Conditional Probability and it's examples

Examples: Three cards are drawn at random from a pack of 52 cards find the probability of getting. 1. 2 ace, 2. 2 spades 3. one spead, one club, & one diamond. 4. Two face cards. 5. at least one king

Soln.

Mutually Exclusive Event → They cannot & occur → Simultaneously ⟹ independent event

$P(A)$, $P(B)$.

$P(A \cap B) = \phi$.

Axioms:

1. $0 \le P(A) \le 1$.
2. $P(S) = 1$. → Certain events
3. For a finite or countably infinite no. of pairwise mutually exclusive events $A_1, A_2, A_3, \ldots, A_n$ of S

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) \cup P(A_2) \cup \cdots \cup P(A_n)$$

$$\Rightarrow P(A_1) + P(A_2) + P(A_3) + \cdots + P(A_n)$$

Conditional probability. Let S be a sample space of a random experiment E and considering A and B be two events connected with E such that $P(A) > 0$. Then the probability of occurrence of B on the hypothesis of the event A has ~~as~~ actually occurred is called conditional probability of B.

* for two events A and B connected with a random experiment E then the probability of their simultaneous occurrence $= P(A)$ & (conditional probability of B on the hypothesis of the event A).

$$\boxed{P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right)}$$

$$P(B/A) = \boxed{\frac{P(A \cap B)}{P(A)}} \qquad \because P(A) \neq 0$$

*Soln. 1. Total no. of cards - 52. → (2 ace + 1 other card)

3 cards are drawn at random.

Let A be the event of getting 2 ace,

$P(A) =$ (out of 4 ace cards 2 are drawn) ⟹ 4_{C_2} ways cards can be drawn.

Total no. of sample space $= 52$.

3 cards are drawn 52_{C_3} ways.

$(52-4) = 48$ other cards are remaining.

one card is taken out of 48 cards in 48_{C_1} ways.

$$P(A) = \frac{4_{C_2} \times 48_{C_1}}{52_{C_3}}$$

2\. 2 spades: Let B be the event, 13 spades cards out of which 2 are drawn randomly in 13_{C_2} ways. remaining cards $= (52-13) = 39$ cards out of which one is selected randomly in 39_{C_1} ways.

$$P(B) = \frac{13_{C_2} \times 39_{C_1}}{52_{C_3}}$$

3\. one spead, one club, one diamond. $\dfrac{13_{C_1} \times 13_{C_1} \times 13_{C_1}}{52_{C_3}}$

4\. Let the event be D of getting two face cards. Total face cards $= 12$ out of which 2 are selected randomly in 12_{C_2} ways.

Now ~~xes~~ from rest of the cards one is selected at random in $(52-12) = 40_{C_1}$ ways.

Hence $P(D) = \dfrac{12_{C_2} \times 40_{C_1}}{52_{C_3}}$.

5\. at least one king. $\{1 - P(\text{no king})\}$ = at least one king

$$\left(1 - \frac{4_{C_0} \times 48_{C_3}}{52_{C_3}}\right) = ?$$

probability of no king

$$\left(\frac{4_{C_0} \times 48_{C_3}}{52_{C_3}}\right)$$

at least → (minimum one exists. may exists more than that)

no king

$1 - (\text{no king})$

$$1 - \left(\frac{4_{C_0} \times 48_{C_3}}{52_{C_3}}\right)$$

H_K, S_K, C_K, D_K

$4_{C_1} + 4_{C_2} + 4_{C_3}$