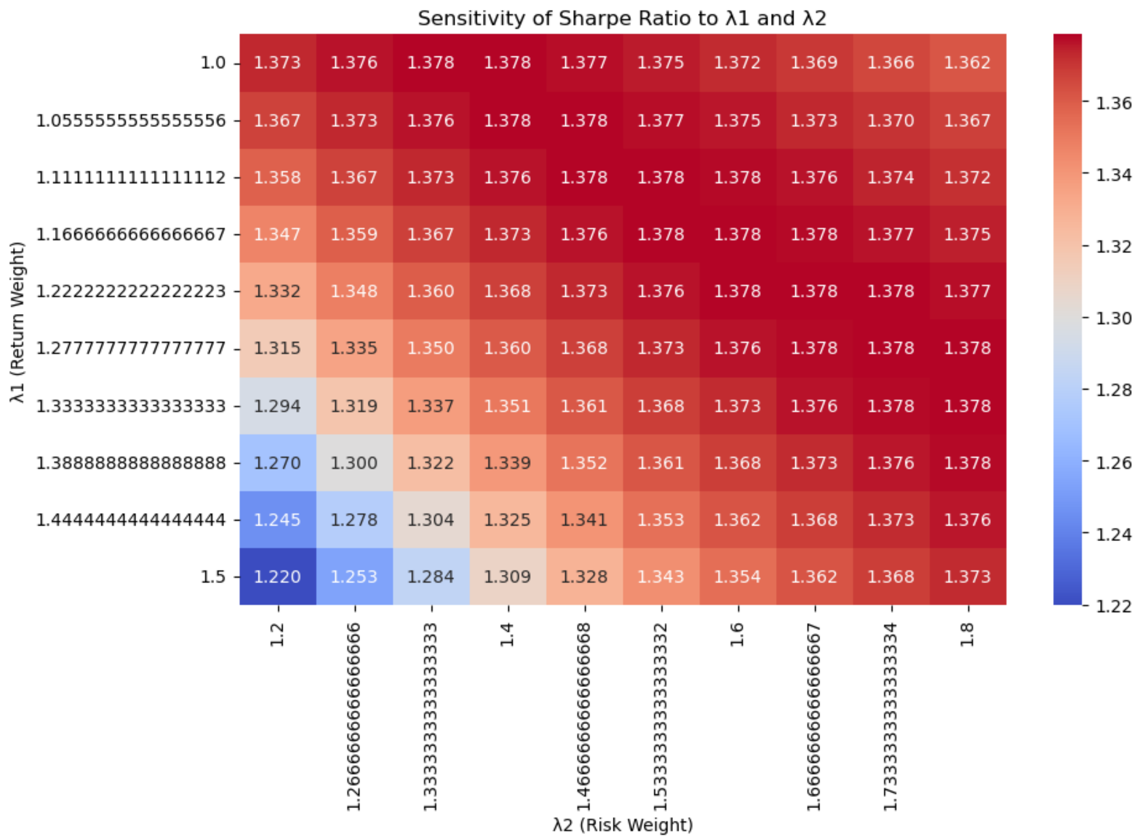
Objective:

- Examine the influence of varying **λ1 (return weight)** and **λ2 (risk weight)** on the Sharpe Ratio

Findings:

- The **optimal Sharpe Ratios** are observed near **λ1 = 1.2** and **λ2 = 1.4**, signifying these parameters effectively balance returns and risks
- The heatmap showcases **consistency** in performance metrics across a wide parameter range, highlighting the **robustness** of the optimization method

This sensitivity analysis validates the selected hyperparameters and reinforces their suitability for constructing an optimized portfolio



Portfolio Weights and Metrics

The performance metrics—**Sharpe Ratio (1.362)**, **CVaR (-0.016),** and **MDD (0.211)** demonstrate the optimized portfolio's superior risk-adjusted returns, reduced tail risk, and controlled drawdowns

High Allocations:

MSFT (21.51%), **DBC** (28.51%), **TLT** (29.13%), and **GLD** (19.08%)

- **MSFT**: Strong returns and moderate volatility
- **DBC**: Stability during market turbulence
- **TLT**: Mitigates risks in downturns
- **GLD**: Hedge against inflation and low volatility

Low or Zero Allocations:

AAPL (0.00%), **GOOGL** (0.00%), and **ETH-USD** (0.00%)

- **Redundant Exposure**: High correlation with **MSFT** reduces diversification
- **High Volatility**: Poor risk-return trade-offs compared to stabilizers like **TLT** and **GLD**

Minimal Allocations:

- **BTC-USD** (1.34%) and **VNQ** (0.43%)
  - Limited diversification benefits due to higher risks or lower returns

Asset	Weight (%)
MSFT	21.51
TLT	29.13
DBC	28.51
GLD	19.08
BTC-USD	1.34
VNQ	0.43
AAPL	0.00

Asset-Metrics and Correlation Matrix

Asset	Mean Return	Volatility	Sharpe Ratio
AAPL	0.244	0.323	0.726
MSFT	0.327	0.298	1.065
GOOGL	0.201	0.308	0.619
TLT	0.064	0.159	0.338
GLD	0.096	0.142	0.606
BTC-USD	0.339	0.676	0.486
ETH-USD	0.297	0.844	0.339
DBC	0.171	0.185	0.871

Asset-Level Metrics:

- **AAPL** and **GOOGL**: Moderate Sharpe Ratios and high correlation with **MSFT**
- **ETH-USD**: High volatility with a low Sharpe Ratio, making it less attractive

Correlation Matrix for Excluded Assets:

- **GOOGL and MSFT**: High correlation (**0.796**) indicates redundancy, making **GOOGL** unnecessary
- **ETH-USD and BTC-USD**: High correlation (**0.792**) suggests overlapping exposure, reducing diversification benefits

	AAPL	GOOGL	ETH-USD
MSFT	0.760139	0.795721	0.307748
TLT	-0.132143	-0.118684	-0.017219
GLD	0.089636	0.103245	0.117941
VNQ	0.545074	0.522964	0.241436
BTC-USD	0.256595	0.258277	0.792032
DBC	0.217861	0.232849	0.132181

Rolling-Window Backtesting

Objective:

- **Assess the adaptability of the portfolio under changing market conditions** using rolling-window analysis

Findings:

1. **Stable Metrics**: Sharpe Ratio, CVaR, and MDD remained robust across rolling windows, even during market volatility
2. **Dynamic Weights**: Portfolio weights adjusted to market conditions, increasing stabilizing assets like **TLT** and **GLD** during downturns, while limiting allocations to high-volatility assets like **BTC-USD** and **ETH-USD**

The optimized portfolio demonstrated resilience and adaptability to varying financial climates

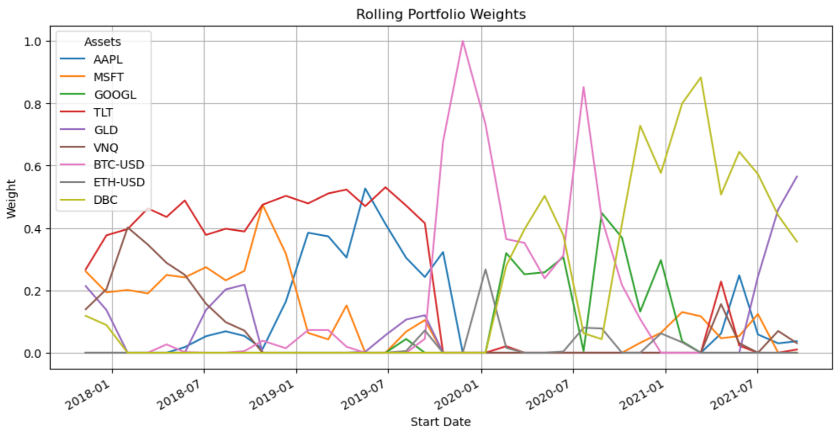


Fig: Rolling Portfolio Weights

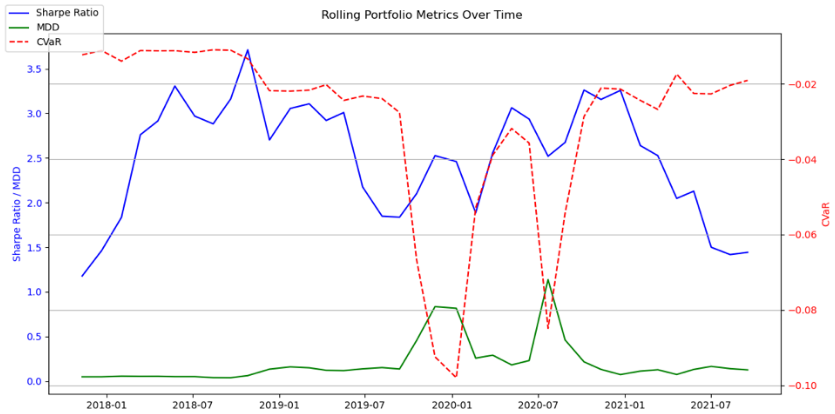


Fig: Rolling Portfolio Metrics Over Time

Discussion and Conclusion

Key Takeaways:

- The optimized portfolio demonstrated superior risk-adjusted returns (Sharpe Ratio: **1.362**) compared to Markowitz and baseline portfolios
- Assets like **AAPL**, **GOOGL**, and **ETH-USD** were excluded due to high correlations or unfavorable risk-return trade-offs

Challenges:

- Balancing computational complexity and scalability for larger datasets
- Addressing real-world constraints like transaction costs

Future Work:

- Extend to real-time portfolio adjustments with transaction costs and liquidity constraints
- Explore machine learning-based models for further optimization

References

1. Historical financial data was sourced using the Yahoo Finance API.

2. Nocedal, J., & Wright, S. J. (2006). Numerical Optimization. Springer. (Reference for the optimization algorithms)

3. https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html

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5. Rockafellar, R. T., & Uryasev, S. (2000). Optimization of Conditional Value-at-Risk. Journal of Risk, 2(3), 21–41. (Source for CVaR calculations.)

Thank You!!!