Probability.

For any two events A and B, ~~P(A) =~~ Caliber

Prove that $P(A) = P(A\cap B) + P(A\cap B^c)$

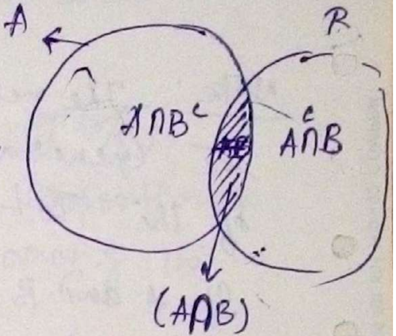
and $P(B) = P(A\cap B) + P(A^c\cap B)$.

For the events A and B,

$(A\cap B)$ and $(A\cap B^c)$ are mutually exclusive,

and $A = \cancel{A\cap B \cup (A\cap B^c)} \cdot (A\cap B) \cup (A\cap B^c)$.

$$P(A) = P\{(A\cap B) \cup (A\cap B^c)\}$$
$$= P(A\cap B) + P(A\cap B^c).$$



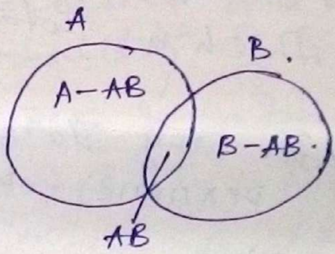
Similarly, $(A\cap B)$ and $(A^c\cap B)$ are mutually exclusive,

and. $B = (A\cap B) \cup (A^c\cap B)$.

$P(B) = P(A\cup B) + P(A^c\cap B)$.

6) For any two events A and B, P.T.

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$.



clearly. $(A\cap B)$ and $(A\cap B^c)$ are mutually exclusive, and

$(A\cap B) \cup (A\cup B^c) = A$.

$\therefore P(A) = P(A\cap B) \cup P(A\cap B^c)$

$= P(A\cap B) + P(A\cap B^c)$

$\therefore P(A\cap B) = P(A) - P(A\cap B^c)$. ——— (i)

Slly. the event $(A\cap B)$ and $(A^c\cap B)$ are mutually exclusive,

$(A\cap B) \cup (A^c\cap B) = B$.

$P(A\cap B) + P(A^c\cap B) = P(B)$. ——— (ii)

$\therefore P(A\cap B) = P(B) - P(A^c\cap B)$.

$\therefore P(A) - P(A\cap B^c) = P(B) - P(A^c\cap B)$.

Again $\therefore$ $(A\cap B)$, $(A\cap B^c)$ and $(A^c\cap B)$ are pairwise mutually exclusive.

$\therefore P(A\cap B) + P(A\cap B^c) + P(A^c\cap B) = P(A\cup B)$

$\therefore P(A) + P(B) = P(A\cap B) + P(A\cap B^c) + P(A^c\cap B)$ ——— (iii)

$=$

$\prod$

$\frac{\prod}{\prod}$ (i2)

$\therefore$ Now, from (i) & (ii) and (iii).

~~(P∩B) = P(A)~~.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note: The results proved above, is called the Generalised forms of the Theorems of the Total Probability.

If A and B mutually exclusive, then $P(A \cap B) = 0$.

then above theorem reduces to

$P(A \cup B) = P(A) + P(B)$.

Which is Total Probability.

Th. For any three events A, B, C P.T.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

Taking.

~~P(A∪B∪C∪~~ ..

$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum P(A) - \sum P(AB) \ldots$

$= \sum_{i=0}^{n} P(A_1) - \sum P(A_1 \cap A_2) + \sum P(A_1 \cap A_2 \cap A_3)$

$\cdots + (-1)^{n-1} \prod P(A_1 \cap A_2 \cdots \cap A_n)$.

If $A_1, A_2, \ldots A_n$ are independent

$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum P(A_1) - \prod P(A_1) \cdot P(A_2)$

$+ \prod P(A_1) P(A_2) \cdot P(A_3) - \cdots$

$+ (-1)^{n-1} \prod P(A_1) P(A_2) \cdots P(A_n)$

Probability :

Let S be the Sample Space of a random experiment E and A and B be two events connected with E. Such that,

$P(A) > 0$. Then the Probability of occurrance of event B on the hypothesis the event A has actually occurred is called the conditional Probability of B.

Given A and denoted by $P(B/A)$.

clearly $P(B/A)$ represents the Proportions of the event pts in event E among the events pts in the event A.

Th of. Compound Probability :

for two events A and B connected with A random experiment E, (The Probability of their Simultaneous occurrance) = $P(A)$ & conditional Probability of B.

on the hypothesis that the event A has actually occured.

Symbolically. $P(A \cap B) = P(A) \cdot P(B/A)$.

Note : 1) Let A and B be two events, Connected with a random exp. then the cond. probability of the event B when it is known that the event A has occured, is denoted by $P(B/A)$ and is defined by,

$$P(B/A) = \left\{ \frac{P(A \cap B)}{P(A)} \right\}, \quad \because P(A) \neq 0.$$

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

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2) The results of the Theorm of compound probability can also be written as follows:

$P(A \cap B) = P(B) \cdot P(A/B)$.

Provided $P(B) \neq 0$.

3) If Probability of $P(A) \neq$, $P(B) \neq 0$.

Then $P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$.

Independent & dependent event:

Two events A and B. ~~two~~ to be independent iff $P(A \cap B) = P(A) \cdot P(B)$.

also, ~~P(B/A)~~ $P(B/A) = P(B)$ & $P(A/B) = P(A)$.

Some important results and formula:

1. For n events $A_1, A_2, \ldots, A_n$. Prove that $P(A_1 \cup A_2 \cup \cdots \cup A_n) \leq P(A_1) + P(A_2) + \cdots + P(A_n)$.

(Booles inequality).

for any two events A_1 and A_2 we have:

~~P(A1)~~. $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

Since $P(A_1 \cap A_2) \geq 0$.

it follows that,

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ —— (i)

Again, $P(A_1) \cup P(A_2) \cup P(A_3) = P(D \cup A_3)$

where $D = A_1 \cup A_2$

$\therefore P(A_1 \cup A_2 \cup A_3) \leq P(D) + P(A_3)$.

or, $P(A_1 \cup A_2 \cup A_3) \leq P(A_1 \cup A_2) + P(A_3)$

$\leq P(A_1) + P(A_2) + P(A_3)$

ingeneral $P(A_1 \cup A_2 \cup \cdots \cup A_n) \leq P(A_1) + P(A_2) + \cdots + P(A_n)$

For any two events A and B. P.T.

$P(A\cap B) \geq P(A) + P(B) - 1.$

Pf For any two events A and B,

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

clearly. $P(A\cup B) \leq 1$

it is clear that $P(A)+P(B) - P(A\cap B) \leq 1$

i.e. $P(A) + P(B) - 1 \leq P(A\cap B)$

i.e. $P(A\cap B) \geq P(A) + P(B) - 1.$

(3) For any two events A and B. P.T.

$\cancel{P(A)}$ $P(A\cap B) \leq P(A) \leq P(A\cup B).$

Soln. clearly. $(A\cap B)$ and $(A\cap B^c)$ are mutually exclusive, and

$$(A\cap B) \cup (A\cap B^c) = A.$$

$\therefore$ $P(A\cap B) + P(A\cap B^c) = P(A).$

Since. $P(A\cap B^c) \geq 0$. it follows that,

$P(A\cap B) \leq P(A).$ —— (1)

Again the event A. and $(A^c\cap B)$ are mutually exclusive, and.

$$A\cup (A^c\cap B) = A\cup B.$$

$$P(A) + P(A^c\cap B) = P(A\cup B).$$

Since. $\cancel{P(A^c \cup}$ $P(A^c\cap B) \geq 0$, it follows that,

$P(A) \leq P(A\cup B)$ —— (2)

Combining (1) and (2) we get,

$$P(A\cap B) \leq P(A) \leq P(A\cup B).$$

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4) If A and B two indepent events P.T the events i) A^c & B^c ii) A and B^c iii) A^c and B also independent.

we have, $P(A^c \cap B^c) = P(A \cup B)^c$
(by demorgans law)

$P(A \cup B)^c = 1 - P(A \cup B)$.

$= 1 - \{P(A) + P(B) - P(A \cap B)\}$.

$= 1 - P(A) - P(B) + P(A \cap B)$.

$= \{1 - P(A)\} + P(A \cap B) - P(B)$.

$= \{1 - P(A)\} - P(B)\{1 - P(A)\}$.

$= \{1 - P(A)\}\{1 - P(B)\}$

$= P(A^c) \cdot P(B^c)$.

Therefore: $P(A^c \cap B^c) = P(A^c) \cdot P(B^c)$.

Hence A^c and B^c are independent.

ii) clearly event $(A \cap B)$ and $(A \cap B^c)$ are mutually exclusive.

and, $A = (A \cap B) \cup (A \cap B^c)$.

$P(A) = P(A \cap B) + P(A \cap B^c)$.

$\therefore P(A \cap B^c) = P(A) - P(A \cap B)$

$= P(A) - P(A) \cdot P(B)$

Since A & B are independent

$= P(A)(1 - P(B))$

$= P(A) \cdot P(B^c)$.

Which shows that, A and B^c are independent.

ily. event $(B \cap A)$ & $(B \cap A^c)$
are mutually exclusive.

$$(B \cap A) \cup (B \cap A^c) = B.$$

$$P(B) = P(B \cap A) + P(B \cap A^c).$$

$$P(B \cap A^c) = P(B) - P(B \cap A).$$

$$= P(B) - P(B)\,P(A).$$ Since A & B are independent,

$$= P(B)\,(1 - P(A))$$

$$= P(B) \cdot P(A^c).$$

Which shows that, A^c, B are independent.

3) For two independent events. A & B. P.T.

$$P(A \cup B) = 1 - P(A^c) \cdot P(B^c).$$

Sol^n Since the event. A & B are independent.
event A^c and B^c are also independent.

Now, $P(A \cup B) = 1 - P(A \cup B)^c$

$$= 1 - \cancel{P(A^c) \cap} \; P(A^c \cap B^c).$$

$$= 1 - P(A^c) \cdot P(B^c).$$

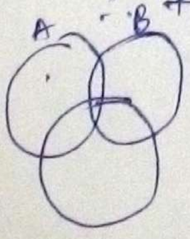
Since A^c & B^c are independent, Hence proved.

* If A, B, C are mutually independent event
the P.T. the event, A & $(B \cup C)$ are also independent
event.

Proof $P(A \cap (B \cup C)) = P\{(A \cap B) \cup (A \cap C)\}$. (By distribution law).

$\cancel{P(A \cap B}$

$$= P(A \cap B) + P(A \cap C) - P\{(A \cap B) \cap (A \cap C)\}$$



$$= P(A \cap B) + P(A \cap C) - P\{A \cap (B \cap C)\}$$

$$= \cancel{P(A \cap B) + P(A \cap C)} =$$

$$= P(A)\,P(B) + P(A) \cdot P(C) - P(A) \cdot P(B \cap C)$$

$P\{A\cap(B\cup C)\} = P(A)\cdot P(B) + P(A)\cdot P(C) - P(A)P(B)P(C)$

$= \cancel{P(A)\cdot P(B)}$

$= P(A)\{P(A) + P(B) - P(B\cap C)\}$

$= P(A)\, P(B\cup C)$.

This shows that ~~B~~ A and $(B\cup C)$ are independent.

*) For two independent events A and B, if $P(A)\neq 0$, $P(B)\neq 0$, then A and B can not be mutually exclusive.

By problem, A and B are independent event

$\therefore P(A\cap B) = P(A)\cdot P(B)$. $\because P(A)\neq$, $P(B)\neq 0$.

$\therefore P(A\cap B)\neq \phi$.

i.e. event A & B are not mutually exclusive.

*Th For any three events A, B, C if $A\subseteq B$, P.T. $P(A/C)\leq P(B/C)$.

Proof By problem: ~~A is a~~

$A\subseteq B$,

$(A\cap C)\subseteq(B\cap C)$.

$\therefore P(A\cap C)\leq P(B\cap C)$. $\because$ when $A\subseteq B$ then $P(A)\leq P(B)$

$P(C)\cdot P(A/C)\leq P(C)\cdot P(B/C)$

$P(A/C)\leq P(B/C)$. Since $P(C)>0$

[BTS)] A dice is thrown n times, show that probability of even number of sixes is $\frac{1}{2}\{1+(\frac{2}{3})^n\}$.

Here p (Probability of getting success) $= \frac{1}{6}$.

$q =$ (Probability of failure) $= 5/6$.

so, required Probability $= P_0 + P_2 + P_4 + P_6 + \cdots$

(i.e. even no of sixes.).

Now, we can write,

$P_0 = {}^nC_0\, p^0 q^{n-0}$, $P_2 = {}^nC_2\, p^2 q^{n-2}$, $P_4 = {}^nC_4\, p^4 q^{n-4}$,

and $P_6 = {}^nC_6\, p^6 q^{n-6}$.

$\therefore$ required Probability $= P_0 + P_2 + P_4 + P_6 + \cdots$

$= {}^nC_0\, p^0 q^{n-0} + {}^nC_2\, p^2 q^{n-2} + {}^nC_4\, p^4 q^{n-4} + {}^nC_6\, p^6 q^{n-6} + \cdots$

Now, we have,

$(q+p)^n = q^n + {}^nC_1\, q^{n-1} p + {}^nC_2\, q^{n-2} p^2 + {}^nC_3\, q^{n-3} p^3 + \cdots$

$\cdots + {}^nC_n\, p^n$ ——— (1)

Again,

$(q-p)^n = q^n - {}^nC_1\, q^{n-1} p + {}^nC_2\, q^{n-2} p^2 - {}^nC_3\, q^{n-3} p^3 + \cdots$

$\cdots + (-1)^n\, {}^nC_n\, p^n$. ——— (2)

$\therefore$ adding (1) and (2) we get,

$(q+p)^n + (q-p)^n = 2q^n + 2\,{}^nC_2\, q^{n-2} p^2 + 2\,{}^nC_4\, p^4 q^{n-4} + 2\,{}^nC_6\, p^6 q^{n-6} + \cdots$

$\frac{1}{2}\{(q+p)^n + (q-p)^n\} = \{q^n + {}^nC_2\, q^{n-2} p^2 + {}^nC_4\, q^{n-4} p^4 + {}^nC_6\, q^{n-6} p^6 + \cdots\}$

$\therefore \{q^n + {}^nC_2\, p^2 q^{n-2} + {}^nC_4\, p^4 q^{n-4} + {}^nC_6\, p^6 q^{n-6} + \cdots\}$

$= \frac{1}{2}\{(\frac{1}{6}+\frac{5}{6})^n + (\frac{5}{6}-\frac{1}{6})^n\}$

$= \frac{1}{2}\{1 + (\frac{4}{6})^n\}$

$\therefore$ Here the required probability $= \frac{1}{2}\{1 + (\frac{2}{3})^n\}$

(4) Given: $P(A) = 3/8$, $P(B) = 5/8$, $P(A \cup B) = 3/4$
find $P(A/B)$, $P(B/A)$.
are the two events A and B independent.

Sol:

$P(A) = 3/8$, $P(B) = 5/8$, $P(A \cup B) = 3/4$.

$$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B).$$

$$= 3/8 + 5/8 - 3/4$$

$$= \frac{8-6}{8} = 2/8 = 1/4$$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{5/8} = \frac{1}{4} \times \frac{8}{5} = 2/5.$$

$$\text{and } P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{3/8} = \frac{1}{4} \times \frac{8}{3} = 2/3$$

bility of Solving a Problem by 3 students by A, B, C are $2/7$, $3/8$, $1/2$, respectively. If all of them try independently find the probability that the problem is solved, and find also the probability that the problem could not be solved.

Solⁿ

Probability of Solving a Problem by A is $P(A) = 2/7$.

Now the Probability that A cannot solve the problem, $\bar{A} = \left(1 - 2/7\right) = 5/7$.

Similarly, for respective events,

$P(B) = 3/8$.

$P.$ (B cannot solve) $= 1 - 3/8 = 5/8$.

$P(C) = 1/2$

P (C cannot solve this) $= 1 - 1/2 = 1/2$

∴ The Probability that A, B, C cannot solve the problem, $= \frac{5}{7} \times \frac{5}{8} \times \frac{1}{2}$ (Since A, B, C independent)

$= \frac{25}{112}$

∴ Therefore the Probability that they can solve the problem $= 1 - \frac{25}{112}$

$= \frac{112 - 25}{112} = \frac{87}{112}$

unbiased dies are ~~rool~~ rolled together
find the odds in favour of getting two digits
sum of which is 7.

$\left.\begin{matrix}6+1\\3+5\\4+3\end{matrix}\right| \times 2$

6 times.

If two dies ~~rool~~ rolled together
then the ~~no. of~~ Total no. of likely
event pts in the sample space is,

$6^2 = 36$.

Let. A be the event that getting two
digits sum of which is 7. then the
total no of event pts in events A.

ie $\{(1,6).\ (2,5)\ (3,4)\ (4,3),(5,2)\ (6,1)\}$

is six. (6).

Therefore the Probability of $A = \frac{6}{6^2} = \frac{1}{6}$.

Therefore the odd in favour of event A

are. $\frac{1}{6-1} = \frac{\text{occurance.}}{\text{non occurance}} = m : (m-n)$

$= \frac{1}{5}$.

2. What is the Probability that if A fair coin is tossed
Q. six times. we get i) exactly two heads.
ii) at least two heads.

If a fair coin is tossed six times. then the
total no of equally likely in the event space
is 2^6

Now, the two heads comes out of 6 heads.
is 6_{C_2} ways.

$\therefore$ the probability of ~~good~~ getting exactly
2 heads $= \frac{6_{C_2}}{2^6}$

$= \frac{15}{64}$.

Let B and C denotes two events respectively of getting no heads and one head.

Then B contains $^6C_0$ equally likely event pts.

i.e. $^6C_0 = 1$.

and C contains the no. of equally likely event pts. contain in C $^6C_1 = 6$.

Now, $\therefore$ the $P(B) = \frac{1}{2^6} = \frac{1}{64}$

and $P(C) = \frac{6}{2^6} = \frac{6}{64}$

Since the event B and C are mutually exclusive then Probability $(B \cup C)$

$= P(B) + P(C) = \frac{1}{64} + \frac{6}{64} = \frac{7}{64}$.

Now, Probability of getting atleast two heads

$= 1 - P(B \cup C)$

$= 1 - \frac{7}{64}$

$= \frac{57}{64}$.

Q. balance dice are thrown together, find the ability of getting different digit on the four dies.

If four balanced dice are thrown together there are 6^4 equally likely event pts in the Sample Space.

Let A be the event of getting different digit of four dice. to find the no. of equally likely event pts Containing in A. we note that the 1st die may have Six different outcomes each of which can be combined with five different out comes of second die, in 6×5 different ways.

Similarly, the total no of equally likely event pts in different digits in four dies. in, $(6 \times 5 \times 4 \times 3)$ ways.

$\therefore$ The probability of A. $= \dfrac{6 \times 5 \times 4 \times 3}{6^4}$

$= \dfrac{5 \times \cancel{4}^{2} \times 3}{\cancel{2}\, \cancel{6} \times \cancel{6} \times 6_{3}}$

$= \dfrac{5}{18}$,

* A bag contains 7 red and 5 white balls, 4 balls are drawn at random, what is the Probability that i) all of them are red.

ii) two of them would be red, and two of white.

Soln.

i) $\dfrac{{}^{7}C_4}{{}^{12}C_4}$, ii) $\dfrac{{}^{7}C_2 \cdot {}^{5}C_2}{{}^{12}C_4}$.

*) 3 Cards are drawn at random. from a of 52 Cards. find the Probability of getting
i) 2 ace
ii) 2 Speads.
iii) one Speads, club, one diamond.
iv) Two face Cards.
v) at least one kind,

Soln. 3 Cards are drawn at random from a well Shuffled pack of 52 Cards.

~~Probability~~ of Now. the event Space. is ~~500~~ ~~500~~ $^{52}C_3$.

is equally likely event pts in the Sample Space $^{52}C_3$.

i) Let. A be the event. of getting 2 ace,

$\therefore$ Probability, of $A = {}^4C_2$

and from rest one Card, rest of, the Cards. $(52-4 \text{ ace}) = 48$ cards.

now, of getting 1 from 48, is $^{48}C_1$

$$\frac{^4C_2 \times {}^{48}C_1}{^{52}C_3}.$$

ii) Let B be the event of getting 2 Speads.

$52-13=39$

$$P(B) = \frac{^{13}C_2 \times {}^{39}C_1}{^{52}C_3}.$$

getting

7_c

Let C be the event of getting 1 Spead, 1 club 1 diamon.

Probability of one Speads = $^{13}C_1$

" " " club = $^{13}C_1$

diamond = $^{13}C_1$

$$\therefore P(C) = \frac{^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{^{52}C_3}$$

iv) Let D be the event of getting 2 face Cards.

$$\text{Now, } P(D) = \frac{^{12}C_2 \times {}^{(52-12)}C_1}{^{52}C_3}$$

$$= \frac{^{12}C_2 \times {}^{40}C_1}{^{52}C_3}$$

v) Let E be the event of getting one kind.

~~of kind~~.

~~event of getting no king = (52-4)C~~.

event of getting no king out of 52 Cards,

= $\dfrac{\text{event of getting no king} \times \text{event of 3 Cards out of 52}}{\text{Total event Space}}$

$$= \frac{^{4}C_0 \times {}^{48}C_3}{^{52}C_3},$$

$\therefore$ P(Getting at least one king)

$$= \left\{1 - \frac{^{4}C_0 \times {}^{48}C_3}{^{52}C_3}\right\}.$$

* In a pac of 10 watches. 3 are known to be defective, If ~~a~~ two watches are selected at random. from the pac, what is the probability that at least one is defective,

Solⁿ

Here total no of sample space $= {}^{10}C_2$

Now, without defective watches. the sample space $= {}^{7}C_2$

Now, Probability.

Two watches are selected at random from the pac of 10 watches in ${}^{10}C_2$ ways.

$\therefore$ the total no of equally likely event pts ~~= 10~~ in the sample space $= {}^{10}C_2$

The no. of non defective watches $= (10-3) = 7$ watches.

Now, let A be the event in getting non-defective watches, ~~for~~

then the total no of equally likely pts in A $= {}^{7}C_2$

$(A) = \frac{^7C_2}{^{10}C_2}$

Thus the Probability of getting at least one is defective watch $\left(1 - \frac{^7C_2}{^{10}C_2}\right)$.

=

40% of the Student in a Class are girls. if 60% and 70% boys and girls respectively of the class pass a certain test. What is the probability that a randomly selected Student from the class. will have passed the test,

$\frac{\cancel{60}}{\cancel{100}}$ 1/60

Sol^n.

~~The no. Student p~~

The no boys passed = ~~30~~. ~~60%~~ of 60%

= 36.

" " girls. passed = 70% of 40 Student.

= 28.

Total passed Student = (36+28) = 64.

now of getting one passed Student = $^{64}C_1$

and Total no of event Space

= $^{100}C_1$.

∴ Probability of randomly Selected one Student will have passed

$= \frac{^{64}C_1}{^{100}C_1} = \frac{64}{100} = \frac{16}{25}$

* Show that the function is given by,

$$f(x) = x \qquad 0 < x < 1$$
$$= k - x \qquad 1 < x < 2$$
$$= 0 \qquad \text{elsewhere},$$

is a probability density function for a suitable value of the constant k. Calculate the probability that the random variable lies betn. $1/2$ and $3/2$

* Find the value of the constant k such that,

$$f(x) = kx(1-x) \qquad 0 < x < 1$$
$$= 0 \qquad \text{elsewhere},$$

is a probability density function, construct the distribution function and compute.

Soln. ~~Def~~ from def. of probability density func.

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = k\int_0^1 x(1-x). \quad \text{Since } 0 < x < 1$$

$$= k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= k\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= k/6$$

or, $k/6 = 1$

$k = 6$.

Now, the distribution function $F(x)$ is in $-\infty < x < 0$. (from def.) and here $f(x) = 0$

in $0 \le x \le 1$

$$\therefore F(x) = 6\int_0^x x(1-x)\,dx = x^2(3-2x)$$

and in $1 < x < \infty$ $\quad F(x) = 6\int_0^1 x(1-x)\,dx = 1$

$$\therefore P(X > 1/2) = \int_{1/2}^{\infty} f(x)\,dx = 6\int_{1/2}^{1} x(1-x)\,dx = 1/2$$

$$\therefore P(X > 1/2) = 1 - P(X \le 1/2) = 1 - F(1/2) = 1 - 1/2 = 1/2$$

...ameter of an electric cable is assumed to be ...nuous random variable with probability density ...ction, $f(x) = 6x(1-x) \quad 0 \le x \le 1$

i) verify that above is p.d.f.

ii)

Sol^n Now, we have from the definition of p.d.f.

from 1st cond.: $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 6(1-x)x\,dx$. Since $0 \le x \le 1$

$= \int_0^1 (6x - 6x^2)\,dx$

$= (3x^2 - 2x^3)_0^1 = 3 - 2 = 1.$

and. here from second cond.:

$f(x) = 6x(1-x) \ge 0$ for $0 \le x \le 1$

Hence the given function is p.d.f.

ii) find the Mean & variance.

Mean $= \int_{-\infty}^{\infty} x f(x)\,dx.$

$= \int_0^1 x \cdot 6x(1-x)\,dx = \int_0^1 (6x^2 - 6x^3)\,dx.$

$= \left(2x^3 - \frac{3}{2}x^4\right)_0^1$

$= 2 - 3/2 = \frac{1}{2}$

Variance $= \int_{-\infty}^{\infty} (x-\bar{x})^2 f(x)\,dx.$ where $\bar{x}$ = mean.

$= \int_0^1 \left(x - \frac{1}{2}\right)^2 \cdot 6x(1-x)\,dx.$

$= \int_0^1 \left(x^2 - x + \frac{1}{4}\right)(6x - 6x^2)\,dx$

$= \int_0^1 \left(12x^3 - 6x^4 - \frac{15}{2}x^2 + \frac{3}{2}x\right)dx.$

$= \left[3x^4 - \frac{6}{5}x^5 - \frac{5}{2}x^3 + \frac{3}{4}x^2\right]_0^1$

$= \frac{1}{20}.$

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* The joint Probability density function of two random variables X and Y is $k(1-x-y)$ inside the triangle formed by the axes and the line $x+y=1$ and '0' elsewhere. Find the value of k and $P(X<\frac{1}{2}, Y>\frac{1}{4})$. Find also the marginal distributions of X, Y, and determine whether the random variables are independent or not.

Sol^n From the given question,

$$f(x,y) = k(1-x-y) \quad \text{for } x>0,\ y>0,\ x+y<1$$
$$= 0 \quad \text{elsewhere.}$$

Again from Joint Probability density function,

$$1 = \int_c^d\int_a^b f(x,y)\,dx\,dy. = P(a<x\le b, c<Y\le d).$$

$$\therefore 1 = k\int_0^1\int_0^{1-y}(1-x-y)\,dx\,dy.$$

Since $x>0, y>0$ and $x+y<1$
$x<1-y$
$\therefore 0<x\le 1-y.$

$$= \frac{1}{2}k\int_0^1 (1-y)^2\,dy.$$

$x-\frac{x^2}{2}-xy$

$x^2-2xy-2x$
$(1-y)^2$

$$= \frac{1}{6}k.$$

$x-\frac{x^2}{2}-xy$
$(1-y)-\frac{(1-y)^2}{2}-(1-y)y$
-0
$\frac{1}{2}(1-y)(2(1-y)-(1-y$
$=\frac{1-y}{2}\cdot(2-2y-1+y-2$
$=\frac{1-y}{2}(1-3y).$

$\therefore k=6.$

Given Condition, $P(X<\frac{1}{2}, Y>\frac{1}{4})$.

$$\therefore P(X<\tfrac{1}{2}, Y>\tfrac{1}{4}) = 6\int_0^{1/2}\int_{1/4}^{1-x}(1-x-y)\,dx\,dy.$$

and $x+y<1$
$y<1-x$

$$= 3\int_0^{1/2}\left(\frac{3}{4}-x\right)^2 dx = \frac{13}{32}$$

$$F_x'(x) = 6\int_0^{1-x} (1-x-y)\,dy.$$

$$= 6\left[y - xy - \frac{y^2}{2}\right]_0^{1-x}$$

$$= 6\left[1-x-x+x^2 - \frac{(1-x)^2}{2}\right]$$

$$= 3\left[2-2x-2x+2x^2-1+2x-x^2\right].$$

$$= 3\left[x^2-2x+1\right]$$

$$= 3(1-x)^2 \qquad 0<x<1$$

Slly, $f_y(y) = 6\int_0^{1-y} (1-x-y)\,dy = 3(1-y)^2 \qquad 0<y<1$

Since $f(x,y) \neq f_x(x)\, f_y(y)$.

So, X, Y are independent.

$f_x(x) = \frac{d}{dx}\{F_x(x)\}$

$= F_x'(x)$

$f_y(y) =$

$F(x,y) = \int$

* If the Probability of n mutually independent $P_1, P_2, \ldots, P_n$, Then show that the Probability at least one of the event will occur is

$$1-(1-P_1)(1-P_2)\cdots(1-P_n).$$

* From an urn containing n balls, any number of ball are drawn. show that probability of drawing an even number of balls is

$$\frac{2^{n-1}-1}{2^n-1}$$

* If a dice is thrown n times, show that the Probability of an even no: of sixes is,

$$\frac{1}{2}\left\{1+\left(\frac{2}{3}\right)^n\right\}.$$

* What is the chance of a ~~leap year~~. 53 sundays in a leap-year,

In

Now. two consecutive days of a week can be selected from 7 days. of a week. in a following different ways.

Mon - tues
tues - wed
wed - Thurs.
Thurs - Fri
Fri - Sat
Sat - Sun.
Sun - Monday

Let. A denotes the event That a leap-year selected at random, 53 sundays.

clearly A contains two equally likely event pts and the sample space contains 7 equally likely event pts.

The required chance, $P(A) = \frac{2_{C_4}}{7_{C_4}}$

$= (2/7)$.

(*). If 30 days dates are named at random what are the Probability that 5 of them will be sunday.

Soln

Let the event be A.

then $P(A) = \frac{{}^{30}C_5 \times 6^{25}}{7^{30}}$.

Sample Space $= 7^{30}$.

(*) Three events A, B, C are mutually exclusive and exhaustive, If $P(B) = \frac{3}{2} P(A)$.

and $P(C) = \frac{1}{3} P(B)$

find $P(C) = ?$

Soln we have, $P(C) = \frac{1}{3} P(B)$

$= \frac{1}{3} \times \frac{3}{2} P(A)$.

$P(C) = \frac{1}{2} P(A)$, $\Rightarrow P(A) = 2 P(C)$.

$\therefore \frac{1}{2} P(A) = \frac{1}{3} P(B)$

$P(A) = \frac{2}{3} P(B) = \frac{2}{\cancel{3}} \times \cancel{3} \times P(C)$

$= 2 P(C)$.

$\therefore$ ~~P(C) = ,~~

$\therefore P(B) = 3 \times P(C)$.

Since A, B, C are exhaustive,

$(A \cup B \cup C) = S$

$\therefore P(A \cup B \cup C) = P(S) = 1$.

$P(A) + P(B) + P(C) = 1$ Since A, B, C are mutually exclusive.

$2 P(C) + 3P(C) + P(C) = 1$

$6 P(C) = 1 \Rightarrow P(C) = \frac{1}{6}$.

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* Can the following represents the measures of probability.

i) $P(A) = 0.4$, $P(A^c) = 0.52$ (no)

* ii) $P(A) = 0.38$, $P(B) = 0.30$, $P(C) = 0.32$ (Yes)

iii) $P(A) = 0.5$, $P(B) = 0.4$, $P(B \cup C) = 0.2$

iv) $P(A \cap B) = 0.1$, $P(B) = 0.4$, $P(C) = 0.5$

v) $P(A \cup B) = 0.6$, $P(B) = 0.2$, $P(C) = 0.3$.

Where $(A \cup B \cup C) = S$ is the ~~the~~ Sure event and, A, B, C are mutually exclusive.

iii)
$P(B \cup C) = 0.2$
$P(B) + P(C) = 0.2$
$P(C) = 0.2 - 0.4$ = negative.
impossible.

iv)
$P(A \cap B) = 0.1$, Since mutually exclusive. $P(A \cap B) = \phi$
$P(A) \cdot P(B) = 0.1$
~~$P(A) = 0.1 + 0.4 = 0.8$~~
~~$P(A) + P(B) + P(C) = 0.5 + 0.4 + 0.5 > 1$~~
~~not possible.~~

v)
$P(A \cup B) = 0.6$.
$P(A) + P(B) = 0.6$
$P(A) = 0.6 - 0.2 = 0.4$.
$\therefore P(A) + P(B) + P(C) = 0.4 + 0.2 + 0.3$
$= 0.9 < 1$ not possible.

$P(A) = 2/3$. $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{6}$. find. $P(A \cup B)$

$P(A/B)$, $P(B/A)$, $P(A \cap B^c)$, $P(A^c \cap B^c)$, $P(A^c \cup B^c)$.

State whether the events A, B are.

a) equally likely

b) mutually exclusive

c) exhaustive.

d) independent.

$P(A) = 3/8$, $P(B) = 5/8$, and $P(A \cup B) = 3/4$.

find. $P(A/B)$, $P(B/A)$.

are the two events A, & B independent.

* Two unbiased dices are thrown, find the Probability. Sum of the faces equal ~~and~~ or exceeds 10.

If two unbiased dices are thrown. the no. of equally likely event pt is $6^2 = 36$.

Let, A, B, C denote the events, that the Sum of the faces equals to 10, 11, 12 respectively.

clearly events A Contains three equally likely events pts say, (4,6), (5,5), (6,4).

event B contains, two equally likely event pts, (5,6), (6,5)

and event C Contains of one event pt. namely (6,6)

∴ we have $P(A) = 3/36$, $P(B) = 2/36$ $P(C) = 1/36$

Since A, B, C are mutually exclusive.

∴ ~~P(Sum of the faces)~~.

∴ $P(A \cup B \cup C) = \frac{3}{36} + 2/36 + \frac{1}{36} = 6/36 = \frac{1}{6}$.

* a no is chosen at random from the first 50 +ve integers. find the Probability that the chosen no: is divisible by 4 /or 5.

Sol^n A no: is chosen at random from the 50 +ve integers. ie $^{50}C_1$ ways.

$= 50$ different ways.

Let, A and B, denotes the event that the chosen no is divisible by 4 and 5 respectively. $A = 4, 8, \ldots 48 = 12$

$B = 5, 10, \ldots 50 = 10$

Now, $P(A) = \frac{12}{50}$ $\quad P(B) = \frac{10}{50}$

$(A \cap B) = 2$

$\therefore P(A \cap B) = 2/50$ (events 20, 40)

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{12}{50} + \frac{10}{50} - \frac{2}{50}$

$= 2/5$.

(*) Probability of solving a Problem by 3 students by, A, B, C are, $2/7$, $3/8$, $1/2$ respectively. If all of them try independently, find the probability that the problem is solved, and find also the probability the problem could not be solved.

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~~x, y, z denotes respective events~~

A bag contains 6 red, and 4 white balls 2 balls are drawn at random one after another, find the probability that both the drawn balls are white when i) first drawn balls is not replaced before second drawing.

ii) 1st drawn ball is replaced before 2nd drawing.

Soln. Total balls in the bag. $6+4 = 10$. one ball can be drawn in $^{10}C_1$ ie 10 different ways.

Let A be the event, ie the 1st drawn ball is white, cly. A contains. $^4C_1 = 4$ equally likely event pts.

$\therefore P(A) = \frac{4}{10} = 2/5$.

Let B denotes the event that the second drawn ball is white,

i) If the 1st ball is white and if it is not replaced before 2nd drawing. then the

$P(\text{occurance of } B) = \text{given } A = P(B/A)$.

$= \frac{^3C_1}{^9C_1} = \frac{1}{3}$.

Now,

$P\left(\begin{matrix}\text{Simultaneous occurance} \\ \text{of } A \text{ and } B\end{matrix}\right) = P(A \cap B)$

$= P(A) \cdot P(B/A)$.

$= \frac{2}{5} \times \frac{3}{9} = \frac{2}{15}$.

ii) If the 1st ball is replaced before 2nd drawing, then the occurance of B doesnot depends on the occurance or non-occurance of A, ie A, B independent,

then $P(\text{Both balls are white}) = P(A \cap B) = P(A) \cdot P(B)$
$= \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

Imp

** If $A_1, A_2, A_3, \ldots, A_n$ are independent events such that $P(A_i) = 1 - q_i \quad i = 1, 2, \ldots, n$.

Prove that $P(A_1 \cup A_2 \cup \cdots \cup A_n) = \{1 - q_1 \cdot q_2 \cdot q_3 \cdots q_n\}$.

Sol:

By the Problem. $A_i = 1 - q_i \quad i = 1, \ldots, n$.

$$P(A_i^c) = 1 - (1 - q_i) = q_i \quad \text{"}$$

Since event $A_1, A_2, \ldots, A_n$ are independent

& Hence $A_1^c, A_2^c, \ldots, A_n^c$ are independent.

Now,
$$\begin{aligned}
P(A_1 \cup A_2 \cup \cdots \cup A_n) &= 1 - P(A_1 \cup A_2 \cup \cdots \cup A_n)^c \\
&= 1 - P(A_1^c) \cap P(A_2^c) \cap \cdots \cap P(A_n^c) \\
&= 1 - P(A_1^c) \cdot P(A_2^c) \cdots P(A_n^c) \\
&= 1 - q_1 \cdot q_2 \cdots q_n .
\end{aligned}$$

very Important

If Probability of n mutually independent event be $P_1, \ldots P_n$, then show that the Probability that at least one of the event will occur is

$1-(1-P_1)(1-P_2)\cdots(1-P_n)$. Ans

Let $A_1, A_2, A_3, \ldots A_n$, be n mutually independent events.

$\therefore P(A_1)=P_1,\ P(A_2)=P_2 \ldots\ldots P(A_n)=P_n$.

Now $P(A_1^c)=1-P_1,\ P(A_2^c)=1-P_2, \ldots P(A_n^c)=1-P_n$.

Through $A_1, A_2, \ldots, A_n$ are mutually independent events so that, $A_1^c, A_2^c, \ldots, A_n^c$ are also mutually independent events,

$\therefore P(A_1^c \cap A_2^c \cap \ldots \cap A_n^c) = P(A_1^c)\, P(A_2^c) \ldots P(A_n^c)$

Let E be the event that at least one of the event occurs, and E^c = be the event that none of the event $A_1, A_2, \ldots A_n$ occurs,

$\therefore P(E^c) = P(A_1^c)\, P(A_2^c) \ldots P(A_n^c)$.

$\therefore P(E) = 1-P(E^c) = 1-P(A_1^c)\, P(A_2^c) \ldots P(A_n^c)$

$= 1-(1-P_1)(1-P_2)\ldots(1-P_n)$.

$\therefore P(E) = \left(\text{Probability that at least one of the event occurs}\right) = 1-(1-P_1)(1-P_2)\ldots(1-P_n)$.

* . From an urn containing n balls any number of are drawn. Show that probability of drawing an even number of balls is

$$\frac{2^{n-1}-1}{2^n-1}.$$

Solⁿ. Here according to the question, the random experiment is drawn, with respect of any number of balls.

Now the Total number of Cases $= {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$.

So that, (2^n-1)

Let A = event of drawing even number of balls.

So that the number of favourable cases for the event $A = {}^nC_2 + {}^nC_4 + \cdots + {}^{2m}C_{2m}$

$n = 2m$

$= (2^{n-1}-1)$

Therefore the Probability of drawing of even no of balls $= \left(\frac{2^{n-1}-1}{2^n-1}\right)$.

$\therefore P(A) = \frac{(2^{n-1}-1)}{(2^n-1)}$. A.

${}^{2m}C_2 + {}^{2m}C_4 + \cdots + {}^{2m}C_{2m}$

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n$

Put $x=1$.

$2^n = 1 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$

$\Rightarrow 2^n - 1 = {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$

$(1+x)^{2m} = {}^{2m}C_0 + \cdots$

$\frac{2^n}{2} = 1 + {}^nC_2 + {}^nC_4 + \cdots$