Chapter 2 – Semiconductor Physics

Objective

Upon completion of this chapter you will be able to:

- Understand the band theory of solids like conductors, semiconductors and insulators.
- Understand different classification of semiconductors like intrinsic and extrinsic.
- Understand electron gas theory
- Determine the electron and hole concentration in a semiconductor
- Determine various parameters like conductivity, mobility
- Understand conduction mechanism i.e. drift and diffusion currents
- Determine the effect of temperature on charge carrier concentration and conductivity.
- Determine hall voltage developed in a semiconducting material.

Introduction

In the last chapter, we discussed about the Atomic Structure and studied about Bohr's Model which determined the energy of electrons in various orbits. The energy depends on which orbit is the electron in and it can only have discrete values and not a continuum of values. This principle is known as Quantization of energy. Pauli's exclusion principle states that two electrons cannot have same quantum state. When atoms interact the energy levels of electrons splits into a band of allowed energy levels. This is basic principle of Band Theory which is first step towards the study of semiconductor behavior.

Semiconductors can be classified as the materials in which the resistivity lies between that of conductors and insulators. Semiconductors are so popular in modern day era due to the fact that their conductivity can be modulated by addition of impurity called as "Doping".

Energy Band Theory

When identical atoms are kept far away from each other then there is no interaction between the atoms and due to that they have identical energy levels. As they are brought closer to each other they begin to interact and we say a bond is formed when the energy of the system is minimum. This is due to the fact that point of minimum energy is the most stable point and such a distance is called as "Interatomic Distance" or "Bond Length".

As Pauli's Exclusion Principle states that two electrons cannot have same quantum state so when atoms interact their energy levels split into multiple energy levels due to interaction energy between the atoms. This leads to formation of band of energy levels.

Let us take an example of Carbon Atom,
Atomic Number (Z) = 6
Electron Configuration: $1s^2$ $2s^2$ $2p^2$
The electrons in outermost shell of an atom are known as
Valence Electrons.
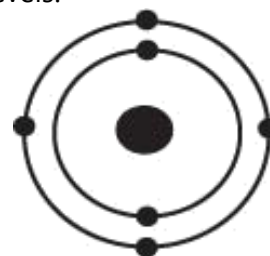
Thus, the outermost orbit here has two orbitals or two energy levels i.e. 2s and 2p.

When two Carbon atoms are brought closer together then the outermost orbit breaks into two energy levels. Each energy level has multiple states in agreement with Pauli's Exclusion Principle that no two electrons can have same quantum state.
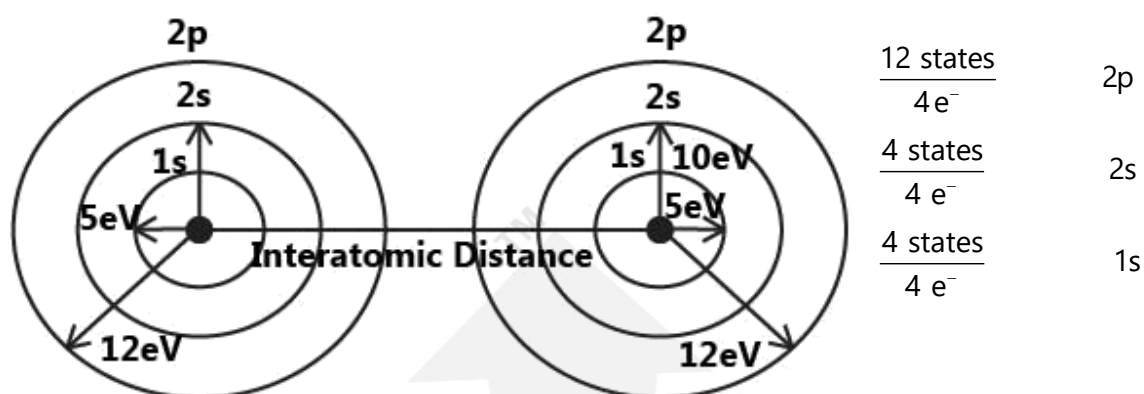

For stability, each atom wants to fulfill ns^2np^6 i.e. $8\ e^-$ in outer orbit.
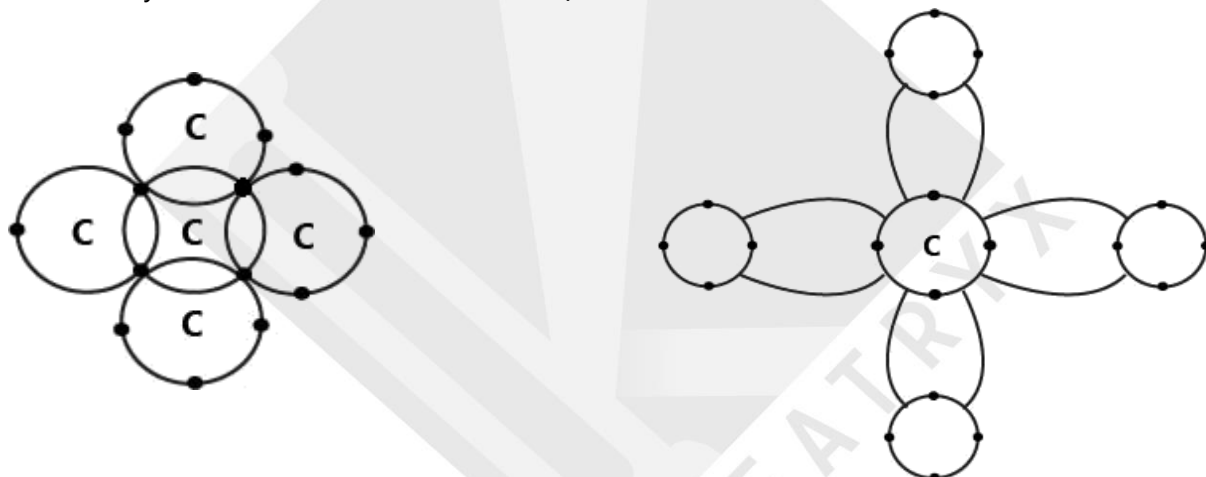
Because of interaction between the atom the outer most energy level which was earlier 12 eV is now been deviated by either 11.99 eV or 12.01 eV.
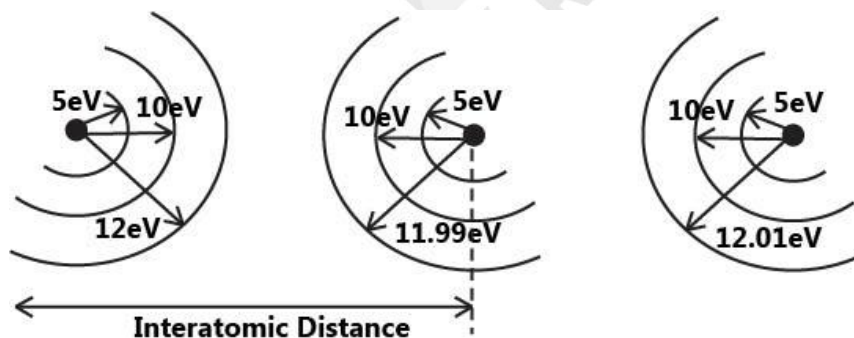

The deviation in energy at the outer most energy level is due to bonding & as they are brought more close the deviation in energy is found more & more.

2

Lattice Constant
for covalent elements
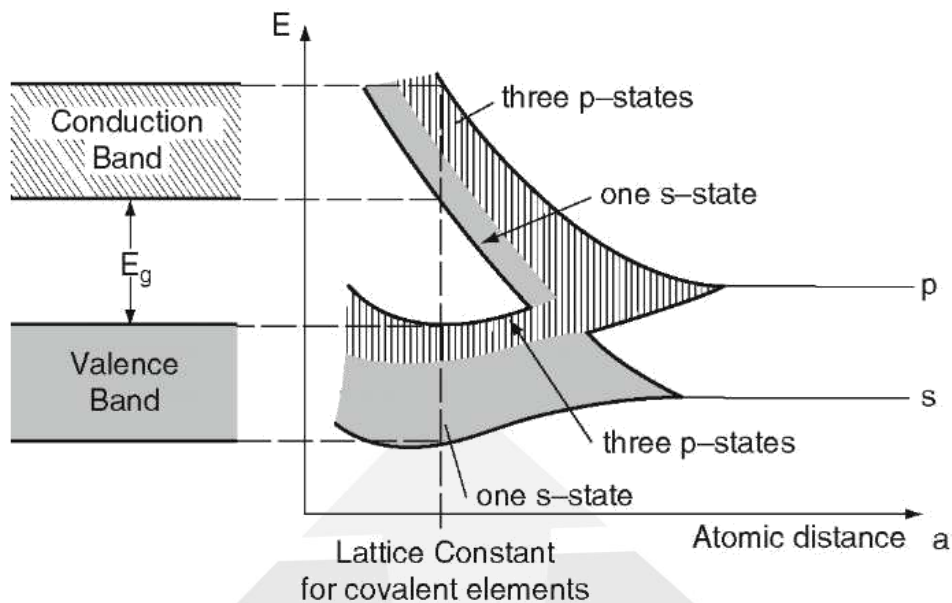
If N atoms are combined together then there are 3N states due to p-orbital in the Conduction Band and N states due to s-orbital in the conduction band. Thus, there are a total of 4N states in the conduction band. By similar logic there are 4N states in the Valence Band.

Since, there are 4 valence electrons in each carbon atom so there are a total of 4N electrons in the outermost orbit of N atoms and they occupy the 4N energy states in the valence band and thus conduction band remains empty.
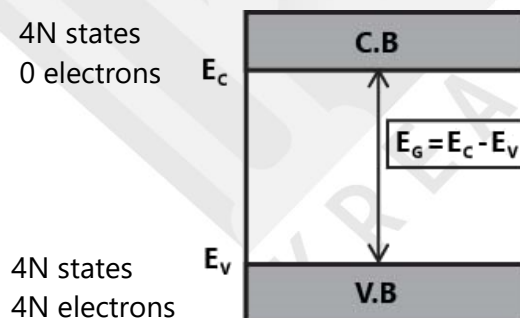
Valence Band

Electrons in the outermost shell of an atom are called as Valence Electrons and the energy band containing these electrons is known as Valence Band. This is the highest occupied energy level by an electron and the Valence Band can never be empty, it is either completely filled or partially filled.

Conduction Band

These are unfilled lowest energy band. Once an electron becomes free from Valence Band i.e.

it gets ionized then it enters Conduction Band and it becomes a part of conduction. It may completely empty or partially filled but never completely filled.

Forbidden Gap

Separation between Conduction Band and Valence Band is known as Forbidden Gap. This gap is equal to energy required to make valence electrons free. If valence electrons are tightly bound to nucleus then higher is the energy gap in the forbidden gap. If electrons are lightly attached to nucleus forbidden gap is small. There are no energy levels in the forbidden gap so no electron exists in this gap and so electron is forbidden to enter this gap.

Metal, Semiconductors and Insulators

Insulator

Band Gap $\left(E_G\right)$ is high and generally greater than 3eV. It means after application of voltage there is need of large amount of energy to make valence electron free. So, in this type of materials conduction is very poor and the value of resistivity is very high.

For ideal case, conductivity is 0 and resistivity is infinite. Eg. Mica, Diamond, Glass, Wood

In general resistivity is of the order is $10^7 \,\Omega-\text{cm}\left(10^{12}\,\Omega-\text{cm for mica}\right)$

Metal

All electrons of valence band are available for conduction in the conduction band. It means very high conductivity. Eg. Silver (best conductor of electricity), Gold, Copper etc.

Number of free electrons in metal are of the order of $10^{28}\,/\text{cm}^3$

Resistivity is of the order of $0.5\,\Omega-\text{cm}$

Semiconductor

It has conductivity between metal and insulator. It means that resistivity is between $0.5\,\Omega-\text{cm}$ and $10^7\,\Omega-\text{cm}$. Energy Gap is of the order of 1eV. Eg. Silicon, Germanium, Gallium Arsenide etc.

The only criteria to decide whether a material behaves as semiconductor or not is the energy gap. Some of the semiconducting materials are:

IV Group: Si, Ge

III + V Group: GaAs, GaP

II + VI Group: CdS

IV + IV Group: SiC

Tertiary semiconductor: GaAsP

GaAs is direct band gap semiconductor and emits light in Infrared region and is used for high speed applications in opto-electronics.

Out of all these semiconductors the most important ones are Si and Ge.

Note: The Band diagram of all three type of materials is shown below

| **Insulator** | **Metal** | **Semiconductor** |
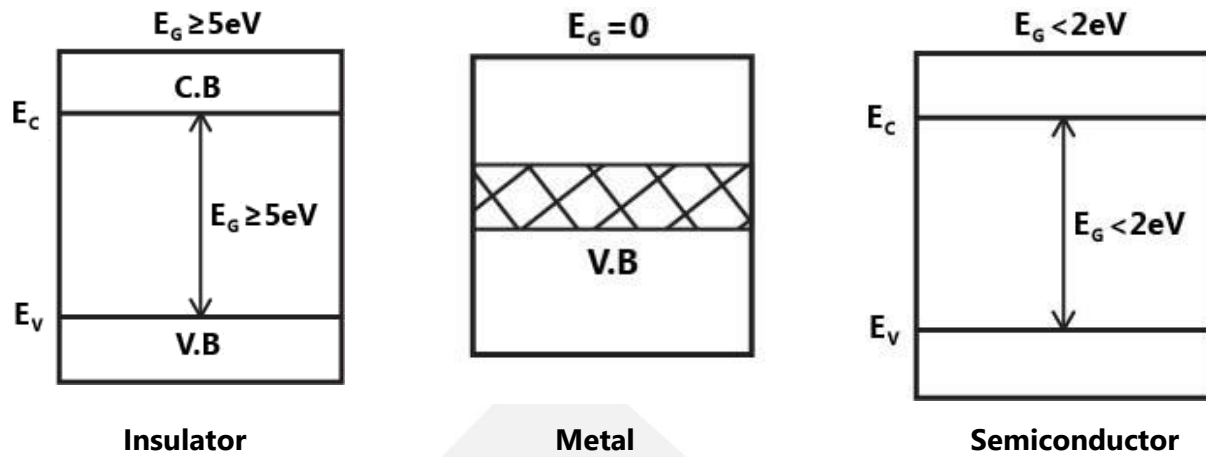
Properties of Si and Ge

Property	Ge	Si
Atomic Number	32	14
Atomic Weight	72.6	28.1
Density (gm/cm^3)	5.32	2.33
Dielectric Constant	16	12
Band Gap at 0K (eV)	0.785	1.21
Band Gap at 300K (eV)	0.72	1.102
Intrinsic Concentration at 300K $(/cm^3)$	2.5×10^{13}	1.5×10^{10}
Intrinsic Resistivity at 300K $(\Omega - cm)$	45	230000
μ_n at 300K $(cm^2/V-s)$	3800	1300
μ_p at 300K $(cm^2/V-s)$	1800	500
D_n at 300K (cm^2/s)	99	34
D_p at 300K (cm^2/s)	47	13
Electron Effective Mass	0.55m	1.08m
Hole Effective Mass	0.37m	0.56m

Note: Both Si and Ge have Diamond Cubic Lattice Structure

Important Data for GaAs

Band Gap at 300K = 1.43 eV

Intrinsic Concentration at 300K = 2×10^6

Electron Mobility = 8500 cm^2 / V − s

Hole Mobility = 400 cm^2 / V − s

Density = 5.32 gm / cm^3

Dielectric Constant = 13.1

Solved Examples

Problem: Why Si is preferred over Ge?

Solution: There are 2 main reasons for Si being the preferred Semiconductor over Ge:

1. SiO$_2$ is an insulator which becomes an important part of fabrication during isolation.
2. In Si, the outermost electrons are in 3rd shell whereas in Ge they are in 4th shell so Si is more stable at room temperature.

Some other differences between Si and Ge are:

1. Power handling capability of Si is higher than Ge
2. Si is easily available on earth
3. In Si, the operating temperature range is -60 to 175^{0}C but for Ge it is -60 to 75^{0}C.
4. Leakage current is of the order of nA for Si while it is of the order of mA for Ge.
5. Conductivity of Si is more temperature sensitive than Ge.

Effect of temperature on Conduction

At T = 0K, the semiconductor behaves as an Insulator and all the electrons are bounded to Nucleus by Electrostatic force of attraction. Thus, an external force needs to be applied in order to free electrons from the hold of nucleus and make them available for conduction. By providing thermal energy we can release the electron from the atom so that it becomes free and then material behaves as Semi-conductor. Thus, conductivity of semiconductor will increase with temperature.

In semiconductors the energy gap reduces as temperature increases and hence the conductivity of sample also increases.

$E_G = E_{G0} - \beta T$

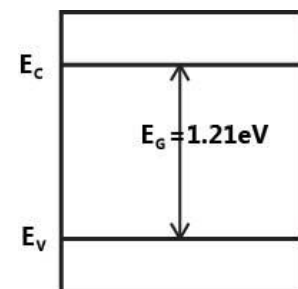where E_{G0} is Band Gap at 0K temperature

Let us take example of Si semiconductor

$E_{G0} = 1.21 eV$

At 300K, $E_G = 1.1 eV$

In Si the energy gap decreases by 3.6×10^{-4} eV / K rise in temp

$E_G(T) = 1.21 - 3.6 \times 10^{-4}$ T eV

For the case of Germanium,

$E_{G0} = 0.785\,eV$

At 300K, $E_G = 0.72\,eV$

In case of germanium energy gap decreases by $2.23 \times 10^{-4}\ eV/K$.

$E_G(T) = 0.785 - 2.23 \times 10^{-4}\ T\quad eV$

Here T is temp is Kelvin

Since, $E_G = E_{G0} - \beta T$

$Si \rightarrow E_{G0} = 1.21$, $\beta = 3.6 \times 10^{-4}\ eV/K$

$Ge \rightarrow E_{G0} = 0.785$, $\beta = 2.23 \times 10^{-4}\ eV/K$

Concept of Hole

If sufficient thermal energy is applied, then covalent bonds will be broken and electron will become free. So, there is free space in valence band which is termed as hole. So, hole is nothing but absence of electrons.

Due to thermal energy if one electron is leaving the valence band, then there will be one hole in the band and one electron is added in the conduction band. Thus, Number of electrons in the Conduction Band is same as Number of electrons in Valence Band.

So, hole is basically a concept not a physical particle. It always carries a positive charge and exists in valence band only. If there is movement of electrons within valence band only then there is no change of holes.

Classification of Semiconductors

Based on the number of elements we can classify the semiconductor into two categories:

1. Elementary Semiconductor: Eg. Si, Ge
2. Compound Semiconductor: More than 1 elements are used to make semiconductor. SiGe are used to make substrate in MOSFETs. It has high mobility due to which device operates at high frequency.

Some examples of compound semiconductors are:

- Two element (Binary) Compound Semiconductors:
  - GaAs, GaP, InP, InN (III and V group compounds)
  - ZnS, ZnSi, CdS, CdSi (II and IV group compounds)
  - SiC, SiGe (IV and IV group compounds)
- Three element (Tertiary) Compound Semiconductors:
  - GaAsP, AlGaAs
- Four Elements (Quaternary) Compound Semiconductors:
  - GaAlAsP, InGaAsP

Solved Examples

Problem: Why LED gives light but not normal PN junction diode?

Solution: As in LED the energy is released in the form of light but in case of PN Junction diode, it is released in the form of heat it just depends on the type of material used & even depend on direct band gap & indirect band gap.

LED has direct band-gap and so it gives light

PN Junction has indirect band gap and it releases energy in heat form.

Atomic Concentration

It represents no of atoms per cubic volume.

$AC = \dfrac{A_0 d}{A}$ atoms / cm^3

$A_0 = 6.023 \times 10^{23}$ molecules / mole

$d \rightarrow$ density

$A \rightarrow$ atomic weight

The atomic concentration of some of the common semiconductors are given as,

Atomic concentration for Si $\rightarrow 5 \times 10^{22}$ Si atoms / cm^3

Atomic concentration for Ge $\rightarrow 4.4 \times 10^{22}$ Ge atoms / cm^3

These values have been as calculated as shown below:

For Si,

Atomic Number = 14

Atomic weight = 28.1

Density $= 2.33$ gm / cm^3

$AC = \dfrac{A_0 d}{A}$ atoms / cm^3

$= \dfrac{6.023 \times 10^{23} \times 2.33}{28.1}$ atoms / cm^3 $= 4.99 \times 10^{22}$ atoms / cm^3

For Ge,

Atomic number = 32

Atomic weight = 72.6

density $= 5.32$ gm / cm^3

$AC = \dfrac{A_0 d}{A}$ atoms / cm^3 $= \dfrac{6.023 \times 10^{23} \times 5.32}{72.6}$

$AC = 4.4 \times 10^{22}$ atoms / cm^3

The energy gap of Si is more than Ge because the interatomic distance of Si is less than Ge i.e. why energy gap is more.

When no electric field electron have random (zig zag) motion & they cancel out the effect of each other by flowing in opposite direction

Thus, $I = 0$

If electric field is applied by applying a potential difference across the specimen, then whenever electron collides with ion in the lattice structure then its velocity is reduced to zero due to inelastic nature of collision. Then the kinetic energy of electron is converted to heat energy and that is why any material gets heated as current flow through it.

Then due to applied electric field electron experiences a force and again picks up speed in a direction opposite to electric field. This speed depends on applied voltage and average distance between successive collisions called as Mean Free Path.

Electron Gas Theory

According to electron gas theory of a metal, electron are in continuous motion their direction of motion being changed. Due to random direction of motion they collide with the ions and the average distance between two collisions is called as mean free path.

Drift Velocity or Drift Speed

Electron speed has two components:

- The first component is due to thermal energy of electron which is due to temperature of the specimen. This component is random and thus all electrons cancel the effect of each other and it does not impact the current in the sample. At absolute zero temperature this component is reduced to zero.

- At each inelastic collision with an ion an electron loses velocity or speed & finally it moves with a certain steady velocity or speed where it continuous to move with that velocity or speed. Such a velocity or speed obtained by the electrons under the influence of applied electric field is called a drift velocity or drift speed. This component is due to applied electric field.

Drift velocity or speed $\propto$ Applied Electric Field
$v \propto E$
The proportionality constant between drift velocity and electric field is called as mobility.
$v = \mu E$ where $\mu \longrightarrow$ mobility (constant)

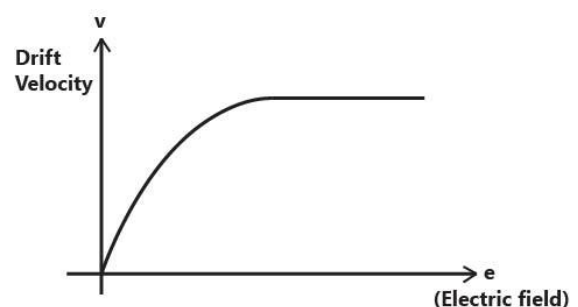Where, $\mu = \dfrac{v}{E}$

Units: $\dfrac{cm^2}{V - sec}$ or $\dfrac{m^2}{V - sec}$

Thus, mobility can be defined as drift velocity per unit applied electric field.
As the electric field increases the drift velocity increases but after a certain point it saturates i.e. Drift Velocity does not increase further as Electric Field is increased.

Mobility

It is defined as the ratio of drift velocity and the electric field intensity. It depends on:

(i) Temperature (T)

(ii) Electric field (E)

The force on an electron due to applied Electric Field is,

$F = eE$

By Newton's Second law of Motion

$$F = m^* a = m^* \frac{dv}{dt}$$

Here, m* refers to effective mass of a particle.

Thus, $eE = m^* \frac{dv}{dt}$

If the time between two collisions is τ

Then velocity gained by electron in this time is, $v = \frac{eE\tau}{m^*}$

This velocity is the drift velocity of the particle.

Thus, mobility can be expressed as,

$$\mu = \frac{e\tau}{m^*}$$

This formula can be expressed separately for holes and electrons as,

Mobility of holes, $\mu_p = \frac{e\tau_p}{m^*_p}$

Mobility of electrons, $\mu_n = \frac{e\tau_n}{m^*_n}$

The value of effective mass of holes and electrons for different materials are given as,

	Ge	Si	GaAS
m_n	0.55 m	1.08 m	0.067 m
m_p	0.37 m	0.56 m	0.48 m

Effect of temperature

• At temperatures above absolute zero the ions in the crystal lattice possess some thermal energy and vibrate about their mean positions. This causes interaction of electrons with ions due to which electrons are not able to move freely through the material.

• This scattering is called as Lattice Scattering or Phonon Scattering.

• As temperature increases this vibration increases and thus scattering increases and mobility reduces. Thus, the relation between mobility due to lattice scattering and temperature is,

$$\mu_L \propto T^{-3/2}$$

- The second phenomenon that affects the mobility is Ionized Impurity Scattering. Impurities are added in semiconductor to alter their conductivity and these impurities are ionized at room temperature. This results in Coulombic Interaction between electrons and ionized impurities which causes scattering of electron.

- As temperature increases the thermal energy of electrons increases and thus their speed increases and electrons spend less time in the vicinity of the impurity of the impurity and scattering is less so mobility increases with temperature.

$$\mu_I \propto \frac{T^{3/2}}{N_I}$$

Here, N_I refers to concentration of impurity and if this concentration increases the scattering increases and mobility reduces.
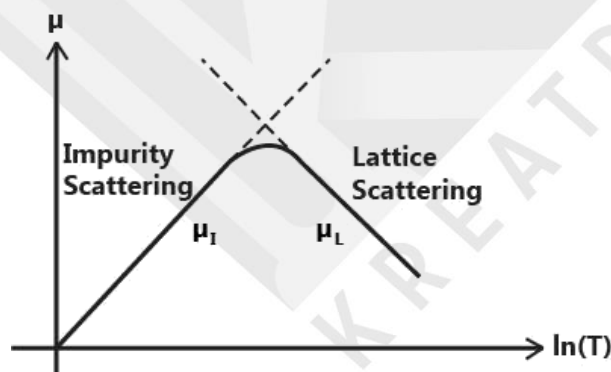
The total mobility μ can be expressed as combination of these two values,

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

- If it is having n no. of scattering effects each with mobility μ_1 , μ_2 , μ_3 ....μ_n

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3} + ...$$

- At low temperatures impurity scattering dominates and at higher temperature lattice scattering dominates. The curve shown below depicts the variation of mobility with temperature,

The above graph is drawn for all semiconductors (in general) for electronic devices the operating temp is 100 K to 400 K.

Thus, $\mu \propto \dfrac{1}{T^m} \implies \mu \propto T^{-m}$

For Si,

$m = 2.5$ for electron $\left[\text{i.e. } \mu_n \propto \dfrac{1}{T^{2.5}} \right]$

$m = 2.7$ for holes $\left[\text{i.e. } \mu_p \propto \dfrac{1}{T^{2.7}} \right]$

For Ge,

$$m = 1.66 \text{ for electron } \left[\text{i.e. } \mu_n \propto \frac{1}{T^{1.66}} \right]$$

$$m = 2.33 \text{ for holes } \left[\text{i.e. } \mu_p \propto \frac{1}{T^{2.33}} \right]$$
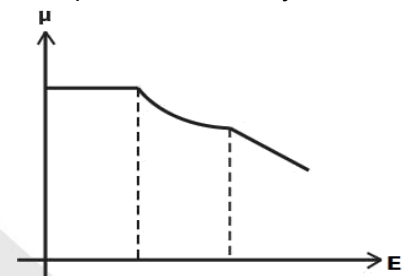
Effect of Electric Field

For all calculations purpose, we assume mobility independent of Electric Field but practically mobility varies with Electric Field. This mobility is known as Field Dependent Mobility.

Mobility with respect to electric field E

(i) μ const for $E < 10^3 \, V/cm$

(ii) $\mu \propto E^{-\frac{1}{2}}$ for $10^3 \, V/cm < E < 10^4 \, V/cm$

(iii) $\mu \propto E^{-1}$ for $E > 10^4 \, V/cm$

The standard values of mobility of electrons and holes for different materials are,

Germanium (Ge)

$$\mu_n = 3900 \, cm^2/V-s = 0.39 \, m^2/V-s$$
$$\mu_p = 1900 \, cm^2/V-s = 0.19 \, m^2/V-s$$

Silicon (Si)

$$\mu_n = 1350 \, cm^2/V-s = 0.135 \, m^2/V-s$$
$$\mu_p = 480 \, cm^2/V-s = 0.048 \, m^2/V-s$$

Gallium Arsenide (GaAs)

$$\mu_n = 8500 \, cm^2/V-s = 0.85 \, m^2/V-s$$
$$\mu_p = 400 \, cm^2/V-s = 0.04 \, m^2/V-s$$

The ratio of electron and hole mobility is,

In Silicon

$$\frac{\mu_n}{\mu_p} = \frac{1300}{500} = 2.6$$

In Germanium

$$\frac{\mu_n}{\mu_p} = \frac{3800}{1800} = 2.11$$

Thus, electron mobility is much higher than hole mobility therefore n type semiconductor prefers p type semiconductors. High mobility required for higher conductivity & current. High mobility means high frequency of operation, propagation delay is less.

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n = electron concentration , elec / cm^3 or elec / m^3

N = total number of electrons

N = n × volume
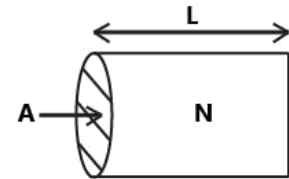
$n = \dfrac{N}{volume}$

Let Atomic Concentration (AC) of Si = 1000 si atom / cm^3

Then electron concentration n = ? assuming one electron is free per atom

n = AC × number of free electron / atom

$\quad$ = 1000 × 1 ele / atom = 1000 ele / cm^3

Solved Examples

Problem: The specific gravity of tungsten is 18.8 g / cm^3 & its atomic weight is 184. If each atom is having 2 free electron calculation electron concentration.

Solution: Concentration of free electron is,

n = AC × no of free e$^-$ / atom

$AC = \dfrac{A_0 d}{A} = \dfrac{6.023 \times 10^{23} \times 18.8}{184} = 6.1539 \times 10^{22}$

$n = 6.1539 \times 10^{22} \times 2 = 1.23078 \times 10^{23}$ ele / cm^3

Problem: A small concentration of minority carriers are injected into a semiconductor crystal at 1 point on electric field of 10 V / cm is applied across the crystal & this moves the minority carriers a distance of 1 cm in 20 μsec. The mobility of the minority carriers is?

Solution: Since the charge carriers travel a distance of 1 cm in 20 μsec, the drift velocity is given by,

$v_d = \dfrac{1}{20 \times 10^{-6}} = 50000$ cm / sec

Mobility, $\mu = \dfrac{v_d}{E}$

E = 10 V / cm

$\mu = \dfrac{50000}{10} = 5000$ cm^2 / v − sec

Problem: Three scattering mechanism exist in a semiconductor. If only 1st mechanism is present the mobility would be 500 cm^2 / V − sec it only 2nd mechanism were present the mobility would be 750 cm^2 / V − sec if only 3rd mechanism were present $\mu = 1500$ cm^2 / V − sec . Determine net mobility.

Solution: $\dfrac{1}{\mu_{net}} = \dfrac{1}{\mu_1} + \dfrac{1}{\mu_2} + \dfrac{1}{\mu_3}$

$$\frac{1}{\mu_{net}} = \frac{1}{500} + \frac{1}{750} + \frac{1}{1500}$$

$$\mu_{net} = 250 \, cm^2 / V - sec$$

Problem: For a sample of GaAS scattering time is $T_{sc} = 10^{-13} \, sec$. An electron effective mass is $m_n = 0.067m$. If an electric field of $1KV / cm$ is applied the drift velocity produced is?

Solution: Mobility of electron is given by,

$$\mu_n = \frac{e \, T_{sc}}{m_n} = \frac{1.6 \times 10^{-19} \times 10^{-13}}{0.067 \times 9.1 \times 10^{-28}} \, cm / sec$$

$$\mu_n = 2.624 \times 10^{-4}$$

Drift Velocity, $v_d = 1000 \times \mu_n = 0.262424 \, cm / sec$

Conduction in Metals

In metals, we use electron sea model in which we assume there is a sea of free electrons in which positive metal ions are embedded at lattice points. We assume that there is no interaction between the metal ions and electrons.

If the metal sample has a length 'L' and electron take 'T' time to travel the length 'L' then,

Drift velocity $v = \dfrac{L}{T}$ ....(1)

Current due to electrons flows in a direction opposite to due to negative charge of electrons.

If total number of electrons in the sample is 'N'

$$\text{Current} \, (I) = \frac{Q}{T} = \frac{N \, e}{T}$$

$$\text{Current density } J = \frac{I}{A} = \frac{Ne}{T.A} = \frac{Ne}{\frac{L}{v}.A} = \frac{N}{LA} ev$$

$J = nev = ne\mu E$ ......(2)

This is because drift velocity $v = \mu E$

By Ohm's Law,

$J = \sigma E$ ....(3)

Comparing equations (2) and (3)
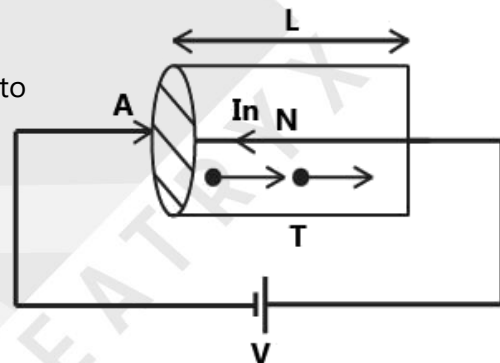
$\sigma = ne\mu$

Here, σ is called as conductivity of the metal

(i) Conductivity of a Metal (σ)

$\sigma = ne\mu$

Units: $(\Omega - cm)^{-1}$ or $(\Omega - m)^{-1}$ or S / cm or S / m or mho / cm or mho / m

Conductivity of a metal is directly proportional to electron concentration, mobility of electron & charge.

(ii) Resistivity of a Metal or conductor

$$\rho = \frac{1}{\sigma} = \frac{1}{ne\mu}$$

Units: $\Omega - cm$ or $\Omega - m$

(iii) Current density of a metal

$$J = \sigma E$$

Where $\sigma = ne\mu E = nev$

(iv) Current through a metal –

$$I = J.A = \sigma \varepsilon A = ne\mu E.A$$
$$I = nevA$$

Conductivity of a semiconductor

$$\sigma_{S.C} = \sigma_n + \sigma_p = ne\mu_n + pe\mu_p$$
$$\sigma_{S.C} = e\left(n\mu_n + p\mu_p\right)$$

Solved Examples

Problem: A specimen on a metal has a square cross – section area of $3mm \times 3mm$ & its length 5 cm. When a potential difference of 1V applied across its length gives a current of 6 mA. Calculate electron concentration (n), drift velocity & number of electron?

Solution: Assume $\mu = 1300\,cm^2 / v - sec$

Area $= 3mm \times 3mm$

$L = 5\,cm$

$V = 1V$

$I = 6mA$

$R = \dfrac{V}{I} = 0.1666 \times 10^3 = 166.66\,\Omega$ or

$n.p = ni^2$

$E = \dfrac{V}{L} = \dfrac{1}{5 \times 10^{-2}} = 20$ or 0.2000 V / cm

$I = ne\mu EA$

$n = \dfrac{I}{e\mu EA}$

$n = \dfrac{6 \times 10^{-3}}{1.6 \times 10^{-19} \times 1300 \times 0.2 \times 9 \times 10^{-2}}$

$n = 1.6025 \times 10^{15} / cm^3$

Drift Velocity, $v = \mu\varepsilon = 1300 \times \dfrac{1}{5} = 260 \, cm/sec$

Number of electrons, $N = n \times volume = 1.6025 \times 10^{15} \times 3 \times 3 \times 10^{-2} \times 5 = 1.44 \times 10^{14} \times 5$

$$N = 7.2 \times 10^{14}$$

Problem: An n type Si bar 0.1 cm long & $100 \, \mu m^2$ cross sectional area has a majority carrier concentration of $5 \times 10^{20} / m^3$ & carrier mobility is $0.13 \, m^2/V-sec$. If charge of an electron is $1.6 \times 10^{19} C$ then resistance of bar is?

Solution: Neglecting the effect of minority carriers on conductivity,

$$\sigma = \frac{1}{\rho} = (ne\mu)$$

$$\rho = \frac{1}{ne\mu} = \frac{1}{5 \times 10^{20} \times 1.6 \times 10^{-19} \times 0.13} = 0.09615$$

Resistance of bar is, $R = \dfrac{\rho L}{A}$

$$R = \frac{0.096 \times 0.1 \times 10^{-2}}{100 \times 10^{-6} \times 10^{-6}} = 0.9615 \times 10^6 \simeq 1 \times 10^6 = 1 M\Omega$$

Problem: The resistivity of an uniformly doped n type Si sample is 0.5 Ω-cm. If electron mobility is $1250 \, cm^2/V-sec$ and charge of an electron is $1.6 \times 10^{-19}C$. Find Donor Concentration?

Solution: $\rho = 0.5 \, \Omega cm$

Majority Carrier Concentration, $n = \dfrac{1}{e\mu\rho} = \dfrac{1}{1.6 \times 10^{-19} \times 1250 \times 0.5} = 10^{16} / cm^3$

Approximately, $n = N_D$

$N_D = 10^{12} / cm^3$

Problem: A heavily doped n type semiconductor has the following data:

Hole-electron mobility ratio $0.4 = \dfrac{\mu_P}{\mu_n}$

Doping concentration, $N_D = 4.2 \times 10^8 \, atoms/m^3$

Intrinsic concentration, $n_i = 1.5 \times 10^4 / m^3$

The ratio of the conductance of the n type semiconductor to that of intrinsic semiconductor of the same material under the same temp is given by?

Solution: Since donor concentration is much higher as compared to intrinsic concentration, $n \approx N_D = 4.2 \times 10^8 \, atoms/m^3$

Neglecting the effect of minority carriers on conductivity,

$\sigma_n = n \mu e$

Intrinsic Conductivity, $\sigma_i = (n_i \mu_n + p_i \mu_P) e = n_i (\mu_n + \mu_P) e$

Ratio of conductivities, $\dfrac{\sigma_n}{\sigma_i} = \dfrac{n \mu_n e}{n_i (\mu_n + \mu_P) e} = \dfrac{n \mu_n e}{n_i (\mu_n + 0.4 \mu_n) e}$

$\dfrac{\sigma_n}{\sigma_i} = \dfrac{n}{1.4 n_i} = \dfrac{4.2 \times 10^8}{1.4 \times 1.5 \times 10^4} = 2 \times 10^4$

Problem: Free electron density in a conductor is $\dfrac{1}{1.6} \times 10^{22} / cm^3$ & electron mobility is $10 \, cm^2 / V - s$. Then resistivity of material is?

Solution: $n = \dfrac{1}{1.6} \times 10^{22} / cm^3$

$\mu = 10 \, cm^2 / v - sec$

$e = 1.6 \times 10^{-19} C$

Conductivity, $\sigma = \dfrac{1}{\rho} = n \mu e$

Resistivity, $\rho = \dfrac{1}{n \mu e} = \dfrac{1}{\dfrac{1}{1.6} \times 10^{22} \times 10 \times 1.6 \times 10^{-19}} = 10^{-4} \Omega - cm$

Problem: A Si sample "A" is doped with $10^{18} \, atoms / cm^3$ of Boron another sample "B" of identical dimensions is doped with $10^{18} \, atoms / cm^3$ of phosphorous. The ratio of electron to hole mobility is 3 then ratio of conductivity is?

Solution: Acceptor Ion Concentration in sample "A" $N_A = 10^{18} / cm^3$

Donor Ion Concentration in sample "B" $N_D = 10^{18} / cm^3$

Ratio of mobility, $\dfrac{\mu_n}{\mu_P} = 3$

Neglecting Minority Carrier Concentration,

Ratio of conductivity, $\dfrac{\sigma_A}{\sigma_B} = \dfrac{p \, e \, \mu_P}{n \, e \, \mu_n} = \dfrac{10^{18} e}{10^{18} e} \times \dfrac{1}{3} = \dfrac{1}{3}$

Problem: A Si n type sample doped at 10^{18} per cm^3 having a cross sectional area $A = 10^{-6} cm^2$ and length $L = 10 \mu m$. If the electron mobility is $\mu_n = 800 \, cm^2 / V - s$ and resistance of sample is $R = 10 K\Omega$. Determine the doping efficiency?

Solution: Resistivity of sample is, $\rho = \dfrac{RA}{l} = \dfrac{10^4 \times 10^{-6}}{10 \times 10^{-4}} = 10\,\Omega - cm$

$\sigma = \dfrac{1}{\rho} = 0.1$

Electron Concentration, $n = \dfrac{\sigma}{e\mu_n} = \dfrac{0.1}{800\,cm^2 / V - sec \times 1.6 \times 10^{-19}}$

$n = 7.8125 \times 10^{17} \; / cm^3$

Doping efficiency $= \dfrac{n}{N_D} \times 100 = \dfrac{7.8125 \times 10^{17}}{10^{18}} \times 100 = 78.125\%$

Semiconductor

(i) Good conductor of electricity is a metal & bad conductor of electricity is an insulator.

(ii) The electrical properties of a semiconductor lies in between metal or conductor & insulator

(iii) Semiconductor acts like an insulator at 0K $(\sigma = 0)$ & it acts like a conductor at room temperature i.e. the electrical properties (conductivity) increases as temperature increases.

(iv) The resistivity or resistance of a semiconductor is infinite at 0K

(v) At temperature increases the resistivity or resistance decreases so semiconductors have negative temperature coefficient.
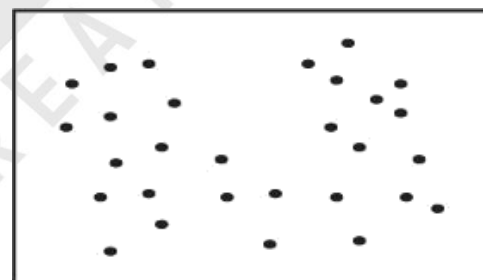
Eg. Si, Ge

Most of metals and semiconductors are crystalline in nature.

Crystallinity means that the arrangement of atoms in the specimen. The pattern of point at which each point an atom is placed is known as lattice.
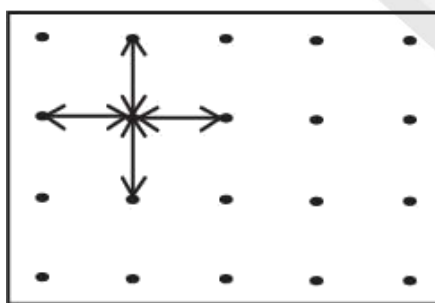
Crystalline Surface

Amorphous Surface

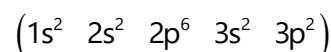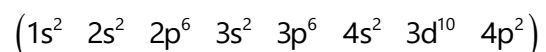If the atoms are arranged in a random manner then the solid is known as Amorphous Solid.

The atomic structure of Silicon and Germanium is as shown below,

Si: 14   Ge: 32

$\left(1s^2 \;\; 2s^2 \;\; 2p^6 \;\; 3s^2 \;\; 3p^2\right)$ $\left(1s^2 \;\; 2s^2 \;\; 2p^6 \;\; 3s^2 \;\; 3p^6 \;\; 4s^2 \;\; 3d^{10} \;\; 4p^2\right)$
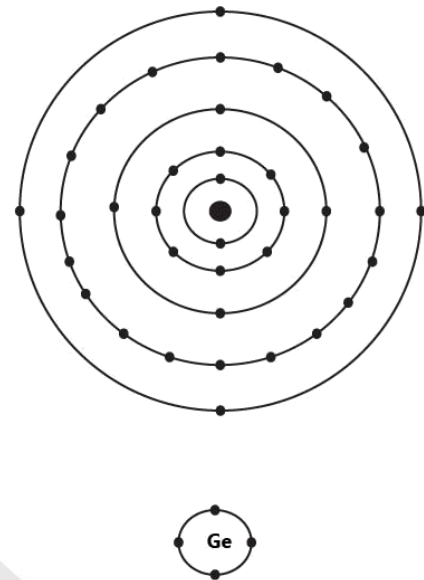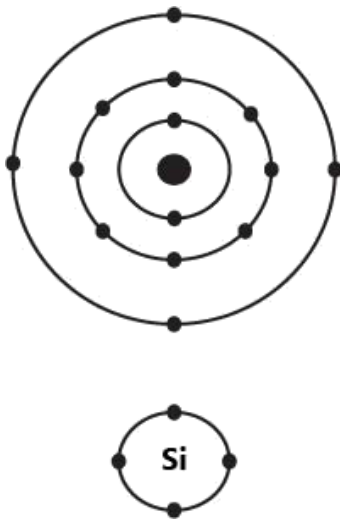
When the Si atoms form a crystal the structure looks like as shown below,
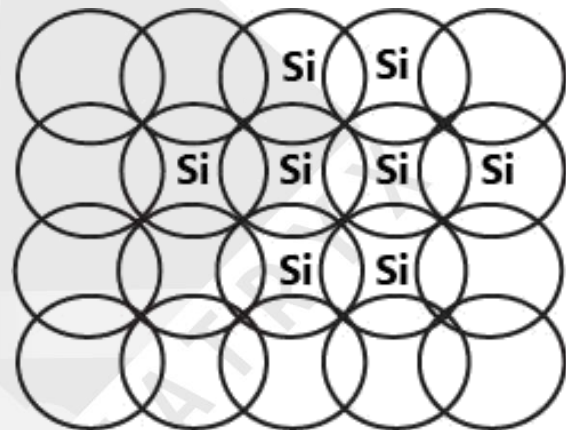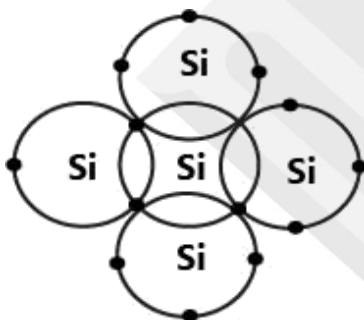
- Si & Ge are having crystalline structure & tetrahedral shape
- Silicon & Germanium are having diamond structure or face centered cubic structure (FCC).
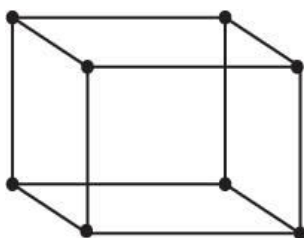
Simple Cubic Lattice   Body Centered Cubic (BCC)   Face Centered Cubic (FCC)
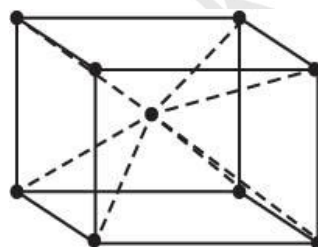
In semiconductors conductivity or current is due to electrons & holes because in semiconductors electrons & holes exist

(i) Conductivity of a Semiconductor (σ)

The conductivity was earlier derived as, $\sigma = n\mu e$

Thus, conductivity of electrons is given by, $\sigma_n = n\mu_n e$

Similarly, conductivity of holes is given by, $\sigma_p = p\mu_p e$

Total conductivity, $\sigma_{s.c} = \sigma_n + \sigma_p = n\mu_n e + p\mu_p e = \left(n\mu_n + p\mu_p\right)e$

(ii) Resistivity of a Semiconductor (ρ)

$$\rho = \frac{1}{\sigma} = \frac{1}{\left(\eta\mu_\eta + \rho\mu_\rho\right)e}$$

(iii) Current density (J)

$$J = \sigma E = \left(\eta\mu_\eta + \rho\mu_\rho\right)eE$$

(iv) Current through a Semiconductor (I)

$$I = JA = \sigma\varepsilon A = \left(\eta\mu_n e + \rho\mu_\rho e\right)EA$$
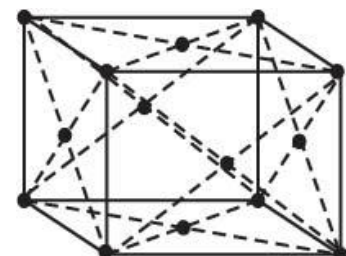
Since, mobility of electron is more than mobility of holes, the conductivity of electrons is more than holes and thus current due to electrons is more than current due to holes.

The total current in any semiconductor material,

$$I_{Total} = I_n + I_p$$

$$I_n > I_p \text{ as } \left(\mu_n > \mu_p\right)$$

$$\sigma_n > \sigma_p$$

Solved Examples

Problem: Why $\mu_\rho < \mu_\eta$?

Solution: Because Valence Band is close to nucleus, its interaction with the nucleus is more that is why electron movement in Valence Band is less that movement of electron in Conduction Band.

Actually, there is no movement of holes as holes do not exist but we consider the movement of electron which will create hole after its movement from one position thus it seems that holes are moving in opposite direction of electrons but actually it does not happens.

Energy Density

Energy density represents number of free electron per cubic volume whose energies lies in the interval dE.

For distribution of various particles among available energy states, certain rules are defined,

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1. Maxwell Boltzmann Statistics

In this case the particles are distinguishable from each other and there is no limit for the number of particles allowed in each energy state. So, average distribution of non-interacting particles over various energy levels in thermal equilibrium is governed by Maxwell Boltzmann Statistics. This distribution is generally used at higher temperature where there is negligible interaction among different particles.

2. Bose Einstein Principle

Particles are not distinguishable and there is a limit for number of particles in each energy state. It means non-distinguishable particles may occupy a set of discrete energy states. Best example of this type of particles is "Boson". These particles do not follow "Pauli's Exclusion Principle".

According to Pauli's Exclusion Principle two particles cannot co-exist in same energy state at same location. In case of Boson, spins are of integer

$$\left(+1, \ +2, \ +3….. \ \text{i.e.} \ 2\times\frac{1}{2}, \ 4\times\frac{1}{2}, \ 6\times\frac{1}{2} \right)$$

3. Fermi Dirac Distribution

In this case, particles are not distinguishable and there is only one particle available in each energy state. It follows Pauli's Exclusion Principle according to which two electrons may not have same quantum state. These type of particles are known as "Fermions" which may be electron, proton and nucleus. Fermions always have spin of half the integer value.

For any further analysis we will assume Fermi-Dirac Distribution. Suppose,
dn = number of free electron per cubic volume with energy between E to E+dE
$d\rho$ = density of the electrons

$dn = \rho_E dE$

$\rho_E = N(E)f_E$

$N(E)$ = density of the state (4 N states)

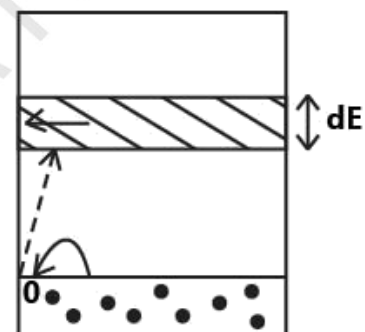$N(E) = \gamma.E^{1/2}$

$\gamma = \dfrac{4\pi}{h^3}(2m)^{3/2}(e)^{3/2}$ = constant

$f_E \rightarrow$ Fermi – Dirac probability Function

$f_E = \dfrac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$

E_F = Fermi Energy Level

It represents the probability of occupancy on an energy level E by an electron.

K= Boltzmann constant $K = 8.62 \times 10^{-5}$ ev / K

$K = 1.38 \times 10^{-23}$ J / K

Fermi level (E_F)
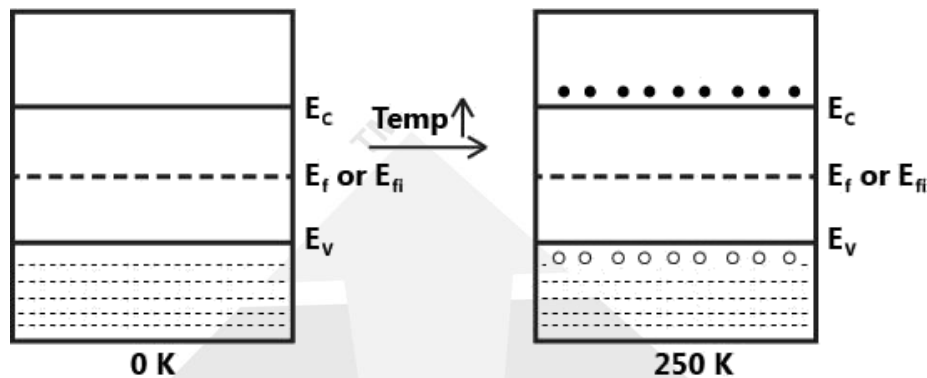
At absolute zero temperature,

If, $E > E_F$

$$f_E = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} = \frac{1}{1+\exp(\infty)} = 0$$

If, $E < E_F$

$$f_E = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} = \frac{1}{1+\exp(-\infty)} = 1$$

So, Fermi Level can be defined as highest energy level in energy band below which all the states are filled by the electrons (in the valence band) {at 0K}

At any other temperature,

If, $E = E_F$

$$f_E = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} = \frac{1}{1+\exp(0)} = 0.5$$

If any particle occupies a particular energy state with probability 0.5 at any temperature, then that energy state is known as fermi level.

For holes, the Fermi Dirac Distribution Function is, $1 - f_E$

Probability of finding electron is 50% & probability of finding holes is also 50%

{only in intrinsic semiconductor at temperature other than 0K}

In case of intrinsic semiconductor, Fermi Level lies midway between conduction and valence band.

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Fermi Dirac Probability Function

It represents probability of occupancy of energy states by the electrons in the conduction band.

$$f_E = \frac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)}$$
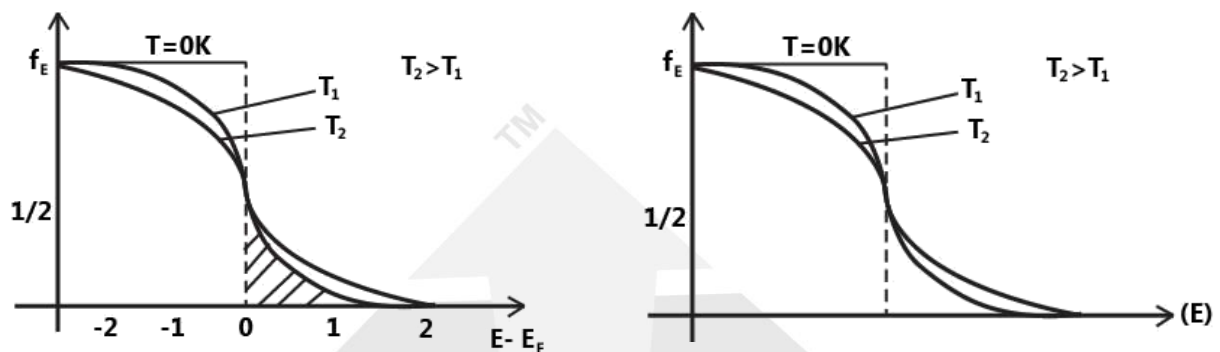
At any temp T (other than OK)

Sum of the probability of finding electron in the conduction bands & probability of finding holes in the valence band is always equal to 1.

$$P(n) + P(p) = 1$$
$$P(p) = 1 - P(n)$$

If the probability of finding electron in the conduction band is $P(n)$ then the probability of finding holes in the valence band $1 - P(n)$

Sum of the Fermi Dirac probability function for Electron & holes is equal to 1.

If the Fermi Dirac probability function for electron & in the conduction band is f(E) then Fermi Dirac probability function for holes in the valence band is $1 - f(E)$

$$f_n(E) + f_p(E) = 1$$
$$f_p(E) = 1 - f_n(E)$$

Fermi Dirac probability function is symmetrical about Fermi Level.

Electron Concentration in Conduction Band

The number of electrons lying in the energy band E to E+dE are,

$dn = \rho_E dE$

Thus, total concentration of electrons in Conduction Band is,

$$n = \int_{E_C}^{\infty} \rho_E dE$$

$$n = \int_{E_C}^{\infty} N(E) f(E) dE \qquad \ldots.(1)$$

E corresponds to energy of a particular state

$$N(E) = \gamma \left(E - E_C\right)^{\frac{1}{2}}$$
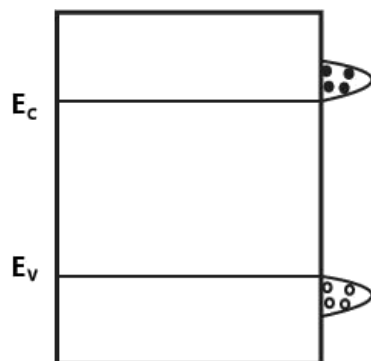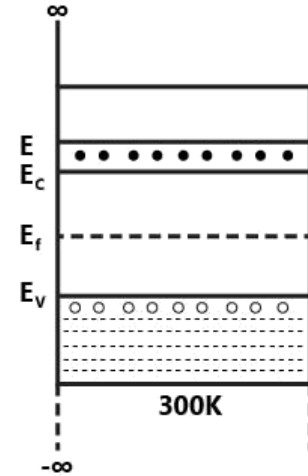
In conduction band energy of state is more than the fermi level. So, $e^{(E-E_F)/KT} >>> 1$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}} = e^{-(E-E_F)/KT}$$

$$\frac{\overline{KT}}{q} = K\,T = \frac{T}{\dfrac{1}{K}} = \frac{T}{\dfrac{1}{8.62 \times 10^{-5}\ ev\,/\,K}} = \frac{T}{11600}$$

At 300 K

$$K\,T = \frac{300}{11600} = 0.2586\ ev \simeq 26\,meV$$

$KT = 26\,meV$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/26meV}} = \frac{1}{1 + e^{40(E-E_F)}}$$

$$f(E) = e^{-40(E-E_F)}$$

Substitute the value of fermi distribution in equation 1

$$n = \int_{E_C}^{\infty} \gamma \left(E - E_C\right)^{\frac{1}{2}} e^{-(E-E_F)/KT} dE$$

$$n = \int_{E_C}^{\infty} \gamma \left(E - E_C\right)^{\frac{1}{2}} e^{-(E-E_F)/KT} dE$$

Let, $E - E_C = KT\,x$

$E = E_C + KT\,x$

$$n = \int_{E_C}^{\infty} \gamma \left(KT\,x\right)^{\frac{1}{2}} e^{-\left((E_C + KT\,x) - E_F\right)/KT} dE$$

$dE = K\,T\,dx$

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$$n = \gamma (KT)^{\frac{3}{2}} e^{-(E_C - E_F)/KT} \int x^{\frac{1}{2}} \cdot e^{-x} dx$$

$$\int x^{\frac{1}{2}} \cdot e^{-x} dx = \left| \frac{1}{2} \right. \quad \rightarrow \text{Gamma function}$$

$$\left| \frac{1}{2} \right. = \frac{\sqrt{\pi}}{2}$$

The value of constant $\gamma = \dfrac{4\pi}{h^3} (2m)^{3/2} (e)^{3/2}$

$$n = \frac{4\pi}{h^3} (2m)^{\frac{3}{2}} (e)^{\frac{3}{2}} (KT)^{\frac{3}{2}} \; e^{-(E_C - E_F)/KT} \left(\frac{\sqrt{\pi}}{2} \right)$$

$$n = 2 \cdot \left(\frac{2\pi m_n KT}{h^2} \right)^{\frac{3}{2}} \left(1.6 \times 10^{-19} \right)^{\frac{3}{2}} e^{-(E_C - E_F)/KT}$$

$$n = N_C \; e^{-(E_C - E_F)/KT}$$

N_C is called as Effective density of state function in the conduction band

$$N_C = 2 \left[\frac{2\pi m_n KT}{h^2} \right]^{\frac{3}{2}} \left(1.6 \times 10^{-19} \right)^{\frac{3}{2}}$$

$$N_C = 2 \left[\frac{2\pi m_n \bar{K} T}{h^2} \right]^{\frac{3}{2}} \qquad \left(\bar{K} = Ke \right)$$

$$N_C = 2 \left[\frac{2\pi m_n \bar{K} T}{h^2} \right]^{\frac{3}{2}} = 2 \left[\frac{2\pi \bar{K}}{h^2} \right]^{\frac{3}{2}} \left(m_n \; T \right)^{\frac{3}{2}} = 2 \left[\frac{2\pi \bar{K} \times 9.1 \times 10^{-31}}{h^2} \right]^{\frac{3}{2}} \left(\frac{m_n}{m} T \right)^{\frac{3}{2}}$$

$$N_C = 2 \left[\frac{2\pi \times 1.38 \times 10^{-23} \times 9.1 \times 10^{-31}}{\left(6.626 \times 10^{-34} \right)^2} \right]^{\frac{3}{2}} \left(\frac{m_n}{m} \right)^{\frac{3}{2}} (T)^{\frac{3}{2}}$$

So, the value of effective density of states function is,

$$N_C = 4.8186 \times 10^{21} \left(\frac{m_n}{m} \right)^{\frac{3}{2}} T^{\frac{3}{2}} / m^3$$

$$N_C = 4.8186 \times 10^{15} \left(\frac{m_n}{m} \right)^{\frac{3}{2}} T^{\frac{3}{2}} / cm^3$$

Hole Concentration in Valence Band

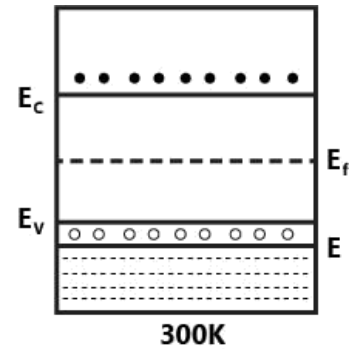The fermi dirac distribution for holes in Valence Band is given by,

$$f_P (E) = 1 - f(E)$$

Hole Concentration in Valence Band is given by,

$$p = \int_{-\infty}^{E_V} N(E)\left[1 - f(E)\right] dE$$

$$N(E) = \gamma\left(E_V - F\right)^{\frac{1}{2}}$$

Fermi Dirac Function is, $f(E) = \dfrac{1}{1 + e^{(E - E_F)/KT}}$

$$1 - f(E) = 1 - \frac{1}{1 + e^{(E - E_F)/KT}} = \frac{e^{(E - E_F)/KT}}{1 + E^{(E - E_F)/KT}}$$

The energy of states in Valence Band is less than Fermi Energy. So, $e^{(E - E_F)/KT} \ll 1$

$$1 - f(E) = e^{(E - E_F)/KT}$$

$$1 - f(E) = e^{-(E_F - E)/KT}$$

$$p = \gamma\int_{-\infty}^{E_V}\left(E_V - E\right)^{\frac{1}{2}} e^{-(E_F - E)/KT} dE = \gamma\int_{-\infty}^{E_V}\left(E_V - E\right)^{\frac{1}{2}} e^{-(E_F - E)/KT} dE$$

Let $E_V - E = KT\,x$

So, $E = E_V - KT\,x$

As E goes from $-\infty$ to E_V

So, the variable x goes from $+\infty$ to 0

$$\rho = \gamma\int_{\infty}^{0}\left(KT\,x\right)^{\frac{1}{2}} e^{-(E_F - E_V + KT_x)/KT} dE$$

Hole Concentration, $p = \gamma\displaystyle\int_{0}^{\infty}\left(KTx\right)^{\frac{1}{2}} e^{-(E_F - E_V)/KT} e^{-x(KT)} dx$

$$p = \gamma\left(KT\right)^{\frac{3}{2}} e^{-(E_F - E_V)/KT}\int_{0}^{\infty} x^{\frac{1}{2}} e^{-x} dx$$

Substituting the value of $\gamma = \dfrac{4\pi}{h^3}\left(2m_p\right)^{3/2}\left(e\right)^{3/2}$

$$p = \frac{4\pi}{h^3}\left(2m_p\right)^{\frac{3}{2}}\left(1.6\times10^{-19}\right)^{\frac{3}{2}}\left(KT\right)^{\frac{3}{2}} e^{-(E_F - E_V)/KT}\cdot\frac{\sqrt{\pi}}{2}$$

$$p = 2\left(\frac{2\pi m_p KT}{h^2}\right)^{\frac{3}{2}}\left(1.6\times10^{-19}\right)^{\frac{3}{2}} e^{-(E_F - E_V)/KT}$$

$$p = N_V\, e^{-(E_F - E_V)/KT}$$

N_V = Effective density of the state function in the valence band.

$$N_V = 2\left[\frac{2\pi m_p \bar{K} T}{h^2}\right]$$

Hole Concentration, $p = 4.82 \times 10^{21} \left(\dfrac{m_p}{m} \right)^{\frac{3}{2}} T^{\frac{3}{2}} \Big/ m^3$

$p = 4.82 \times 10^{15} \left(\dfrac{m_p}{m} \right)^{\frac{3}{2}} T^{\frac{3}{2}} \Big/ cm^3$

Types of Semiconductor

Semiconductors can be categorized into two categories:

• **Intrinsic Semiconductor:** These are the materials in which there is no impurity added and all atoms in the crystal belong to a single element.

• **Extrinsic Semiconductor:** Impurity atoms are added to intrinsic semiconductor in order to increase the conductivity. This process is known as Doping and the impurity atoms added are known as dopants. These are of two types p-type and n-type.

Doping is of four types:

- Heavily Doped (1 impurity atom in $10^2 - 10^3$ atoms)
- Moderately Doped (1 impurity atom in $10^6 - 10^7$ atoms)
- Lightly Doped (1 impurity atom in $10^{10} - 10^{11}$ atoms)
- Intrinsic (1 impurity atom in more than 10^{11} atoms)

Mass Action Law

According to mass action law the product of electron & holes remains constant at fixed temperature. This is applicable to intrinsic as well as extrinsic semiconductor.

Suppose the intrinsic concentration, $ni = 4$

For intrinsic semiconductor,

$n = n_i = 4$
$p = p_i = 4$ $n \cdot p = n_i^2$

For n-type semiconductor,

$n_n = 8 > n_i$
$p_n = 2 < n_i$ $n_n \cdot p_n = n_i^2$

As number of electron increases, the number of holes reduces but their product remains same.

For p-type semiconductor,

$n_p = 2 < n_i$
$p_p = 8 > n_i$ $n_p p_p = n_i^2$

As number of holes increases, the number of electrons reduces but their product remains same.

The intrinsic concentration increases as temperature increases so the product of number of holes and electrons increases.

Solved Examples

Problem: The intrinsic carrier density at 300 K is $1.5 \times 10^{10} / cm^3$. For n type Si doped to 2.25×10^{15} atoms $/ cm^3$ the equilibrium election & hole densities are?

Solution: Intrinsic Carrier Concentration, $n_i = 1.5 \times 10^{10} / cm^3$

For an n-type semiconductor $n \simeq N_D = 2.25 \times 10^{15} / cm^2 \quad (\because N_D >> n_i)$

At Thermal Equilibrium,

$n_i^2 = n.p$

$p = \dfrac{n_i^2}{n} = \dfrac{2.25 \times 10^{20}}{2.25 \times 10^{15}} = 10^5 / cm^3$

Problem: Two pure specimen of a semiconductor material are taken. One is doped with $10^{18} / cm^3$ donor concentration & other is doped with $10^{16} / cm^3$ of acceptor concentration. The minority carrier density in 1st specimen is $10^7 / cm^3$. What is the minority carrier density in other specimen?

Solution: Assuming the dopant concentration is much higher than intrinsic concentration,

Electron concentration in 1st specimen, $n_1 = 10^{18} / cm^3$

Hole Concentration in 2nd specimen, $p_2 = 10^{16} / cm^3$

Minority Carrier Hole Concentration in 1st specimen, $p_1 = 10^7 / cm^3$

Since, both specimens are of same material so both must have same intrinsic concentration at same temperature.

By Mass Action Law, $n_1 P_1 = n_2 P_2 = n_i^2$

Minority Carrier Concentration in 2nd specimen, $n_2 = \dfrac{10^{18} \times 10^7}{10^{16}}$

$n_2 = 10^9 / cm^3$

Problem: A Si sample is uniformly doped with $10^{16} P - atom / cm^3$ and $2 \times 10^{16} B - atom / cm^3$ If all the dopants are fully ionized. The material behaves as?

Solution: Since acceptor ion concentration is higher than donor ion concentration,

$N_D = 10^{16} / cm^3$

$N_A = 2 \times 10^{16} / cm^3$

Hole Concentration, $p = N_A - N_D$

$p = 2 \times 10^{16} - 1 \times 10^{16}$; $p = 10^{16} / cm^3$

Intrinsic Concentration, $n_i = 1.5 \times 10^{10} / cm^3$

Electron Concentration, $n = \dfrac{n_i^2}{p} = \dfrac{2.25 \times 10^{20}}{10^{16}} = 2.25 \times 10^4 / cm^3$

Since, holes are majority carriers the material behaves as p-type semiconductor.

Problem: In Ge semiconductor at $T = 300K$ that donor concentration is $N_D = 5 \times 10^{15} / cm^3$ & $N_A = 0$. The thermal equation concentration of electron & hole are?

Solution: $n_i = 2.5 \times 10^{13} / cm^3$

In n-type semiconductor, $n \simeq N_D = 5 \times 10^{15} / cm^3$

$$p = \frac{n_i^2}{n} = \frac{(2.5 \times 10^{13})^2}{5 \times 10^{15}}$$
$$p = 1.25 \times 10^{11} / cm^3$$

Intrinsic Concentration (nᵢ)

Intrinsic concentration is also called as electron concentration or hole concentration in an intrinsic semiconductor.

According to mass action law
$$n.p = n_i^2$$
Or $n_i^2 = n.p$

Here, n = electron concentration

p = hole concentration

$$n_i^2 = N_C e^{-(E_C - E_F)/KT} . N_V \ e^{-(E_F - E_V)/KT}$$
$$n_i^2 = N_C \ N_V \ e^{(E_V - E_C)/KT}$$
$$n_i^2 = N_C \ N_V \ e^{-E_G/KT}$$

Since effective density of state is constant thus N_C & N_V is also constant

$$e^{E_G/KT} = \frac{N_C N_V}{n_i^2}$$

$$E_G = (kT) \ln\left(\frac{N_C N_V}{n_i^2}\right)$$

In equation,
$$n_i^2 = N_C N_V \ e^{-E_G/KT}$$

$$n_i^2 = \left(4.82 \times 10^{21}\right)\left(\frac{m_n}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}} . \ 4.82 \times 10^{21} \left(\frac{m_p}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}} \ e^{-E_G/KT} \Big/ \left(m^3\right)^2$$

$$n_i^2 = 2.33 \times 10^{43} \left(\frac{m_n m_p}{m^2}\right)^{\frac{3}{2}} T^3 \ e^{-E_G/KT} \Big/ \left(m^3\right)^2$$

$$n_i^2 = N T^3 \ e^{-E_G/KT} \Big/ \left(m^3\right)^2$$

$$N = const = 2.33 \times 10^{43} \left(\frac{m_n m_p}{m^2}\right)^{\frac{3}{2}}$$

$$n_i^2 = 2.33 \times 10^{31} \left[\frac{m_n m_p}{m^2} \right]^{\frac{3}{2}} e^{-E_G/KT} . T^3 \Big/ (cm^3)^2$$

$$n_i^2 = N \, e^{-(E_G/KT)} . T^3 \Big/ (cm^3)^2$$

$$N = 2.33 \times 10^{31} \left(\frac{m_n m_p}{m^2} \right)^{\frac{3}{2}}$$

The variation of Energy Gap with respect to temperature is,

$E_G = $ Energy $gap = E_{G0} - \beta T$

$E_G = E_{G0} - \beta T$

So, $e^{-E_G/KT} = e^{-(E_{G0} - \beta T)/KT} = e^{-E_{G0}/KT} \, e^{\beta/K}$

$e^{-E_G/KT} = e^{\beta/K} \, e^{-E_{G0}/KT}$

Put in above equation,

$$ni^2 = 2.33 \times 10^{43} \left(\frac{m_n m_p}{m^2} \right)^{\frac{3}{2}} e^{\beta/K} . T^3 \, e^{-E_{G0}/KT} \Big/ (m^3)^2$$

$$n_i^2 = A_o T^3 \, e^{-E_{G0}/KT} \Big/ (m^3)^2$$

$$A_o = 2.33 \times 10^{43} \left(\frac{m_n m_p}{m^2} \right)^{\frac{3}{2}} e^{\beta/K}$$

Use above equation when it is mentioned that E_{G0} is energy gap at 0K.

$$n_i^2 = A_o T^3 \, e^{-E_{G0}/KT} \Big/ (cm^3)^2$$

$$A_o = 2.33 \times 10^{21} \left(\frac{m_n m_p}{m^2} \right)^{\frac{3}{2}} . e^{\beta/K}$$

Factors affecting Intrinsic Concentration

Intrinsic Concentration in a semiconductor depends on,

(i) Temperature

From the expression for Intrinsic Concentration,

$n_i^2 \propto T^3$

$n_i \propto T^{\frac{3}{2}}$

Thus, as temperature increases intrinsic concentration increases.

(ii) **Energy Gap**

From the expression for Intrinsic Concentration,

$$n_i^2 \propto e^{-E_G/KT}$$

As energy gap increases, the intrinsic concentration decreases.

Intrinsic concentration of Si at Room temperature

$$n_i^2 = 2.33 \times 10^{43} \left(\frac{m_n m_p}{m^2} \right)^{\frac{3}{2}} . T^3 \, e^{-E_G/KT}$$

For Silicon,

Effective mass of electron, $m_n = 1.08m$

Effective mass of holes, $m_p = 0.56m$

Intrinsic Concentration, $n_i = 1.5 \times 10^{10} \, / \, cm^3$

For Germanium,

Effective mass of electron, $m_n = 0.55m$

Effective mass of holes, $m_p = 0.37m$

Intrinsic Concentration, $n_i = 2.5 \times 10^{19} \, / \, m^3$

Note: Intrinsic Concentration is more in Ge as compared to Si due to lesser energy gap in Ge.

Concept of Effective Mass

Movement of particles in periodic potential will be very different than its movement in vacuum. During Periodic Potential Movement, due to collisions the mass may change and that mass is known as Effective Mass (m*). If there are more obstructions due to band structure then effective mass is high and if there is less obstruction then effective mass is less.

So, value of effective mass depends on the curvature of the band and it can be calculated using "Cyclotron Resonance Experiment". Electron reside in Conduction Band and CB has more regular structure than Valence Band. Valence Band is more affected by nucleus of the atoms and adjacent valence electrons and other irregularities. Due to this effective mass of electrons is less than holes.

Based on Quantum Physics, $m^* = \dfrac{\hbar^2}{\left(\dfrac{d^2E}{dk^2} \right)}$

Effective Mass is neither a scalar nor a vector but it is a Tensor Quantity.

Value of effective mass is positive near the bottom of Conduction Band and negative near the top of Valence Band.

Parameters of Intrinsic Semiconductor

Conductivity

$$n = p = n_i = p_i$$

$$\sigma_i = \left(n_i\ \mu_n + n_i\ \mu_p\right)e$$

$$\sigma_i = n_i\left(\mu_n + \mu_p\right)e\ \left(\Omega\,cm\right)^{-1}$$

Resistivity

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{n_i\left(\mu_n + \mu_p\right)e}\ \left(\Omega - cm\right)$$

Current Density

$$J_i = \sigma_i E = n_i\left(\mu_n + \mu_p\right)eE$$

Current

$$J_i = J_i A = n_i\left(\mu_n + \mu_p\right)eEA$$

Solved Examples

Problem: Calculate intrinsic concentration for Ge at 400 K

Solution: $ni^2 = 2.33 \times 10^{43} \times \left(\dfrac{m_n m_p}{m^2}\right)^{\frac{3}{2}} T^{\frac{3}{2}}\, e^{-E_G/KT}\Big/\left(m^3\right)^2$

$$n_i^2 = 2.33 \times 10^{43} \times \left(\frac{0.55m \times 0.37m}{m^2}\right)^{\frac{3}{2}} .\left(400\right)^{\frac{3}{2}} e^{-E_G/KT}\Big/\left(m^3\right)^2$$

$$n_i^2 = 2.33 \times 10^{31} \left(0.2035\right)^{\frac{3}{2}} e^{\left(-0.785 + 2.23 \times 10^{-4} \times 400\right)/KT} \times 400^3 \Big/\left(cm^3\right)^2$$

$$n_i^2 = 2.33 \times 10^{31} \left(0.2035\right)^{\frac{3}{2}} e^{\frac{-0.6958}{0.03448}\Big/K} \times \left(400\right)^3$$

$$n_i^2 = 2.33 \times 10^{31} \times \left(0.2035\right)^{\frac{3}{2}} \times 1.7219 \times 10^{-9} \times 64 \times 10^6$$

$$n_i^2 = 2.357 \times 10^{29} / \left(cm^3\right)^2$$

$$n_i = 4.8584 \times 10^{14} / \left(cm^3\right)^2$$

Problem: Calculate intrinsic resistivity & conductivity of a Ge and Si at room temperature.

Solution: Intrinsic Conductivity is, $\sigma_i = n_i\left(\mu_n + \mu_p\right)e$

For Ge at $T = 300K$

$n_i = 2.5 \times 10^{13} / cm^3$

$$\mu_n = 3800\,cm^2\,/\,v-sec$$

$$\mu_p = 1800\,cm^2\,/\,v-sec$$

$$\sigma_i = n_i\left(\mu_n + \mu_p\right)e = 0.0224\left(\Omega - cm\right)^{-1}$$

$$\rho_i = \frac{1}{\sigma_i} = 44.6428\ \Omega - cm \simeq 45\Omega - cm$$

For Si at $T = 300K$

$$n_i = 1.5 \times 10^{10}\,/\,cm^3$$

$$\mu_n = 1300\,cm^2\,/\,V-sec$$

$$\mu_p = 500\,cm^2\,/\,V-sec$$

$$\sigma i = ni\left(\mu_n + \mu_p\right) \times e = 1.5 \times 10^{10}\left(1300 + 500\right) \times 1.6 \times 10^{-19} = 4.32 \times 10^{-6}\left(\Omega\ cm\right)^{-1}$$

$$\rho_i = \frac{1}{\sigma_i} = 231481.4815\ \Omega\ cm = 231.5K\ \Omega\ cm \simeq 230\ K\ \Omega\ cm$$

Conclusion: From this we note that the conductivity of Si & Ge at room temp is very less i.e. why we move toward extrinsic semiconductor where σ double for every 7°C rise in temperature whereas conductivity in Ge increase by 6% per degree increase in temperature.

Problem: An intrinsic semiconductor with $E_G = 1eV$ has a carrier concentration N at temperature 200 K. Another intrinsic semiconductor has the same value of carrier concentration N at temp 600 K. What is the energy gap value for the 2nd semiconductor?

Solution: Since, $n_i^2 = NT^3\,e^{-E_g/KT}$

Intrinsic Concentration, $n_i = \sqrt{N}\ T^{\frac{3}{2}}\,e^{-E_g/2KT}$

Ratio of Intrinsic Concentration, $\dfrac{n_{i1}}{n_{i2}} = \sqrt{\dfrac{N}{N}}\ \dfrac{T_1^{\frac{3}{2}}\ e^{-\frac{1}{2K\times300}}}{T_2^{\frac{3}{2}}\ e^{-\frac{E_g}{2K\times600}}}$

Given, $n_{i1} = n_{i2}$

$$T_2^{\frac{3}{2}} \times e^{-E_g/2K\times600} = e^{-\frac{1}{2K\times200}} \times T_1^{\frac{3}{2}}$$

Thus, $e^{-\frac{Eg}{2K\times600}} = \left(\dfrac{T_1}{T_2}\right)^{\frac{3}{2}} \cdot e^{-\frac{1}{2K\times200}} = 0.1924 \cdot e^{-\frac{1}{2K\times200}}$

So, $\dfrac{-E_g}{2K \times 600} = -30.6525$

$$E_g = 3.17\,eV$$

$$E_g = 3\,eV$$

Problem: Two semiconductor material have exactly same properties except that material A has an energy band gap of 1 eV and material B has an energy gap band gap of 1.2 eV. The ratio of intrinsic concentration of material A to that of material B is?

Solution: Since, $n_i^2 = nT^3 e^{-\frac{Eg}{KT}}$

The ratio of intrinsic concentration of two materials,

$$\frac{n_{i1}^2}{n_{i2}^2} = \frac{nT^3 e^{-\frac{Eg_1}{KT}}}{nT^3 e^{-\frac{Eg_2}{KT}}} = e^{\frac{(Eg_2 - Eg_1)}{KT}}$$

$$\frac{n_{iA}^2}{n_{iB}^2} = e^{(E_{GB} - E_{GA})/KT}$$

$$\frac{n_{iA}}{n_{iB}} = \sqrt{e^{(E_{GB} - E_{GA})/KT}}$$

$$\frac{n_{iA}}{n_{iB}} = \sqrt{e^{(1.2-1)/KT}}$$

$$\frac{n_{iA}}{n_{iB}} = 46.81$$

Problem: In Intrinsic GaAS the electron & hole mobility are $0.85 \, m^2 / V - sec$ and $0.04 \, m^2 / V - sec$ respectively & corresponding effective mass are $0.068 m_0$ & $0.5 m_0$. Where m_0 is the rest mass of an electron and $m_0 = 9.11 \times 10^{-28} g$. If the energy gap of GaAS 300 K is 1.43 eV. Calculate intrinsic carrier concentration & conductivity.

Solution: $n_i^2 = 2.33 \times 10^{31} \left[\frac{m_n m_p}{m^2} \right]^{\frac{3}{2}} T^3 \cdot e^{-E_G/KT} / \left(cm^3 \right)^2$

$n_i^2 = 2.33 \times 10^{31} \times \left(0.068 \times 0.5\right)^{\frac{3}{2}} \times 300^3 \times e^{-1.43/KT}$

$n_i^2 = 5.12 \times 10^{12} / \left(cm^3\right)^2$

Intrinsic Concentration, $n_i = 2.26 \times 10^6 / cm^3 = 2.26 \times 10^{12} / m^3$

Conductivity, $\sigma_i = n_i \left(\mu_n + \mu_p \right) e = 2.26 \times 10^{12} \left(0.85 + 0.04\right) \times 1.6 \times 10^{-19} \left(\Omega - m\right)^{-1}$

$\sigma_i = 3.2184 \times 10^{-7} \, S / m$

Problem: The intrinsic resistivity of Ge at room temp is 0.47 Ω-m the electron & hole mobility at room temp are 0.39 & $0.19 m^2 / V - sec$. Calculate density of electron in the intrinsic Si & also calculate drift velocity of these charge carriers for $E = 10 kV / m$. Find intrinsic concentration.

Solution: Electron Mobility, $\mu_n = 0.39 m^2 / V - sec$

Hole Mobility, $\mu_p = 0.19\, m^2 / V - sec$

Resistivity, $\rho_i = 0.47\, \Omega - m$

Intrinsic Conductivity, $\sigma = n_i \left(\mu_n + \mu_p \right) e$

Intrinsic Concentration, $n_i = \dfrac{\dfrac{1}{\rho}}{\left(\mu_n + \mu_p \right) e} = \dfrac{\dfrac{1}{0.47}}{\left(0.39 + 0.19 \right) \times 10^{-19} \times 1.6} = 2.29 \times 10^{19} / m^3$

Drift Velocity, $v_d = \mu E = \left(\mu_n + \mu_p \right) E$

Drift Velocity of electrons, $v_d = \mu_n E = 0.39 \times 10000 = 3900\, m / sec$

Drift Velocity of holes, $v_d = \mu_p E = 0.19 \times 10000 = 1900\, m / sec$

Effect of Temperature

Conductivity of an Intrinsic Semiconductor is given by,

$\sigma_i = n_i \left(\mu_n + \mu_p \right) e$

Conductivity is proportional to intrinsic carrier concentration as well as mobility.

Intrinsic Concentration increases with temperature whereas mobility reduces with temperature.

• Mobility reduces due to lattice scattering effect in a semiconductor. But rate of increase of Intrinsic Concentration is higher as compared to rate to decrease in mobility.

• Thus, overall conductivity increases with increase in temperature.

• So, resistivity and resistance reduce with temperature and intrinsic semiconductors are said to possess negative temperature coefficient.

• So, these material are used to make thermistor in which resistance reduces with increase in temperature. Si and Ge are not generally used as they are too sensitive to impurities.

• Rate of change of conductivity with temperature is,

$\dfrac{d\sigma}{dT} = \dfrac{3}{2T} + \dfrac{E_{G0}}{2KT^2}$

• Conductivity of Ge increases by 6% per degree rise in temperature.

$\dfrac{d\sigma}{dT} = \dfrac{3}{2 \times 300} + \dfrac{0.785}{2 \times 8.862 \times 10^{-5} \times 300^2} \simeq 6\%$

• Conductivity of Si increases by 8% per degree rise in temperature.

$\dfrac{d\sigma}{dT} = \dfrac{3}{2 \times 300} + \dfrac{1.21}{2 \times 8.862 \times 10^{-5} \times 300^2} \simeq 8\%$

• Temperature sensitivity of Si is more than Ge

Effect of Temperature on Metals

In case of Metals $\sigma = n\mu e$

- In metals n remains constant as all atoms are assumed to be ionized.
- With increase in temperature mobility reduces due to reduction in mean free path because of more collisions.
- Conductivity of metals reduce with increase in temperature.
- With increase in temperature resistivity increases and resistance also increases.
- So, metals have positive temperature coefficient.
- Resistance of a Cu metal increases by 0.4% per degree rise in temperature.

To increase conductivity of intrinsic semiconductor we either increase temperature but that would also increase the leakage current which is undesirable. So, we tend to go for the other option which is to introduce the impurities called as Dopants and this process is known as Doping. The semiconductors thus formed are known as Extrinsic Semiconductors.

Extrinsic Semiconductors

When impurities are added to modulate the conductivity of intrinsic semiconductors they are known as Extrinsic Semiconductors.

Thus, Impurity added Intrinsic Semiconductors are known as Extrinsic Semiconductors.

Depending on the type of impurity atom added i.e. the group to which that particular element belongs they are classified as n-type and p-type semiconductors.

n-type Semiconductors

When 5th group or pentavalent impurities are added to an intrinsic semiconductor, then it is called as n-type semiconductor.

Pentavalent impurities are Phosphorous (P), Arsenic (As), Antimony (Sb), Bismuth (Bi) (mostly we prefer phosphorous)

5th group impurities are having 5 valence electrons but 4th group elements like Si and Ge have only 4 valence electrons and so 5th group elements have 1 extra electron and are called as Donors.
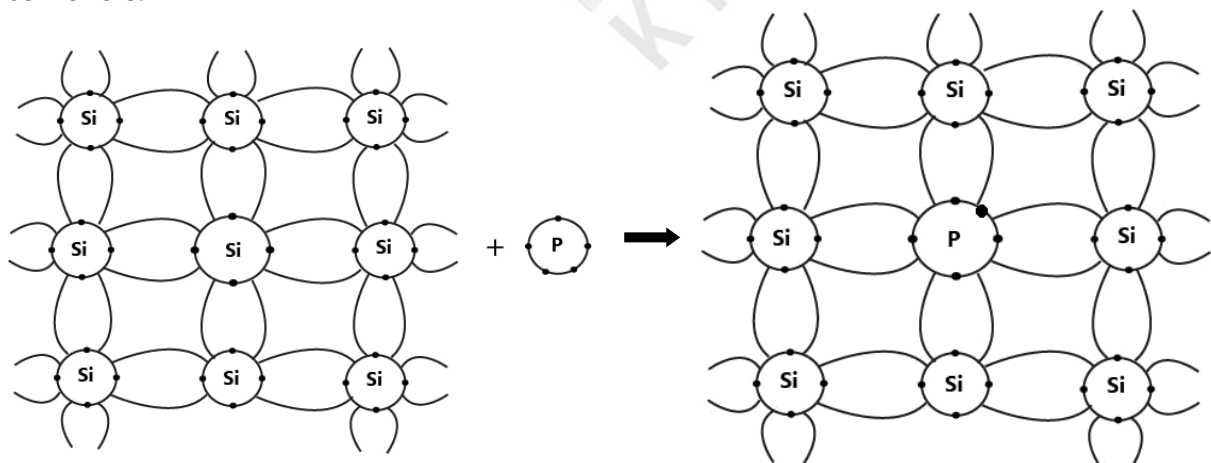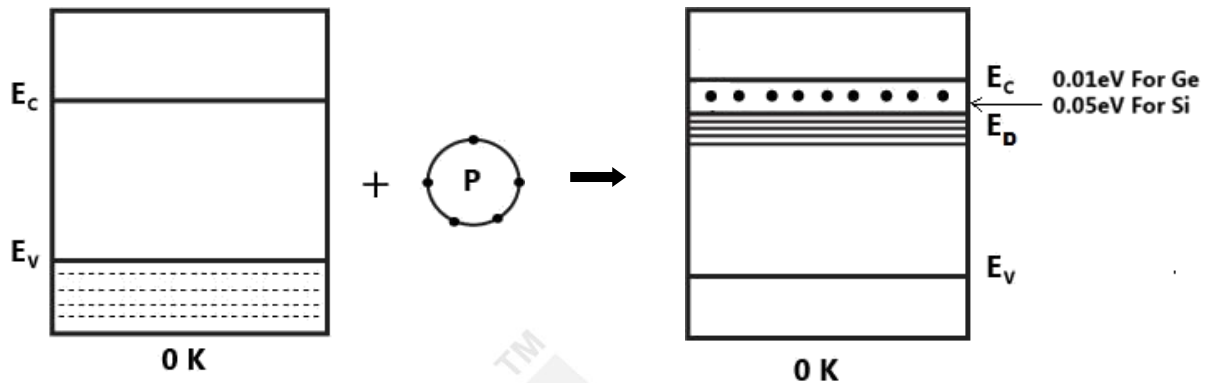

- Here P has 5 valence electron it forms bonding with 4 Si electrons but 1 electron is extra as it has to acquire ns^2np^6 i.e. octet that is why 5th group elements are called donors as it is ready to donate its 1 electron to have 8 electrons in valence shell.
- If there are 'n' number of Si atom are there then Si atoms remains 'n' in number but 1 pentavalent element (i.e. P) is added to it so there is no replacement.
- Initially, the pentavalent impurity is electrically neutral so after donating one electron it attains a positive charge and becomes a positively charged ion.
- When impurity is added then an energy band or a collection of energy states corresponding to impurity atoms appear near the conduction band.
- Let donor concentration i.e. number of donor atoms per unit volume be N_D
- Let $N_D = 1000$ donor atom, we will have 1000 free electron (free electron here means it is not bonded it is still with the atom) But we require electrons which are free & which moved to the CB. So at 0K the electrons are still in Donor Band and no electrons have progressed to Conduction Band. So, it cannot conduct any current and hence n-type semiconductor at 0K behaves as an Insulator.
- Electrons require 0.05 eV energy for Si and 0.01 eV energy for Ge to move from donor energy band to conduction band. As temperature increases electrons get this energy and begin to conduct.
- When an electron from donor band moves to conduction band it leaves a positively charged ion behind which has a complete octet and thus no electron deficiency and hence no hole is created. So, there is a creation of one positive and one negative charge and so the sample remains electrically neutral.
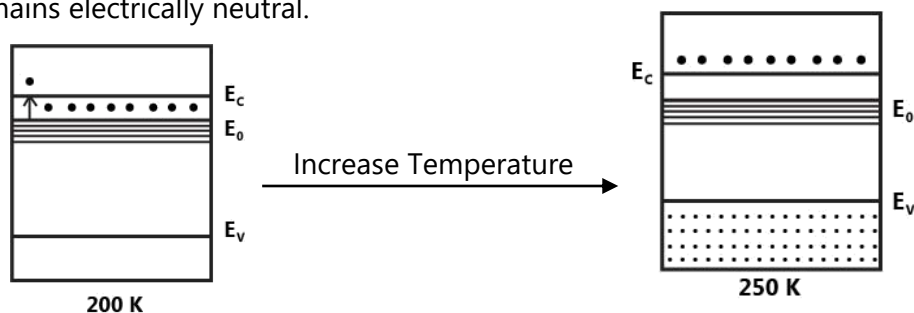
Increase Temperature

- If we further increase the temperature then electrons start moving from Valence Band into the Conduction Band i.e. Band to Band (BB) Transition.
- These electrons cannot move to donor energy level from Valence Band as there we have all stable atoms so all energy states are filled.
- In a BB Transition when an electron moves from Valence to Conduction Band, a hole is also left behind in the Valence Band in contrast to transition from Donor energy level to Conduction Band.
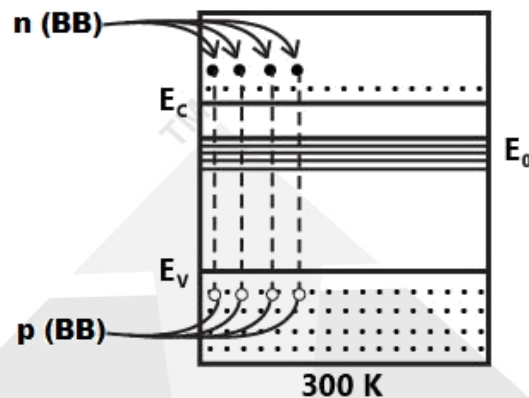
- The total positive charge in the sample is due to holes in the valence band and the positive donor ions.

Concentration of donor atoms = N_D

Concentration of holes in Valence Band = p

Total positive charge = $p + N_D$

Negative charge is due to electrons which may come from Donor atoms or from BB transition.

Concentration of electrons = Total negative charge = n

The sample is electrically neutral so,

Positive Charge = Negative Charge

$n = p + N_D$

- In n type Semiconductor electron concentration is always more than the hole concentration. This is because the electrons are contributed both by Donor atoms and BB Transition whereas holes are contributed only by BB Transition. Since energy gap between donor level and conduction band is small, number of electrons from donor level is much higher as compared to those generated from BB Transition.

- By Mass Action Law, $n.p = n_i^2$

$n.(n - N_D) = n_i^2$

$n^2 - nN_D - n_i^2 = 0$

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Solving this Quadratic Equation we get,

$$n = \frac{N_D}{2} + \sqrt{\left(\frac{N_D}{2}\right)^2 + n_i^2}$$

Usually, donor concentration is much more than intrinsic concentration $n_i <<< N_D$

Thus, $n \approx N_D$

By Mass Action Law, $p = \dfrac{n_i^2}{n} \approx \dfrac{n_i^2}{N_D}$

So, as donor concentration increases the electron concentration increases and hole concentration decreases.

- In n type Semiconductor electron are the majority carriers & holes are the minority carriers. Majority Carriers mean that they are higher in number.
- In n type Semiconductor conductivity (current) is mainly due to electrons

$$\sigma = \left(n\mu_n + p\mu_P\right)e \qquad p\mu_P <<< n\mu_n$$

- In n type Semiconductor electron concentration is greater than intrinsic concentration & hole concentration is less than intrinsic concentration.
- Hole concentration is less than intrinsic concentration because some of the electrons are recombining with the holes.

p type semiconductor

When 3rd group or trivalent impurities are added to an intrinsic semiconductor, then it is called as p-type semiconductor.

Pentavalent impurities are Boron (B), Aluminum (Al), Gallium (Ga), Indium (In), Thallium (Tl) (mostly we prefer Boron)

3rd group impurities are having 3 valence electrons but 4th group elements like Si and Ge have only 4 valence electrons and so 3rd group elements need 1 extra electron to achieve ns^2np^6 configuration and are called as Acceptors.
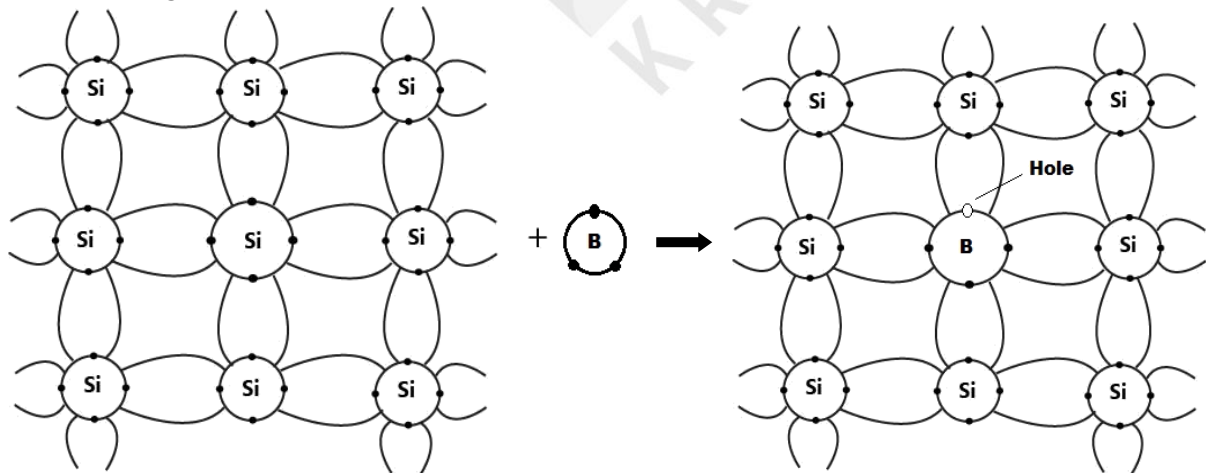
- If there are 'n' number of Si atom are there then Si atoms remains 'n' in number but 1 trivalent element (i.e. B) is added to it so there is no replacement.
- Initially, the trivalent impurity is electrically neutral so after accepting one electron it attains a negative charge and becomes a negatively charged ion.

- When impurity is added then an energy band or a collection of energy states corresponding to impurity atoms appear near the valence band.
- Let acceptor concentration i.e. number of donor atoms per unit volume be N_A
- Let $N_A = 1000$ acceptor atom, we will have 1000 holes i.e. 1000 vacancies. At 0K no electron goes from Valence Band into Acceptor Band and thus no hole is created in the Valence Band. So at 0K p type semiconductor behaves as an Insulator.

- Electrons require 0.05 eV energy for Si and 0.01 eV energy for Ge to move from valence band to acceptor energy band. As temperature increases electrons get this energy and begin to move and we can say holes begin to move from acceptor energy band to valence band.

- When an electron from valence band moves to acceptor energy level it creates a negatively charged ion which has a complete octet and thus no electron deficiency and hence no free electron is created. So, there is a creation of one positive hole and one negatively charged ion and so the sample remains electrically neutral.


If temperature is increased the electrons move from Valence Band to Acceptor Energy Level or holes move from Acceptor Energy level to Valence Band.

If intrinsic Semiconductor then Si would required 1.1 eV but due to doping it only require 0.05 eV.
This increases conductivity.

- If we further increase the temperature then electrons start moving from Valence Band into the Conduction Band i.e.
Band to Band (BB) Transition.
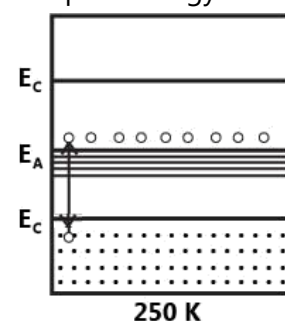
- These electrons cannot move to acceptor energy level from Valence Band as there we have all stable atoms so all energy states are filled.
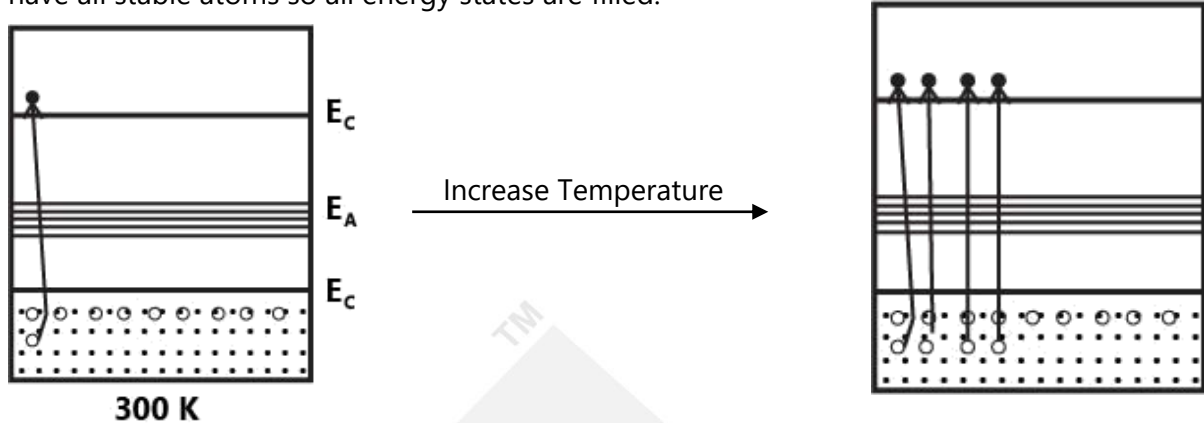
- In p type semiconductors holes are generated due to transition of electrons from Valence Band to Acceptor energy level and BB Transition.
- Electrons in Conduction Band are only due to BB Transition so number of holes in Valence Band are more than number of electrons in Conduction Band.
- The total positive charge in the sample is due to holes in the valence band.

The total negative charge is due to negatively charged acceptor ions and electrons in the Conduction Band.

Concentration of donor atoms = N_A

Concentration of electrons in Conduction Band = n

Total negative charge = $n + N_A$

Concentration of holes = Total positive charge = p

The sample is electrically neutral so,

Positive Charge = Negative Charge

$p = n + N_A$

- Semiconductor is electrically neutral i.e. total positive charge is equal to the total negative charge and this is applicable to intrinsic as well as extrinsic conductors.

- By Mass Action Law, $n.p = n_i^2$

$p.(n - N_A) = n_i^2$

$p^2 - pN_A - n_i^2 = 0$

Solving this Quadratic Equation we get,

$$p = \frac{N_A}{2} + \sqrt{\left(\frac{N_A}{2}\right)^2 + n_i^2}$$

Usually, donor concentration is much more than intrinsic concentration $n_i <<< N_A$

Thus, $p \approx N_A$

By Mass Action Law, $n = \dfrac{n_i^2}{p} \approx \dfrac{n_i^2}{N_A}$

So, as donor concentration increases the hole concentration increases and electron concentration decreases.

- In p type Semiconductor holes are the majority carriers & electrons are the minority carriers.
- In p type Semiconductor conductivity (current) is mainly due to holes

$\sigma = (n\mu_n + p\mu_p)e \qquad p\mu_p >>> n\mu_n$

- In p type Semiconductor hole concentration is greater than intrinsic concentration & electron concentration is less than intrinsic concentration.
- Electron concentration is less than intrinsic concentration because some of the electrons are recombining with the holes.

Representation of Extrinsic Semiconductor
n-type

p-type
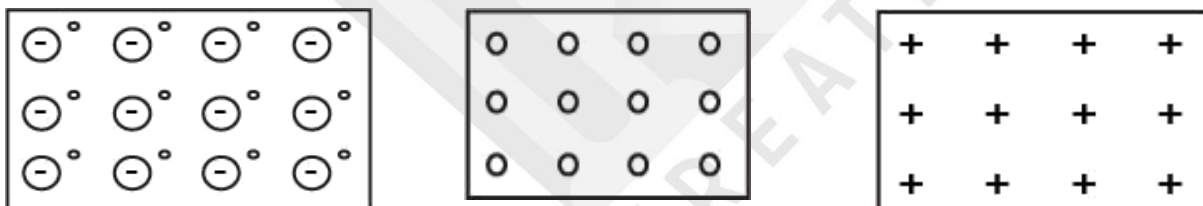
Parameters of Extrinsic Semiconductors
Conductivity
For n-type semiconductors, $n = N_D >> p$

$\sigma_n = (n\mu_n + p\mu_p)e = N_D\mu_n e$

For p-type semiconductors, $p = N_A >> n$

$\sigma_p = (n\mu_n + p\mu_p)e = N_A\mu_p e$

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Resistivity

For n-type semiconductors,

$$\rho_n = \frac{1}{\sigma_n} = \frac{1}{(n\mu_n + p\mu_P)e} = \frac{1}{N_D\mu_n e}$$

For p-type semiconductors,

$$\rho_p = \frac{1}{\sigma_p} = \frac{1}{(n\mu_n + p\mu_P)e} = \frac{1}{N_A\mu_p e}$$

Note: Usually, there is a doping of 1 impurity atom in 10^8 Si atoms.

Thus, $N_A (\text{or} N_D) = AC \times \dfrac{1}{10^8}$

Where AC is the atomic concentration of Si atoms.

If the donor concentration is added to the extent of 1 in 10^8 germanium atom to an intrinsic Germanium, its conductivity multiplies by a factor of 12.

If $N_D = 1$ in 10^7 Ge atoms

$\sigma_n = 120\,\sigma_i$

If $N_D = 1$ in 10^6 Ge atoms

$\sigma_n = 1200\,\sigma_i$

Position of Fermi Level in n type semiconductor

Fermi Energy Level represents the highest filled energy state at T=0K.

For intrinsic semiconductors,

$$n = n_i = N_C \exp\left(-\frac{E_C - E_{Fi}}{KT}\right)$$

Where, N_C is the density of states in the Conduction Band.

E_C is the energy of Conduction Band edge

E_{Fi} is the Fermi Energy for Intrinsic Semiconductors.

For an extrinsic semiconductor,

$$n = N_C \exp\left(-\frac{E_C - E_F}{KT}\right)$$

Where, E_F is the Fermi Energy for Extrinsic Semiconductors

For a n-type semiconductor, $n = N_D = N_C \exp\left(-\dfrac{E_C - E_F}{KT}\right)$

Therefore, position of the Fermi level with reference to edge of the conduction band

$$E_C - E_F = KT \ln\left(\frac{N_C}{N_D}\right)$$

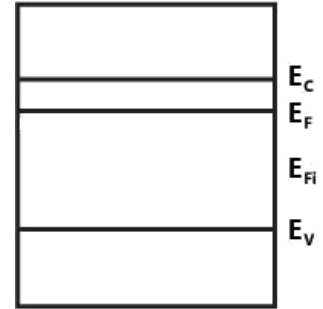**Case-1:** If $KT \ln\left(\dfrac{N_C}{N_D}\right) = 0$ i.e. $N_D = N_C$

$E_F = E_C$ i.e. E_F coincide with edge of the conduction band

Case-2: If $KT \ln\left(\dfrac{N_C}{N_D}\right) > 0$ i.e. $N_D < N_C$

$E_F < E_C$ i.e. Fermi Level lies below edge of the conduction band

Case-3: If $KT \ln\left(\dfrac{N_C}{N_D}\right) < 0$ i.e. $N_D > N_C$

$E_F > E_C$ i.e. Fermi Level lies above edge of the conduction band

From the above equations,

$$\frac{n}{n_i} = \exp\left(\frac{E_F - E_{Fi}}{KT}\right)$$

$$E_F - E_{Fi} = KT \ln\left(\frac{n}{n_i}\right)$$

$$E_F = E_{Fi} + KT \ln\left(\frac{n}{n_i}\right)$$

For n-type semiconductor, $n = N_D$

$$E_F = E_{Fi} + KT \ln\left(\frac{N_D}{n_i}\right)$$

From the band diagram,

$$E_F - E_{Fi} = \frac{1}{2}E_G - \left(E_C - E_F\right)$$

$$E_C - E_F = \frac{1}{2}E_G - \left(E_F - E_{Fi}\right)$$

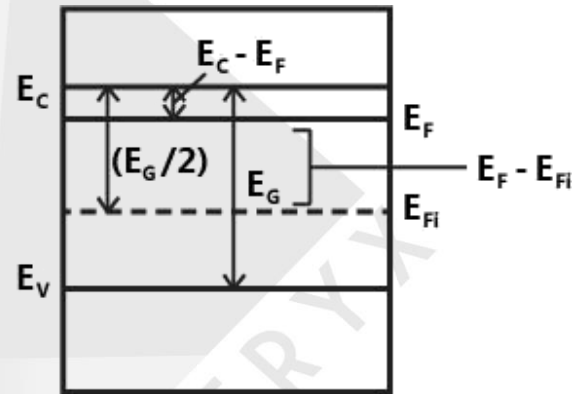As doping concentration increases the Fermi levels moves upward.
In n type Semiconductor

$$E_C - E_F = KT \ln\left(\frac{N_C}{N_D}\right)$$

$$N_C = 4.82 \times 10^{21}\left(\frac{m_n}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}}$$

Here Temperature constant so, N_C is constant

As Doping Concentration increases $E_C - E_F = KT \ln\left(\dfrac{N_C}{N_D}\right)$ decreases

Since, energy of conduction band edge is constant so Fermi Level increases as the difference reduces.

So, with increase in doping concentration the fermi level moves upwards at any fixed temperature.

As temperature increases, $N_C = 4.82 \times 10^{21} \left(\dfrac{m_n}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}} \Big/ cm^3$ increases. Thus, density of states

increases with temperature.

$$E_C - E_F = KT \ln\left(\frac{N_C}{N_D}\right)$$

So, with increase in temperature the difference between conduction band edge and fermi level increases.
Since, conduction band energy is constant the fermi level moves downwards.

Once E_F coincide with E_{Fi} it becomes intrinsic semiconductor.

So as temperature increases the semiconductor tends to become intrinsic semiconductor. Therefore at very high temp extrinsic semiconductor becomes intrinsic semiconductor.
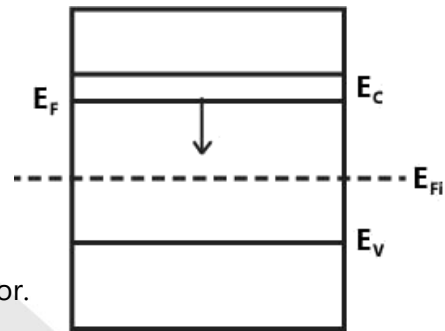
Position of Fermi Level in p type semiconductor

For intrinsic semiconductors,

$$p = n_i = N_V \exp\left(-\frac{E_{Fi} - E_V}{KT}\right)$$

Where, N_V is the density of states in the Conduction Band.

E_V is the energy of Valence Band edge

E_{Fi} is the Fermi Energy for Intrinsic Semiconductors.

For an extrinsic semiconductor,

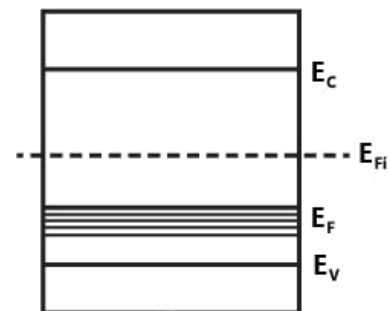$$p = N_V \exp\left(-\frac{E_F - E_V}{KT}\right)$$

Where, E_F is the Fermi Energy for Extrinsic Semiconductors

Dividing the above two equations,

$$\frac{p}{n_i} = \exp\left(\frac{E_{Fi} - E_F}{KT}\right)$$

For a p-type semiconductor, $p = N_A = N_V \exp\left(-\dfrac{E_F - E_V}{KT}\right)$

Therefore, position of the Fermi level with reference to edge of the conduction band

$$E_F - E_V = KT \ln\left(\frac{N_V}{N_A}\right)$$

$$E_F = E_V + KT \ln\left(\frac{N_V}{N_A}\right)$$

$$N_A = const = 4.82 \times 10^{21} \left(\frac{m_p}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}} \big/ cm^3$$

N_A = Acceptor Concentration

N_A = A.C × number of acceptor impurity atoms / Si or Ge atoms

Case-1: If $KT \ln\left(\frac{N_V}{N_A}\right) = 0$ i.e. $N_A = N_V$
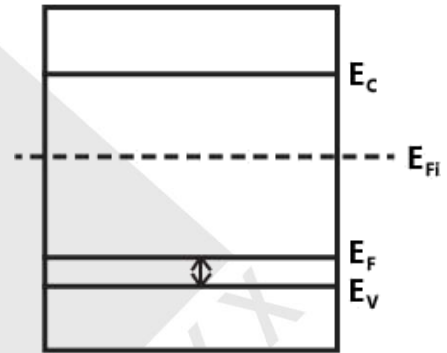
$E_F = E_V$ i.e. E_F coincide with edge of the valence band

Case-2: If $KT \ln\left(\frac{N_V}{N_A}\right) > 0$ i.e. $N_A < N_V$

$E_F > E_V$ i.e. Fermi Level lies above edge of the valence band

Case-3: If $KT \ln\left(\frac{N_V}{N_A}\right) < 0$ i.e. $N_D > N_C$

$E_F < E_V$ i.e. Fermi Level lies below edge of the valence band

For p-type semiconductor, $p = n_i \exp\left(\frac{E_{Fi} - E_F}{KT}\right)$

$$\ln \frac{p}{n_i} = (E_{Fi} - E_F) / KT$$

Since, $p = N_A$ for p-type semiconductor

$$E_{Fi} - E_F = KT \ln\left(\frac{p}{n_i}\right) = KT \ln\left(\frac{N_A}{n_i}\right)$$

The position of fermi level in terms of band gap is given by,

$$E_{Fi} - E_F = \frac{1}{2} E_G - (E_F - E_V)$$

$$E_F - E_V = \frac{1}{2} E_G - (E_{Fi} - E_F)$$

As Doping Concentration increases $E_F - E_V = KT \ln\left(\frac{N_V}{N_A}\right)$ decreases

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Since, energy of valence band edge is constant so Fermi Level decreases as the difference reduces.

So, with increase in doping concentration the fermi level moves downwards at any fixed temperature.

With increase in temperature, N_V increases and so the difference $E_F - E_V = KT \ln\left(\dfrac{N_V}{N_A}\right)$ increases and the fermi level moves away from valence band edge towards the intrinsic fermi level. At very high temperature material becomes intrinsic semiconductor.

Solved Examples

Problem: If the effective mass of an electron is $\dfrac{1}{2}$ of its true mass at room temp, how far from the edge of the CB is the Fermi level? Is E_F above or below E_C? Assume Donor Concentration $N_D = 4.4 \times 10^{14}$?

Solution: Density of states in Conduction Band, $N_C = 4.82 \times 10^{15} \left(\dfrac{m_n}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}} \Big/ cm^3$

$$N_C = 4.82 \times 10^{15} \left(\dfrac{0.5m}{m}\right)^{\frac{3}{2}} (300)^{\frac{3}{2}} \Big/ cm^3 = 8.85 \times 10^{18} \, / \, cm^3$$

$$E_C - E_F = KT \ln\left(\dfrac{N_C}{N_D}\right)$$

$$E_C - E_F = 0.02586 \times \ln\left(\dfrac{8.85 \times 10^{18}}{4.4 \times 10^{14}}\right) = 0.256 \, eV$$

E_F is at a distance of 0.256 eV from the edge of CB and E_F is below E_C as $E_C - E_F$ is positive

Problem: Repeat previous problem for the doping concentration corresponds to 1 in 10^3 Ge atom. Assume atomic concentration of Ge as $= 4.4 \times 10^{22} / \, cm^3$

Solution: The doping concentration of Ge can be calculated as,

$$N_D = A.C. \times \dfrac{\text{Number of Impurity Atoms}}{\text{Number of Ge Atoms}} = 4.4 \times 10^{22} \times \dfrac{1}{10^3} = 4.4 \times 10^{19}$$

$$E_C - E_F = KT \ln\left(\dfrac{N_C}{N_D}\right) = 0.02586 \ln\left(\dfrac{8.85 \times 10^{18}}{4.4 \times 10^{19}}\right)$$

$$E_C - E_F = -0.04147$$

$$E_C - E_F = -0.04147 \, eV$$

Fermi level E_F is at a distance of 0.0414 eV from the edge of CB and it lies above CB.

Problem: In the previous problem at what doping concentration E_F coincide with E_C ?

Solution: When Fermi Level coincides with CB $E_C = E_F$

$$KT \ln\left(\frac{N_C}{N_D}\right) = 0$$

$$\frac{N_C}{N_D} = 1$$

$$N_C = N_D$$

$$N_D = 8.85 \times 10^{18} \, / \, cm^3$$

N_D = AC × number of donor impurity atom / Ge atom

No. of donor imp atom/ Ge atom $= \dfrac{8.85 \times 10^{18}}{4.4 \times 10^{22}} = 2.011 \times 10^{-4} \simeq \dfrac{2}{10^4}$

2 impurity donor atom per 10^4 Ge atom

Problem: In n type Si donor concentration is 1 atom per 2×10^8 Si atom. Assume that effective mass of an electron equals its true mass at what temp will Fermi level coincide with the edge of the conduction band.

Solution: Effective Mass of Electron $m_n = m$

Donor Concentration, $N_D = \dfrac{5 \times 10^{22}}{2 \times 10^8} = 2.5 \times 10^{14} \, / \, cm^3$

$$E_C - E_F = KT\ln\left(\frac{N_C}{N_D}\right)$$

$$N_C = 4.82 \times 10^{15} \left(\frac{m_n}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}}\Big/cm^3 = 4.82 \times 10^{15} \left(\frac{m}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}}\Big/cm^3$$

When fermi level coincides with Conduction Band, $KT\ln\dfrac{N_C}{N_D} = 0$

$$N_C = N_D$$

Therefore, $2.5 \times 10^{14} = 4.82 \times 10^{15} \times T^{\frac{3}{2}}$

$$T^{\frac{3}{2}} = \frac{2.5 \times 10^{14}}{4.82 \times 10^{15}}$$

$$T = 0.139 \simeq 0.14K$$

Problem: In an n-type semiconductor Fermi level lies 0.3 eV below the conduction band and at 300 K. If temperature is increased to 330 K, find the new position of Fermi level. Assume density of states as constant.

Solution: Since $E_C - E_F = KT\ln\left(\dfrac{N_C}{N_D}\right)$

$$0.3\,eV = KT\ln\left(\frac{N_C}{N_D}\right)$$

$$\ln\left(\frac{N_C}{N_D}\right) = \frac{0.3}{8.62\times10^{-5}\times300}$$

$$\frac{N_C}{N_D} = 109199.0971$$

Density of States, $N_C = 4.82\times10^{15}\left(\frac{m_n}{m}\right)^{\frac{3}{2}}T^{\frac{3}{2}}\Big/cm^3$

Since, it is given that density of states remains constant so neglecting the effect of temperature.

$$E_C - E_F = 8.85\times10^{-5}\times330\times\ln\left(\frac{N_C}{N_D}\right) = 0.34\,eV$$

Problem: In an n type semiconductor Fermi level lies 0.2 eV below the conduction band at room temperature. If the concentration of donor atoms is increased by a factor of 4, find new position of fermi level with respect to conduction band. Assume $KT = 0.025\,eV$

Solution: At room temperature, $E_C - E_F = 0.2\,eV = KT\ln\left(\frac{N_C}{N_D}\right)$

$N = N_C\,e^{-(E_C-E_F)/KT}$

Initially, $N_{D1} = N_C\,e^{-(E_C-E_{F1})/KT}$

$N_{D1} = N_C e^{-0.2/0.025} = N_C\,e^{-8}$

New concentration, $N_{D2} = N_C\,e^{-(E_C-E_{F2})/KT}$

Since, $N_{D2} = 4N_{D1}$

$4N_C\,e^{-8} = N_C\,e^{-(E_C-E_{F2})/0.025}$

$4 = e^{-\frac{(E_C-E_{F2})}{0.025}+8}$

$1.3862 = -\left(\frac{E_C-E_{F2}}{0.025}\right) + 8$

Then, $E_C - E_{F2} = 0.1653\,eV$

So, by increasing donor concentration the fermi level moves closer to the Conduction Band edge.

Problem: A Si sample is doped with Boron with a concentration of $4\times10^{17}\,atoms\,/\,cm^3$. Assume intrinsic concentration of Si to be $1.5\times10^{10}\,/\,cm^3$. Assume the value of $\frac{KT}{q}$ to be 26

mV at 300 K. Determine the position of fermi level in doped Si sample with the intrinsic fermi level.

Solution: Acceptor Concentration, $N_A = 4 \times 10^{17}$ atoms / cm^3

Since, impurity concentration is much higher then intrinsic concentration, $p \approx N_A$

$n_i = 1.5 \times 10^{10} / cm^3$

$p = n_i e^{(E_{Fi} - E_F)/KT}$

Therefore, $N_A = n_i e^{(E_{Fi} - E_F)/KT}$

$\dfrac{E_{Fi} - E_F}{KT} = \ln\left(\dfrac{N_A}{n_i}\right) = \ln\left(\dfrac{4 \times 10^{17}}{1.5 \times 10^{10}}\right) = 17.098$

$E_{Fi} - E_F = 17.098 \times 0.026$ eV

$E_{Fi} - E_F = 0.444$ eV

Problem: The mobility of electron & holes in Si sample are $0.125 \, m^2 / V-sec$ and $0.048 \, m^2 / V-sec$ respectively. Determine conductivity of intrinsic Si at 27℃ if the intrinsic carrier concentration is $1.6 \times 10^{16} / m^3$.

When it is doped with 10^{23} P atoms / m^3. Determine hole concentration, conductivity & position of the Fermi level relative to the intrinsic fermi level.

Solution: The intrinsic conductivity $\sigma_{In} = n_i (\mu_n + \mu_P)e$

$\sigma_{In} = 1.6 \times 10^{16} (0.125 + 0.048) \times 1.6 \times 10^{-19} = 3.68 \times 10^{-4}$

Intrinsic Concentration, $n_i = 1.6 \times 10^{16} / cm^3$

Due to Phosphorous as impurity, the material is n-type

$N_d = 10^{23}$ (n type)

Hole Concentration, $p = \dfrac{n_i^2}{N_d} = 2.56 \times 10^9 / m^3$

Conductivity, $\sigma = (n\mu_n + p\mu_P)e = (10^{23} \times 0.125 + 2.56 \times 10^9 \times 0.048)e$

$\sigma = 2 \times 10^3 \, S / cm$

Position of Fermi Level, $(E_F - E_{Fi}) = KT \ln\left(\dfrac{n}{ni}\right) = KT \ln\left(\dfrac{N_d}{ni}\right)$

$(E_F - E_{Fi}) = 0.026 \times \ln\left(\dfrac{10^{23}}{1.6 \times 10^{16}}\right) = 0.4068$ eV

So Fermi Level lies above Intrinsic Fermi Level

Problem: A Si bar is doped with 10^{17} As atom / cm^3. What is the equilibrium hole concentration at 300 K. Where is the Fermi level of the sample located relative to intrinsic Fermi level. It is known that $n_i = 1.5 \times 10^{10} / cm^3$

Solution: Donor Concentration, $N_d = 10^{17}$

Intrinsic Concentration, $n_i = 1.5 \times 10^{10}$

Temperature $T = 300K$

Hole Concentration, $p = \dfrac{n_i^2}{n} = \dfrac{n_i^2}{N_d} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{10^{17}}$

$p = 2250 / cm^3$

Electron Concentration, $n = n_i e^{(E_F - E_{Fi})/KT}$

$10^{17} = 1.5 \times 10^{10} e^{(E_F - E_{Fi})/KT}$

$E_F - E_{Fi} = 0.406\,ev$

Problem: In Si at $T = 300K$ if the Fermi energy level is $0.22\,eV$ above the valence band energy. Calculate hole concentration.

Solution: The difference in energy levels, $E_F - E_V = 0.22\,ev$ $\qquad$ $T = 300K$

Hole concentration is given by,

$p = N_V\, e^{-(E_F - E_V)/KT}$

Effective Mass of Hole in Si, $m_p = 0.56m$

$p = 4.82 \times 10^{15} \times \left(\dfrac{m_p}{m}\right)^{\frac{3}{2}} T^{\frac{3}{2}} . e^{-0.22/8.62 \times 10^{-6} \times 300}$

$p = 4.82 \times 10^{15} \times \left(0.56\right)^{\frac{3}{2}} 300^{\frac{3}{2}} . e^{-0.22/8.62 \times 10^{-6} \times 300} = 2.1199 \times 10^{15} / cm^3$

$p = 2.12 \times 10^{15} / cm^3 = 2.12 \times 10^{21} / m^3$

Problem: A Si wafer is doped with $10^{15} P\, atom / cm^3$. Find the carrier concentration & Fermi level at 300K. Assume $N_C = 2.8 \times 10^{19} / cm^3$

Solution: The difference in conduction band and fermi energy level,

$E_C - E_F = KT \ln\left(\dfrac{N_C}{N_D}\right)$

$E_C - E_F = 0.026 \times \ln\left(\dfrac{2.8 \times 10^{19}}{10^{15}}\right) = 0.266\,eV$

For n-type semiconductor $n = N_D = 10^{15}$

Intrinsic Concentration, $n_i = 1.5 \times 10^{10} / cm^3$

Hole Concentration, $p = \dfrac{n_i^2}{n} = \dfrac{\left(1.5 \times 10^{10}\right)^2}{10^{15}} = 2.25 \times 10^5 / cm^3$

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Problem: For a particular semiconductor the effective mass of electron is $m_n = 1.4m$. If $E_C - E_F = 0.25\,eV$, determine effective density of states in the conduction band & concentration of electron in conductor at $T = 300K$

Solution: Given, $E_C - E_F = 0.25\,ev$

Density of states in conduction band, $N_C = 4.82 \times 10^{15} \left(\dfrac{mn}{m}\right)^{\frac{3}{2}} \times T^{\frac{3}{2}}$

$N_C = 4.82 \times 10^{15} (1.4)^{\frac{3}{2}} \times 300^{\frac{3}{2}}$

$N_C = 4.1487 \times 10^{19} \,/\,cm^3$

Electron Concentration, $n = N_C\, e^{-(E_C - E_F)/KT}$

$n = 4.1487 \times 10^{19} \times e^{-0.25}$

$n = 2.621 \times 10^{15} \,/\,cm^3$

Problem: The effective masses of electron & holes in Ge are $m_n = 0.55m$, $m_p = 0.37m$. Find the position of the intrinsic Fermi level in Ge at 300 K.

Solution: Fermi Level, E_F or $E_{Fi} = \dfrac{E_C + E_V}{2} - \dfrac{KT}{2} \ln\left(\dfrac{N_C}{N_V}\right)$

$\dfrac{N_C}{N_V} = \left(\dfrac{m_n}{m_p}\right)^{\frac{3}{2}} = \left(\dfrac{0.55m}{0.37m}\right)^{\frac{3}{2}} = 1.8123$

$E_F - E_{Fi} = \dfrac{kT}{2} \ln\left(\dfrac{N_C}{N_V}\right) = \dfrac{0.026}{2} \times \ln(1.8123) = 7.729 \times 10^{-3}\,eV$

Position $= 7.729 \times 10^{-3}\,eV$

Problem: For a particular semiconductor material
$N_c = 1.5 \times 10^{18} \,/\,cm^3$, $N_V = 1.3 \times 10^{19} \,/\,cm^3$ $E_G = 1.43\,eV$ at $T = 300K$
(a) Determine position of the intrinsic Fermi level with respect to center of band gap.
(b) What is the position of the Fermi level with respect to top of the valence band E_V
(c) Find the intrinsic carrier concentration of the SC at $T = 300K$

Solution: The position of fermi level with respect to center of band gap
$E_F - \left(\dfrac{E_C + E_V}{2}\right) = -\dfrac{KT}{2} \ln\left(\dfrac{N_c}{N_V}\right) = -\dfrac{0.026}{2} \times \ln\left(\dfrac{1.5 \times 10^{18}}{1.3 \times 10^{19}}\right) = 0.028\,eV$

$\dfrac{E_G}{2} = \dfrac{1.43}{2} = 0.715\,eV$

Position of Fermi level with respect to top of $VB = 0.715 + 0.028 = 0.743\,eV$

For intrinsic semiconductor, $n_i^2 = N_C N_V e^{\frac{-E_g}{kT}} = 1.95 \times 10^{37} \times e^{\frac{-1.43}{0.026}} = 2.534 \times 10^{13}$

Intrinsic Concentration, $n_i = 5.034 \times 10^6 / cm^3$

Problem: Find the concentration of holes & electron in p type Ge at 300 K if the σ is $100(\Omega - cm)$. Assume that σ due to electron is negligible as compared to holes.

Determine position of the Fermi level with respect to edge of the conduction band.

Assume for Ge $N_V = 6.0 \times 10^{19} / cm^3$, $\mu_p = 800 \, cm^2 / V - \sec$ and $E_G = 0.72 \, ev$ at 300K

Solution: Conductivity, $\sigma = 100(\Omega cm)^{-1}$

Since electron conductivity is neglected, $\sigma = P\mu_p e$

$100 = p \times 1800 \times 1.6 \times 10^{-19} = 3.47 \times 10^{17} / cm^3$

Hole Concentration, $p = N_V e^{-(E_F - E_V)/KT}$

$(E_F - E_V) = KT \ln\left(\frac{N_V}{p}\right) = 0.026 \times \ln\left(\frac{6 \times 10^{19}}{3.47 \times 10^{17}}\right)$
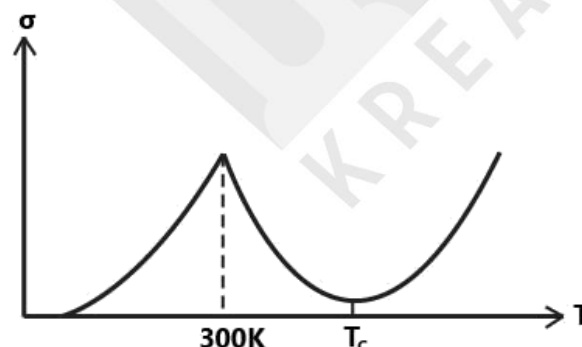
$E_F - E_V = 0.1339 \, eV$

$E_F \text{ wrt } CB = E_G - (E_F - E_V) = 0.72 - 0.134 = 0.586 \, eV$

Effect of Temperature

At high temperatures, the distance between fermi level and Valence Band edge reduces in p-type semiconductor and the distance between fermi level and Conduction Band edge reduces in n-type semiconductor.

So, an extrinsic semiconductor tends to become intrinsic semiconductor.

A heavily doped extrinsic semiconductor tends to behave like metals.

Compensated Semiconductor

A compensated semiconductor is the one which has both donor and acceptor impurity atoms in the same region. A compensated semiconductor can be formed by diffusing

acceptor impurities in the n-type semiconductor or by diffusing donor impurities in the p-type semiconductor.

If $N_A < N_D$ n type Semiconductor

If $N_A > N_D$ p type Semiconductor

If $N_A = N_D$ intrinsic Semiconductor

Case (i)

If $N_D > N_A$ $\Rightarrow$ n type

$n = N_D - N_A$ for $N_D - N_A \gg n_i$

$n = \left(\dfrac{N_D - N_A}{2}\right) + \sqrt{\left(\dfrac{N_D - N_A}{2}\right)^2 + n_i^2}$ otherwise

Case (ii)

If $N_A > N_D$ p type

$p = N_A - N_D$ $N_A - N_D \ggg ni$

$p = \dfrac{N_A - N_D}{2} + \sqrt{\left(\dfrac{N_A - N_D}{2}\right)^2 + n_i^2}$ otherwise

Case (iii)

If $N_A = N_D$ Intrinsic semiconductors

Note: Doping Concentration in Extrinsic Semiconductor ranges from 10^{13} to 10^{18} impurity atoms $/cm^3$

Solved Examples

Problem: In Ge semiconductor at $T = 300K$ the acceptor concentration is $10^{13} / cm^3$ donor concentration is $N_D = 0$. The thermal equation concentration of electron & hole are?

Solution: Intrinsic Concentration, $n_i = 2.5 \times 10^{13} / cm^3$

$N_A = 10^{13} / cm^3$

$p = \dfrac{N_A}{2} + \sqrt{\dfrac{N_A}{2} + n_i^2} = 5 \times 10^{12} + \sqrt{\left(5 \times 10^{12}\right)^2 + \left(25 \times 10^{12}\right)^2} = 5 \times 10^{12} + 10^{12} \times \sqrt{650}$

$p = 3.049 \times 10^{13} / cm^3$

$n = \dfrac{n_i^2}{p} = 2.049 \times 10^{13} / cm^3$

Problem: A sample of Ge is doped to the extent of 10^{14} donor atom/cm^3 & 5×10^{13}. Acceptor atom/cm^3. At 300 K resistivity of intrinsic Ge is $60\,\Omega\,cm$. If the applied electric field is $2V/cm$. Find the total conduction current density.

Assume $\dfrac{\mu_p}{\mu_n} = \dfrac{1}{2}$; $n_i = 2.5 \times 10^{13}/cm^3$

Solution: Donor Concentration, $N_D = 10^{14}/cm^3$

Acceptor Concentration, $N_A = 5 \times 10^{13}/cm^3$

$N_D - N_A = 5 \times 10^{13}$

Electron Concentration, $n = \dfrac{N_D - N_A}{2} + \sqrt{\left(\dfrac{N_D - N_A}{2}\right)^2 + n_i^2}$

$n = \dfrac{5 \times 10^{13}}{2} + \sqrt{\left(\dfrac{5 \times 10^{13}}{2}\right)^2 + \left(2.5 \times 10^{13}\right)^2}$

$n = 6.03 \times 10^{13}/cm^3$

$p = \dfrac{n_i^2}{n} = \dfrac{\left(2.5 \times 10^{13}\right)^2}{\left(6.03 \times 10^{13}\right)} = 1.037 \times 10^{13}/cm^3$

Given, $\mu_n = 2\mu_p$

Intrinsic Conductivity, $\sigma = n_i\left(\mu_n + \mu_p\right)e$

$\sigma_i = 3\mu_p n_i e$

Hole Mobility, $\mu_p = \dfrac{\sigma_i}{3n_i e} = \dfrac{1}{3 \times \rho_i n_i e} = \dfrac{1}{3 \times 60 \times 2.5 \times 10^{13} \times 1.6 \times 10^{-19}}$

$\mu_p = 1389\,cm^2/v-sec$

$\mu_n = 2778\,cm^2/v-sec$

Conduction Current Density, $J = \left(n\mu_n + p\mu_p\right)e\,E$

$J = \left(1389 \times 6.03 \times 10^{13} + 1.037 \times 10^{13} \times 2778\right)E e$

$J = 10^{13} \times 1.6 \times 10^{-19} \times 2 \times \left(1389 \times 6.03 + 1.037 \times 2778\right) = 0.036\,A/cm^2$

Problem: A thermistor is to be made from a GaAS doped n-type the resistor is to have a value of $2K\Omega$, R length is to be $20\,\mu m$, $A = 10^{-6}\,cm^2$ & doping efficiency is known to be 90%. If $\mu_n = 8000\,cm^2/v-sec$. Determine the doping concentration required.

Solution: Doping Efficiency, $\dfrac{n}{N_D} \times 100 = 90$

$\dfrac{n}{N_D} = 0.9 \quad => \quad n = 0.9N_D$

Electron Mobility, $\mu_n = 8000\,cm^2\,/\,v - sec$

Resistance, $R = \rho\dfrac{l}{A}$

$2 \times 10^3 = \dfrac{\rho \times 20 \times 10^{-4}}{10^{-6}}$

$\rho = 1\,\Omega - cm$

$\rho = \dfrac{1}{\sigma} = 1$

Electron Conductivity, $\sigma_n = \mu_n\,n\,e$

$1 = 0.9 \times 8000 \times 1.6 \times 10^{-19} \times N_D$

$N_D = 8.68 \times 10^{14}\,/\,cm^3$

Problem: A 100Ω resistor is to be made at a room temperature in a rectangular Si bar of 1 cm in length & $1\,mm^2$ area by doping with P atom. The electron mobility in Si at room temp is $1350\,cm^2\,/\,V - s$. Calculate dopant density required.

Solution: Resistivity of the material is,

$\rho = \dfrac{R \times A}{L} = \dfrac{10^{-2} \times 100}{1} = 1\,\Omega - cm$

$\sigma = \dfrac{1}{\rho} = 1\,S\,/\,cm$

Electron Concentration, $n = \dfrac{1}{\rho e \mu} = \dfrac{1}{1.6 \times 10^{-19} \times 1350} \simeq 4.629 \times 10^{15}\,/\,cm^3$

$\rho = 4.629 \times 10^{21}\,/\,m^3$

Problem: Pure Si has an electrical resistivity of $3000\,\Omega - m$. If the free carrier concentration is $1.1 \times 10^6\,/\,m^3$. Electron mobility is 3 times that of hole mobility. Calculate mobility of electron & hole mobility.

Solution: Resistivity, $\rho = 3000\,\Omega m$

Since the sample is pure Si, it behaves as intrinsic semiconductor and electron and hole concentrations will be equal

$n_i = 1.1 \times 10^6$

Conductivity, $\sigma_i = n_i\left(\mu_n + \mu_p\right)e$

$\dfrac{1}{3000} = 1.6 \times 10^{-19} \times 1.1 \times 10^6 \left(\mu_n + \mu_p\right)$

$\mu_n + \mu_p = \dfrac{1}{3000 \times 1.76 \times 10^{-13}}$

Since, $\mu_n = 3\mu_p$

$$4\,\mu_p = \frac{1}{3000 \times 1.76 \times 10^{-13}}$$

$$\mu_p = 4.7348 \times 10^8 \, m \, t^2 / v - sec$$

$$\mu_n = 1.420 \times 10^9 \, m^2 / v - sec$$

Problem: A P type material has an acceptor concentration of $N_A = 10^{16} / cm^3$ & its intrinsic carrier concentration is $1.48 \times 10^{10} / cm^3$ the hole & electron mobility are $0.05 m^2 / V - sec$ & $0.13 m^2 / V - sec$ respectively. Calculate resistivity of the material.

Solution: Hole Mobility, $\mu_p = 0.05 m^2 / v - sec$

Electron Mobility, $\mu_n = 0.13 m^2 / v - sec$

Acceptor Concentration, $N_A = 1 \times 10^{16} / cm^3$

Intrinsic Concentration, $n_i = 1.48 \times 10^{10} / cm^3$

In p-type semiconductor, $p \approx N_A = 10^{16} / cm^3$

$$n = \frac{\left(1.48 \times 10^{10}\right)^2}{10^{16}} = 21904 / cm^3$$

Conductivity, $\sigma = \left(n\mu_n + p\mu_P\right)e = \left(21904 \times 0.13 + 10^{16} \times 0.05\right) \times 1.6 \times 10^{-19} \times 10^4$

$\sigma = 0.8 S / cm$

Resistivity, $\rho = 1.25 \Omega - cm$

Classification of Extrinsic Semiconductor based on Doping

Based on doping concentration the semiconductors can be classified in the following four categories,

(i) Lightly doped Semiconductor : Doping Concentration : 10^{13} to 10^{14} impurity atoms / cm^3

(ii) Moderately doped Semiconductor : Doping concentration : 10^{15} to 10^{16} imp atoms / cm^3

(iii) Heavily dope Semiconductor : Doping concentration : 10^{17} to 10^{18} imp atoms / cm^3

(iv) Degenerative Semiconductor : Doping concentration > 10^{18} imp atoms / cm^3
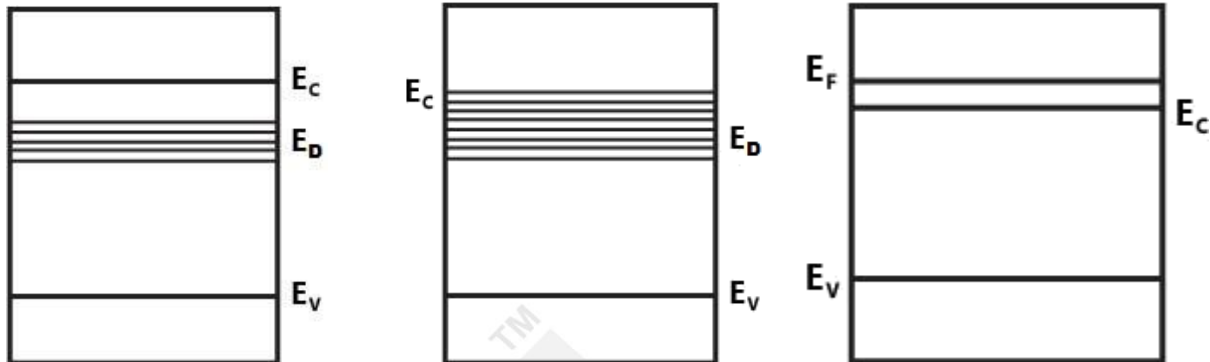
Photodiodes and Avalanche Photodiodes are an application of Lightly Doped Semiconductors.
Zener Diode is an application of Heavily Doped Semiconductors.
Tunnel Diode is an application of Degenerate Semiconductor.
Normal PN Junction Diode is an application of Moderately Doped Semiconductor.

Degenerate Semiconductors

- As the doping concentration increases the energy gap between Donor Energy Level and the Conduction Band reduces. At some value of doping concentration equal to density of states the donor energy level coincides with the Conduction Band edge.

- If the doping concentration is greater than 10^{18} impurity atoms$/\,cm^3$ or if the donor energy band coincide with the edge of the conduction band or if the Fermi level lies inside the conduction band, those type of semiconductor are called as Degenerative Semiconductors.

- Degenerate Semiconductor are using to make a special type of devices like tunnel diodes.

- Degenerative Semiconductor are not using make a normal electronic devices.

Charge Density in a Semiconductor

The negative charge in a semiconductor is due to electrons and acceptor impurity which on accepting electrons become negatively charged.

The positive charge in a semiconductor is due to holes and donor impurity which on donating electrons become positively charged.

Total negative charge $=\left(n+N_A\right)e$

Total positive charge $=\left(p+N_D\right)e$

According to charge neutrally principle, semiconductor is electrically neutral

Total negative charge = Total positive charge

This will be applicable to intrinsic as well as extrinsic Semiconductor

$n+N_A=p+N_D$

Let n type Semiconductor; where $N_A=0$

$n=p+N_D$

In n type semiconductor electron are the majority carriers & holes are the minority carriers.

$n\simeq N_D \qquad \therefore n>>p$

Number of electrons in n type Semiconductor $n_n\simeq N_D \qquad \therefore n_n>>>p_n$

$n_n.p_n=n_i^{\,2}$

$N_D \cdot p_n = n_i^2$

$p_n = \dfrac{n_i^2}{N_D} = \dfrac{n_i^2}{n_n}$

Let P type Semiconductor where, $N_D = 0$

$p = n + N_A$

$p \simeq N_A \qquad \therefore p >>> n$

Number of holes in p type semiconductor $p_p \simeq N_A \qquad\qquad p_p >>> n_p$

$n_p \cdot p_p = n_i^2$

$N_a \cdot n_p = n_i^2$

$n_p = \dfrac{n_i^2}{N_A} = \dfrac{n_i^2}{p_p}$

Current Flow in Semiconductors

There are two major phenomenon responsible for current conduction in any semiconductor:

- Drift Current
- Diffusion Current

Drift Current

The current due to flow of charge carriers under the influence of Electric Field is known as Drift Current.

Electric Field is given by, $E = \dfrac{V}{L}$

Where, V is the potential difference across the ends of sample
L is the length of sample

Current depends on Area so we consider $J = \dfrac{I}{A}$ to make the consistency as J always remain constant.

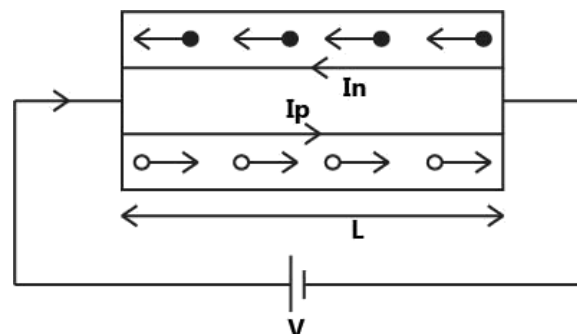Similarly we always consider $\dfrac{C}{A}$ as it remain constant instead of only capacitance as C depends on Area.

Drift current density due to electron ($J_{n(drift)}$)

$J_{n(drift)} = n\mu_n eE$

Drift current density due to holes ($J_{P(drift)}$)

$J_{p(drift)} = p\mu_p e\, E$

Total drift current density $J_{(drift)} = J_{n(drift)} + J_{P(drift)}$

$$J_{(drift)} = n\,\mu_n\,e\,E + P\,\mu_P\,e\,E = \left(n\,\mu_n + p\,\mu_P\right)e\,E$$

Total drift current $I_{drift} = J_{drift}\,A = \left(n\mu_n + p\mu_P\right)e\,E\,A$

Diffusion Current

Diffusion is a process by which any material travels from a region of high concentration to a region of low concentration or we can say the flow is due to concentration gradient.
This is best illustrated by flow of liquid between two containers as shown below,
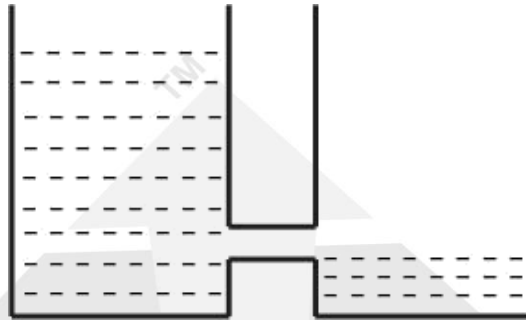
Diffusion current density in semiconductors is directly proportional to concentration gradient. Concentration Gradient is due to non-uniform doping i.e. the concentration is not same at all points which causes charge carriers to flow from a region of high concentration to a region of low concentration.

(i) $J_{p(diff)} \propto \dfrac{dp}{dx}$

(ii) $J_{n(diff)} \propto \dfrac{dn}{dx}$

Case – 1   Let $P_1 = 100$ $P_2 = 60$
Case – 2   Let $P_1 = 100$ $P_2 = 50$

In case (2) current will flow more as the difference is large between P_1 & P_2

Diffusion current density due to holes $J_{p(diff)}$

$$J_{p(diff)} = -e\,D_P\,\frac{dp}{dx}$$

D_P = diffusion const for holes $e = 1.6 \times 10^{-19} c$

$D_P = 47\ cm^2 / sec$ for Ge

$D_P = 13\ cm^2 / sec$ for Si

$$\frac{dp}{dx} = \frac{P_2 - P_1}{x_2 - x_1}$$ (means we are getting negative as $P_2 < P_1$)

Current flows in the direction of holes and since holes will flow from direction of high concentration to low concentration and in that direction concentration gradient is negative so negative sign is used.

                                 www.kreatryx.com
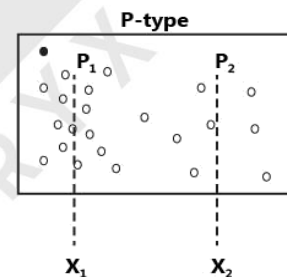
Diffusion current density due to electrons $J_{n(diff)}$

$$J_{n(diff)} = e\,D_n \frac{dn}{dx}$$

D_n : diffusion current for electron

D_n = 99 cm^2 / sec for Ge

D_n = 34 cm^2 / sec for Si

Current flows opposite to the direction of flow of electrons and electrons will flow from higher concentration to lower concentration so negative sign cancels out and there is no negative sign in the formula.

Total diffusion current density J_{diff}

$$J_{diff} = J_{n\,diff} + J_{p\,diff}$$

$$J_{diff} = e\,D_n \frac{dn}{dx} - e\,D_p \frac{dp}{dx}$$

Total diffusion current $I_{diff} = J_{diff} . A$

$$I_{diff} = \left(e\,D_n \frac{dn}{dx} - e\,D_p \frac{dp}{dx} \right) A$$

Total current density $J = J_{drift} + J_{diff}$

$$J = \left(n\mu_n + p\mu_p \right)eE + \left(e\,D_n \frac{dn}{dx} - eD_p \frac{dp}{dx} \right)$$

Total current $\quad I = I_{drift} + I_{diff} = J . A$

$$I = \left(n\mu_n + p\mu_p \right)eEA + \left(e\,D_n \frac{dn}{dx} - eD_p \frac{dp}{dx} \right) A$$

Mostly concentration diffusion is very small i.e. why most of the cases we take $I_{diffusion} = 0$ we try to neglect diffusion current & consider I_{drift} only.

In PN diode & BJT (due to existence of junction) total $\simeq$ diffusion current there I_{drift} is almost zero. As electric field is zero (almost).

In FET there is no junction , so total current $\simeq$ drift current

Einstein's Relation

At a constant temperature, ratio of diffusion constant & mobility remains constant

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

n type:

$$\frac{D_n}{\mu_n} = V_T = \frac{\overline{K}T}{q} = KT = \frac{T}{11600} = 0.02586$$

$$\mu_n = \frac{D_n}{0.02586} \simeq 39\,D_n$$

P type:

$$\frac{D_P}{\mu_P} = V_T = \frac{\overline{K}T}{q} = KT = \frac{T}{11600} = 0.02586$$

$$\mu_P = \frac{D_P}{0.02586} \simeq 39\,D_P$$

Therefore, at room temperature $\mu = 39D$

For Si and Ge, the diffusion coefficient for electrons is,

Ge: $D_n = \dfrac{\mu_n}{39} = \dfrac{3800}{39} \simeq 99\,cm^2\,/\,sec$

Si: $D_n = \dfrac{\mu_n}{39} = \dfrac{1300}{39} \simeq 39\,cm^2\,/\,sec$

For Si and Ge, the diffusion coefficient for holes is,

Ge: $D_p = \dfrac{\mu_p}{39} = \dfrac{1800}{39} \simeq 47\,cm^2\,/\,sec$

Si: $D_p = \dfrac{\mu_p}{39} = \dfrac{500}{39} \simeq 13\,cm^2\,/\,sec$

Solved Examples

Problem: The diffusion constant for holes in Si is $13\ cm^2\,/\,sec$. What is the diffusion current if the gradient of the hole concentration is $\dfrac{dp}{dx} = -2 \times 10^{14}$. Assume unit area whenever not mentioned in question.

Solution: Diffusion Coefficient, $D_P = 13\,cm^2\,/\,sec$

$$\frac{dp}{dx} = \left(-2 \times 10^{14}\right) holes\,/\,cm^3\,/\,cm$$

Diffusion Current due to holes, $I_{diff} = eD_P \dfrac{dp}{dx}\,A = 1.6 \times 10^{-19} \times 13 \times 2 \times 0^{14} \times 1$

$$I_{diff} = 4.16 \times 10^{-4} = 0.0416\,mA$$

Problem: The electron concentration in a sample of uniformly doped n type Si at 300 K varies linearly from $10^{17} / cm^3$ at $x = 0$ to $6 \times 10^{16} / cm^2$ at $x = 2\mu m$. If electric charge is $1.6 \times 10^{-19} C$ & diffusion constant $D_n = 35 \, cm^2 / sec$. Determine the diffusion current density in Si?

Solution: $n_1 = 10^{17} / cm^3$ at $x = 0$

$n_2 = 6 \times 10^{19} / cm^3$ at $x = 2$

$\dfrac{dn}{dx} = \left(\dfrac{n_2 - n_1}{x_2 - x_1} \right) = \dfrac{6 \times 10^{16} - 10^{17}}{2 \times 10^{-4}} = -2 \times 10^{20} / cm^4$

Diffusion Current Density, $J = eD_n \dfrac{dn}{dx}$

$J = 1.6 \times 10^{-19} \times 35 \times -4 \times 10^{20} = -1120 \, A / cm^2$

Problem: Assuming that electron mobility in intrinsic Si is $1500 \, cm^2 / V - sec$ at room temp & the corresponding thermal voltage equivalent of temperature $V_T = 25.9 \, mV$. What is the approximate value of the electron diffusion constant?

Solution: Electron Mobility, $\mu_n = 1500 \, cm^2 / v - sec$

By Einstein's Relation, $\dfrac{D_n}{\mu_n} = V_T$

Diffusion Coefficient, $D_n = 38850 \times 10^{-3} = 38.85 \, cm^2 / sec$

Problem: In an n type semiconductor at $T = 300K$ the electron concentration varies uniformly from $2 \times 10^{18} / cm^3$ to $5 \times 10^{17} / cm^3$ over a distance 1.5 mm & the diffusion current density is $360 \, A / cm^2$. Find the mobility of electron.

Solution: Diffusion Current Density, $J = eD_n \dfrac{dn}{dx}$

$10^{18} \times \dfrac{(0.5 \cdot 2)}{1.5 \times 10^{-1}} \times 1.6 \times 10^{-19} \times D_n = 360$

Diffusion Coefficient, $D_n = 2.25 \times 10^{20} \times 10^{-18} = 2.25 \times 10^2 \simeq 225 \, cm^2 / sec$

By Einstein's Relation, $\mu_n = 39 D_n = 8.775 \times 10^3 \, cm^2 / v - sec$

Problem: The Si sample with unit cross sectional area shown below as thermal equation following information is gain $T = 300K$ electronic charge $1.6 \times 10^{-19} C$ Thermal voltage $V_{th} = 26 \, mV$, Electron mobility $\mu = 1350 \, cm^2 / V - sec$?

Determine the magnitude of electric field at $x = 0.5\mu m$.

Determine magnitude of the electron drift current density at $x = 0.5\mu m$?
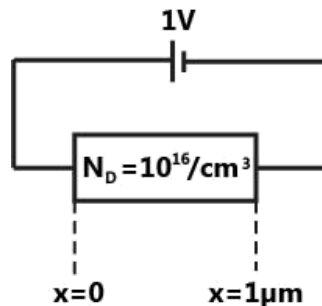
Solution: Electric Field, $E = \dfrac{V}{L} = \dfrac{1}{10^{-6}} = 10^6 \text{ V / m or } 10^4 \text{ V / cm or } 10 \text{ kV / cm}$

Drift Current Density, $J_{drift} = n\mu_n e\, E = 10^{16} \times 1350 \times 1.6 \times 10^{-19} \times 10^4$

$J_{drift} = 2.16 \times 10^{-6} \times 10^{10} = 2.16 \times 10^4 = 21600 \text{ A / cm}^2$

Carrier Generation and Recombination

When temperature is increases the valence electrons gets ionized and electron-hole pair is generated.

When electrons travel back from Conduction Band to Valence Band then they will recombine with the holes and electron-hail pair will disappear and this process is known as recombination.

The mean time for which electrons and hole exist before recombination takes place is known as lifetime of electrons and holes respectively.

τ_n = mean lifetime of electrons

τ_p = mean lifetime of holes

Generation and Recombination process occurs simultaneously.

The recombination rate is proportional to the concentration of charge carriers,

$$\dfrac{dp}{dt} \propto -p$$

$$\dfrac{dn}{dt} \propto -n$$

Negative sign indicates that concentration of charge carriers decreases due to recombination.

$$\dfrac{dp}{dt} = -\dfrac{p}{\tau_p}$$

$$\dfrac{dn}{dt} = -\dfrac{n}{\tau_n}$$

If generation rate "g" is also considered

$$\frac{dp}{dt} = g - \frac{p}{\tau_p}$$

$$\frac{dn}{dt} = g - \frac{n}{\tau_n}$$

Usually, we are more interested in change in concentration of minority carriers as even a small change in concentration will be more in terms of percentage. Assume in a n type semiconductor we have 1000 electrons and 5 holes then if 50 electron hole pairs are generated then electrons becomes 1050 and holes become 55 so number of holes increases by 10 times. Thus, minority carrier concentration will be considered henceforth.

Assuming at t=0 $p = p_0$ and no temperature is increased

$$\frac{dp}{dt} = 0 = g - \frac{p_0}{\tau_p}$$

Therefore, $g = \dfrac{p_0}{\tau_p}$

At any other temperature, $\dfrac{dp}{dt} = g - \dfrac{p}{\tau_p} = -\dfrac{p - p_0}{\tau_p}$

Assuming $p' = p - p_0$

$$\frac{dp'}{dt} = \frac{dp}{dt}$$

So, above equation can be expressed as,

$$\frac{dp'}{dt} = -\frac{p'}{\tau_p}$$

The solution of this differential equation is, $p' = p'(0) e^{-\frac{t}{\tau_p}}$

Here, p' represents the excess charge carriers and as we can see from this equation that excess charge carriers decay exponentially.

So, $p(t) = p'(0) e^{-\frac{t}{\tau_p}} + p_0$

So, after generation is switched off in an n-type semiconductor minority carriers follow this relation.

Similarly, in a p-type semiconductor

$$n(t) = n'(0) e^{-\frac{t}{\tau_n}} + n_0$$

Continuity Equation

The concentration of charge carriers is not only affected by generation and recombination processes but also by flow of current through the material.

$$I_p = \frac{dq_p}{dt}$$

$$J_p = \frac{I_p}{A} = \frac{dq_p}{Adt}$$

$$\frac{dJ_p}{dx} = \frac{dq_p}{Adxdt}$$

$$\frac{dJ_p}{dx} = \frac{d}{dt}\left(\frac{q_p}{Adx}\right)$$

Adx can be considered as differential volume and charge per unit volume is charge density which is "e" times the charge carrier density.

$$\frac{dJ_p}{dx} = -e\frac{dp}{dt}$$

Negative sign indicates that charge concentration reduces due to flow of current.

$$\frac{dp}{dt} = -\frac{1}{e}\frac{dJ_p}{dx}$$

So, the generation and recombination equation can be rewritten as,

$$\frac{dp}{dt} = g - \frac{p}{\tau_p} - \frac{1}{e}\frac{dJ_p}{dx}$$

Substitute the value of current density in terms of drift and diffusion components,

$$\frac{dp}{dt} = g - \frac{p}{\tau_p} - \frac{1}{e}\frac{d}{dx}\left(p\mu_p eE - eD_p\frac{dp}{dx}\right)$$

Since, $g = \frac{p_0}{\tau_p}$

$$\frac{dp}{dt} = \frac{p_0 - p}{\tau_p} - \mu_p\frac{d(pE)}{dx} + D_p\frac{d^2p}{dx^2}$$

Similarly for electrons in p-type semiconductor

$$\frac{dn}{dt} = \frac{n_0 - n}{\tau_n} - \mu_n\frac{d(nE)}{dx} - D_n\frac{d^2n}{dx^2}$$

Case-1: If concentration is independent of distance and E=0

$$\frac{dp}{dt} = \frac{p_0 - p}{\tau_p}$$

This same case we discussed earlier.

Case-2: If concentration is independent of time and E = 0

$$\frac{dp}{dt} = 0 = \frac{p_0 - p}{\tau_p} + D_p \frac{d^2p}{dx^2}$$

So, $D_p \dfrac{d^2p}{dx^2} = \dfrac{p - p_0}{\tau_p}$

$$\frac{d^2p}{dx^2} = \frac{p - p_0}{L_p{}^2} \quad \text{where } L_p = \sqrt{D_p \mu_p}$$

L_p is known as Diffusion Length for holes

Assume $p' = p - p_0$

So, $\dfrac{d^2p'}{dx^2} = \dfrac{p'}{L_p{}^2}$

The solution of this differential equation is,

$$p'(x) = k_1 e^{-x/L_p} + k_2 e^{x/L_p}$$

The second term must be zero as due to recombination the excess concentration of holes at infinite distance must be 0.

$$p'(\infty) = 0$$

So, $p'(x) = k_1 e^{-x/L_p}$

At x = 0 $\;p'(x) = k_1 = p'(0)$

$$p(x) = p_0 + p'(0) e^{-x/L_p}$$

Diffusion Length is the distance in semiconductor at which the diffusion current density becomes 1/e of the original value.

Hole Diffusion Current Density

Diffusion current density due to holes is, $J_p = -eD_p \dfrac{dp}{dx}$

Since, $p(x) = p_0 + p'(0) e^{-x/L_p}$

$$\frac{dp(x)}{dx} = -\frac{p'(0)}{L_p} e^{-x/L_p}$$

$$J_p = -eD_p\left(-\frac{p'(0)}{L_p}e^{-x/L_p}\right) = \frac{eD_p p'(0)}{L_p}e^{-x/L_p}$$

Since, $p(0) = p_0 + p'(0)$

$$J_p = \frac{eD_p\left(p(0) - p_0\right)}{L_p}e^{-x/L_p}$$

So, Hole Diffusion Current decreases with increase in distance.

Electron Diffusion Current Density

Diffusion current density due to electron is, $J_n = eD_n\dfrac{dn}{dx}$

Since, electrons and holes are generated together. So, $p - p_0 = n - n_0$

Therefore, $\dfrac{dp}{dx} = \dfrac{dn}{dx}$

So, $J_n = eD_n\dfrac{dp}{dx}$

Since, $J_p = -eD_p\dfrac{dp}{dx}$

So, $J_n = -\left(\dfrac{D_n}{D_p}\right)J_p$

In general, $\left(\dfrac{D_n}{D_p}\right) = 2.11$ for Ge and 2.61 for Si.

Calculation of Drift Current density

Since holes concentration is insignificant in n-type semiconductor so hole drift current may be neglected.

Assuming low voltage is applied so total voltage and current density can be assumed to be zero.

$$J_n(\text{diff}) + J_p(\text{diff}) + J_n(\text{drift}) + J_p(\text{drift}) = 0$$

Since hole drift current is neglected

$$J_n(\text{diff}) + J_p(\text{diff}) + J_n(\text{drift}) = 0$$

$$J_n(\text{drift}) = -\left(J_n(\text{diff}) + J_p(\text{diff})\right)$$

$$J_n\left(\text{drift}\right) = \left(\frac{D_n}{D_p} - 1\right)J_p\left(\text{diff}\right)$$

So, magnitude of electron drift current will almost be same as hole diffusion current and its value will also vary with distance.

Note: The same expressions can also be obtained for a p-type semiconductor as,

Concentration of electrons, $n\left(x\right) = n_0 + n'\left(0\right)e^{-x/L_n}$

Diffusion Length of electrons, $L_n = \sqrt{D_n \mu_n}$

Electron Diffusion Current Density, $J_n = \dfrac{eD_n\left(n\left(0\right) - n_0\right)}{L_n}e^{-x/L_n}$

Hole Diffusion Current Density, $J_p = -\left(\dfrac{D_p}{D_n}\right)J_n$

Hole Drift Current Density, $J_p\left(\text{drift}\right) = \left(\dfrac{D_p}{D_n} - 1\right)J_n\left(\text{diff}\right)$

Solved Examples

Problem: The minority carrier lifetime & diffusion constant in a semi-conducting material or $100\,\mu\sec$ & $100\,cm^2/\sec$ respectively. The diffusion length is ?

Solution: Diffusion Length, $L = \sqrt{\tau\,D}$

$L = \sqrt{100 \times 10^{-6} \times 100} = 0.1\,cm$

Problem: Electron mobility & life time in a semiconductor at room temperature $0.36\,m^2/v - \sec$ & $340\,\mu\sec$ respectively. The diffusion length of electron is?

Solution: Diffusion Coefficient, $D_n = \mu_n \times \dfrac{kT}{q} = 0.36 \times 0.026 = 9.36 \times 10^{-3}\,m^2/\sec$

Diffusion Length, $L_n = \sqrt{D_n \tau_n} = \sqrt{340 \times 10^{-6} \times 9.36 \times 10^{-3}} = 1.77 \times 10^{-3}\,m$

$L_n = 1.77\,mm$

Problem: A semiconductor is irradiated with light such that carriers are uniformly generated throughout its volume. The semiconductor is n type with $N_D = 10^{19}/cm^3$. If the excess electron concentration in the steady state is $\Delta n = 10^{15}/cm^3$ & minority carrier life time $T_p = 10\,\mu\sec$. The generation rate due to irradiation is?

Solution: Under Steady State, Generation Rate = Recombination Rate

$$\text{Generation rate} = \frac{\Delta n}{T_P} = \frac{10^{15}}{10 \times 10^{-6}} = 10^{20} \, e - hpair \, / \, cm^3 \, / \, sec$$

Hall Effect

It states that when a current carrying or semiconductor is placed in transverse magnetic field, then an induced voltage is generated which is perpendicular to both B and I.

Hall effect can also applied for metal but it gives very small values but in case of Semiconductor it gives large value.
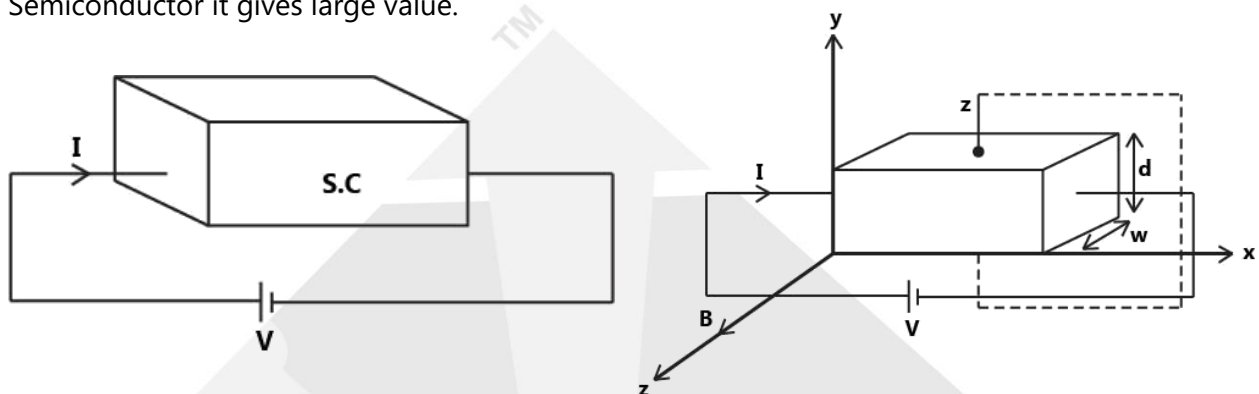
When a specimen of Semiconductor or metal carrying a current of I in the positive X direction placed in a magnetic field which is in positive Z direction. An electric field is induced which is perpendicular to both I & B in the negative y direction.

- It is based to find whether a Semiconductor is n type or p type
- To find carrier concentration
- By simultaneously measuring the conductivity mobility can be calculated.

If the semiconductor is n-type, the majority carrier electrons are flowing in the negative x direction. The electrons will experience a magnetic force (Lorentz Force)

$$F = -e\left(\vec{v} \times \vec{B}\right) = -evB\left(-\vec{a_x} \times \vec{a_z}\right) = evB\left(-\vec{a_y}\right)$$

So electrons accumulate at the lower surface and an electric field is induced in the negative y-direction. So the polarity of voltage induced is top surface positive and bottom surface negative.

If the semiconductor is p-type, the majority carrier holes are flowing in the positive x direction. The electrons will experience a magnetic force (Lorentz Force)
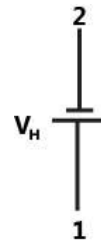
$$F = e\left(\vec{v} \times \vec{B}\right) = evB\left(\vec{a_x} \times \vec{a_z}\right) = evB\left(-\vec{a_y}\right)$$

So holes accumulate at the lower surface and an electric field is induced in the positive y-direction. So the polarity of voltage induced is top surface negative and bottom surface positive.
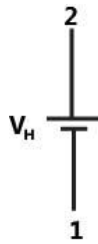
n-type: p-type:

The induced electric field opposes the motion of charge carriers due to magnetic field and under steady state, no more charge carriers accumulate and there is no net force on the charges.

$$E = \frac{V_H}{d} \qquad ....(1)$$

$$V_H = E \cdot d \qquad ....(2)$$

Force on charge due to electric field = eE

Force due to magnetic field = evB

Under steady state, $eE = evB$

Induced Electric Field, $E = vB \qquad ....(3)$

Current Density,

$$J_n = ne\mu_n E = \rho v_n$$

$$J_p = pe\mu_p E = \rho v_p$$

Where, v_n is the drift velocity of electrons

v_p is the drift velocity of holes

ρ is the charge density in semiconductor

The charge density for n-type and p-type semiconductor is,

n-type: $\rho = ne$

p-type: $\rho = pe$

Current Density, $J = \rho v$

The area perpendicular to current direction $= wd$

Current Density $J = \dfrac{I}{wd} = \rho v = J \qquad ....(4)$

So, $v = \dfrac{J}{\rho}$

From equation (2)

$$V_H = Ed = Bvd$$

$$V_H = \frac{BJd}{\rho}$$

$$V_H = \frac{BJd}{\rho} = \frac{BI}{w\rho}$$

$$V_H = E\,d = B\,v\,d = \frac{B\,J\,d}{\rho} = \frac{B\,I}{w\,\rho}$$

So, Hall Voltage $V_H \propto I$

Also, $V_H \propto \dfrac{1}{\rho}$

Where, $\rho = ne$

For n type, $e = -1.6 \times 10^{-19}\,C$

For p type, $e = 1.6 \times 10^{-19}\,C$

Hence, Hall Voltage will come out to be negative for n type semiconductors and positive for p type semiconductors.

Note: In case of intrinsic semiconductor, electrons and holes are present in equal number so when both accumulate on the lower surface there should be no net charge but since mobility of electrons is higher than holes so it behaves as n type semiconductor.

Hall Coefficient or Hall Constant $\left(R_H\right)$

It is the reciprocal of charge density i.e.

$$R_H = \frac{1}{\rho}\ , \ cm^3\,/\,C \ \ or \ \ m^3\,/\,C$$

For n type:

$$R_H = \frac{1}{\rho} = \frac{1}{ne}$$

$$n = \frac{1}{R_H \cdot e} = \text{electron concentration}$$

For p type:

$$R_H = \frac{1}{\rho} = \frac{1}{pe}$$

$$p = \frac{1}{R_H \cdot e} = \text{Hole concentration}$$

Hall coefficient is negative for n type semiconductors & it is positive for p type semiconductors.

Since in metal electron concentration is more than electron concentration in semiconductor i.e. why semiconductors gives large hall coefficient value whereas metals give smaller hall coefficient value.

Since, Hall Voltage $V_H = \dfrac{B\,I}{\rho\,w}$

$$\frac{1}{\rho} = \frac{V_H\,w}{B\,I}$$

Hall Coefficient, $R_H = \dfrac{1}{\rho} = \dfrac{V_H w}{BI}$ cm^3 / C or m^3 / C

Hence, $R_H \propto V_H \propto \dfrac{1}{\rho}$

$R_H \propto V_H \propto \dfrac{1}{ne \text{ or } pe}$

Conductivity

For electrons, $\sigma_n = ne\mu_n = \rho\mu_n$

For holes, $\sigma_p = Pe\mu_p = \rho\mu_p$

So, conductivity $\sigma = \rho\mu$

Mobility, $\mu = \sigma\dfrac{1}{\rho} = \sigma \cdot R_H$

For Electrons, $\mu_n = \sigma_n R_H$

For Holes, $\mu_p = \sigma_p R_H$

Applications

- It can determine type of semiconductor of Semiconductor whether it is n-type or p-type. Hall Coefficient is negative for n-type and positive for p-type.

- By seeing the sign of Hall Coefficient, we cannot distinguish between n-type and intrinsic semiconductor.

- It is used to determine the concentration of charge carriers.

- It is also used for calculation of mobility of majority carriers.

- For determination of mobility of minority carriers "Haynes Shockley" experiment is used.

- It can be used to measure magnetic flux density by measuring current and voltage by ammeter and voltmeter respectively.

$B = \dfrac{V_H ne W}{I}$

- Hall Effect confirms the validity of concept that it is possible for two independent charge carriers to exist in a semiconductor.

- It can be used as Hall Multiplier

$V_H = \dfrac{BI}{neW}$

$B \propto (Input)_1$

$I \propto (Input)_2$

$V_H \propto (Input)_1 (Input)_2$

- **Hall Angle:** It is the ratio of magnitude of current density in y direction to magnitude of current density in z-direction.

$$\tan\theta = \frac{J_y}{J_z}$$

Note: The effect of temperature is negligible on Hall Coefficient so it remains nearly constant with temperature.

Solved Examples

Problem: In specimen of P type Si has a resistivity of $300\,K\Omega cm$ magnetic flux density $B = 0.1\,wb/m^2$ & $d = w = 6\,mm$. The measured values of hall voltage & currents are 60 mv & 10 µA .Calculate mobility of the carrier.

Solution: Hall Coefficient is, $R_H = \dfrac{V_H W}{BI}$

$$R_H = \frac{60 \times 10^{-3} \times 6 \times 10^{-3}}{0.1 \times 10 \times 10^{-6}}$$

$$R_H = 360\,m^3/c$$

Mobility $\mu = \sigma_n R_H = \dfrac{1}{300 \times 10^3 \times 10^{-2}} \times 360 = 0.12\,m^2/v-sec = 1200\,cm^2/v-sec$

Problem: The Hall constant in a p type Si bar is $5 \times 10^3\,cm^3/C$. The hole concentration in the bar is given by?

Solution: Hall Coefficient, $R_H = 5 \times 10^3 = \dfrac{1}{\rho}$

Charge density, $\rho = 2 \times 10^{-4}$

For p type semiconductor, $\rho = pe = 2 \times 10^{-4}$

Hole concentration, $p = 1.25 \times 10^{15}/cm^3$

Problem: An n-type Ge sample is 2 mm wide and 2.2 mm thick. If a current of 10 mA is passed in the sample in x-direction and a magnetic field of 0.1T is directed perpendicular to the current flow in the z-direction & developed hall voltage is 1 mV. Calculate hall coefficient & electron concentration.

Solution: Hall Coefficient, $R_H = \dfrac{V_H W}{BI} = \dfrac{10^{-3} \times 2 \times 10^{-3}}{0.1 \times 10 \times 10^{-3}} = 2 \times 10^{-3}\,m^3/C$

Charge Concentration, $ne = \dfrac{1}{R_H}$

$$n = \frac{1}{eR_H} = \frac{1}{1.6 \times 10^{-19} \times 2 \times 10^{-3}} = 3.125 \times 10^{21} \, / \, m^3$$

Minimum Conductivity

Conductivity in a semiconductor is $\sigma = (n\mu_n + p\mu_p)e$

From mass action law

$$n.p = n_i^2$$

So, $n = \dfrac{n_i^2}{P}$

Therefore, $\sigma = \left(\dfrac{n_i^2}{p} \mu_n + \mu_p \right) e$

$$\frac{d\sigma}{dp} = \left(\frac{-n_i^2}{p^2} \mu_n + \mu_p \right) e$$

For minimum conductivity, $\dfrac{d\sigma}{dp} = 0$

$$\frac{n_i^2}{p^2} \mu_n = \mu_p$$

Therefore, $p = \sqrt{n_i^2 \dfrac{\mu_n}{\mu_p}} = n_i \sqrt{\dfrac{\mu_n}{\mu_p}}$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{n_i \sqrt{\dfrac{\mu_n}{\mu_p}}} = n_i \sqrt{\frac{\mu_p}{\mu_n}}$$

Therefore, $\sigma_{min} = \left(n_i \sqrt{\mu_n \mu_p} + n_i \sqrt{\mu_n \mu_p} \right) e = 2n_i \sqrt{\mu_n \mu_p}$

Solved Examples

Problem: A semiconductor has the following parameters

$\mu_n = 7500 \, cm^2 / V - sec$, $\mu_p = 300 \, cm^2 / V - sec$ & $n_i = 3.6 \times 10^{12} / cm^3$

When conductivity is minimum the hole concentration is?

Solution: As derived earlier, $p = n_i \sqrt{\dfrac{\mu_n}{\mu_p}}$

$$p = 3.6 \times 10^{12} \sqrt{\frac{7500}{300}} = 18 \times 10^{12} / cm^3$$

Minimum Conductivity is given by,

$$\sigma_{min} = 2 \times 3.6 \times 10^{12} \sqrt{7500 \times 300} \times 1.6 \times 10^{-19} = 1.728 \times 10^{-3} / cm^3$$