→ Properties of X-OR gate :-

i) $A \oplus 1 = \bar{A}$

ii) $A \oplus 0 = A$

iii) $A \oplus A = 0$

iv) $A \oplus \bar{A} = 1$

v) if $A \oplus B = C$, $B \oplus C = A$ and $A \oplus C = B$

then

$$A \oplus B \oplus C = 0$$

vi) $AB \oplus AC = A(B \oplus C)$

We know, $A \oplus B = A\bar{B} + \bar{A}B \quad -(1)$

• $A \oplus 1 = \bar{A}$

→ So, using eq (1),

$$A \oplus 1 = A \cdot \bar{1} + \bar{A} \cdot 1$$

$$= A \cdot 0 + \bar{A} \cdot 1$$

$$= 0 + \bar{A}$$

$$= \bar{A}$$

- $A \oplus 0 = A$

$\rightarrow$ Using eq. (1)

$$A \oplus 0 = A \cdot \bar{0} + \bar{A} \cdot 0$$
$$= A \cdot 1 + 0$$
$$= A + 0$$
$$= A$$

- $A \oplus A = 0$

$\rightarrow$ Using eq (1)

$$A \oplus A = A \cdot \bar{A} + \bar{A} \cdot A$$
$$= 0 + 0 \quad [\bar{A} \cdot A = 0 = A \cdot \bar{A}]$$
$$= 0$$

- $A \oplus \bar{A} = 1$

$\rightarrow$ Using eq (1)

$$A \oplus \bar{A} = A \cdot \bar{\bar{A}} + \bar{A} \cdot \bar{A}$$
$$= A \cdot A + \bar{A} \cdot \bar{A} \quad [A \cdot A = A \mid \bar{A} \cdot \bar{A} = \bar{A}]$$
$$= A + \bar{A}$$
$$= 1$$

• If $A \oplus B = C$, $B \oplus C = A$ & $A \oplus C = B$ then

$A \oplus B \oplus C = 0$

⇒ $A \oplus B \oplus C = \underbrace{A \oplus B}_{\text{same}} \oplus \underbrace{A \oplus B}_{\text{same}}$ $(A \oplus B = C)$

$= 0$ (as $A \oplus A = 0$)

$=$

• $AB \oplus AC = A(B \oplus C)$

⇒ $AB \oplus AC = AB \cdot \overline{AC} + \overline{AB} \cdot AC$

$= AB(\bar{A} + \bar{C}) + (\bar{A} + \bar{B})AC$

$[\overline{AB} = \bar{A} + \bar{B}]$ rule

$= A\bar{A}B + AB\bar{C} + \bar{A}AC + A\bar{B}C$

$= 0 + AB\bar{C} + 0 + A\bar{B}C$

$= A(B\bar{C} + \bar{B}C)$

$= A(B \oplus C)$

→ Boolean algebra :-

→ Complementation law :-

- $\bar{0} = 1$
- $\bar{1} = 0$
- if $A = 0$, then $\bar{A} = 1$
- if $A = 1$, then $\bar{A} = 0$
- $\bar{\bar{A}} = A$

→ Commutative law :-

- $A + B = B + A$
- $A \cdot B = B \cdot A$

→ Associative law :-

- $(A + B) + C = A + (B + C)$
- $(A \cdot B) \cdot C = A(B \cdot C)$

→ Distributive law :-

- $A(B + C) = AB + AC$

→ Transposition law :-

- $AB + \bar{A}C = (A+C)\cdot(\bar{A}+B)$

Proff:-

$$(A+C)\cdot(\bar{A}+B)$$

$$= A\cdot\bar{A} + C\cdot\bar{A} + AB + BC$$
$$= 0 + C\bar{A} + AB + BC$$
$$= C\bar{A} + AB + BC\cdot 1 \quad [BC = BC\cdot 1]$$
$$= C\bar{A} + AB + BC(A+\bar{A})$$
$$= C\bar{A} + AB + ABC + \bar{A}BC$$

$$= AB + ABC + \bar{A}C + \bar{A}CB$$
$$= AB(1+C) + \bar{A}C(1+B)$$
$$= AB\cdot 1 + \bar{A}C\cdot 1$$
$$[1+C = 1,\ 1+A = 1,\ 1+B = 1]$$
$$= AB + \bar{A}C$$

→ De Morgan's theorem :-

- Law 1

$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

- Law 2

$$\overline{AB} = \bar{A} + \bar{B}$$

→ Consensus theorem:-

- $AB + \bar{A}C + BC = AB + \bar{A}C$

Proof:-

$AB + \bar{A}C + BC$

$= AB + \bar{A}C + BC \cdot 1$

→ Some other important rule for simplification

- $A \cdot \bar{A} = 0$ • $A + \bar{A} = 1$
- $A \cdot A = A$ • $\bar{A} \cdot \bar{A} = \bar{A}$ • $(1 + A) = 1$
- $(1 + \bar{A}) = 1$

Q. Simplify :-

i) $A + [B + \bar{C}(\overline{AB + A\bar{C}})]$

$\Rightarrow A + [B + \bar{C}(\overline{AB} \cdot \overline{A\bar{C}})]$

$\Rightarrow A + [B + \bar{C}((\bar{A} + \bar{B}) \cdot (\bar{A} + \bar{\bar{C}}))]$

$= A + [B + \bar{C}((\bar{A}+\bar{B}) \cdot (\bar{A}+C))]$

$= A + [B + \bar{C}(\bar{A}\bar{A} + \bar{A}C + \bar{B}\bar{A} + \bar{B}C)]$

$= A + [B + \bar{C}(\bar{A} + \bar{A}C + \bar{B}\bar{A} + \bar{B}C)]$

$= A + [B + \bar{A}\bar{C} + 0 + \bar{A}\bar{B}\bar{C} + 0]$

$= A + [B + \bar{A}\bar{C}(1 + \bar{B})]$

$= A + [B + \bar{A}\bar{C}]$

$= A + B + \bar{A}\bar{C}$

2. $(B+BC)(B+\bar{B}C)(B+D)$

$\rightarrow (BB + B\bar{B}C + BBC + BC \cdot \bar{B}C)(B+D)$

$= (B + 0 + BC + 0)(B+D)$

$= B(1+C)(B+D)$

$= B(B+D)$

$= BB + BD$

$= B + BD = B(1+D) = B$

~~$B(B+\bar{B}C)(B+D)$~~

~~$=(B+0)(B+D)$~~

3. $(\overline{A+\overline{BC}})(A\bar{B}+ABC)$

$\rightarrow (\bar{A}\cdot\overline{\overline{BC}})(A\bar{B}+ABC)$

$= (\bar{A}\cdot BC)(A\bar{B}+ABC)$

$= \bar{A}BC(A\bar{B}+ABC)$

$= \bar{A}BCA\bar{B} + \bar{A}BCABC$

$= 0 + \bar{A}BCABC$

$= 0 + 0 = 0$

→ Boolean ~~function~~ expression and their representation:-

- SOP:- Sum of products
- POS:- Product of sum
- canonical form:-
  - SSOP (standard sum of products)
  - SPOS (standard product of sum)