BAB 7 TRIGONOMETRI

A Satuan Sudut

Satuan yang biasanya digunakan untuk mengukur sudut adalah **derajat** dan **radian**.

Sudut $\frac{1}{2}$ putaran $= 180^\circ = \pi$ radian

Sudut 1 putaran $= 360^\circ = 2\pi$ radian

Nilai pendekatan $\pi = 3{,}14$ atau $\pi = \frac{22}{7}$

$1^\circ \approx \frac{2\pi}{360}$ radian $= \frac{6{,}28}{360}$ radian $= 0{,}0017$ radian

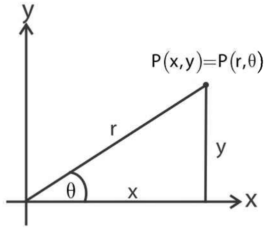
$1 \text{ radian} = \frac{180^\circ}{\pi} = \frac{180^\circ}{3{,}14} \approx 57{,}3^\circ$ atau $57^\circ 18'$

Rumus untuk mengubah satuan derajat ke radian dan sebaliknya adalah:

$$\theta^\circ = \left(\theta \times \frac{\pi}{180}\right) \quad \text{dan} \quad p \text{ radian} = \left(p \times \frac{180}{\pi}\right)^\circ$$

B Koordinat Titik Kutub

Letak suatu titik pada bidang $X - Y$ dapat disajikan dalam koordinat Cartesius, yaitu (x, y) atau dalam koordinat kutub, (r, θ°), seperti terlihat pada gambar di bawah ini:



Letak suatu titik P dalam koordinat Cartesius dapat diubah ke koordinat kutub, atau sebaliknya dengan menggunakan hubungan:

$$P(x, y) \rightarrow p(r, \theta^\circ)$$

dengan: $r = \sqrt{x^2 + y^2}$; θ° ditentukan dari $\tan \theta^\circ = \frac{y}{x}$

$$P(r, \theta^\circ) \rightarrow P(x, y)$$

dengan: $x = r \cos\theta^\circ$ dan $y = r \sin\theta^\circ$

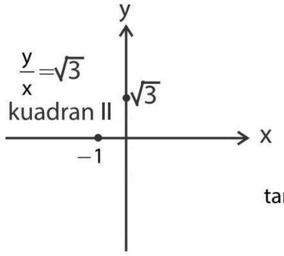
Jadi, dapat dituliskan $P(r \cos\theta^\circ, r\sin\theta^\circ)$

Contoh

1. Koordinat kutub dari titik $A(-1, \sqrt{3})$ adalah ....

   Pembahasan:

   Titik $A(-1, \sqrt{3})$, berarti $x = -1$ dan $y = \sqrt{3}$.



$r = \sqrt{x^2 + y^2}$

$= \sqrt{1+3}$

$= \sqrt{4} = 2$

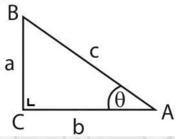
$\tan \theta^\circ = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

titik A berada di kuadran II, maka $\theta = 120^\circ$

Jadi, koordinat kutubnya adalah $A(2, 120^\circ)$.

C Perbandingan Trigonometri

Pada setiap segitiga siku-siku, berlaku perbandingan trigonometri berikut.



$\sin\theta^\circ = \frac{a}{c}\left(\text{demi} = \frac{\text{depan}}{\text{miring}}\right)$ $\quad \operatorname{cosec} \theta^\circ = \frac{c}{a} = \frac{1}{\sin\theta^\circ}$

$\cos\theta^\circ = \frac{b}{c}\left(\text{sami} = \frac{\text{samping}}{\text{miring}}\right)$ $\quad \sec \theta^\circ = \frac{c}{b} = \frac{1}{\cos\theta^\circ}$

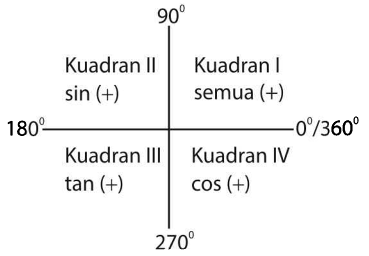
$\tan\theta^\circ = \frac{a}{b}\left(\text{desa} = \frac{\text{depan}}{\text{samping}}\right)$ $\quad \operatorname{cotan} \theta^\circ = \frac{b}{a} = \frac{1}{\tan\theta^\circ}$

Nilai perbandingan trigonometri untuk sudut-sudut istimewa diperlihatkan pada tabel di bawah ini:

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos θ	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	~
cosec θ	~	2	$\sqrt{2}$	$\frac{2}{3}\sqrt{3}$	1
sec θ	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2	~
Cotan θ	~	$\sqrt{3}$	1	$\frac{1}{3}\sqrt{3}$	0

D Sudut Berelasi

Sumbu koordinat membagi bidang koordinat menjadi empat bagian yang disebut kuadran.



1. Rumus Perbandingan Trigonometri Sudut di Kuadran I

$\sin(90-\theta)^\circ = \cos\theta^\circ$ $\quad$ $\cot an(90-\theta)^\circ = \tan\theta^\circ$

$\cos(90-\theta)^\circ = \sin\theta^\circ$ $\quad$ $\sec(90-\theta)^\circ = \text{cosec}\,\theta^\circ$

$\tan(90-\theta)^\circ = \cot an\,\theta^\circ$ $\quad$ $\text{cosec}(90-\theta)^\circ = \sec\theta^\circ$

2. Rumus Perbandingan Trigonometri Sudut di Kuadran II

$\sin(180-\theta)^\circ = \sin\theta^\circ$

$\cos(180-\theta)^\circ = -\cos\theta^\circ$

$\tan(180-\theta)^\circ = -\tan\theta^\circ$

$\cot an(180-\theta^\circ) = -\cot an\,\theta^\circ$

$\sec(180-\theta^\circ) = -\sec\theta^\circ$

$\text{cosec}(180-\theta^\circ) = \text{cosec}\,\theta^\circ$

3. Rumus Perbandingan Trigonometri Sudut di Kuadran III

$\sin(180+\theta)^\circ = -\sin\theta^\circ$

$\cos(180+\theta)^\circ = -\cos\theta^\circ$

$\tan(180+\theta)^\circ = \tan\theta^\circ$

$\cot an(180+\theta)^\circ = \cot an\,\theta^\circ$

$\sec(180+\theta)^\circ = -\sec\theta^\circ$

$\text{cosec}(180+\theta)^\circ = -\text{cosec}\,\theta^\circ$

4. Rumus Perbandingan Trigonometri Sudut di Kuadran IV

$\sin(360-\theta)^\circ = -\sin\theta^\circ$

$\cos(360-\theta)^\circ = \cos\theta^\circ$

$\tan(360-\theta)^\circ = -\tan\theta^\circ$

$\cot an(360-\theta)^\circ = -\cot an\,\theta^\circ$

$\sec(360-\theta)^\circ = \sec\theta^\circ$

$\text{cosec}(360-\theta)^\circ = -\text{cosec}\,\theta^\circ$

atau dengan **sudut negatif**, yaitu:

$\sin(-\theta)^\circ = -\sin\theta^\circ$

$\cos(-\theta)^\circ = \cos\theta^\circ$

$\tan(-\theta)^\circ = -\tan\theta^\circ$

$\cot an(-\theta)^\circ = -\cot an\,\theta^\circ$

$\sec(-\theta)^\circ = \sec\theta^\circ$

$\text{cosec}(-\theta)^\circ = -\text{cosec}\,\theta^\circ$

5. Rumus Perbandingan Trigonometri Sudut Lebih dari 360°

$\sin(\theta+n\cdot 360)^\circ = \sin\theta^\circ$

$\cos(\theta+n\cdot 360)^\circ = \cos\theta^\circ$

$\tan(\theta+n\cdot 360)^\circ = \tan\theta^\circ$

$\text{cosec}(\theta+n\cdot 360)^\circ = \text{cosec}\,\theta^\circ$

$\cot an(\theta+n\cdot 360)^\circ = \cot an\,\theta^\circ$

E Hubungan Perbandingan Trigonometri

1. Hubungan antara Perbandingan-perbandingan Trigonometri

$\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$ $\quad$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\quad$ $\cot an\theta = \dfrac{\cos\theta}{\sin\theta}$

$\tan\theta = \dfrac{1}{\cot an\,\theta}$

2. Identitas Trigonometri

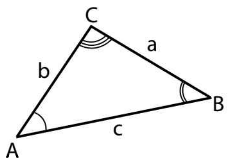
$\cos^2\theta + \sin^2\theta = 1$

$1+\tan^2\theta = \sec^2\theta$

$1+\cot an^2\theta = \text{cosec}^2\,\theta$

F Aturan Sinus - Cosinus

1. Aturan Sinus



Pada setiap segitiga ABC berlaku aturan sinus, yaitu:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Aturan sinus digunakan jika diketahui 3 unsur yang secara berurutan, yaitu:

a. sisi – sudut – sudut (ss – sd –sd)
b. sisi – sisi – sudut (ss – ss – sd)
c. sudut – sisi – sudut (sd – ss – sd)

2. Aturan Kosinus

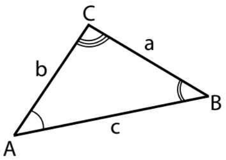
Pada setiap segitiga ABC berlaku aturan kosinus, yaitu:

$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Aturan kosinus digunakan jika diketahui 3 unsur secara berurutan, yaitu:

a. sisi – sudut – sisi (ss – sd – ss)
b. sisi – sisi – sisi (ss – ss – ss)

3. Luas Segitiga



$$L = \frac{1}{2}ab\sin C$$

$$L = \frac{1}{2}ac\sin B$$

$$L = \frac{1}{2}bc\sin A$$

4. Rumus-rumus Trigonometri

a. Rumus Trigonometri untuk Jumlah Dua Sudut dan Selisih Dua Sudut

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\cdot\tan\beta}$$

b. Rumus Trigonometri untuk Sudut Rangkap

$$\sin 2\theta = 2\sin\theta\cdot\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

c. Rumus Perkalian Sinus dan Kosinus

$$\sin\alpha\cdot\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$$

$$\cos\alpha\cdot\sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right)$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$$

$$\sin\alpha\cdot\sin\beta = -\frac{1}{2}\left(\cos(\alpha+\beta) - \cos(\alpha-\beta)\right)$$

d. Rumus Penjumlahan dan Pengurangan Sinus dan Kosinus

$$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cdot\cos\frac{1}{2}(\alpha-\beta)$$

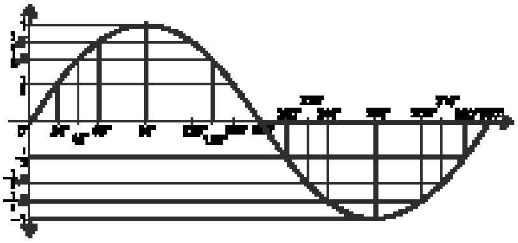
$$\sin\alpha - \sin\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cdot\sin\frac{1}{2}(\alpha-\beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cdot\cos\frac{1}{2}(\alpha-\beta)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha+\beta)\cdot\sin\frac{1}{2}(\alpha-\beta)$$

G Grafik Fungsi Trigonometri

1. Grafik Fungsi y = sin x, x ϵ [0°, 360°]



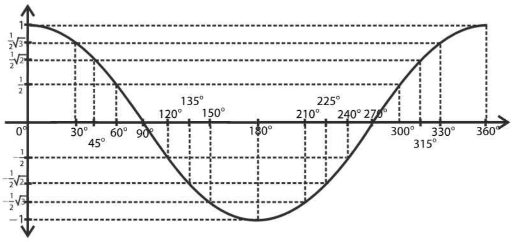
Nilai maksimum = 1
Nilai minimum = –1

Amplitudo $= \frac{1}{2}(\text{max} - \text{min})$

Periode = 2π radian

Fungsi sin $(x + k\cdot 2\pi)$ = sin x, kϵ bilangan bulat.

2. Grafik Fungsi y = cos x, x ϵ [0°,360°]



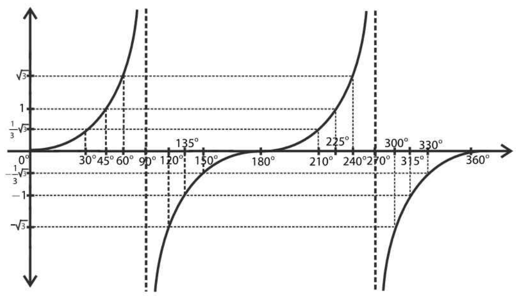
Nilai maksimum = 1
Nilai minimum = –1

Amplitudo = $\frac{1}{2}(\text{max} - \text{min})$

Periode = 2π radian

Fungsi cos $(x + k\cdot 2\pi)$ = cos x, k ϵ bilangan bulat.

3. Grafik Fungsi y = tan x, x ϵ [0°,360°]



Nilai maksimum = + ~
Nilai minimum = – ~

Amplitudo = $\frac{1}{2}(\max - \min)$

Periode = π radian

Fungsi tan $(x + k\cdot\pi)$ = tan x, k ϵ bilangan bulat.

4. Nilai Maksimum dan Minimum Fungsi Sinus dan Cosinus

a. $(\sin \alpha^\circ)$ maksimum = 1, untuk $\alpha = 90 + n\cdot 360$
   $(\sin \alpha^\circ)$ minimum = −1, untuk $\alpha = 270 + n\cdot 360$
   Jadi, $-1 \le \sin \alpha^\circ \le 1$ untuk $\alpha \in R$

b. $(\cos \alpha^\circ)$ maksimum = 1, untuk $\alpha = n\cdot 360$
   $(\cos \alpha^\circ)$ minimum = −1, untuk $\alpha = 180 + n\cdot 360$
   Jadi, $-1 \le \cos \alpha^\circ \le 1$ untuk $\alpha \in R$

e. tan α° tidak mempunyai nilai maksimum juga minimum.

CONTOH SOAL DAN PEMBAHASAN

1. Jika $\sin\left(\frac{2a+\pi}{2}\right) = \frac{3}{5}$ maka nilai dari $\sin(a-\pi)+\cos(-a) = \ldots$

A. $-\frac{1}{5}$ B. $-\frac{3}{5}$ C. $\frac{1}{5}$ D. $\frac{3}{5}$ E. $\frac{4}{5}$

Pembahasan SMART:

$\sin\left(\frac{2a+\pi}{2}\right) = \frac{3}{5}$

$\rightarrow \sin\left(a+\frac{\pi}{2}\right) = \frac{3}{5}$

$\rightarrow \sin(a)\cos\left(\frac{\pi}{2}\right) + \cos(a)\sin\left(\frac{\pi}{2}\right) = \frac{3}{5}$

$\rightarrow \sin(a)\cdot 0 + \cos(a)\cdot 1 = \frac{3}{5}$

$\rightarrow \cos(a) = \frac{3}{5}$

$\rightarrow \sin(a) = \frac{4}{5}$

Sehingga, diperoleh:

$\sin(a-\pi)+\cos(-a)$

$= \sin(a)\cos(\pi) - \cos(a)\sin(\pi) + \cos(a)$

$= \sin(a-\pi)+\cos(-a) = -\frac{4}{5}+\frac{3}{5} = -\frac{1}{5}$

Jawaban: A

2. Fungsi $y = \cos 2x - \sqrt{3}\sin 2x + 1$ memotong sumbu x untuk interval $\pi \le x \le 2\pi$, himpunan penyelesaian absisnya adalah ....

A. $\left\{\frac{7\pi}{6}, \frac{4\pi}{3}\right\}$ D. $\left\{\frac{4\pi}{3}, \frac{3\pi}{2}\right\}$

B. $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ E. $\left\{\frac{3\pi}{2}, \frac{5\pi}{3}\right\}$

C. $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}\right\}$

Pembahasan SMART:

ingat! ingat!

$a.\sin x + b\cos x = k.\cos(x-\alpha)$

$k = \sqrt{a^2+b^2}$ dan

$\alpha = \frac{a}{b}$ (kuadran α tergantung nilai a dan b)

$y = \cos 2x - \sqrt{3}\sin 2x + 1$

$k = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = 2$

$\tan \alpha = \frac{-\sqrt{3}}{1}$ (α kuadaran IV)

$\alpha = 300^\circ$

Maka $y = \cos 2x - \sqrt{3}\sin 2x + 1$

$= 2\cos\left(2x - 300^\circ\right) + 1$

Memotong sumbu X, artinya y = 0

$y = 2\cos\left(2x-300^\circ\right)+1 = 0$

$\Rightarrow \cos\left(2x-300^\circ\right) = -\frac{1}{2} = \cos 120^\circ$

Penyelesaian (1):

$2x - 300^\circ = 120^\circ \pm k\cdot 360^\circ$

$\Rightarrow 2x = 420^\circ \pm k\cdot 360^\circ$

$\Rightarrow x = 210^\circ \pm k\cdot 180^\circ$

$\Rightarrow x = 210^\circ$

Penyelesaian (2):

$2x - 300^\circ = -120^\circ \pm k\cdot 360^\circ$

$\Rightarrow 2x = 180^\circ \pm k\cdot 360^\circ$

$\Rightarrow x = 90^\circ \pm k\cdot 180^\circ$

$\Rightarrow x = 270^\circ$

$HP = \left\{210^\circ, 270^\circ\right\} = \left\{\frac{7\pi}{6}, \frac{3\pi}{2}\right\}$

Jawaban: C

3. Diketahui sin A + sin B = 1 dan cos A + cos B = $\sqrt{\frac{5}{3}}$.
Nilai (A − B) = ...

A. $\frac{1}{3}$ D. $\frac{1}{2}\sqrt{3}$

B. $\frac{1}{2}$ E. 1

C. $\frac{1}{2}\sqrt{2}$

Pembahasan SMART:

ingat! ingat!

$\sin^2 A + \cos^2 A = 1$

$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$

Diketahui:

- $\sin A + \sin B = 1$

  $\Rightarrow (\sin A + \sin B)^2 = \sin^2 A + \sin^2 B + 2\sin A.\sin B$

  $\Rightarrow 1 = \sin^2 A + \sin^2 B + 2\sin A.\sin B$

- $\cos A + \cos B = \sqrt{\frac{5}{3}}$

  $\Rightarrow (\cos A + \cos B)^2 = \cos^2 A + \cos^2 B + 2\cos A.\cos B$

  $\Rightarrow \frac{5}{3} = \cos^2 A + \cos^2 B + 2\cos A.\cos B$

Sehingga:

$$\begin{array}{l} \sin^2 A + \sin^2 B + 2\sin A.\sin B = 1 \\ \cos^2 A + \cos^2 B + 2\cos A.\cos B = \frac{5}{3} \\ \hline 1+1+2(\sin A.\sin B + \cos A.\cos B) = 1\frac{5}{3} \end{array} +$$

$\Rightarrow 2(\cos(A - B)) = 1\frac{5}{3} - 2$

$\Rightarrow \cos(A - B) = \frac{2}{3} : 2$

$\Rightarrow \cos(A - B) = \frac{1}{3}$

Jawaban: A

4. Jika $\cos(x+10^\circ) = a$ dengan $0^\circ < x < 30^\circ$, maka nilai $\cos(2x+65^\circ)$ adalah ...

A. $-\frac{1}{2}\sqrt{2}\left((2a^2-1) - 2a\sqrt{1-a^2}\right)$

B. $\frac{1}{2}\sqrt{2}\left((2a^2-1) - 2a\sqrt{1-a^2}\right)$

C. $\frac{1}{2}\sqrt{2}\left((2a^2-1) + 2a\sqrt{1-a^2}\right)$

D. $\frac{1}{2}\sqrt{2}\left((2a^2-1) + a\sqrt{1-a^2}\right)$

E. $\frac{1}{2}\sqrt{2}\left((2a^2-1) - a\sqrt{1-a^2}\right)$

Pembahasan SMART:

ingat! ingat!

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

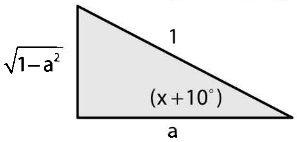
ingat! ingat!

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Soal ini dapat dikerjakan dengan menciptakan bentuk $(2x+65^\circ)$ dari $(x+10^\circ)$, yaitu :

$(2x+65^\circ) = (2(x+10^\circ) + 45^\circ)$

Diketahui bahwa $\cos(x+10^\circ) = a$, maka dapat direpresentasikan pada segitiga siku-siku berikut



Jadi, $\sin(x+10^\circ) = \sqrt{1-a^2}$

Sehingga:

$\cos(2x+65^\circ) = \cos(2(x+10^\circ)+45^\circ)$

$= \cos(2(x+10^\circ))\cos 45^\circ - \sin(2(x+10^\circ))\sin 45^\circ$

$= (2\cos^2(x+10^\circ) - 1)\cos 45^\circ$

$\quad - 2\sin(x+10^\circ)\cos(x+10^\circ)\sin 45^\circ$

$= (2a^2-1)\frac{1}{2}\sqrt{2} - 2\cdot\sqrt{1-a^2}\cdot a\cdot\frac{1}{2}\sqrt{2}$

$= \frac{1}{2}\sqrt{2}\left((2a^2-1) - 2a\sqrt{1-a^2}\right)$

Jawaban: B

5. Nilai dari $\sin 6^\circ - \sin 42^\circ - \sin 66^\circ + \sin 78^\circ$ adalah ....

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

Pembahasan SMART:

1) $\sin 6^\circ - \sin 42^\circ - \sin 66^\circ + \sin 78^\circ$

$= -(\sin 66^\circ - \sin 6^\circ) + \sin 78^\circ - \sin 42^\circ$

$= -2\cdot\frac{1}{2}\cdot\cos 36^\circ \sin 30^\circ + 2\cos 60^\circ \sin 18^\circ$

$= -2\cdot\frac{1}{2}\cdot\cos 36^\circ + 2\cdot\frac{1}{2}\cdot\sin 18^\circ$

$= -\cos 36^\circ + \sin 18^\circ$

$= -(1 - 2\sin^2 18^\circ) + \sin 18^\circ$

$= 2\sin^2 18^\circ + \sin 18^\circ - 1$

2) Bila $x = 18^\circ$, maka $5x = 90^\circ$

Berlaku:

$\sin 3x = \cos 2x$

$\sin(2x + x) = \cos 2x$

$\sin 2x\cdot\cos x + \cos 2x\cdot\sin x = \cos 2x$

$\sin 2x\cdot\cos x = \cos 2x(1 - \sin x)$

$$\frac{\sin 2x}{\cos 2x} = \frac{1-\sin x}{\cos x}$$

$$\frac{2\sin x.\cos x}{\cos^2 x - \sin^2 x} = \frac{1-\sin x}{\cos x}.\frac{1+\sin x}{1+\sin x}$$

$$\frac{2\sin x.\cos x}{\cos^2 x - \sin^2 x} = \frac{1-\sin^2 x}{\cos x(1+\sin x)}$$

$$\frac{2\sin x.\cos x}{\cos^2 x - \sin^2 x} = \frac{\cos^2 x}{\cos x(1+\sin x)}$$

$2\sin x + 2\sin^2 x = \cos^2 x - \sin^2 x$

$3\sin^2 x + 2\sin x = \cos^2 x$

$3\sin^2 x + 2\sin x = 1 - \sin^2 x$

$4\sin^2 x + 2\sin x - 1 = 0$

Dengan rumus a – b – c:

$$\sin x = \frac{-2 \pm \sqrt{2^2 - 4.4.(-1)}}{2.4} = \frac{-2 \pm 2\sqrt{5}}{2 \cdot 4}$$

Karena $x = 18°$ di kuadran I, maka $\sin x = -\frac{1}{4} + \frac{1}{4}\sqrt{5}$

- Dengan demikian, diperoleh:

$2\sin^2 18° + \sin 18° - 1$

$= 2.\left[-\frac{1}{4}\left(1-\sqrt{5}\right)\right]^2 + \frac{1}{4}\sqrt{5} - \frac{1}{4} - 1$

$= \frac{2}{16}\left(1-2\sqrt{5}+5\right) + \frac{1}{4}\sqrt{5} - \frac{1}{4} - 1$

$= \frac{3}{4} - \frac{1}{4}\sqrt{5} + \frac{1}{4}\sqrt{5} - \frac{1}{4} - 1$

$= -\frac{1}{2}$

Jawaban: B

6. Jika $\cos A = \frac{3}{5}$ dan $p < A < 2p$ maka nilai

$\frac{\sin A}{\cos A} - \frac{1}{\sin A} = ...$

A. $-\frac{1}{2}$

B. $-\frac{1}{12}$

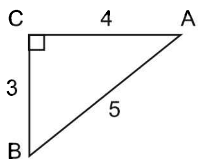
C. $\frac{1}{12}$

D. $\frac{4}{5}$

E. 2

Pembahasan:

$p < A < 2p$ karena $\cos A = \frac{3}{5}$ maka terletak kuadran IV



$\cos A = \frac{3}{5}$; $\sin A = -\frac{4}{5}$; $\tan A = -\frac{4}{3}$

$$\frac{\sin A}{\cos A} - \frac{1}{\sin A} = -\frac{4}{3} - \frac{1}{-\frac{4}{5}} = -\frac{4}{3} + \frac{5}{4} = -\frac{1}{12}$$

Jawaban: B

7. Jika $\cos^2 x = \sqrt{3}\sin x$, maka $\sin x = ...$

A. $\frac{1-2\sqrt{3}}{2}$

B. $\frac{1-\sqrt{3}}{2}$

C. $\frac{2-\sqrt{3}}{2}$

D. $\frac{\sqrt{7}+\sqrt{3}}{2}$

E. $\frac{\sqrt{7}-\sqrt{3}}{2}$

Pembahasan:

$\cos^2 x = \sqrt{3}\sin x$

$1 - \sin^2 x = \sqrt{3}\sin x$

$0 = \sin^2 x + \sqrt{3}\sin x - 1$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin x_{1,2} = \frac{-\sqrt{3} \pm \sqrt{\left(\sqrt{3}\right)^2 - 4.1.(-1)}}{2.1}$$

$\sin x_{1,2} = \frac{-\sqrt{3} \pm \sqrt{7}}{2}$ (pilih yang positif yang memenuhi)

$\sin x = \frac{-\sqrt{3} + \sqrt{7}}{2}$

Jawaban: E