Probability :

(*) **Random experiment** : Such an experiment in which we know the set of all different possible results or out comes and which is such that, it is impossible to ~~pre~~ predict which one of the set will occure at any particular performance of the experiment is called a random experiment.

(*) **Event** : The results or out comes of a random experiment connected with the experiment, is called an event.

Two types of events are there.

i) **Simple or elementary event** :
An event connected with the experiment which cannot be decomposed is called a simple or elementary event.

ii) **Compound or composit event** :
An simple event is the out comes of a random experiment which cannot be further decomposed whereas compound event is an event connected with the experiment is the out comes of a random experiments which can be decomposed. further into simple events.

For example :

Let A and B two events of even faces. and multiples of 3 respectively. in the random experiment of throwing an unbiased die.
clearly event A occurs, when the result of the experiment is 2 or 4 or 6
Similarly. the event B occurs if the out comes of the experiments occurs 3 or 6.

(*) **mutually exclusive events** : Two events A and B connected with the random experiment ~~E~~ E is said to be mutually exclusive, if they cannot occur simultaneously.
Symbollically. event A and B are mutually exclusive when $A \cap B = \phi$ or $P(A \cap B) = 0$ where ϕ is the impossible event.

Impossible and Certain events :

In any random experiments we can think an event which is logically impossible i.e. cannot occure any performance of the experiment such an event is called an impossible event. An impossible event is denoted by ϕ and probability of occurance of an impossible event is zero i.e. $P(\phi) = 0$.

Again, in any random experiment we can think of an event which is sure to occure at every performance of the experiment, such an event will be called a certain or sure event, A certain event is usually denoted by S and probability of occurance of a certain event is 1 i.e. $P(S) = 1$.

Complementary event :

Let A be an event connected with a random experiment E, the event of non-occurangce of A is called the event complementary to A, & is denoted by $\bar{A}$ or A'.

Equally likely event :

Two or more event's is said to be equally likely event if after taking into consideration all relevant evidance none can be expected in performance to another.

The events A and B are equally likely.

if $P(A) = P(B)$.

Exhaustive events :

A set of events connected with a random experiment is said to be exhaustive if at least one of the set is sure to occure at every performance of the experiment.

Simple events connected with a random experiments always constitutes the exhaustive set of events.

Sample Space or Event Space:

A Sample event Connected with a random experiment E is called a Sample pt or event pt and the set of all possible event pt. is called Sample Space or event Space of E, and is usually denoted by S.

Note: i) If two Coins are tossed simultaneously or one Coin is tossed two times of succession, then the sample space S contains of $4 = 2^2$ event pts.

ii) If one Coin tossed three times in succession or 3 Coins tossed simultaneously then sample Space S consists of $8 = 2^3$ event pts.

HHH, HHT, HTT, THT, TTH, HTH, TTT, THH.

iii) Thus if a Coin tossed n times in succession or n Coins tossed at a time, then the sample space of random experiment $= 2^n$ event pts.

Note: In general the sample of random experiment of throwing n die simultaneously contains 6^n event points.

or A die is thrown n times in succession then the event pts of sample space is 6^n.

* Classical definition of probability:

Suppose the sample space S of a random experiment E contains a finite no. of event pts say $n(S)$, all of which are known to be equally likely or mutually symmetrical, If $m(A)$ no. of event pts of this $n(S)$ event pts are contained in the event A connected with E then the ratio $\{m(A)/n(S)\}$ is called the probability of occurrance of the event A. and is denoted by the symbol $P(A)$.

Therefore $P(A) = \dfrac{m(A)}{n(S)}$

$$P(A) = \frac{\left(\text{No. of equally likely event pts contained in the event A.}\right)}{\left(\text{Finite no. of equally likely event pts contained in the Sample Space } S(E)\right)}$$

Important Note :

i). For the impossible event A connected with E $m(A) = 0$, then the probability of occurrence of an impossible event A is $P(A) = \frac{0}{n(S)} = 0$.

ii) For the ~~cer~~ certain event A, connected with E we have $m(A) = n(S)$

Hence the probability of occurrence of a certain event is $P(A) = \frac{n(S)}{n(S)} = 1$

iii) For an event A connected with E we have

$0 \le m(A) \le n(S)$.

or. $\frac{0}{n(S)} \le \frac{m(A)}{n(S)} \le 1$

or, $0 \le P(A) \le 1$

i.e It is evident that the probability of ~~occurring~~ occurrance of an ~~element~~ event is a positive proper fraction.

iv) If an event A connected with E contains $m(A)$ no. of equally likely event pts then the event complementary of A

i.e A^c contained $(n(S) - m(A))$ no: of equally likely event pts.

Therefore we get,

$$P(A^c) = \frac{n(S) - m(A)}{n(S)}$$

$$= 1 - \frac{m(A)}{n(S)}$$

$$\therefore P(A^c) = 1 - P(A).$$

$$\therefore P(A^c) + P(A) = 1.$$

{(Prob. of non occurance) + (Prob of occurrance) = 1}.

v) If the probability of occurrance of an event A be m/n then the ratio $\{m : (n-m)\}$ is often called the odd in favour of the event A.

And the ratio $\{(n-m) : m\}$ is called the odds in against of the event A.

If the odds in favour of the event A are a:b, then the probability of the occurrence of A is $P(A) = \frac{a}{a+b}$.

If the odds against ~~on~~ an event B are a:b then the probability of occurrence of B.

i.e. $P(B) = \frac{b}{a+b}$.

Th. of Total probability or additive Law of probability.

Statement: The probability of occurrence of any one of n pair wise mutually exclusive events $A_1, A_2, \ldots, A_n$ is equals to the sum of the probabilities of the events Symbolically,

$$\left\{ P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n) \right\}.$$

Note: 1) If the events A and B are mutually exclusive then the probability of occurrence of A or B = $P(A \cup B) = P(A) + P(B)$.

2) If the events $A_1, A_2, \ldots, A_n$ are exhaustive then $A_1 \cup A_2 \cup \cdots \cup A_n$ represents the same events.

i.e. $A_1 \cup A_2 \cup \cdots \cup A_n = S$

$\therefore P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(S) = 1$.

$P(A_1) + P(A_2) + \cdots + P(A_n) = 1$.

Since the events $A_1, A_2 \ldots A_n$ are pair wise mutually exclusive.

3) Let A be an event connected with E, and A^c is the event complementary ~~of~~ to A, then $A \cup A^c = S$

i.e. $P(A \cup A^c) = P(S) = 1$.

i.e. $P(A) + P(A^c) = 1$ Since A, A^c are mutually exclusive.

Axioms of Mathematical Probability:

Let S be the sample space of the random experiment E and A be any event connected with E i.e. $A \subseteq S$ a real no. P(A) associate with A is called the probability of event A if the following axioms are satisfied.

axiom. 1. for any event A, $P(A) \geq 0$.

axiom. 2. for certain event S, $P(S) = 1$

axiom. 3. for a finite or countably infinite no of pair wise mutually exclusive events $A_1, A_2, \ldots, A_n$ of , $P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

Deduction of Some Results from the axiom.

1) Prove that the probability of an impossible event is zero.

$P(\phi) = 0$

Proof. For any event A, $A \cap \phi = \phi$

∴ event A and ϕ are mutually exclusive.

again we have $A \cup \phi = A$.

$$P(A \cup \phi) = P(A)$$
$$P(A) + P(\phi) = P(A)$$
$$P(\phi) = P(A) - P(A) = 0.$$

2) If A^c is the complementary event of the event A

Prove that $P(A^c) = 1 - P(A)$.

Soln. We know that A and A^c are mutually exclusive,

Since A^c is the complementary to A then the events A and A^c are mutually exclusive.

and $A \cup A^c = S$ where S is the sure event

Therefore, $\cancel{P(A \cap A^c) = P(S)}$. $P(A \cup A^c) = P(S)$

or. $\cancel{P(A \cap A^c) = 1}$. ∴ $P(A) + P(A^c) = 1$

$\cancel{P(A) + P(A^c)}$. $P(A^c) = 1 - P(A)$.

3) For any event A prove that $0 \leq P(A) \leq 1$.

Proof If A^c is the event complementary to A

Then A & A^c are mutually exclusive

and $A \cup A^c = S$ where S is the Sure event

$$P(A \cup A^c) = P(S)$$
$$P(A) + P(A^c) = 1 \qquad P(A^c) = \{1 - P(A)\}$$

Since by axiom (1) $P(A^c) \geq 0$.

$$\{1 - P(A)\} \geq 0$$
$$1 \geq P(A). \quad \text{——— (i)}$$

Also by axiom (1) again $P(A) \geq 0$. ——— (ii)

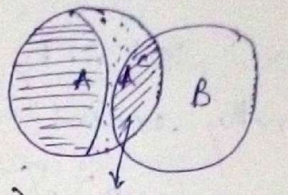
∴ by (i) and (ii)

$$0 \leq P(A) \leq 1$$

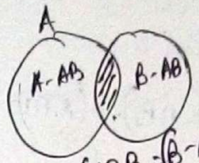
4) If $A \subseteq B$ Prove that $P(A) \leq P(B)$.

$A \cup (A^c \cap B)$

Proof Since $A \subseteq B$; $A \cap (A^c \cap B) = \phi$.



i.e. event A and $(A^c \cap B)$ are mutually exclusive.



and $A \cup (A^c \cap B) = B$.

$A^c \cap B = (B - AB)$ $\quad (A^c \cap B) \cap A = \phi$.

$A \cup B - AB$

$A \cup B$

$P\{A \cup (A^c \cap B)\} = P(B)$

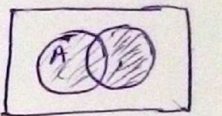
$P(A) + P(A^c \cap B) = \cancel{P(B)}$ $P(A \cup B)$ $\Rightarrow P(A^c \cap B) = \{P(B) - P(A)\}$.

i.e. $\cancel{P(A) + P(A^c)}$..

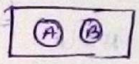
now by axiom (i) $P(A^c \cap B) \geq 0$.

$\therefore$ $P(B) - P(A) \geq 0$

$P(B) \geq P(A)$.



$A \cup (A^c \cap B) = A \cup B$



i.e. $P(A) \leq P(B)$.

5). For any two events A and B prove that $P(A) = P(A \cap B) + P(A \cap B^c)$.

and. $P(B) = P(A \cap B) + P(A^c \cap B)$.

$P(A) + P(A^c \cap B) = P(A \cup B)$

$\cancel{P(A)} + P(A^c \cap B) = \cancel{P(A)} + P(B)$

Sol. For the events A and B

$(A \cap B)$ and $(A \cap B^c)$ are mutually exclusive

and $A = (A \cap B) \cup (A \cap B^c)$

$P(A) = P(A \cap B) + P(A \cap B^c)$.

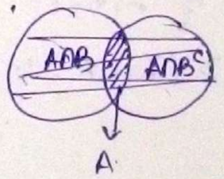
Similarly, $(A \cap B)$ and $(A^c \cap B)$ are mutually exclusive.

$B = (A \cap B) \cup (A^c \cap B)$

$P(B) = P(A \cup B) + P(A^c \cap B)$.

6). For any two events A and B prove that $P(A \cup B) = \{P(A) + P(B) - P(A \cap B)\}$

clearly $(A \cap B)$ and $(A \cap B^c)$ are mutually exclusive.



and. $\cancel{(A \cap B) \cup (A \cap B)}$ and

$(A \cap B) \cup (A \cap B^c) = A$.

$P(A) = P(A \cap B) + P(A \cap B^c)$.

$P(A \cap B) = P(A) - P(A \cap B^c)$. —— (i)

Similarly. the event $(A \cap B)$ and $(A^c \cap B)$ are mutually exclusive. now. $(A \cap B) \cup (A^c \cap B) = B$.

$P(A \cap B) + P(A^c \cap B) = P(B)$

$\therefore$ $P(A \cap B) = P(B) - P(A^c \cap B)$ —— (ii)

~~from (i) & (ii)~~ $\cancel{P(A) - P(B)} = \cancel{P(A \cap B^c) - P(A^c \cap B)}$

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now from (i) and (ii)

$P(A) - P(A \cap B^c) = P(B) - P(A^c \cap B)$. —— (iii)

again B, $(A \cap B)$, $(A^c \cap B)$ and $(A \cap B^c)$ are pair wise mutually exclusive.

$\therefore$ $(A \cup B) = (A \cap B) + (A \cap B^c) + (A^c \cap B)$

$P(A \cup B) = P(A \cap B) + P(A \cap B^c) + P(A^c \cap B)$. —— (iv)

from (i) (ii) (iii) & (iv).

$\cancel{P(A \cup B) = P(A) + P(A \cap B^c) + P(B) + P(A \cap B)}$ $\cancel{+P(A \cap B^c)}$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note: The results proved above is called the generalised form of the Theorem of the Total Probability.

If A and B mutually exclusive,

Then $P(A \cap B) = 0$.

then above theorem reduces to,

$P(A \cup B) = P(A) + P(B)$.

Which is Total Probability.

Th. For any three ~~elements A~~, events A, B, C prove that

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C)$
$+ P(A \cap B \cap C)$.

Taking,

$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum P(A_1) - \sum P(A_1 \cap A_2) + \sum P(A_1 \cap A_2 \cap A_3)$
$- \cdots + (-1)^{n-1} \prod P(A_1 \cap A_2 \cdots \cap A_n)$

If $A_1, A_2 \cdots A_n$ are independent,

$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum P(A_1) - \prod P(A_1) \cdot P(A_2)$
$+ \prod P(A_1) P(A_2) \cdots P(A_3) - \cdots$
$\cdots + (-1)^{n-1} \prod P(A_1) P(A_2) \cdots P(A_n)$.

// Conditional Probability.

Let S be the sample space of a random experiment E and A and B be two events connected with E such that $P(A) > 0$. then the probability of occurance of event B on the hypothesis the event A has actually occurred is called the conditional probability of B.

Given A and denoted by $P(B/A)$, clearly $P(B/A)$ represents the proportions of the event pts. in event E among the events pts in the event A.

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Th of Compound Probability :

for two events A and B connected with a random experiment E, (The Probability of their simultaneous occurrance) := P(A) & Conditional Probability of B on the hypothesis that the event A has actually occurred.

Symbolically . $P(A \cap B) = P(A) \cdot P(B/A)$.

Note : i) Let A and B be two events connected with a random experiment then the conditional probability of the event B. when it is known that the event A has occurred is denoted by $P(B/A)$. and is defined by.

$$P(B/A) = \left\{ \frac{P(A \cap B)}{P(A)} \right\} \quad \because P(A) \neq 0,$$

2) Now the results of the theorems of Compound Probability can also be written as follows,

$$P(A \cap B) = P(B) \cdot P(A/B)$$

Provided $P(B) \neq 0$.

Independent and Dependent event :

Two events A and B are said to be independent if and only iff $P(A \cap B) = P(A) \cdot P(B)$.

also $P(B/A) = P(B) \cdot P(A/B) = P(A)$.

Some important results and ~~probability~~ formula.

1) for n events $A_1, A_2, \ldots, A_n$ Prove that

$P(A_1 \cup A_2 \cup \cdots \cup A_n) \leq P(A_1) + P(A_2) + \cdots + P(A_n)$.

(Booles inequality).

Soln Now for any two events A_1 and A_2 we have

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$.

Since $P(A_1 \cap A_2) \geq 0$ by axiom.

it follows that,

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$. ——— (i)

Again $P(A_1) \cup P(A_2) \cup P(A_3) = P(D \cup A_3)$
where $D = A_1 \cup A_2$

$\therefore P(A_1 \cup A_2 \cup A_3) \leq P(D) + P(A_3)$

or, $P(A_1 \cup A_2 \cup A_3) \leq P(A_1 \cup A_2) + P(A_3)$
$\leq P(A_1) + P(A_2) + P(A_3)$.

* in general, $P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup (A_n) \leq P(A_1) + P(A_2) \cdots + P(A_n)$

*2. For any two events A and B, Prove that $P(A \cap B) \geq P(A) + P(B) - 1$

For any two events A and B.
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
Clearly. $P(A \cup B) \leq 1$
$\therefore$ Now it is clear that
$P(A) + P(B) - P(A \cap B) \leq 1$
$P(A) + P(B) \leq P(A \cap B) + 1$
or, $P(A) + P(B) - 1 \leq P(A \cap B)$.
$\therefore P(A \cap B) \geq \{P(A) + P(B) - 1\}$.

3). For any two events A and B Prove that
$P(A \cap B) \leq P(A) \leq P(A \cup B)$.

Soln. Clearly. $(A \cap B)$ and $(A \cap B^c)$ are mutually exclusive
and. $(A \cap B) \cup (A \cap B^c) = A$.
$P(A \cap B) + P(A \cap B^c) = P(A)$.
Since $P(A \cap B^c) \geq 0$, by axiom (1) it follows that
$P(A \cap B) \leq P(A)$ —— (1)
again the event A and $(A^c \cap B)$ are mutually exclusive, and, $A \cup (A^c \cap B) = A \cup B$.
$P(A) + P(A^c \cap B) = P(A \cup B)$.
Since $P(A^c \cap B) \geq 0$. it follows that,
Thus. $P(A) \leq P(A \cup B)$ —— (2)
now Combining (1) and (2) we get
$P(A \cap B) \leq P(A) \leq P(A \cup B)$.

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1) If A and B two independent events prove that the events i) A^c & B^c ii) A and B^c iii) A^c and B also independent.

Sol i) A^c and B^c are independent:

we have $P(A^c \cap B^c) = P(A \cup B)^c$

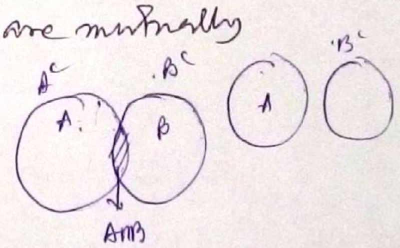
By de-morgans law,

$$P\{(A \cup B)^c\} = 1 - P(A \cup B)$$
$$= 1 - \{P(A) + P(B) - P(A \cap B)\}$$
$$= 1 - P(A) - P(B) + P(A \cap B)$$
$$= \{1 - P(A)\} - \cancel{P(B)}\cancel{\{1 - P(A \cap B)^c\}}\ P(B) + P(A \cap B)$$
$$= P(A^c) - P(B)\{1 - P(A)\}$$ Since $P(A \cap B) = P(A)P(B)$
$$= P(A^c) - P(B)\ P(A^c)$$
$$= P(A^c) \cdot \{1 - P(B)\}$$
$$= P(A^c) \cdot P(B^c).$$

Therefore. ~~P(A^c∩B^c) = P(A^c).P(B)~~
$P(A \cup B)^c = P(A^c) \cdot P(B^c) = P(A^c \cap B^c)$.
$\therefore P(A^c \cap B^c) = P(A^c) \cdot P(B^c)$. hence proved.

ii) A and B^c are independent.

clearly event $(A \cap B)$ and $(A \cap B^c)$ are mutually exclusive, and



$$A = (A \cap B) \cup (A \cap B^c).$$
$$P(A) = P(A \cap B) + P(A \cap B^c).$$
$$P(A \cap B^c) = P(A) - P(A \cap B)$$
$$= P(A) - P(A) \cdot P(B)$$ Since A, B are independent.
$$= P(A)\{1 - P(B)\}$$
$$\therefore P(A \cap B^c) = P(A) \cdot P(B^c).$$

Which shows that A and B^c are independent.

iii). A^c and B also independent

clearly $(A \cap B)$ and $(A^c \cap B)$ are mutually exclusive
so that $B = (A \cap B) \cup (A^c \cap B)$
$$P(B) = P(A \cap B) + P(A^c \cap B)$$
$$P(A^c \cap B) = P(B) - P(A \cap B) = P(B)\{1 - P(A)\}$$
$$= P(B) \cdot P(A^c).$$

$\therefore$ This shows that B & A^c also independent.

5) For two independent events Prove that, ~~A&B.~~ for A & B, ∴ $P(A\cup B) = 1 - P(A^c)\cdot P(B^c)$.

Sol? Since the event A and B are independent. events Thus A^c and B^c are also independent event

∴ $P(A\cup B) = 1 - P(A\cup B)^c$

$= 1 - P(A^c\cap B^c)$

$= 1 - P(A^c)\cdot P(B^c)$.

Since A^c and B^c are independent

hence proved.

(*) If A, B, C are mutually exclusive events then Prove that the event A & $(B\cup C)$ are also. independent event.

$P\{A\cap(B\cup C)\} = P\{(A\cap B)\cup(A\cap C)\}$ (By distribution law)

$= P(A\cap B) + P(A\cap C) - P\{(A\cap B)\cap(A\cap C)\}$

$= P(A\cap B) + P(A\cap C) - P\{A\cap(B\cap C)\}$

$= P(A\cap B) + P(A\cap C) - P(A)\cdot P(B\cap C)$

$= P(A)P(B) + P(A)P(C) - P(A)\cdot\{P(B)P(C)\}$

$= P(A)\{P(B) + P(C) - P(B)\cdot P(C)\}$

$= P(A)\{P(B) + P(C) - P(B\cap C)\}$

$= P(A)\cdot P(B\cup C)$

$=$ ~~P(A∩(B∪C))~~

This shows that (A) and $(B\cap C)$ are independent events.

Th For two independent events A and B, if $P(A)\neq 0$, $P(B)\neq 0$. then A and B cannot be mutually exclusive.

Soln By problem A & B are ~~mutually~~ independent event

∴ $P(A\cap B) = P(A)\cdot P(B)$. Since $P(A)\neq 0$, $P(B)\neq 0$.

$(A\cap B)\neq \phi$

∴ event A & B are not mutually exclusive.

Th for any two events A, B, C & if $A\subseteq B$, P.T. $P(A/C)\le P(B/C)$.

By problem. $A\subseteq B$.

$(A\cap C)\subseteq(B\cap C)$

$P(A\cap C)\subseteq P(B\cap C)$ i.e. $P(A\cap C)\le P(B\cap C)$ ∵ when $A\le B$ ∴ $P(A)\le P(B)$

$P(C)\cdot P(A/C) \le$ ~~P(B)·P(C/B)~~ $P(C)\cdot P(B/C)$

$P(A/C)\le P(B/C)$ since $P(C)>0$

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Probability - 1 - (A)

(1) A bag contains ~~red~~ 3 red, 6 white and 7 blue balls. What is the probability of that two balls draw at a time one will be white and the other blue.

Soln: Total number of balls $= 3+6+7 = 16$.

Now, 2 balls can be drawn out of 16 balls. in $^{16}C_2 = 120$ ways.

Total no. of possible cases for the event $= n = 120$.

Let A be the event that two balls drawn. One will be white and the other blue.

1 white ball out of 6 white balls and 1 blue ball out of 7 ~~b~~ blue balls may be drawn in $^6C_1 \times {}^7C_1 = 6\times 7 = 42$ ways.

$\therefore$ Total no. favourable cases. $m = 42$

Hence the required probability $P(A) = \frac{m}{n} = \frac{42}{120} = \frac{7}{20}$.

Ex. 65% of the students in a College like football, 34% hockey, ~~A~~ 41% cricket, 10% football and hockey both, 17% football and cricket both, 15% hockey and cricket both and 6% all of three prove that the statement is wrong.

$F \equiv$ event of liking football

$H \equiv$ " " " hockey.

$C \equiv$ " " " cricket

$F\cap H =$ event " " football & hockey both

$H\cap C =$ " " " hockey & cricket "

$C\cap F =$ " " " cricket & football "

$F\cap H\cap C =$ event " " football, hockey & cricket both.

$P(F) = \frac{65}{100}$, $P(H) = \frac{34}{100}$, $P(C) = \frac{41}{100}$.

$P(F\cap H) = \frac{10}{100}$, $P(F\cap C) = \frac{17}{100}$, $P(HC) = \frac{15}{100}$. $P(F\cap H\cap C) = \frac{6}{100}$.

$P(F\cup H\cup C) = P(F)+P(H)+P(C) - P(FH) - P(HC) - P(CF) + P(FHC)$

$= \frac{104}{100} > 1$

Ex. Two Cards are drawn from a pack of 52 Cards. Show that the ev[illegible] of appearance of a King and the appearance of a Heart are independent event.

Sol^n A Cards may be drawn from a pack of 52 Cards. is $^{52}C_1$

out of 4 Kings 1 may be drawn = $^4C_1$

event of appearance of the King = $\frac{^4C_1}{^{52}C_1}$

out of 13 Hearts 1 may be drawn = $^{13}C_1$

event of appearance of the Heart = $\frac{^{13}C_1}{^{52}C_1}$

$\therefore P(A) \cdot P(B) = \frac{^4C_1}{^{52}C_1} \cdot \frac{^{13}C_1}{^{52}C_1}$

$= \frac{1}{52}$

Again the no of cases favorable to the cases.

i.e. Appearance of $(A \cap B) = 1$

~~AEAB~~. $P(AB) = \frac{1}{52}$

$\therefore P(A) \cdot P(B) = P(AB)$

i.e. the events are independent.

Ex. Two candidates A and B appeared for an interview for two vacancies. The probabilities of their selection are 1/4 and 1/3 respectively. What is the probability that only one of them will be selected.

Sol^n the event of A is selected E_1

" " " B " " E_2

$P(E_1) = \frac{1}{4}$, $P(E_1^c) = 3/4$.

$P(E_2) = \frac{1}{3}$, $P(E_2^c) = 2/3$

now, $\{P(E_1 \cap E_2^c) \cup P(E_1^c \cap E_2)\}$ = only one is selected.

$\Rightarrow P(E) \cdot P(E_2^c) + P(E_1^c) \cdot P(E_2)$

$\Rightarrow 5/12$

Ex A bag contain 8 red balls and 5 white balls. 3 balls at a time are drawn twice without replacement. What is the probability that the first 3 balls are white and second three balls drawn are red.

Soln Let A be the event ⇒ the first ~~three~~ 3 drawn balls are white
and B = the second 3 " " " red.

Now the event of joint probability
$= P(AB) = P(A).P(B/A)$.

$n = 8+5 = 13$.

~~For event~~ A. ⇒ 3 out of 13 can be drawn in $^{13}C_3$ ways.

Now, 3 white balls out of 5 white can be drawn in $^5C_3$ ways.

$\therefore P(A) = \frac{^5C_3}{^{13}C_3} = \frac{5}{143}$

Again Let, A has already occurred.
now the bag contains 8 red and 2 white balls.
Total 10, 3 can be drawn in $^{10}C_3$ ways.
out of 10, 3 red balls out of 8 can be drawn $^8C_3$ ways.

$P(B/A) = \frac{^8C_3}{^{10}C_3} = \frac{7}{15}$

Therefore the required probability $P(AB) = P(A).P(B/A)$
$= \frac{5}{143} \cdot \frac{7}{15}$
$= \frac{7}{429}$.

* An urn contains 5 black and 7 red balls, two balls are drawn at a time, find the probability that both balls are of same colour.

Soln

A be the event of black

B be " " " red

the event "both balls are of same colour" can be expressed as both balls are red and or both the balls are white

Hence $P(A \cup B)$. where A and B are mutually exclusive event.

$$P(A) = \frac{5c_2}{12c_2} = \frac{5}{33}$$

$$P(B) = \frac{7c_2}{12c_2} = \frac{7}{22}$$

$$P(A+B) = P(A) + P(B) = \frac{5}{33} + \frac{7}{22} = \frac{31}{66}$$

* Two urn contain respectively 3 white, 7 red and 15 black balls and, 10 white, 9 black, 6 red balls, One ball is drawn from each urn, find the probability that both balls are of same colour.

Soln

Let W_1 denotes that one white ball is choosen from 1st urn

& W_2 " " " " " " " from 2nd urn.

Thus. $W_1 \cap W_2$ denotes that both the drawn balls are white of same colour.

Slly. $(R_1 \cap R_2)$ & $(B_1 \cap B_2)$

Since the events are ~~not mutually~~ independent.

$\therefore$ ~~P(W_1)~~ $P(W_1 \cap W_2) = P(W_1) \cdot P(W_2) = \frac{3c_1}{25c_1} \cdot \frac{10c_1}{25c_1}$

$= \frac{3}{25} \cdot \frac{10}{25} = \frac{30}{625}$

Slly. $P(R_1 \cap R_2) = \frac{7}{25} \cdot \frac{6}{25} = \frac{42}{625}$; $P(B_1 \cap B_2) = \frac{15}{25} \cdot \frac{9}{25} = \frac{135}{625}$

Hence $P(W_1 \cap W_2) + P(R_1 \cap R_2) + P(B_1 \cap B_2) = \frac{30}{625} + \frac{42}{625} + \frac{135}{625}$

$= \frac{207}{625}$

अभियुक्ति

* Two fair dice are thrown indepently. Where three events are defined as. A: Odd face with first dice
B: odd face with second dice.
C: sum of points on two dice is odd.

Are the events A, B and C mutually independent?

Sol^n odd faces 1, 3, 5

$P(A) = \frac{3 \times 6}{36} =$

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