# Leetcode 1911 - Maximum Alternating Subsequence Sum

Example 1 : [4, 2, 5, 3]

Output : 7

Optimal to choose the subsequence [4, 2, 5]

$\Rightarrow 4 - 2 + 5$

$\Rightarrow 7$

Example 2 :-

nums = [5, 6, 7, 8]

output = 8

Optimal to choose the subsequence = [8]

Example 3 :-

nums = [6, 2, 1, 2, 4, 5]

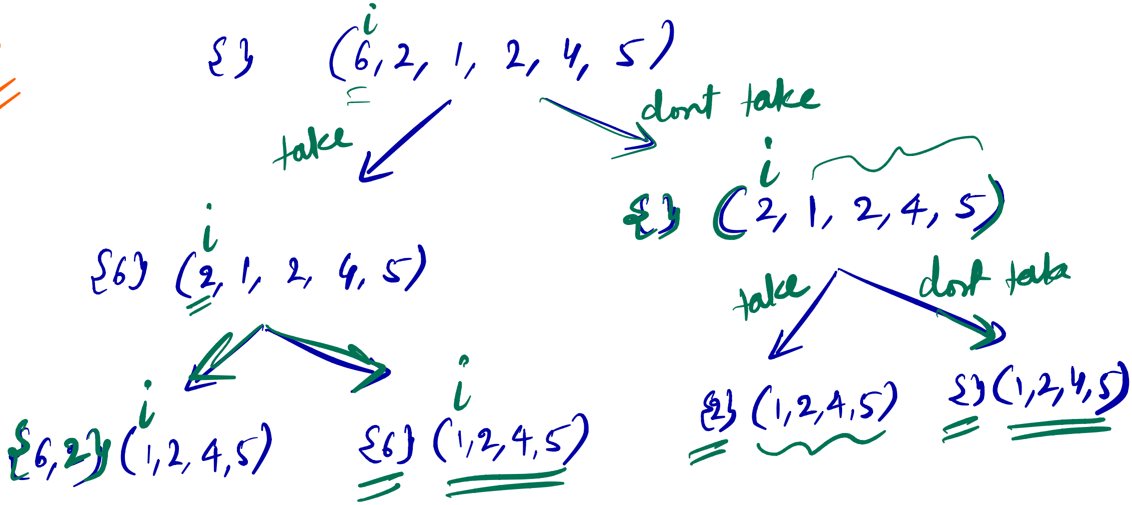
Output = 10

Optimal to choose the subsequence = [6, 1, 5]

$= 6 - 1 + 5$

$= 5 + 5 = 10$

DP




1. This shows we have - overlapping subproblems
2. We can form answer to a larger range by solving for smaller ranges - optimal substructure.

Lets define a dp state :-

dp[i][0] - states the maximum sum we can make using a prefix of length i which is even.

dp[i][1] = states the maximum sum we can make using a prefix of length i which is odd.

(I)

$\longleftarrow$ even $\longrightarrow$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$\longleftarrow$ odd $\longrightarrow$ $\quad i$

$\rightarrow a_1 \quad a_2 \quad a_3 \quad a_4$

$\rightarrow a_1 - a_2 + a_3 - a_4$

$\Rightarrow dp[i][even] = \text{take } a(i) \;\Big|\; \text{dont take } a(i)$

$dp(i-1)(odd) - a(i) \qquad\qquad dp(i-1)(even)$

(II)

$\longleftarrow$ odd $\longrightarrow$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$\longleftarrow$ even $\longrightarrow$

$\Rightarrow dp[i][odd] = \text{take } a(i) \;\Big|\; \text{dont take } a(i)$

$dp(i-1)(even) + a(i) \qquad\qquad dp(i-1)(odd)$

$$dp(i)(even) = \max\big(dp(i-1)(even);\ dp(i-1)(odd) - a(i)\big)$$

$$dp(i)(odd) = \max\big(dp(i-1)(odd);\ dp(i-1)(even) + a(i)\big)$$

$6-2+1=5$

| | | 6 | 2 | 1 | 2 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| even dp[i][0] | 0 | (0, 0-6) = 0 | (0, 6-2) = 4 | (4, 6-1) = 5 | (5, 6-2) = 5 | (5, 7-4) = 5 | 5, 9-5 = (5) |
| odd dp[i][1] | 0 | (0, 0+6) = 6 | (6, 0+2) = 6 | (6, 4+1) = 6 | (6, 5+2) = 7 | (7, 5+4) = 9 | (9, 5+5) = 10 |
| | | i=1 | i=2 | i=3 | i=4 | | |

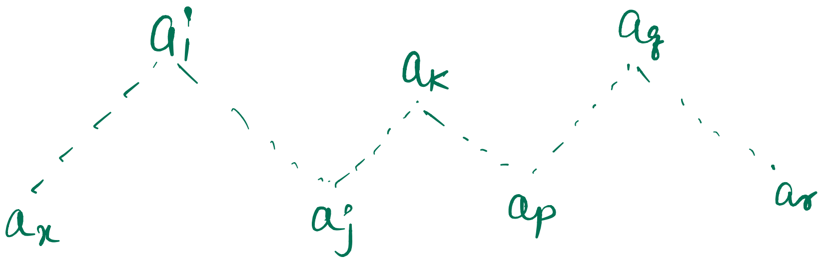
Ans

Time Complexity = $O(n)$

where $n$ is the length of the array.

Space Complexity = $O(n)$

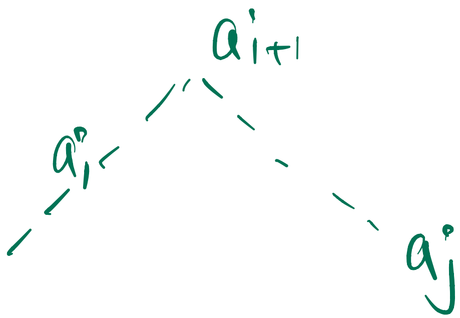
Greedy :-




Solution $\rightarrow (a_i - a_j + a_k - a_p + a_q)$

only take local maxima & local minima

Proof :-




(I) Consider $a_{i+1}$ in ans

$\Rightarrow a_i, a_{i+1}, a_j$

$\Rightarrow (a_i - a_{i+1}) + a_j$

(II) Dont consider $a_{i+1}$ in ans

$\Rightarrow a_i - a_j$

& $a_i - a_j < a_{i+1} - a_j$

as $a_i < a_{i+1}$

assume its better than $a_{i+1} - a_j$

so $\Rightarrow (a_i - a_{i+1}) + a_j > a_{i+1} - a_j$

$\Rightarrow a_i > 2a_{i+1} - 2a_j$

$\Rightarrow a_i/2 > a_{i+1} - a_j$

Hence our assumption was wrong and we find $(a_i - a_{i+1} + a_j) < a_{i+1} - a_j$

Hence we proof it's optimal to take local maximum. and similar for local minimum.