The earlier equation is same as Poisson's equation in electrostatics which is

$$\nabla^2 V = -\rho/\epsilon_0$$

Where V is the electrostatic potential and satisfies

$$V = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho\, dv}{r}$$

In analogy with this we can write

$$\bar{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}\, dv}{r} \quad \text{for volume current}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_S \frac{\vec{K}\, ds}{r} \quad \text{for surface current}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_l \frac{\vec{I}\, dl}{r} \quad \text{for line current.}$$

Problem: A magnetic field $4\times10^{-3}\hat{k}$ tesla exerts a force of $(4\hat{i}+3\hat{j})\times10^{-10}$ N on a particle having charge of 1×10^{-9} C and moving in the xy plane, calculate the velocity of the particle.

Soln: We know the magnetic force

$$\bar{F} = q(\bar{v}\times\bar{B})$$

Here $(4\hat{i}+3\hat{j})\times10^{-10} = 1\times10^{-9}\left[(v_x\hat{i}+v_y\hat{j})\times4\times10^{-3}\hat{k}\right]$

$$= 4\times10^{-12}\left[v_x(-\hat{j})+v_y(\hat{i})\right]$$

$$\therefore v_x = -\frac{3\times10^{-10}}{4\times10^{-12}}\,\text{m/s} = -75\,\text{m/s}$$

$$v_y = \frac{4\times10^{-10}}{4\times10^{-12}}\,\text{m/s} = 100\,\text{m/s}$$

Therefore velocity $\bar{v} = (-75\hat{i}+100\hat{j})$ m/s

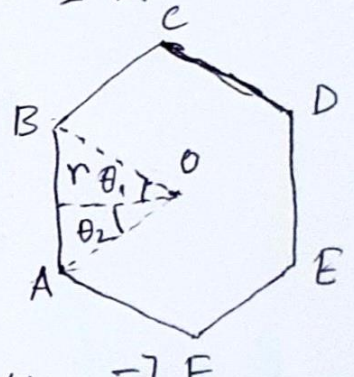
Problem: How many electrons pass through a wire in 1 minute if current passing through the wire is 200 mA.

Soln: We know the current $I = \frac{q}{t}$

$\therefore I = \frac{ne}{t}$ = ~~...~~ $\therefore n = \frac{It}{e} = \frac{200 \times 10^{-3} \times 60}{1.6 \times 10^{-19}}$

$\therefore n = 7.5 \times 10^{19}$ is the number of electrons passing through the wire.

Problem: Calculate the magnetic field at the center of a regular hexagon of side 'a' meter and carrying a current I A.



Soln: The magnetic field at the center O due to the segment AB of the hexagon is $B' = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\theta_1 + \sin\theta_2)$

Here $r = \frac{\sqrt{3}}{2}a$ and $\theta_1 = \theta_2 = 30^\circ$, $\frac{\mu_0}{4\pi} = 10^{-7}$

So $B' = 10^{-7} \times \frac{I}{\frac{\sqrt{3}}{2}a}(\sin 30^\circ + \sin 30^\circ) = 10^{-7} \times \frac{2I}{\sqrt{3}a}\left(\frac{1}{2} + \frac{1}{2}\right)$

$\therefore B' = 10^{-7} \times \frac{2I}{\sqrt{3}a}$ Tesla

$\therefore$ Therefore, total magnetic field at center O of the hexagon is

$B = 6B' = 6 \times 10^{-7} \times \frac{2I}{\sqrt{3}a} = \frac{4\sqrt{3}}{a} \times I \times 10^{-7}$ Tesla

Problem:

A test charge having a magnitude of 0.4C is moving with a velocity of $4\hat{i}-\hat{j}+2\hat{k}$ m/s through an electric field intensity $10\hat{i}+10\hat{k}$ and a magnetic field $2\hat{i}-6\hat{j}-6\hat{k}$ Tesla. Determine the magnitude and direction of the Lorentz force acting on the test charge.

Soln: The Lorentz force is $\bar{F} = q\bar{E} + q(\bar{v}\times\bar{B})$

Here, the electric force is $= q\vec{E} = 0.4(10\hat{i}+10\hat{k})$

and magnetic force $= q(\bar{v}\times\bar{B})$

$$= 0.4\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ 2 & -6 & -6 \end{vmatrix} = 0.4(18\hat{i}+28\hat{j}-22\hat{k})$$

$= \cancel{0.4}[($

Now the Lorentz force

$$\bar{F} = q\bar{E} + q(\bar{v}\times\bar{B})$$
$$= 0.4(10\hat{i}+10\hat{k}) + 0.4(18\hat{i}+28\hat{j}-22\hat{k})$$
$$= 0.4(28\hat{i}+28\hat{j}-12\hat{k})$$

Therefore, the magnitude of the Lorentz force is $0.4\sqrt{(28)^2+(28)^2+(-12)^2} = 16.6N$.

Let the force makes θ angle with x-axis

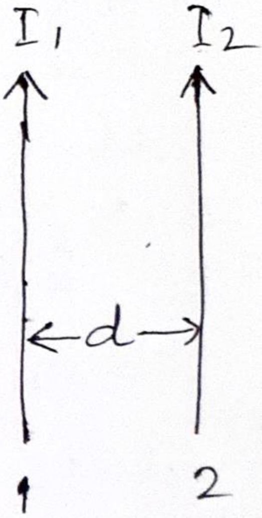
$\therefore \bar{F}.\hat{i} = 0.4(28\hat{i}+28\hat{j}-12\hat{k}).\hat{i} = 0.4\times 28$

or, $F\cos\theta = 0.4\times 28$ or, $\cancel{0.4}\sqrt{(28)^2+(28)^2+(-12)^2}\cos\theta = 28$

$\therefore \cos\theta = \frac{\cancel{28}\,7}{\sqrt{107}}$ or, $\theta = \cos^{-1}\left(\frac{7}{\sqrt{107}}\right) = 47.41^\circ$

$\therefore$ The force acting at 47.41° angle w.r.t x-axis.

Calculate the force of attraction between two long parallel wires, 'd' distant apart carrying I_1, I_2 current.



Let in wires 1 and 2 current I_1 and I_2 flowing respectively

~~Fiel~~ Magnetic field at wire 2 due I_1 current is

$$B_{21} = \frac{\mu_0 I_1}{2\pi d}$$

The force law predicts a force directed toward wire 1 of magnitude

$$F_{21} = I_2 \left(\frac{\mu_0 I_1}{2\pi d}\right) \int dl$$

Total force will be infinite if we integrate dl, but the force per unit length is

$$\boxed{F = \frac{\mu_0}{2\pi}\left(\frac{I_1 I_2}{d}\right)}$$