Magnetostatics

Lorentz force

When a charge Q moves in a magnetic field $\vec{B}$ with a velocity $\bar{v}$, the magnetic force experienced by the charged particle $\vec{F}$ will be perpendicular to the $\bar{B}$ & $\bar{v}$. Thus the combination of the directions is just right for a cross product. Therefore the magnetic force is given by $\bar{F}_{mag} = Q(\bar{v} \times \bar{B})$. When we combine this with the electric force, $\bar{F}_{elec} = Q\bar{E}$, we obtain Lorentz force law

$$\vec{F} = Q\left[\bar{E} + (\bar{v} \times \bar{B})\right].$$

Magnetic force on small current element placed in a magnetic field

The current in a wire is the charge per unit time passing a given point. Therefore, current is measured in Coulomb per second or amperes (A). That is

$$1\,A = 1\,C/s$$

If a line charge λ moving down a wire at a speed v, in time Δt, a segment of length $v\Delta t$ will carry a charge $\lambda v \Delta t$

Thus current a each point will be

$$I = \frac{\lambda v \Delta t}{\Delta t} = \lambda v$$ and $\vec{I}$ is a ~~magnetic~~ vector quantity as $\bar{v}$ is a vector. If we have a segment $\vec{dl}$, then the magnetic force ~~of~~ on the segment of current carrying wire is

$$\vec{F}_{mag} = \int \lambda \, \bar{dl} \, (\bar{v} \times \bar{B}) = \int (\bar{I} \times \bar{B}) \, \bar{dl}$$

Here $\bar{I}$ and $\bar{dl}$ both point the same direction, then we can write

$$\vec{F}_{mag} = I \int (\bar{dl} \times \bar{B})$$ As I is constant along the wire. Therefore, the relation of mag. force is

$$\boxed{\bar{F}_{mag} = I \int (\bar{dl} \times \bar{B})}$$

Magnetic field of a steady current: Biot-Savart law

The magnetic field of a steady line current is given by the Biot-Savart law

$$\bar{B} = \frac{\mu_0}{4\pi}\int \frac{\bar{I}\times\hat{r}}{r^2}\, dl$$

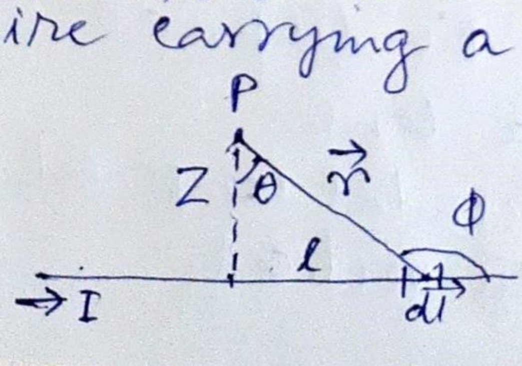
$$\boxed{\bar{B} = \frac{\mu_0 I}{4\pi}\int \frac{\bar{dl}\times\hat{r}}{r^2}}$$

The integration is along the current path, in the direction of flow, $\bar{dl}$ is an element of length of the wire, $\vec{r}$ is the vector from source to the point p where mag. field is measured. μ_0 is the permeability of free space, it's value is $\mu_0 = 4\pi\times10^{-7} N/A^2$. Thus the unit of $\bar{B}$ comes out in Newtons/ampere-meter. It is defined as teslas (T)

$$\therefore\ 1\ \text{tesla} = 1N/(A.m)$$

Application of Biot-Savart law

* Magnetic field at a point z distance from a wire carrying a steady current I.



Here $\vec{dl} \times \hat{r}$ points out of paper and has the magnitude

$$dl \sin\phi = dl \cos\theta$$

also $l = z \tan\theta$, $\therefore dl = \frac{z}{\cos^2\theta} d\theta$, $\frac{z}{r} = \cos\theta$

So, $\frac{1}{r^2} = \frac{\cos^2\theta}{z^2}$

Thus $\bar{B} = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \frac{\cos^2\theta}{z^2} \times \frac{z}{\cos^2\theta} \cos\theta \, d\theta$

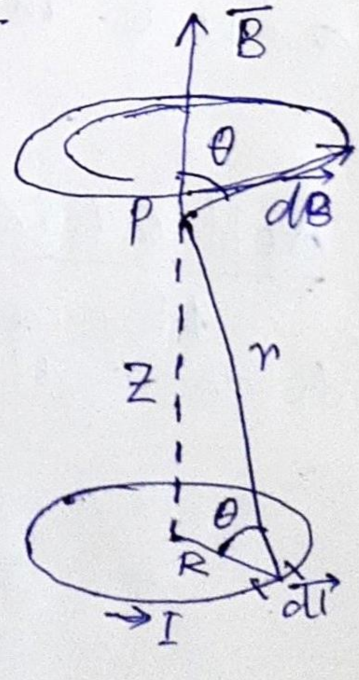
$$= \frac{\mu_0 I}{4\pi z} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \frac{\mu_0 I}{4\pi z}(\sin\theta_2 - \sin\theta_1)$$

for an infinite wire $\theta_1 = -\pi/2, \theta_2 = \pi/2$

So we obtain

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi z}}$$

☆ Magnetic field at a distant point 'z' above the center of a circular loop of radius R carries a steady current I



Let the magnetic field $\vec{dB}$ attributed due to the segment $\vec{dl}$. As we integrate $\vec{dl}$ around the loop, $\vec{dB}$ sweeps out a cone.

The horizontal components cancel and the vertical components combine to give

$$\bar{B} = \frac{\mu_0 I}{4\pi} \int \frac{dl}{r^2} \cos\theta$$

Here
$r^2 = Z^2 + R^2$
$\cos\theta = \frac{R}{r}$

Here $\int dl = 2\pi R$

Hence, $$|\bar{B}| = \frac{\mu_0 I}{4\pi} \left(\frac{\cos\theta}{r^2}\right) 2\pi R$$

$$= \frac{\mu_0 I R}{2} \frac{\cos\theta}{r^2}$$

$$= \frac{\mu_0 I R}{2} \times \frac{R}{r^3}$$

$$\therefore \boxed{\bar{B} = \frac{\mu_0 I R}{2} \times \frac{R}{(R^2 + Z^2)^{3/2}} \hat{k}}$$