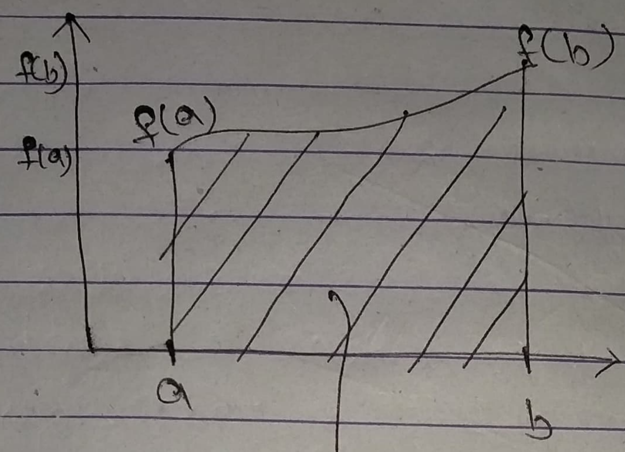
⇒ ~~[illegible]~~ integrals :-



integral → $\int_a^b f(x)\,dx$

Integrals have two types of integrals :-

i) proper integrals :-

Integral of proper curve is called proper integrals.

proper curve -

- exist with in the region/domain.
- well defined in the region.
- Continuous in the region.

ii> improper integrals

Integral which is not proper.

There are two types of improper integrals.

a) Infine integrand

In this type of improper ~~integral~~ integral, ~~which~~ with in the domain at some/one point value of integrand in infinite.

ex.- $\int_a^b f(x)$ where 'c' is a point such that $f(c)$ is ∞, $a<c<b$

b) Infinite interval

In infinite interval one or both the limits of integration are infinite.

- $\int_{-\infty}^{\infty} f(x)$. $\int_{-\infty}^{a} f(x)$. $\int_{a}^{\infty} f(x)$

SO, we can say improper integrals are those in which integrand is infinite ~~whi within~~ ~~[illegible]~~ or infinite for the range of the integration.

→ Convergent :-

These integral which have sensible finite value, are called convergent.

Sensible means integrand has finite value within the limiting range, where integrand must also be finite.

for example,

if $I = \int_a^\infty f(x)\,dx$, where f is continuous

for $x \geq a$, and the limit

$$L = \lim_{t \to \infty} \int_a^t f(x)\,dx$$ exists and

is finite, then I will be convergent and converges to L. Otherwise, I diverges.

We can also understand it, as follow

$$\lim_{x \to a} f(x) = f(a) \to \text{finite quantity}$$

here $x \neq a$ and $f(x) \not> f(a)$

So, in it's limiting range till a all values are converging to f(a) but not equal to f(a), so, f(x) will be convergent and will converge to f(a).

Like this if any Limit/integrand/integral has finit value wrt limiting range and these values and converging to a specific value ~~(like f(a) if a is upper limit)~~ the which is finite then it is convergent and converges to that specific value.

also,

if $\int_a^b f(x)$ exists, where c is point of dis-continuty as

$b > c > a$

then $\lim_{\phi \to 0} \int_a^{c-\phi_1} f(x) + \lim_{\phi_2 \to 0} \int_{c+\phi_2}^{b} f(x)$ will be
(1) (2)

convergent only if both (1) & (2) are convergent, any one of them is divergent then $\int_a^b f(x)$ will be divergent.

→ **Divergent :-**

Those integrals which do not have sensible finite value are called divergent.

If an integrals have infinite value wrt it's limiting range it will be divergent.

ex. -

$$\lim_{x \to \infty} f(x) = f(\infty) \approx \infty$$

here, every value of x approching towards ∞, value of integral approces towards ∞, so, given integral is divergent.