Quantum mechanics
Part - 4

As we have learnt that laws based on classical mechanics fails to explains the radiation spectrum of a black-body.

They fails either for low wavelength or high wavelength of radiation.

Here comes scientist Plank and gives his hypothesis and law which is called Plank's hypothesis and radiation law.

→ **Plank's Hypothesis and Radiation law :-**

Plank examined the data of distribu-tion of energy in black body radiation and suggested that correct result can be obtained if energy of the oscillating electron is taken as discrete rather than continuous.

To explain the experimental observations, the following assumptions were made:

i) Black body radiation contains simple harmonic oscillators of molecular

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dimensions which can vibrate with all possible frequencies.

ii) The frequency of vibration of oscillators is equal to or same as the frequency of radiation.

iii) The oscillators emit or absorb energy in discrete units of light energy called quanta or photons i.e. the oscillator can have only discrete nergy value $E_n = nh\nu$
where,

'n' is integer and
h is planks constant.

iv) The oscillators can emit or absorb radiation energy in packets of $h\nu$.

According to Max Plank, the ~~e~~ energy possesed by an oscillator of given frequency can't ~~take~~ arbitary value from '0' to '∞'. ~~It has energy in discrete values~~ It has total energy in discrete value '$n\epsilon_0$'

where,

n is integer value (+ve) with '0', which indicates no. of discrete energy ϵ_0 in oscillator.

ϵ_0 is called quantum of energy.

$$\epsilon_0 = h\nu$$

So,

$$\boxed{\epsilon_n = nh\nu}$$

where,

$\epsilon_n \rightarrow$ Total discrete energy of oscillator having 'n' number of quantum.

$n \rightarrow$ no. of quantum.

$h \rightarrow$ plank's constant.

$\nu \rightarrow$ frequency with which oscillator is oscillating.

Since, the energy of oscillator is quantised i.e. it can't have just any energy value, it's energy value changes by discrete amount $h\nu$. So, when oscillator looses or gains energy at frequency ν

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, that energy will be integer multiple of $h\nu$.

According to Maxwell-Boltzmann distribution function, the number of oscillators having energy E_n is given by,

$$N_n = N_0 e^{(-E_n/KT)}$$

$$\left[\Rightarrow N_n = N_0 e^{(-nh\nu/KT)} \right]$$

here,

'N_n' can also be classified as no of oscillators having 'n' quanta of energy ($h\nu$).

N_0 is number of oscillator in ground state having zero energy.

So, average energy of a particle vibrating with frequency ν is given by

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} N_n \varepsilon_n}{\sum_{n=0}^{\infty} N_n} = \frac{\sum_{n=0}^{\infty} N_0 \varepsilon_n e^{(-\varepsilon_n/kT)}}{\sum_{n=0}^{\infty} N_0 e^{(-\varepsilon_n/kT)}}$$

$$\Rightarrow \bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} nh\nu\, e^{(-nh\nu/kT)}}{\sum_{n=0}^{\infty} e^{(-nh\nu/kT)}}$$

Let's take $e^{(-h\nu/kT)} = x$

$$\Rightarrow e^{(-nh\nu/kT)} = x^n$$

$$\Rightarrow \bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} h\nu\, n x^n}{\sum_{n=0}^{\infty} x^n}$$

$$\Rightarrow \bar{\varepsilon} = h\nu \frac{\sum_{n=0}^{\infty} n x^n}{\sum_{n=0}^{\infty} x^n}$$

$$\Rightarrow \bar{\varepsilon} = \frac{h\nu x(1+2x+3x^2+4x^3+\dots)}{(1+x+x^2+x^3+\dots)}$$

$$\Rightarrow \bar{\varepsilon} = h\nu x \frac{(1-x)^{-2}}{(1-x)^{-1}} = \frac{h\nu x}{1-x} = \frac{h\nu}{1/x - 1}$$

Putting value of $x = e^{-h\nu/kT}$

$$\Rightarrow \bar{\varepsilon} = \frac{h\nu}{\frac{1}{e^{-h\nu/kT}} - 1}$$

$$\Rightarrow \bar{\varepsilon} = \frac{h\nu}{(e^{h\nu/kT}) - 1}$$

$$\boxed{\Rightarrow \bar{\varepsilon} = \frac{hc/\lambda}{(e^{hc/\lambda kT}) - 1}}$$

now, we know,

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \bar{\varepsilon}\, d\lambda$$

$$\Rightarrow u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}$$

$$u_\lambda d\lambda = \frac{8\pi}{\lambda^4} \bar{\varepsilon}\, d\lambda$$

$$\Rightarrow u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{(hc/\lambda kT)} - 1} d\lambda$$

So,

$$\boxed{E_\lambda = \frac{2\pi hc^2}{\lambda^5} \times \frac{1}{e^{(hc/\lambda kT)} - 1}}$$

where,

E_λ is total radiant heat energy per unit time, per unit area corresponding to wavelength λ at parti-
-cular temp(T).

$h = 6.6256 \times 10^{-34}$ J sec.

This is plank's formula for the intensity distribution of blackbody radiation. It fulfills it's aggrement with whole range of wavelength.

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Case 1 :- high wavelength

$$\lambda \to \infty, \ (hc/\lambda KT) \to 0$$

also, E_λ formula can be written as,

$$E_\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{(1+hc/\lambda KT)-1}$$

as, expansion e^x for first two terms.
because x, here $hc/\lambda KT$ is small

$$\Rightarrow E_\lambda = \frac{2\pi hc^2}{\lambda^5} \times \frac{\lambda KT}{hc}$$

$$\boxed{\Rightarrow E_\lambda = \frac{2\pi c}{\lambda^4} KT}$$

which is identical to Rayleigh-Jeans expression at high wavelength.

Case 2 :- low wavelength

$$\lambda \to 0, \ e^{(hc/\lambda KT)} >> 1$$

$$\boxed{E_\lambda = \frac{2\pi hc^2}{\lambda^5} e^{(-hc/\lambda KT)}}$$

which is identical to Wien's expression of intensity distribution.

$$E_\lambda = \frac{C_1}{\lambda^5} e^{(-C_2/\lambda T)}$$

where,

$$C_1 = 2\pi hc^2 \quad \& \quad C_2 = hc/K$$