Faraday's law of electromagnetic induction

Faraday's law tells whenever the flux (ϕ) of magnetic field through the area bounded by a close conducting loop changes an emf ($\mathcal{E}$) is produced in the loop.

Hence $\mathcal{E} \propto \frac{d\phi}{dt}$

Lenz's law says the direction of induced emf. Thus the law states that if a current flows, it will be in such a direction that the magnetic field it produces tends to contradict the change in flux that induced the emf.

Therefore, we can write $\mathcal{E} = -\frac{d\phi}{dt}$

Integral form of Faraday's law

At any time t, the magnetic flux through a closed coil is ϕ, then by Faraday's law, the induced emf is

$$\mathcal{E} = -\frac{d\phi}{dt}$$

If $\bar{E}$ is the electric field induced in the space, then the induced emf $\mathcal{E}$ around the closed path C is given by line integration of $\bar{E}$. Thus, $\mathcal{E} = \oint_c E \cdot d\bar{l}$

Therefore, we get $\oint \bar{E} \cdot d\bar{l} = -\frac{d\phi}{dt}$

We know the total flux through a closed conductor is the integral of normal component of mag. field B over the surface bounded. Hence,

$$\phi = \oint_s \bar{B} \cdot d\bar{s}$$

$$\therefore \oint_{C} \bar{E}\cdot d\bar{l} = -\oint_{S} \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

This is the integral form of Faraday's law of electromagnetic induction.

Differential form of Faraday's law

Our integral form of Faraday's law is given by

$$\oint_{C} \bar{E}\cdot d\bar{l} = -\oint \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

We can convert the line integral into surface integral using Stoke's theorem, which is

$$\oint_{S} (\nabla\times\bar{E})\cdot d\bar{s} = -\oint_{S} \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

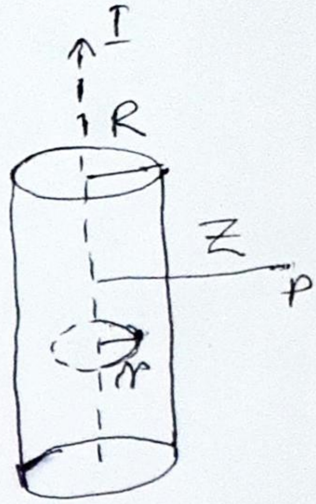
So $$\boxed{\nabla\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

Which is called differential form of Faraday's law of electromagnetic induction.

It says that the ~~etal~~ electric field is no longer a conservative field when the magnetic field varies with time.

Application of Ampere's Circuital Law

Long Straight Cylindrical wire



Let us consider an infinitely long conducting wire of radius R, carrying I as shown in the figure. Suppose the current distribution is uniform throughout the crosssection of the wire. Now applying Ampere's law to an amperian loop of radius r is

$$\oint \bar{B} \cdot \bar{dl} = \mu_0 I_{enc}.$$

Where $I_{enc} = \frac{I}{\pi R^2} \times \pi r^2 = I \frac{r^2}{R^2}$

Therefore, we get

$$\oint \bar{B} \cdot \bar{dl} = I \frac{r^2}{R^2} \mu_0$$

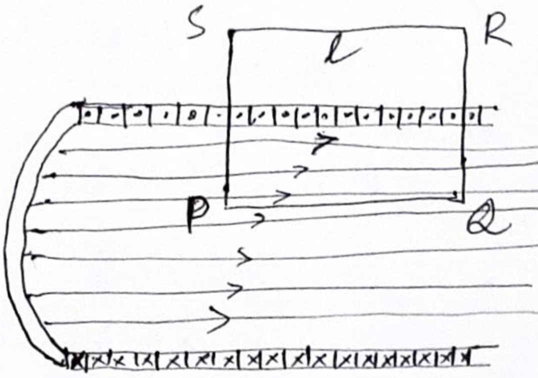
or, $B \times 2\pi r = \mu_0 \frac{I r^2}{R^2}$

$$\therefore \boxed{B = \mu_0 \frac{I r}{2\pi R^2}}$$

which is the magnetic field at an inner point of the conductor at a distance r ~~of~~ from the axis.

Magnetic field inside a long solenoid

When a current I carrying wire is wound tightly on the surface of a cylindrical tube, we get a solenoid. Generally the length (L) of the solenoid is large as compared to the transverse dimension. If N is the total



number of turns over a length L, we get $N/L = n$ as the number of turns per unit length. Keeping the product nI fixed, if we make n very large and corresponding I very small, then we get surface current of value nI over the curved surface of the cylinder. Ampere's law can be easily applied to find out magnetic field $\vec{B}$ inside the solenoid.

We draw a rectangle $PQRS$ of length l. The line PQ is parallel to the solenoid axis and hence parallel to the magnetic field $\vec{B}$ inside the solenoid.

Thus

$$\int_P^Q \bar{B}\cdot\bar{dl} = Bl$$

Along QR, RS and SP, $\bar{B}\cdot\bar{dl}$ is zero as $\bar{B}$ is zero outside or perpendicular to $\bar{dl}$. Thus we get

$$\oint \bar{B}\cdot\bar{dl} = \mu_0 I_{enc}.$$

Here $I_{enc} = nl I$

Where I current is flowing through the wire.

$$\oint \bar{B}\cdot\bar{dl} = \mu_0 n l I$$

or, $B \times l = \mu_0 n l I$

$$\therefore \boxed{B = \mu_0 n I}$$

This equation gives the magnetic field inside the solenoid.