Therefore phase difference between two particles on the wave is

$$\delta = \frac{2\pi}{\lambda}(vt - r_1) - \frac{2\pi}{\lambda}(vt - r_2) = \frac{2\pi}{\lambda}(r_2 - r_1)$$

Now $r_2 - r_1$ is path difference between two particles. So $\boxed{\text{phase difference} = \frac{2\pi}{\lambda} \times \text{path differen}}$

Path difference for particles in same phase

If two particles are in same phase of oscillation, the phase difference $\delta = 2\pi n$, Hence, $2n\pi = \frac{2\pi}{\lambda} \times$ path difference. Therefore, path difference $= n\lambda$, where $n = 0, 1, 2, \ldots$ etc.

Path difference $= 2n \times \frac{\lambda}{2}$ which is even multiple of $\lambda/2$.

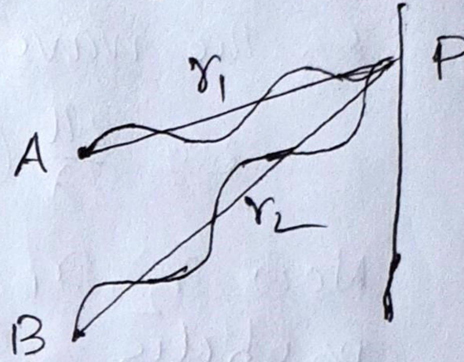
Path difference for particles in opposite phase

If two particles in opposite phase of oscillation, the phase difference $\delta = 2n\pi + \pi$. So, $\cancel{(2n+1)\pi =}$

$$\text{So } (2n+1)\pi = \frac{2\pi}{\lambda} \times \text{path difference}.$$

Hence, path difference $= (2n+1)\frac{\lambda}{2}$, which is odd multiple of $\lambda/2$. If the phase difference is constant with time, then the waves are said to be coherent.

Formulation of Interference Intensity



Let two monochromatic waves of light of wave-length λ from two sources A and B and superpose at a point P of the medium. Let y_1 and y_2 are the displacements of wave from A & B.

Then we can write $y_1 = a \sin \frac{2\pi}{\lambda}(vt - r_1) = a \sin\theta$

$y_2 = b \sin \frac{2\pi}{\lambda}(vt - r_2) = b \sin\left[\frac{2\pi}{\lambda}(vt - r_1) + \delta\right] = b \sin(\theta + \delta)$

So the resultant displacement

$$y = y_1 + y_2 = a\sin\theta + b\sin(\theta+\delta)$$

$$= a\sin\theta + b\sin\theta\cos\delta + b\cos\theta\sin\delta$$

$$= (a + b\cos\delta)\sin\theta + b\sin\delta\cos\theta$$

$$= A\cos\phi\sin\theta + A\sin\phi\cos\theta$$

Let $a + b\cos\delta = A\cos\phi$
$b\sin\delta = A\sin\phi$

$$= A\sin(\theta + \phi)$$

So ~~the~~ $A^2 = b^2\sin^2\delta + a^2 + b^2\cos^2\delta + 2ab\cos\delta$

$$= a^2 + b^2 + 2ab\cos\delta$$

Intensity of light I = square of the amplitude

Hence, $I = A^2 = a^2 + b^2 + 2ab\cos\delta$.

Thus we can get the condition of maximum intensity depending on the value of phase δ.

Condition for maximum intensity

The intensity will be maximum when $\cos\delta = +1$ i.e, phase difference $\delta = 2n\pi$, where, $n = 0, \pm 1, \pm 2 \ldots$

Hence, path difference $= 2n\pi \times \frac{\lambda}{2\pi} = n\lambda$

The maximum intensity $I_{max} = (a+b)^2$

When $a = b$, $I_{max} = 4a^2$

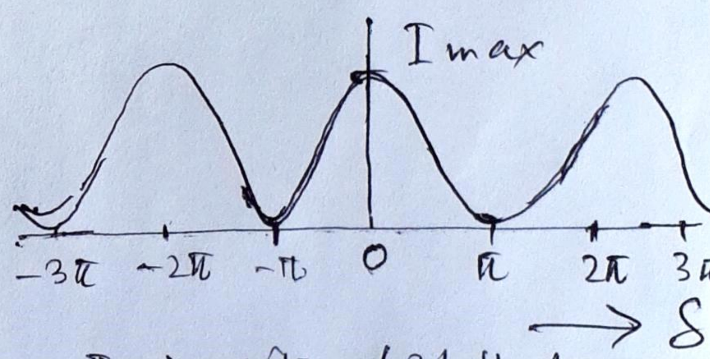
Condition for minimum intensity

When $\cos\delta = -1$, the intensity becomes minimum. i.e $\delta = (2n+1)\pi$, $n = 0, \pm 1, \pm 2 \ldots$

$\therefore$ Path difference $= (2n+1)\pi \times \frac{\lambda}{2\pi} = (2n+1)\frac{\lambda}{2}$

$I_{min} = (a-b)^2$. When $a = b$, $I_{min} = 0$

At the centre, the phase between two waves is zero produce central maxima and ~~mini~~ there are bands of



Intensity distribution

alternative maximum and minimum intensity. So we get fringe pattern due interference of light.