As the wave is propagating in the x-direction, therefore, if

$$\bar{E}(x,t) = \bar{E}_0 e^{i(\bar{k}\cdot\bar{x} - \omega t)} \hat{j}.$$

then

$$\bar{B}(x,t) = \frac{1}{c} \times \bar{E}_0 e^{i(\bar{k}\cdot\bar{x} - \omega t)} \hat{k}.$$

The electromagnetic wave as a whole is said to be polarised along y-direction by convention, we use the direction of $\bar{E}$ to specify the polarisation of an electromagnetic wave.

Electromagnetic wave in a free conducting media

For free space the free charge density ρ_f and free current density J_f bee zero, and every property of E.M. wave we derived was based on this assumption. But in case of conductors (sea water, metals etc.) we can not control the flow of charge and in general $\bar{J}_f$ is not zero. In fact according to Ohm's law, the current density is proportional to the electric field:

$$\bar{J}_f = \sigma \bar{E}$$

Here σ is an emperical constant that varies from metal to ~~bo~~ metal, it is called conductivity of the medium. With this, Maxwell's equations for <u>linear media</u> assume the form

(i) $\nabla \cdot \bar{E} = \rho_f / \epsilon$ (iii) $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

(ii) $\nabla \cdot \bar{B} = 0$ (iv) $\nabla \times \bar{B} = \mu\sigma\bar{E} + \mu\epsilon\frac{\partial \bar{E}}{\partial t}$

Now the continuity equation for the free current $\nabla \cdot \bar{J}_f = -\frac{\partial \rho_f}{\partial t}$. Together with Ohm's law and Gauss' law we get

$$\frac{\partial \rho_f}{\partial t} = -\sigma(\nabla \cdot E) = -\frac{\sigma}{\epsilon}\rho_f$$

$\therefore \boxed{\frac{\partial \rho_f}{\partial t} = -\frac{\sigma}{\epsilon}\rho_f}$ ~~fo~~ from which it follows that $\rho_f(t) = \rho_f(0)\, e^{-\left(\frac{\sigma}{\epsilon}\right)t}$

$$\boxed{\rho_f(t) = \rho_f(0)\, e^{-(\sigma/\epsilon)t}}$$

Thus any initial free charge density $\rho(0)$ dissipal in a characteristic time $\tau = (\epsilon/\sigma)$. This reflects the familiar fact that if one put some free charge on a conductor, it will flow out to the edges. At present we are not interested in this kind of transien behaviour. We will wait for any accumulated free charge to disappear from then on $\rho_f = 0$ and we have

$$\text{(i)}\ \nabla\cdot\bar{E} = 0 \qquad \text{(iii)}\ \nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$$

$$\text{(ii)}\ \nabla\cdot\bar{B} = 0 \qquad \text{(iv)}\ \nabla\times\bar{B} = \mu\sigma\bar{E} + \mu\epsilon\frac{\partial\bar{E}}{\partial t}$$

Applying curl to (iii) and (iv) as before, we obtain modified wave equations for $\bar{E}$ & $\bar{B}$:

$$\nabla^2\bar{E} = \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} + \mu\sigma\frac{\partial\bar{E}}{\partial t}$$

$$\nabla^2\bar{B} = \mu\epsilon\frac{\partial^2\bar{B}}{\partial t^2} + \mu\sigma\frac{\partial\bar{B}}{\partial t}$$

These equations again admit plane-wave solutions:

$$\bar{E}(x,t) = \bar{E}_0 e^{i(\bar{k}\cdot\bar{x} - \omega t)}$$

$$\bar{B}(x,t) = \bar{B}_0 e^{i(\bar{k}\cdot\bar{x} - \omega t)}$$

Substituting the solutions for electric field in the above corresponding wave equation, we get

$$-k^2\bar{E}(x,t) = -\mu\epsilon\omega^2\bar{E}(x,t) - i\sigma\mu\omega\bar{E}(x,t)$$