Problem 1: The aatemp data come from the U.S. Historical Climatological Network. They are the annual mean temperatures (in degrees F) in Ann Arbor Michigan, going back about 150 years.

(a) Fit a linear trend model. Is there a significant linear trend? Interpret your results.
(b) Observations in successive years may be correlated. Fit the model that estimates this correlation. Does this change your opinion about the significance of the trend?

(c) Fit a polynomial model with degree 10 and use backward elimination to reduce the degree of the model. Plot the fitted model on the top of the data. Use this model to predict the temperature in 2001.

(d) Make a cubic spline fit with six basis functions evenly spaced on the data range. Plot the fit in comparison with the previous fit. Does this model fit better than the selected polynomial model?

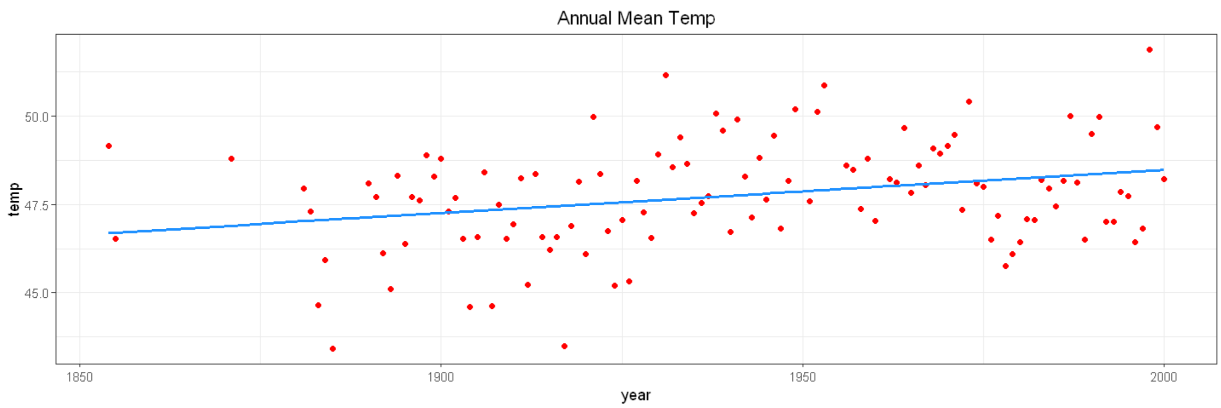
In [382]:
```
# install.packages('gridExtra')
library('faraway')
data(aatemp)
library(ggplot2)
options(repr.plot.width=12, repr.plot.height=4)
head(aatemp, 3)
```

year	temp
1854	49.15
1855	46.52
1871	48.80

(a) Fit a linear trend model. Is there a significant linear trend? Interpret your results.

In [383]:
```
mod = lm(temp ~ year, data = aatemp)
pred = predict(mod, aatemp)
```

In [384]:
```
ggplot(data = aatemp, aes(x = year, y = temp))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point(color='red')+
    labs(title='Annual Mean Temp', x='year', y = 'temp')+
    geom_line(aes(x = year, y = pred), color = 'dodgerblue', size = 1)
```



In [385]:
```
summary(mod)
```

```
Call:
lm(formula = temp ~ year, data = aatemp)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.9843 -0.9113 -0.0820  0.9946  3.5343 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)   
(Intercept) 24.005510   7.310781   3.284  0.00136 **
year         0.012237   0.003768   3.247  0.00153 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.466 on 113 degrees of freedom
Multiple R-squared:  0.08536,	Adjusted R-squared:  0.07727 
F-statistic: 10.55 on 1 and 113 DF,  p-value: 0.001533
```

In [386]:
```
confint(mod)
```

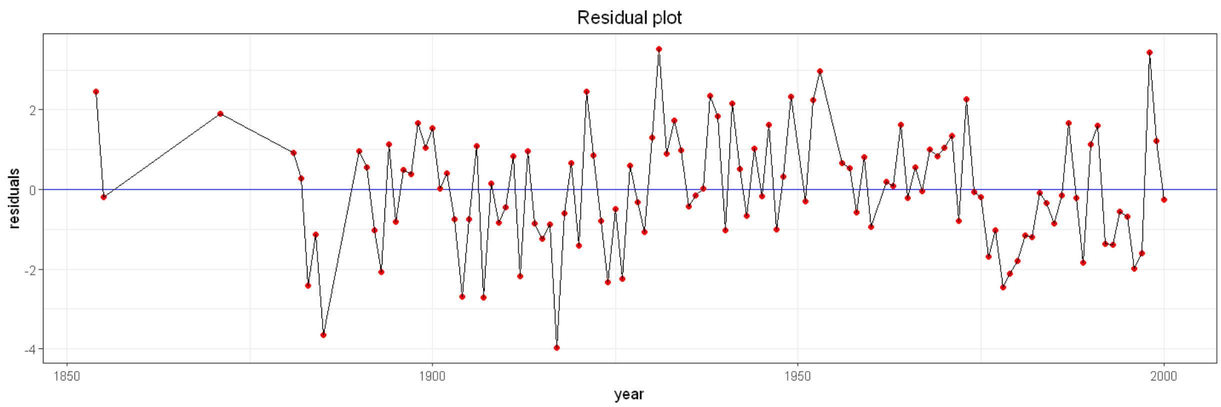
	2.5 %	97.5 %
(Intercept)	9.521535277	38.48948531
year	0.004771599	0.01970293

- Since β coefficient for year is very low, there seems to be a very small linear trend between year and temp.

(b) Observations in successive years may be correlated. Fit the model that estimates this

correlation. Does this change your opinion about the significance of the trend?

In [387]:
```
ggplot(data = aatemp, aes(x = year, y = mod$res))+
    geom_point()+
    geom_hline(yintercept = 0, color = 'blue') +
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    labs(title='Residual plot', x='year', y = 'residuals')+
    geom_point(color='red')+
    geom_line()
```



- The residuals look like auto correlated with time. The value of a residual at a particular point depend upon value of preceding residual.
- Lets confirm it by Derbin-Watson test

In [388]:
```
dwtest(mod)
```

```
	Durbin-Watson test

data:  mod
DW = 1.6177, p-value = 0.01524
alternative hypothesis: true autocorrelation is greater than 0
```

- The low p value of Durbin-Watson test indicates that residuals are correlated. The value of a residual depend upon value of preceding residual.
- Here, we can interpret that, if annual mean temperature of a given year is low, the mean temperature for succeding year changes with respect to the year given. Basically, shift in mean temperature happens gradually rather than sudden spikes.
- Since the null hypothesis of no autocorrelation is failed from Durbin-Watson test, we can fit generalized linear model.

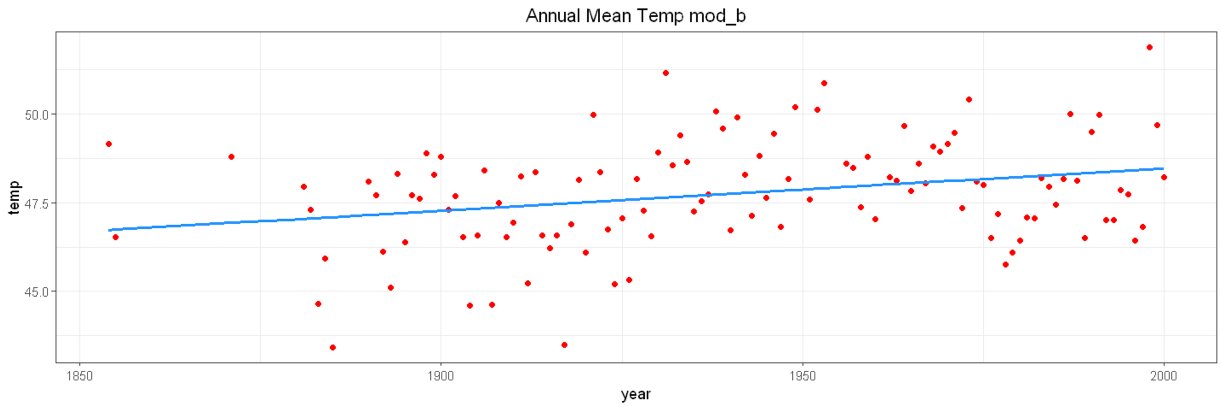
In [389]:
```
library(nlme)
mod_b = gls(temp ~ year, correlation = corARMA(p=1), data = aatemp)
summary(mod_b)
```

```
Generalized least squares fit by REML
  Model: temp ~ year
  Data: aatemp
       AIC      BIC    logLik
  427.2854 438.195 -209.6427

Correlation Structure: AR(1)
 Formula: ~1
 Parameter estimate(s):
      Phi
0.2029313

Coefficients:
                Value Std.Error  t-value p-value
(Intercept) 24.604907  8.935293 2.753677  0.0069
year         0.011931  0.004606 2.590398  0.0108

 Correlation:
     (Intr)
year -1

Standardized residuals:
        Min          Q1         Med          Q3         Max
-2.71297226 -0.62233530 -0.04994469  0.67429317  2.39436044

Residual standard error: 1.472974
Degrees of freedom: 115 total; 113 residual
```

In [390]:
```
intervals(mod_b)
```

```
Approximate 95% confidence intervals

 Coefficients:
                  lower        est.       upper
(Intercept) 6.902479671 24.60490705 42.30733442
year        0.002805908  0.01193074  0.02105558
attr(,"label")
[1] "Coefficients:"

 Correlation structure:
         lower      est.     upper
Phi 0.01264178 0.2029313 0.3790359
attr(,"label")
[1] "Correlation structure:"

 Residual standard error:
   lower     est.    upper
1.284087 1.472974 1.689647
```

In [391]:
```
pred = predict(mod_b, aatemp)
ggplot(data = aatemp, aes(x = year, y = temp))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point(color='red')+
    labs(title='Annual Mean Temp mod_b', x='year', y = 'temp')+
    geom_line(aes(x = year, y = pred), color = 'dodgerblue', size = 1)
```



- 95% confidence interval for β coefficient of year does not include zero. Hence, there is a very small linear relationship with year and temp.

(c) Fit a polynomial model with degree 10 and use backward elimination to reduce the degree of the model. Plot the fitted model on the top of the data. Use this model to predict the temperature in 2001.

In [392]:
```
mod_c = lm(temp ~ year + I(year^2) + I(year^3) + I(year^4)+ I(year^5)+ I(year^6)
           I(year^8)+I(year^9) + I(year^10), data = aatemp)
# mod_c = lm(temp ~ year + I(year)^2)
# round(summary(mod_c)$coef,3)
summary(mod_c)
```

```
Call:
lm(formula = temp ~ year + I(year^2) + I(year^3) + I(year^4) +
    I(year^5) + I(year^6) + I(year^7) + I(year^8) + I(year^9) +
    I(year^10), data = aatemp)

Residuals:
    Min      1Q  Median      3Q     Max
-3.7126 -0.9175 -0.1441  0.9905  3.2313

Coefficients: (5 not defined because of singularities)
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -7.049e+07  2.972e+07  -2.372   0.0194 *
year         1.676e+05  7.047e+04   2.379   0.0191 *
I(year^2)   -1.526e+02  6.396e+01  -2.385   0.0188 *
I(year^3)    6.347e-02  2.653e-02   2.392   0.0185 *
I(year^4)   -1.031e-05  4.299e-06  -2.399   0.0182 *
I(year^5)           NA         NA      NA       NA
I(year^6)           NA         NA      NA       NA
I(year^7)           NA         NA      NA       NA
I(year^8)    1.074e-20  4.432e-21   2.423   0.0170 *
I(year^9)           NA         NA      NA       NA
I(year^10)          NA         NA      NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.4 on 109 degrees of freedom
Multiple R-squared:  0.1955,    Adjusted R-squared:  0.1586
F-statistic: 5.298 on 5 and 109 DF,  p-value: 0.0002141
```

- we can see, from polynomial degree 5 to 10, the coefficients are not significant. Hence, we can eliminate those and fit the lm model upto degree 4.

In [393]:
```
mod_d = lm(temp ~ year + I(year^2) + I(year^3) + I(year^4), data = aatemp)
round(summary(mod_d)$coef,3)
summary(mod_d)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1496861.307	855298.314	1.750	0.083
year	-3085.571	1774.669	-1.739	0.085
I(year^2)	2.385	1.381	1.727	0.087
I(year^3)	-0.001	0.000	-1.716	0.089
I(year^4)	0.000	0.000	1.704	0.091

```
Call:
lm(formula = temp ~ year + I(year^2) + I(year^3) + I(year^4),
    data = aatemp)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.0085 -0.9618 -0.0913  0.9926  3.7370 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)  
(Intercept)  1.497e+06  8.553e+05   1.750   0.0829 .
year        -3.086e+03  1.775e+03  -1.739   0.0849 .
I(year^2)    2.385e+00  1.381e+00   1.727   0.0869 .
I(year^3)   -8.189e-04  4.773e-04  -1.716   0.0890 .
I(year^4)    1.054e-07  6.186e-08   1.704   0.0912 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.431 on 110 degrees of freedom
Multiple R-squared:  0.1522,	Adjusted R-squared:  0.1213 
F-statistic: 4.936 on 4 and 110 DF,  p-value: 0.001068
```

- We can see, none of the coefficients for polynomial model upto 4 are significant at 5% α level. So, lets remove 4th level polynomial term.

In [394]:
```
mod_e = lm(temp ~ year + I(year^2) + I(year^3), data = aatemp)
round(summary(mod_e)$coef,3)
summary(mod_e)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	39590.477	17339.743	2.283	0.024
year	-61.594	26.941	-2.286	0.024
I(year^2)	0.032	0.014	2.291	0.024
I(year^3)	0.000	0.000	-2.296	0.024

```
Call:
lm(formula = temp ~ year + I(year^2) + I(year^3), data = aatemp)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8557 -0.9646 -0.1552  1.0485  4.1538 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)  
(Intercept)  3.959e+04  1.734e+04   2.283   0.0243 *
year        -6.159e+01  2.694e+01  -2.286   0.0241 *
I(year^2)    3.197e-02  1.395e-02   2.291   0.0238 *
I(year^3)   -5.527e-06  2.407e-06  -2.296   0.0236 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.443 on 111 degrees of freedom
Multiple R-squared:  0.1298,	Adjusted R-squared:  0.1063 
F-statistic: 5.518 on 3 and 111 DF,  p-value: 0.001436
```

- Now, we can see all β coefficients are significant at 5% α level.

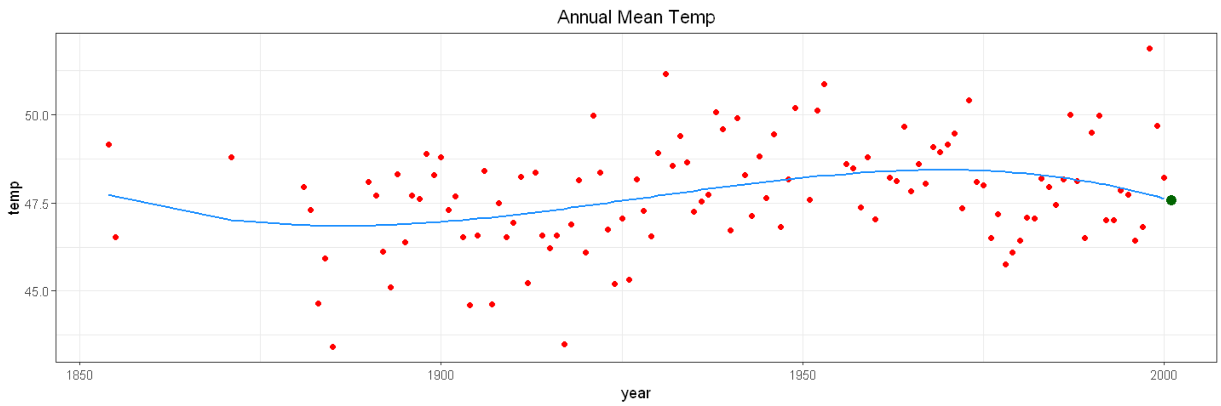
In [404]:
```
RSS = 1.443**2*111
RSS
```

231.129639

In [395]:
```
predict(mod_e, data.frame(year = 2001))
```

1: 47.5662932193882

In [396]:
```
pred = predict(mod_e, aatemp)
ggplot(data = aatemp, aes(x = year, y = temp))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point(color='red')+
    labs(title='Annual Mean Temp', x='year', y = 'temp')+
    geom_line(aes(x = year, y = pred), color = 'dodgerblue', size = 0.75)+
    geom_point(x = 2001, y = predict(mod_e, data.frame(year = 2001)), color = 'da
```



(d) Make a cubic spline fit with six basis functions evenly spaced on the data range. Plot the fit in comparison with the previous fit. Does this model fit better than the selected polynomial model?

- number of basis functions = df = 6
- number of knots(m) = 6 - 4 = 2

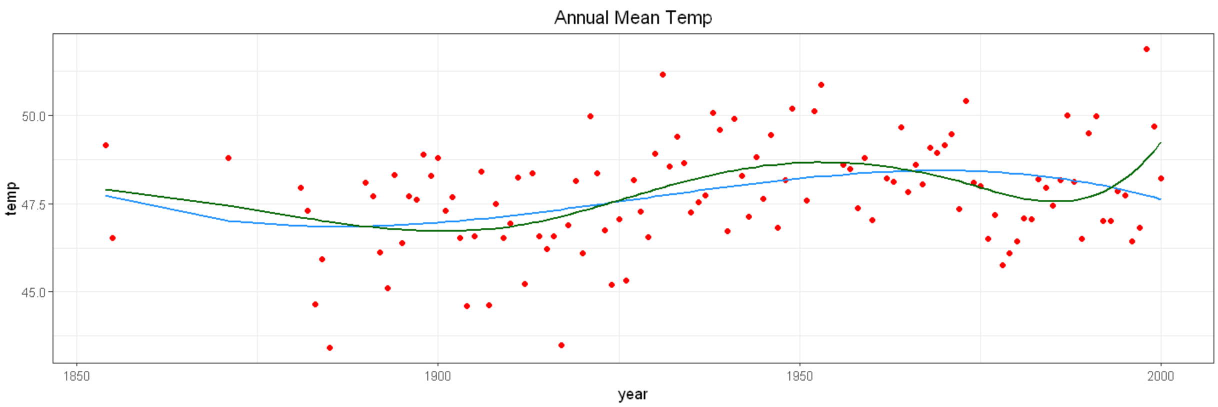
In [426]:
```
library(splines)
mod_f = lm(temp ~ bs(year, df = 5, intercept = FALSE), data = aatemp)
summary(mod_f)
```

```
Call:
lm(formula = temp ~ bs(year, df = 5, intercept = FALSE), data = aatemp)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6461 -0.9101 -0.2006  0.9427  3.3248 

Coefficients:
                                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)                           47.9406     0.9832  48.760   <2e-16 ***
bs(year, df = 5, intercept = FALSE)1  -0.8898     1.6369  -0.544   0.5878    
bs(year, df = 5, intercept = FALSE)2  -2.1812     1.2069  -1.807   0.0735 .  
bs(year, df = 5, intercept = FALSE)3   2.5074     1.3252   1.892   0.0611 .  
bs(year, df = 5, intercept = FALSE)4  -1.8368     1.1956  -1.536   0.1273    
bs(year, df = 5, intercept = FALSE)5   1.2750     1.2123   1.052   0.2953    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.392 on 109 degrees of freedom
Multiple R-squared:  0.2045,	Adjusted R-squared:  0.168 
F-statistic: 5.604 on 5 and 109 DF,  p-value: 0.0001234
```

In [427]:
```
RSS = 1.392**2*109
RSS
```

211.205376

In [401]:
```r
pred = predict(mod_e, aatemp)
pred1 = predict(mod_f, aatemp)
ggplot(data = aatemp, aes(x = year, y = temp))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point(color='red')+
    labs(title='Annual Mean Temp', x='year', y = 'temp')+
    geom_line(aes(x = year, y = pred), color = 'dodgerblue', size = 0.75)+
    geom_line(aes(x = year, y = pred1), color = 'darkgreen', size = 0.75)
```



- Since RSS for cubic spline model is less and R^2 value is higher, cubic spline model fits better.

Problem 2: Consider the infmort data set from the faraway library.

(a) Plot the data and make a brief summary. Which variables are categorical? Which variables are numerical?
(b) Fit a model for the infant mortality in terms of the other variables using the main effects and up to second order interactions. Comment on your results.

(c) Check for unusual observations and potential transformations. Re-fit your model if necessary.

(d) Interpret your model by explaining what the regression parameter estimates mean.

In [604]:
```r
library(faraway)
data(infmort)
```

(a) Plot the data and make a brief summary. Which variables are categorical? Which variables are numerical?

In [605]:
```
head(infmort)
```

	region	income	mortality	oil
Australia	Asia	3426	26.7	no oil exports
Austria	Europe	3350	23.7	no oil exports
Belgium	Europe	3346	17.0	no oil exports
Canada	Americas	4751	16.8	no oil exports
Denmark	Europe	5029	13.5	no oil exports
Finland	Europe	3312	10.1	no oil exports

- There are four data points where mortality values missing, hence lets drop them.

In [606]:
```
infmort = na.omit(infmort)
```

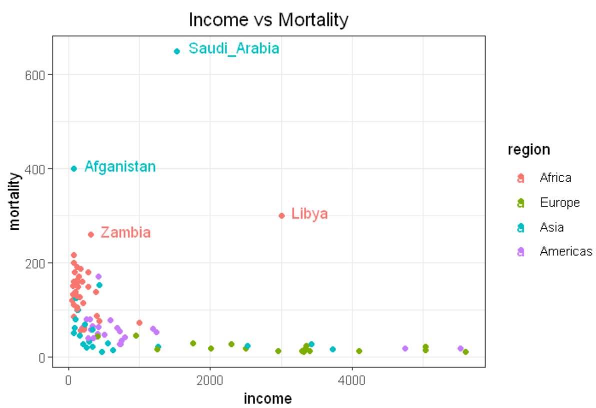
In [607]:
```
str(infmort)
```

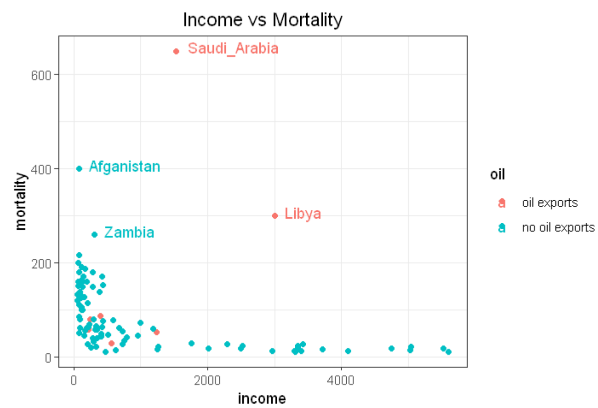
```
'data.frame':	101 obs. of  4 variables:
 $ region   : Factor w/ 4 levels "Africa","Europe",..: 3 2 2 4 2 2 2 2 2 2 ...
 $ income   : num  3426 3350 3346 4751 5029 ...
 $ mortality: num  26.7 23.7 17 16.8 13.5 10.1 12.9 20.4 17.8 25.7 ...
 $ oil      : Factor w/ 2 levels "oil exports",..: 2 2 2 2 2 2 2 2 2 2 ...
 - attr(*, "na.action")= 'omit' Named int  24 83 86 91
  ..- attr(*, "names")= chr  "Iran                  " "Haiti                 " "Laos
" "Nepal                 "
```

In [608]:
```
summary(infmort)
```

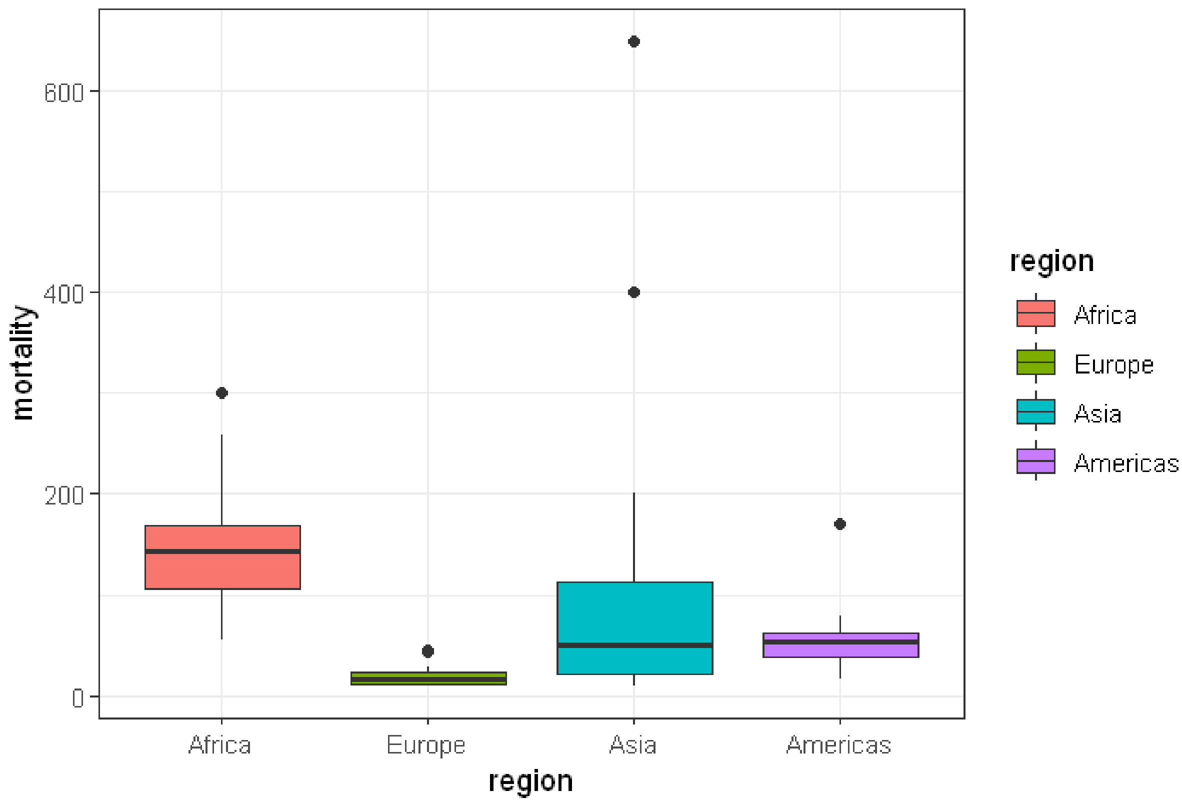
```
      region       income        mortality                   oil    
 Africa  :34   Min.   :  50   Min.   :  9.60   oil exports   : 8  
 Europe  :18   1st Qu.: 130   1st Qu.: 26.20   no oil exports:93  
 Asia    :27   Median : 334   Median : 60.60                      
 Americas:22   Mean   :1022   Mean   : 89.05                      
               3rd Qu.:1191   3rd Qu.:129.40                      
               Max.   :5596   Max.   :650.00                      
```

In [609]:
```
library(gridExtra)
options(repr.plot.width=12, repr.plot.height=4)

plot1 <- ggplot(data = infmort, aes(x = income, y = mortality, color = region))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point()+
    labs(title='Income vs Mortality', x='income', y = 'mortality')+
    geom_text(aes(label=ifelse(mortality>250,as.character(row.names(infmort)),'')

plot2 <- ggplot(data = infmort, aes(x = income, y = mortality, color = oil))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point()+
    labs(title='Income vs Mortality', x='income', y = 'mortality')+
    geom_text(aes(label=ifelse(mortality>250,as.character(row.names(infmort)),'')
grid.arrange(plot1, plot2, nrow = 1)
```

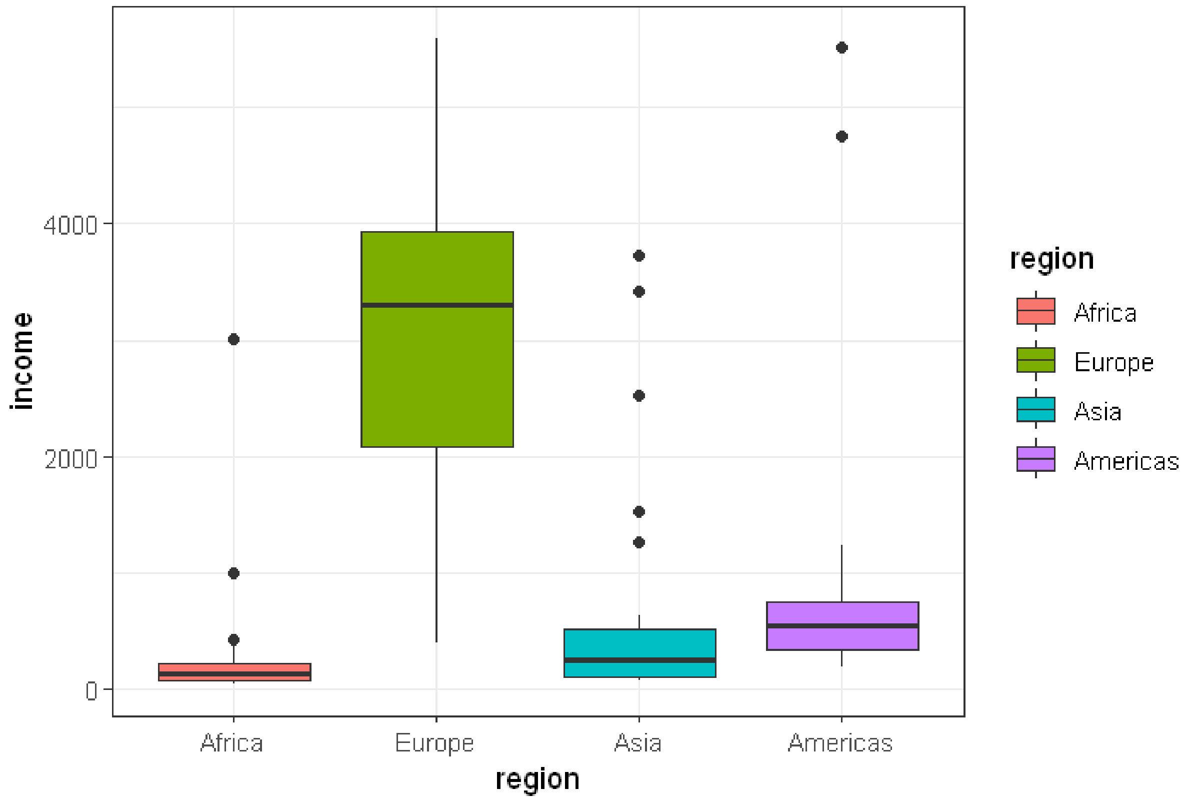




In [610]:
```
options(repr.plot.width=6, repr.plot.height=4)
ggplot(infmort, aes(x = region, y = mortality, fill=region)) +
  geom_boxplot()
```



In [611]:
```
ggplot(infmort, aes(x = region, y = income, fill=region)) +
  geom_boxplot()
```



- Variables Region and Oil are categorical variables
- Variables Income and Mortality are numerical variables
- We can see, as the income increases, mortality decreases. There are however two datapoints which come from oil producing countries that looks like outliers.
- Africa is having highest mortality and lowest income.
- Europe is having lowest mortality and highest income.

(b) Fit a model for the infant mortality in terms of the other variables using the main effects and up to second order interactions. Comment on your results.

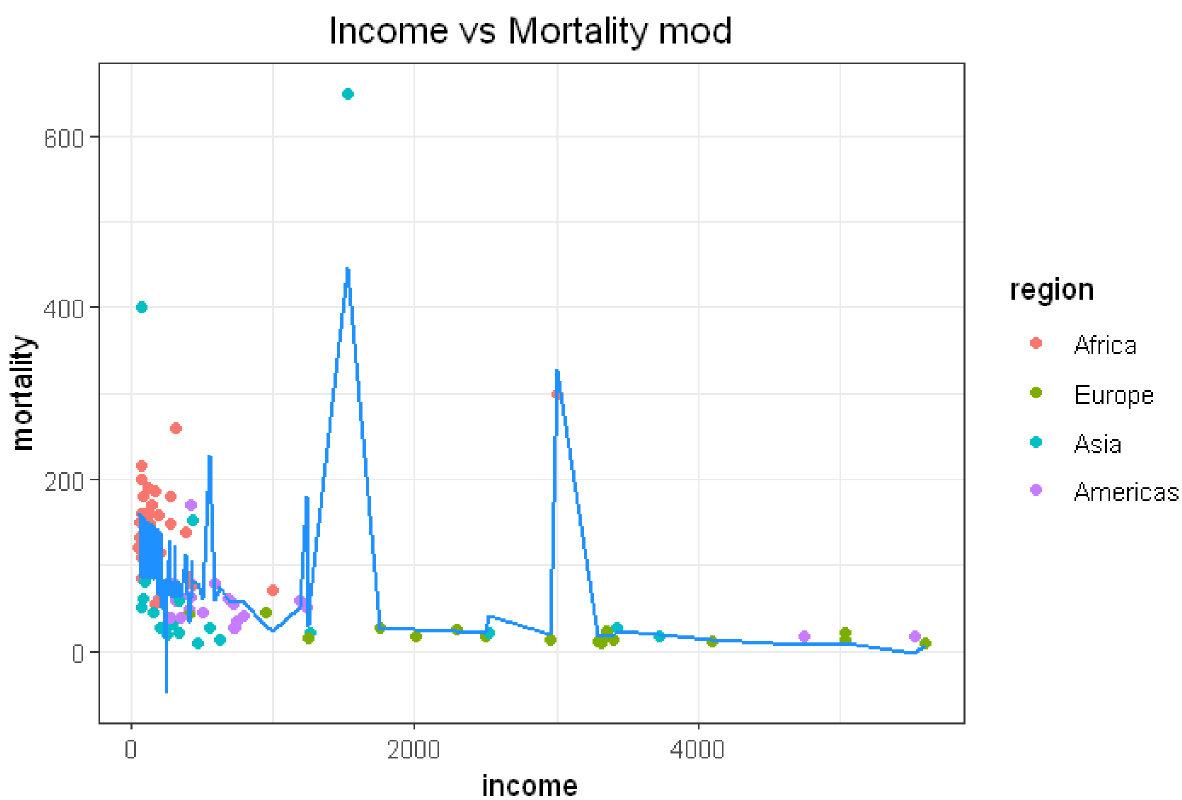
In [612]:
```
names(infmort)
```

'region' 'income' 'mortality' 'oil'

In [613]:
```
mod = lm(mortality ~ income + region + oil + income:region + income:oil + region:
```

In [614]:
```
pred = predict(mod, infmort)
plot1 <- ggplot(data = infmort, aes(x = income, y = mortality, color = region))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point()+
    labs(title='Income vs Mortality mod', x='income', y = 'mortality')+
    geom_line(aes(x = income, y = pred), color = 'dodgerblue', size = 0.75)
plot1
```

```
Warning message in predict.lm(mod, infmort):
"prediction from a rank-deficient fit may be misleading"
```



In [615]:
```
summary(mod)
```

```
Call:
lm(formula = mortality ~ income + region + oil + income:region + 
    income:oil + region:oil, data = infmort)

Residuals:
     Min       1Q   Median       3Q      Max 
-200.606  -23.858   -2.578   15.676  314.797 

Coefficients: (1 not defined because of singularities)
                                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)                        27.88469   48.70131   0.573 0.568383    
income                              0.09935    0.02687   3.697 0.000376 ***
regionEurope                     -133.35340   37.99857  -3.509 0.000707 ***
regionAsia                         74.83990   64.29758   1.164 0.247550    
regionAmericas                   -134.64863   69.44359  -1.939 0.055674 .  
oilno oil exports                 140.12320   48.65760   2.880 0.004984 ** 
income:regionEurope                 0.13887    0.04451   3.120 0.002441 ** 
income:regionAsia                   0.12561    0.04368   2.876 0.005041 ** 
income:regionAmericas               0.13134    0.04397   2.987 0.003641 ** 
income:oilno oil exports           -0.24328    0.04140  -5.876 7.17e-08 ***
regionEurope:oilno oil exports           NA         NA      NA       NA    
regionAsia:oilno oil exports     -156.27099   62.28212  -2.509 0.013915 *  
regionAmericas:oilno oil exports   33.37454   67.81541   0.492 0.623834    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 62.8 on 89 degrees of freedom
Multiple R-squared:  0.5742,	Adjusted R-squared:  0.5216 
F-statistic: 10.91 on 11 and 89 DF,  p-value: 1.825e-12
```

In [618]:
```
round(anova(mod),4)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
income	1	90086.162	90086.162	22.8401	0.0000
region	3	121982.055	40660.685	10.3090	0.0000
oil	1	43936.012	43936.012	11.1394	0.0012
income:region	3	5338.328	1779.443	0.4512	0.7171
income:oil	1	165627.542	165627.542	41.9926	0.0000
region:oil	2	46490.523	23245.261	5.8935	0.0039
Residuals	89	351034.371	3944.206	NA	NA

- We can see income:region is not significant at 5% significance level. Lets drop it.

In [621]:
```
mod = lm(mortality ~ income + region + oil + income:oil + region:oil, data = infm
summary(mod)
round(anova(mod),4)
```

```
Call:
lm(formula = mortality ~ income + region + oil + income:oil +
    region:oil, data = infmort)

Residuals:
    Min      1Q  Median      3Q     Max
-217.07  -27.38   -4.37   18.65  318.04

Coefficients: (1 not defined because of singularities)
                                 Estimate Std. Error t value Pr(>|t|)
(Intercept)                      -9.18489   48.75605  -0.188  0.85099
income                            0.12999    0.02568   5.061 2.12e-06 ***
regionEurope                    -86.34203   26.74772  -3.228  0.00173 **
regionAsia                      181.56071   54.52280   3.330  0.00125 **
regionAmericas                  -22.55581   60.59049  -0.372  0.71055
oilno oil exports               153.22243   50.15122   3.055  0.00294 **
income:oilno oil exports         -0.14263    0.02649  -5.385 5.53e-07 ***
regionEurope:oilno oil exports         NA         NA      NA       NA
regionAsia:oilno oil exports   -242.69014   57.40058  -4.228 5.55e-05 ***
regionAmericas:oilno oil exports -54.69587   63.62170  -0.860  0.39219
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 65.07 on 92 degrees of freedom
Multiple R-squared:  0.5275,    Adjusted R-squared:  0.4864
F-statistic: 12.84 on 8 and 92 DF,  p-value: 3.011e-12
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
income	1	90086.16	90086.162	21.2753	0.0000
region	3	121982.05	40660.685	9.6027	0.0000
oil	1	43936.01	43936.012	10.3762	0.0018
income:oil	1	96435.06	96435.058	22.7747	0.0000
region:oil	2	82499.89	41249.947	9.7419	0.0001
Residuals	92	389555.81	4234.302	NA	NA

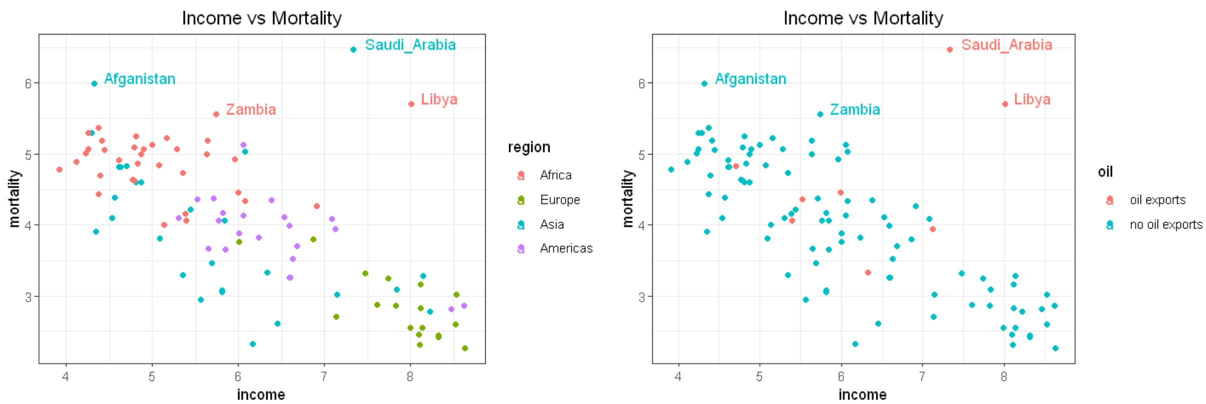
- We can see, not much of the variation in mortality is explained by the model. It is having R^2 value of 0.5275 with large residual standard error of 65.07.

(c) Check for unusual observations and potential transformations. Re-fit your model if necessary.

We will apply log transformation to income as well as mortality

In [295]:

```
options(repr.plot.width=12, repr.plot.height=4)

plot1 <- ggplot(data = infmort, aes(x = log(income), y = log(mortality), color =
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point()+
    labs(title='Income vs Mortality', x='income', y = 'mortality')+
    geom_text(aes(label=ifelse(mortality>250,as.character(row.names(infmort)),'')

plot2 <- ggplot(data = infmort, aes(x = log(income), y = log(mortality), color =
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point()+
    labs(title='Income vs Mortality', x='income', y = 'mortality')+
    geom_text(aes(label=ifelse(mortality>250,as.character(row.names(infmort)),'')
grid.arrange(plot1, plot2, nrow = 1)
```

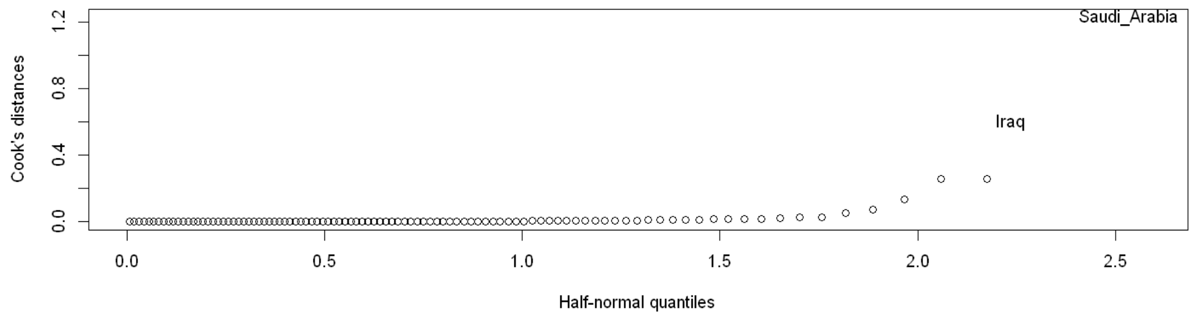


In [623]:
```
mod_b = lm(log(mortality) ~ log(income) + region + oil + log(income):region + log
summary(mod_b)
round(anova(mod_b),4)
```

```
Call:
lm(formula = log(mortality) ~ log(income) + region + oil + log(income):region +
    log(income):oil + region:oil, data = infmort)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.60045 -0.28092 -0.00656  0.29879  1.45482 

Coefficients: (1 not defined because of singularities)
                                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)                        0.6752     1.3167   0.513 0.609348    
log(income)                        0.6288     0.1982   3.173 0.002071 ** 
regionEurope                       1.3934     1.6641   0.837 0.404648    
regionAsia                         2.5569     1.0756   2.377 0.019586 *  
regionAmericas                     1.5482     1.2925   1.198 0.234146    
oilno oil exports                  4.9580     1.2508   3.964 0.000149 ***
log(income):regionEurope          -0.3825     0.2352  -1.627 0.107364    
log(income):regionAsia            -0.3596     0.1563  -2.301 0.023743 *  
log(income):regionAmericas        -0.3234     0.1883  -1.718 0.089325 .  
log(income):oilno oil exports     -0.7780     0.1948  -3.994 0.000133 ***
regionEurope:oilno oil exports         NA         NA      NA       NA    
regionAsia:oilno oil exports      -1.4571     0.4939  -2.950 0.004054 ** 
regionAmericas:oilno oil exports  -0.2917     0.5406  -0.540 0.590789    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5258 on 89 degrees of freedom
Multiple R-squared:  0.7376,    Adjusted R-squared:  0.7051 
F-statistic: 22.74 on 11 and 89 DF,  p-value: < 2.2e-16
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(income)	1	47.0835	47.0835	170.2772	0.0000
region	3	10.7053	3.5684	12.9052	0.0000
oil	1	2.8243	2.8243	10.2142	0.0019
log(income):region	3	1.7052	0.5684	2.0556	0.1118
log(income):oil	1	4.0678	4.0678	14.7110	0.0002
region:oil	2	2.7732	1.3866	5.0147	0.0086
Residuals	89	24.6094	0.2765	NA	NA

In [624]:
```
options(repr.plot.width=12, repr.plot.height=4)
cook = cooks.distance(mod_b)
halfnorm(cook, labs=row.names(infmort), ylab="Cook's distances")
```



- Although there not significant outliers, values of cook's distance for Iraq and Saudi_Arabia are significantly higher than rest of the other datapoints. So, lets remove them and see what effect it will have on our model performance.

In [242]:
```
new_infmort = infmort
# new_infmort = new_infmort[row.names(new_infmort) != 'Libya                    ',]
new_infmort = new_infmort[row.names(new_infmort) != 'Iraq                     ',]
new_infmort = new_infmort[row.names(new_infmort) != 'Saudi_Arabia             ',]
```

In [243]:
```
row.names(new_infmort)
```

```
'Australia      ' 'Austria        ' 'Belgium        ' 'Canada         ' 'Denmark        '
'Finland        ' 'France         ' 'West_Germany   ' 'Ireland        ' 'Italy          '
'Japan          ' 'Netherlands    ' 'New_Zealand    ' 'Norway         ' 'Portugal       '
'South_Africa   ' 'Sweden         ' 'Switzerland    ' 'Britain        ' 'United_States  '
'Algeria        ' 'Ecuador        ' 'Indonesia      ' 'Libya          ' 'Nigeria        '
'Venezuela      ' 'Argentina      ' 'Brazil         ' 'Chile          ' 'Colombia       '
'Costa_Rica     ' 'Dominican_Republic ' 'Greece         ' 'Guatemala      '
'Israel         ' 'Jamaica        ' 'Lebanon        ' 'Malaysia       ' 'Mexico         '
'Nicaragua      ' 'Panama         ' 'Peru           ' 'Singapore      ' 'Spain          '
'Taiwan         ' 'Trinidad_and_Tobago' 'Tunisia        ' 'Uruguay        ' 'Yugoslavia     '
'Zambia         ' 'Bolivia        ' 'Cameroon       ' 'Congo          ' 'Egypt          '
'El_Salvador    ' 'Ghana          ' 'Honduras       ' 'Ivory_Coast    ' 'Jordan         '
'South_Korea    ' 'Liberia        ' 'Moroco         ' 'Papua_New_Guinea '
'Paraguay       ' 'Philippines    ' 'Syria          ' 'Thailand       ' 'Turkey         '
'South_Vietnam  ' 'Afganistan     ' 'Bangladesh     ' 'Burma          ' 'Burundi        '
'Cambodia       ' 'Central_African_Rep' 'Chad           ' 'Dahomey        '
'Ethiopia       ' 'Guinea         ' 'India          ' 'Kenya          ' 'Madagascar     '
'Malawi         ' 'Mali           ' 'Mauritania     ' 'Niger          ' 'Pakistan       '
'Rwanda         ' 'Sierra_Leone   ' 'Somalia        ' 'Sri_Lanka      ' 'Sudan          '
'Tanzania       ' 'Togo           ' 'Uganda         ' 'Upper_Volta    ' 'Southern_Yemen '
'Yemen          ' 'Zaire          '
```

```
In [625]: mod_c = lm(log(mortality) ~ log(income) + region + oil + log(income)*region + log
          # mod_c = lm(log(mortality) ~ log(income)*region*oil, data = new_infmort)
          summary(mod_c)
          round(anova(mod_c),4)
```

```
Call:
lm(formula = log(mortality) ~ log(income) + region + oil + log(income) *
    region + log(income) * oil + region * oil, data = new_infmort)

Residuals:
     Min       1Q   Median       3Q      Max
-1.26138 -0.26401 -0.01274  0.29714  1.42471

Coefficients: (1 not defined because of singularities)
                                 Estimate Std. Error t value Pr(>|t|)
(Intercept)                        1.5832     1.4429   1.097  0.27557
log(income)                        0.4884     0.2190   2.230  0.02831 *
regionEurope                       1.5898     1.5336   1.037  0.30278
regionAsia                         2.9323     1.0202   2.874  0.00509 **
regionAmericas                     1.6810     1.1893   1.413  0.16108
oilno oil exports                  3.8537     1.4970   2.574  0.01174 *
log(income):regionEurope          -0.4222     0.2181  -1.936  0.05617 .
log(income):regionAsia            -0.4219     0.1531  -2.755  0.00714 **
log(income):regionAmericas        -0.3476     0.1737  -2.001  0.04851 *
log(income):oilno oil exports     -0.5979     0.2378  -2.515  0.01376 *
regionEurope:oilno oil exports         NA         NA      NA       NA
regionAsia:oilno oil exports      -1.5084     0.6926  -2.178  0.03212 *
regionAmericas:oilno oil exports  -0.3279     0.4961  -0.661  0.51040
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4816 on 87 degrees of freedom
Multiple R-squared:  0.7691,	Adjusted R-squared:  0.7399
F-statistic: 26.34 on 11 and 87 DF,  p-value: < 2.2e-16
```

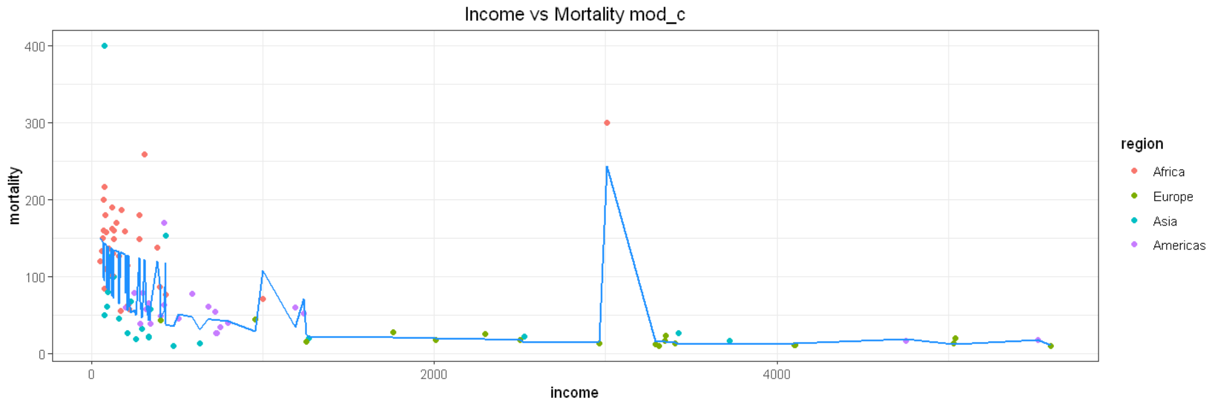
	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(income)	1	50.6662	50.6662	218.4763	0.0000
region	3	10.4303	3.4768	14.9920	0.0000
oil	1	0.8610	0.8610	3.7127	0.0573
log(income):region	3	3.4286	1.1429	4.9281	0.0033
log(income):oil	1	0.7023	0.7023	3.0285	0.0854
region:oil	2	1.1002	0.5501	2.3722	0.0993
Residuals	87	20.1759	0.2319	NA	NA

- After removing these two datapoints, residual standard error has reduced from 0.7376 to 0.7691 and R^2 value increased from 0.5258 to 0.4816.

In [298]:

```
pred = exp(predict(mod_c, new_infmort))
plot1 <- ggplot(data = new_infmort, aes(x = income, y = mortality, color = regior
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point()+
    labs(title='Income vs Mortality mod_c', x='income', y = 'mortality')+
    geom_line(aes(x = income, y = pred), color = 'dodgerblue', size = 0.75)
plot1
```

```
Warning message in predict.lm(mod_c, new_infmort):
"prediction from a rank-deficient fit may be misleading"
```



(d) Interpret your model by explaining what the regression parameter estimates mean.

- Interaction is created by product of any two variables.

$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 * x_2$

- β_0 is intecept
- β_1 is slope when x_2 is zero
- β_2 is the slope when x_1 is zero
- β_3 is the interaction term; change in slope of x_1 when x_2 changes by unity and vice versa
- In the similar way, we can interpret the β values estimated by our model for different interactions.
- The interactions region-Europe and oil-no oil exports, region-America and oil-no oil exports, and log(income) and region-Europe are not significant at 5% α level.
- The residual standard error is 0.4816. 76.91% of the variation in mortality is explained by the model.

Problem 3: Generate 100 observations from the model $y = \sin^3(2\pi x^3) + \varepsilon$, with $\varepsilon \sim N(0, (0.1)^2)$. (The model is described in section 9.5 of the textbook).

(a) Fit regression splines with 12 evenly-spaced knots using $y \sim bs(x, knots = \ldots)$. You need to load the splines package. Display the fit on top of the data.

(b) Compute the AIC for this model.

(c) Compute the Adjusted R^2

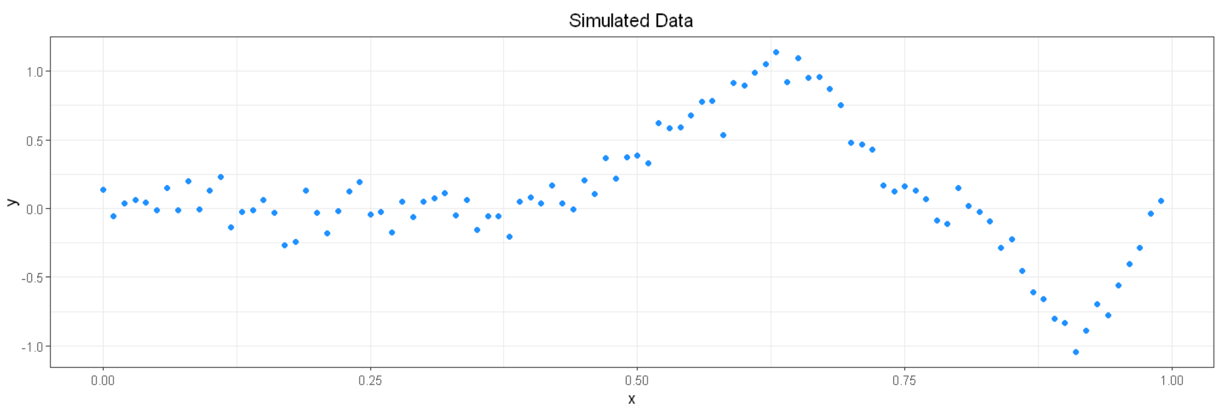
(d) Compute the AIC for all models with a number of knots between 3 and 20 inclusive. Plot the AIC as a function of the number of degrees of freedom. Which model is the best?

(e) Plot the fit for your selected model on top of the data.

Generate 100 observations

In [111]:
```
set.seed(42)
e = rnorm(n = 100, mean = 0, sd = 0.1)
x = seq(0, 0.99, by = 0.01)
y = sin(2*pi*x^3)^3 + e
data = data.frame(x, y)
```

In [112]:
```
options(repr.plot.width=12, repr.plot.height=4)

plot1 <- ggplot(data = data, aes(x = x, y = y))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point(color = 'dodgerblue')+
    labs(title='Simulated Data', x='x', y = 'y')
plot1
```



(a) Fit regression splines with 12 evenly-spaced knots using $y \sim bs(x, knots=...)$. You need to load the splines package. Display the fit on top of the data.

In [113]:
```
library(splines)
```

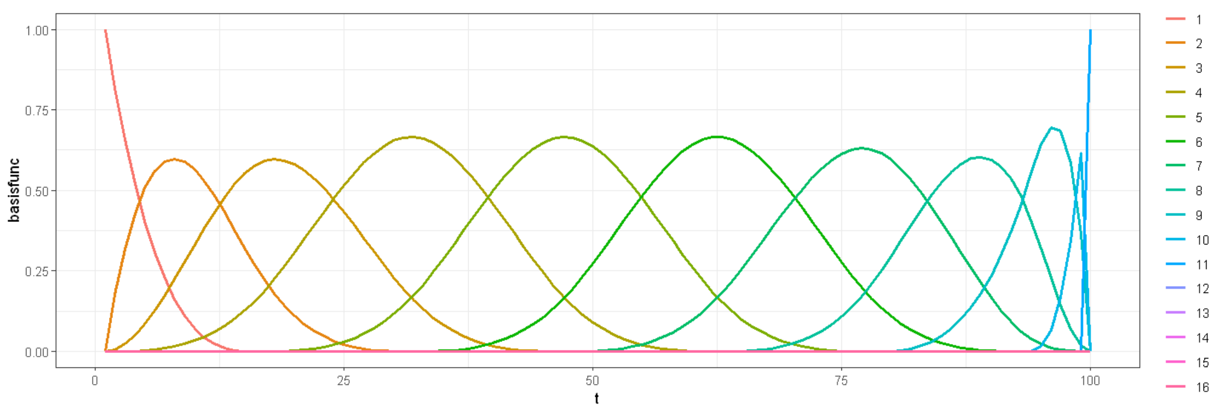
In [114]:
```
m=12;
myknots= 2*(1:m)/(m+1)
myknots
```

0.153846153846154   0.307692307692308   0.461538461538462   0.615384615384615
0.769230769230769   0.923076923076923   1.07692307692308   1.23076923076923
1.38461538461538   1.53846153846154   1.69230769230769   1.84615384615385

In [115]:
```
F = bs(x,knots=myknots, intercept=TRUE) #Using the Intercept option
dim(F)
```

100   16

In [116]:
```
mydf = m+4
n = length(x)
tmpdata = data.frame(t = rep(1:n, mydf),
                     basisfunc=as.vector(F),
                     type=as.factor(rep(1:mydf, each=n)))
ggplot(tmpdata, aes(x=t, y=basisfunc, color=type)) +
  geom_path(size = 1)
#   geom_point(data = data, aes(x = x, y = y),color = 'dodgerblue')
```



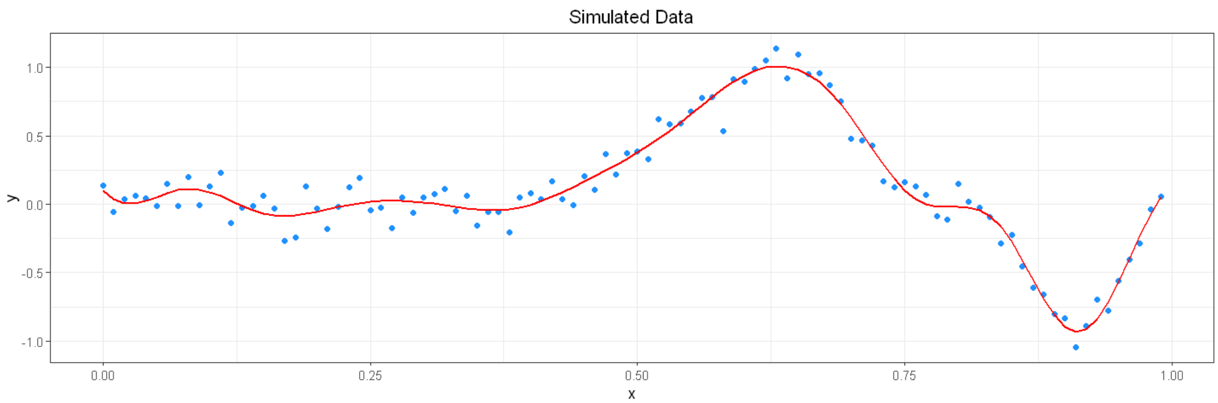
In [117]:
```
mod_using_knots = lm(y ~ bs(x, knots = myknots, intercept = TRUE))
summary(mod_using_knots)
```

```
Call:
lm(formula = y ~ bs(x, knots = myknots, intercept = TRUE))

Residuals:
     Min       1Q   Median       3Q      Max 
-0.33482 -0.06863 -0.00407  0.08639  0.26544 

Coefficients: (6 not defined because of singularities)
                                           Estimate Std. Error t value Pr(>|t|)
(Intercept)                                 0.05893    0.13222   0.446    0.657
bs(x, knots = myknots, intercept = TRUE)1  -0.04326    0.16158  -0.268    0.790
bs(x, knots = myknots, intercept = TRUE)2   0.10771    0.17240   0.625    0.534
bs(x, knots = myknots, intercept = TRUE)3  -0.20294    0.16962  -1.196    0.235
bs(x, knots = myknots, intercept = TRUE)4   0.01653    0.15682   0.105    0.916
bs(x, knots = myknots, intercept = TRUE)5  -0.19558    0.15424  -1.268    0.208
bs(x, knots = myknots, intercept = TRUE)6   1.39757    0.15554   8.985 4.03e-14
bs(x, knots = myknots, intercept = TRUE)7  -0.01946    0.16373  -0.119    0.906
bs(x, knots = myknots, intercept = TRUE)8  -0.80570    0.17536  -4.594 1.42e-05
bs(x, knots = myknots, intercept = TRUE)9  -0.90676    0.18300  -4.955 3.42e-06
bs(x, knots = myknots, intercept = TRUE)10  0.55441    0.26040   2.129    0.036
bs(x, knots = myknots, intercept = TRUE)11       NA         NA      NA       NA
bs(x, knots = myknots, intercept = TRUE)12       NA         NA      NA       NA
bs(x, knots = myknots, intercept = TRUE)13       NA         NA      NA       NA
bs(x, knots = myknots, intercept = TRUE)14       NA         NA      NA       NA
bs(x, knots = myknots, intercept = TRUE)15       NA         NA      NA       NA
bs(x, knots = myknots, intercept = TRUE)16       NA         NA      NA       NA

(Intercept)                                   
bs(x, knots = myknots, intercept = TRUE)1     
bs(x, knots = myknots, intercept = TRUE)2     
bs(x, knots = myknots, intercept = TRUE)3     
bs(x, knots = myknots, intercept = TRUE)4     
bs(x, knots = myknots, intercept = TRUE)5     
bs(x, knots = myknots, intercept = TRUE)6  ***
bs(x, knots = myknots, intercept = TRUE)7     
bs(x, knots = myknots, intercept = TRUE)8  ***
bs(x, knots = myknots, intercept = TRUE)9  ***
bs(x, knots = myknots, intercept = TRUE)10 *  
bs(x, knots = myknots, intercept = TRUE)11    
bs(x, knots = myknots, intercept = TRUE)12    
bs(x, knots = myknots, intercept = TRUE)13    
bs(x, knots = myknots, intercept = TRUE)14    
bs(x, knots = myknots, intercept = TRUE)15    
bs(x, knots = myknots, intercept = TRUE)16    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1322 on 89 degrees of freedom
Multiple R-squared:  0.9226,	Adjusted R-squared:  0.9139 
F-statistic: 106.1 on 10 and 89 DF,  p-value: < 2.2e-16
```

In [118]:
```
mod_a = lm(y ~ bs(x, df = 15, intercept = FALSE), data = data)
summary(mod_a)
```

```
Call:
lm(formula = y ~ bs(x, df = 15, intercept = FALSE), data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.30926 -0.06214  0.01633  0.05692  0.20160 

Coefficients:
                                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)                          0.09976    0.08783   1.136  0.25923    
bs(x, df = 15, intercept = FALSE)1  -0.21117    0.17296  -1.221  0.22553    
bs(x, df = 15, intercept = FALSE)2   0.19561    0.12645   1.547  0.12563    
bs(x, df = 15, intercept = FALSE)3  -0.28894    0.13450  -2.148  0.03457 *  
bs(x, df = 15, intercept = FALSE)4  -0.06883    0.11729  -0.587  0.55893    
bs(x, df = 15, intercept = FALSE)5  -0.06679    0.12295  -0.543  0.58840    
bs(x, df = 15, intercept = FALSE)6  -0.21186    0.11894  -1.781  0.07848 .  
bs(x, df = 15, intercept = FALSE)7   0.09149    0.12081   0.757  0.45100    
bs(x, df = 15, intercept = FALSE)8   0.40649    0.11976   3.394  0.00105 ** 
bs(x, df = 15, intercept = FALSE)9   0.99570    0.12042   8.269 1.76e-12 ***
bs(x, df = 15, intercept = FALSE)10  0.85697    0.12048   7.113 3.49e-10 ***
bs(x, df = 15, intercept = FALSE)11 -0.35700    0.12190  -2.929  0.00438 ** 
bs(x, df = 15, intercept = FALSE)12  0.14626    0.12632   1.158  0.25023    
bs(x, df = 15, intercept = FALSE)13 -1.67488    0.14393 -11.637  < 2e-16 ***
bs(x, df = 15, intercept = FALSE)14 -0.32860    0.14528  -2.262  0.02629 *  
bs(x, df = 15, intercept = FALSE)15 -0.04345    0.12421  -0.350  0.72734    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1014 on 84 degrees of freedom
Multiple R-squared:  0.957,	Adjusted R-squared:  0.9494 
F-statistic: 124.7 on 15 and 84 DF,  p-value: < 2.2e-16
```

```
In [119]: pred = predict(mod_a, data)
          plot1 <- ggplot(data = data, aes(x = x, y = y))+
              geom_point()+
              theme_update(plot.title = element_text(hjust = 0.5))+
              theme_set(theme_bw())+
              geom_point(color = 'dodgerblue')+
              labs(title='Simulated Data', x='x', y = 'y')+
              geom_line(aes(x = x, y = pred), color = 'red', size = 0.75)
          plot1
```



(b) Compute the AIC for this model.

- AIC/BIC

$$AIC : -2 \times loglik_\gamma + 2p_\gamma$$
$$BIC : -2 \times loglik_\gamma + \log(n)p_\gamma$$

where p_γ is the number of predictors included in model γ
For the linear regression model:

$$-2 \times loglik_\gamma = n \log \frac{RSS_\gamma}{n}$$

The lower the AIC/BIC the better. Note that when n is large, adding an additional predictor costs a lot more in BIC than AIC. So AIC tends to pick a bigger model than BIC.

p_γ is number of parameters

```
In [120]: AIC_mod_a = n*log(sum(mod_a$residuals**2)/n) + 2*16
          AIC_mod_a
```

-443.158229993163

(c) Compute the Adjusted $R2$

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1}$$

image source: https://en.wikipedia.org/wiki/Coefficient_of_determination (https://en.wikipedia.org/wiki/Coefficient_of_determination)

$\bar{R}^2$ is adjusted R^2

In [122]:
```
Adj_R_squared_for_model_with_df = 1 - ((1 - 0.957)*(100-1))/(100-15-1)
Adj_R_squared_for_model_with_df
```

0.949321428571429

In [129]:
```
Adj_R_squared_for_model_with_knots = 1 - ((1 - 0.9226)*(100-1))/(100-10-1)
Adj_R_squared_for_model_with_knots
```

0.913903370786517

(d) Compute the AIC for all models with a number of knots between 3 and 20 inclusive. Plot the AIC as a function of the number of degrees of freedom. Which model is the best?

- We need to use knots between 3 and 20. Hence degrees of freedom for the models without intercept = false, will be between 6 to 23.

In [130]:
```
DoFs = 6:23
# DoFs = 7:24
```

In [131]:
```
create_models = function(df){
    mod = lm(y ~ bs(x, df = df, intercept = FALSE), data = data)
    rss = sum(mod$residuals**2)
}
```

In [132]:
```
models_RSS = sapply(DoFs, create_models)
models_RSS
```

2.42252336150288 1.97140662667587 1.87710971960585 1.62166380836979 1.3026093689183 1.42139085977314 1.50104631710189 1.09512048176152 0.822432124785787 0.863801645119028 0.867236719929665 0.83481528199784 0.839077674403963 0.809722840684115 0.796934121570797 0.779250964512705 0.80608287436758 0.769918597800584

In [133]:
```
Calc_AIC_first_term = function(models_RSS){
    n*log(sum(models_RSS)/n)
}
```

In [134]:
```
AICs = sapply(models_RSS, Calc_AIC_first_term) + 2*(DoFs+1)
AICs
```

-358.036047770056 -376.64228743385 -379.543697532077 -392.171752158724 -412.080072640852 -401.353431503772 -393.900777631659 -423.430579976642 -450.065950879271 -443.158229993163 -440.761349208896 -442.57149837562 -440.062218304576 -441.623344785286 -441.215344760039 -441.459230857866 -436.073890595339 -438.664067285305

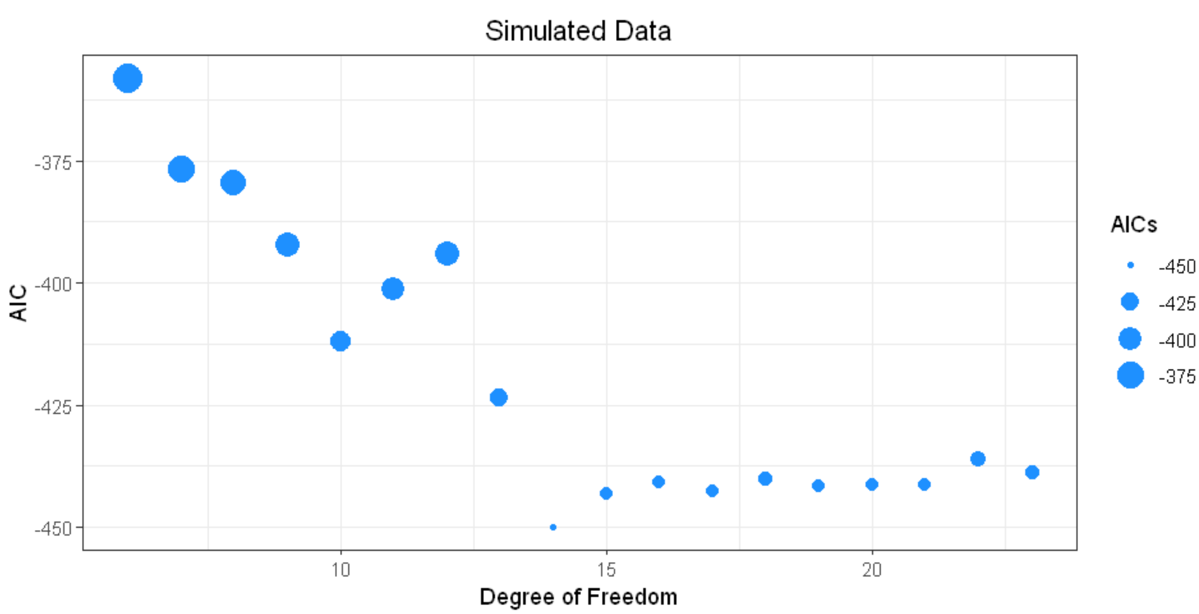
In [135]:
```
which.min(AICs)
```

9

In [136]:
```
DoFs[9]
```

14

In [137]:
```
options(repr.plot.width=8, repr.plot.height=4)

plot1 <- ggplot(data = data.frame(DoFs, AICs), aes(x = DoFs, y = AICs, size = AIC
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point(color = 'dodgerblue')+
    labs(title='Simulated Data', x='Degree of Freedom', y = 'AIC')
plot1
```



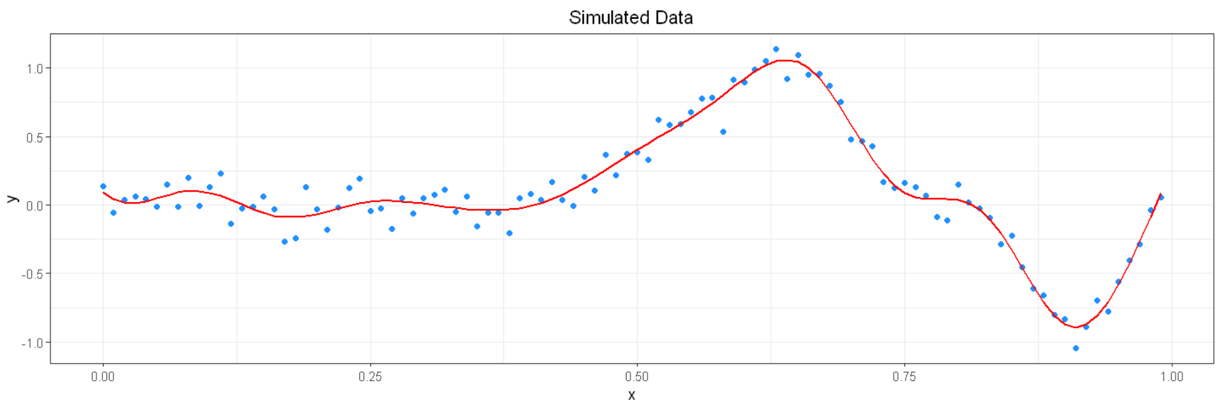
- Model with 14 Degrees of freedom (11 knots) has lowest AIC.

(e) Plot the fit for your selected model on top of the data.

In [138]:
```
final_model = lm(y ~ bs(x, df = 14, intercept = FALSE), data = data)
summary(final_model)
```

```
Call:
lm(formula = y ~ bs(x, df = 14, intercept = FALSE), data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.26586 -0.05829  0.01427  0.05113  0.21274 

Coefficients:
                                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)                          0.090284   0.083543   1.081 0.282893    
bs(x, df = 14, intercept = FALSE)1  -0.174700   0.163641  -1.068 0.288732    
bs(x, df = 14, intercept = FALSE)2   0.188217   0.118269   1.591 0.115226    
bs(x, df = 14, intercept = FALSE)3  -0.311307   0.126884  -2.453 0.016190 *  
bs(x, df = 14, intercept = FALSE)4   0.006656   0.110363   0.060 0.952049    
bs(x, df = 14, intercept = FALSE)5  -0.130496   0.115917  -1.126 0.263432    
bs(x, df = 14, intercept = FALSE)6  -0.151316   0.112041  -1.351 0.180425    
bs(x, df = 14, intercept = FALSE)7   0.308877   0.113888   2.712 0.008090 ** 
bs(x, df = 14, intercept = FALSE)8   0.643346   0.112934   5.697 1.71e-07 ***
bs(x, df = 14, intercept = FALSE)9   1.286963   0.113811  11.308  < 2e-16 ***
bs(x, df = 14, intercept = FALSE)10 -0.325317   0.114682  -2.837 0.005697 ** 
bs(x, df = 14, intercept = FALSE)11  0.238733   0.119036   2.006 0.048085 *  
bs(x, df = 14, intercept = FALSE)12 -1.547986   0.135259 -11.445  < 2e-16 ***
bs(x, df = 14, intercept = FALSE)13 -0.483215   0.136930  -3.529 0.000676 ***
bs(x, df = 14, intercept = FALSE)14 -0.003007   0.118141  -0.025 0.979750    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.09836 on 85 degrees of freedom
Multiple R-squared:  0.9591,	Adjusted R-squared:  0.9523 
F-statistic: 142.3 on 14 and 85 DF,  p-value: < 2.2e-16
```

In [139]:
```
options(repr.plot.width=12, repr.plot.height=4)

pred = predict(final_model, data)
plot1 <- ggplot(data = data, aes(x = x, y = y))+
    geom_point()+
    theme_update(plot.title = element_text(hjust = 0.5))+
    theme_set(theme_bw())+
    geom_point(color = 'dodgerblue')+
    labs(title='Simulated Data', x='x', y = 'y')+
    geom_line(aes(x = x, y = pred), color = 'red', size = 0.75)
plot1
```



In [140]:
```
AIC_final_model = n*log(sum(final_model$residuals**2)/n) + 2*15
AIC_final_model
```

-450.065950879271