**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer - B.

*μ* = 45

*σ* = 8

X̅ = 50

z = (50-45)/8

= 0.625

P (commitment) = 0.7324 (Z value from table)

P (not commit) = 1 – 0.7324

= 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Answer- a. False

z-score for 44 = (44 - 38)/6 = 1(z value from table is 0.84134) => 84.13 %

People above 44 ages = 100 - 84.13 = 15.87% ≈ 63    out of 400

z-score for 38 = (38 - 38)/6 = 0 (z value from table is 0.50000)=> 50%

People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

b. z-score (Employee=30) = (30-38)/6 = -1.333(z value from table is 0.09176)

P(Employee=30) = 0.09176

No. of Employees attending training Program =

=400\*0.09176 = 36.706

Said statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer - if X ∼ N (µ1, σ1^2), and Y ∼ N (µ2, σ2^2) the

X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2) and X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear

Combination of X and Y, then

Z = a(µ1, σ1^2) +b(µ2, σ2^2)

Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2X1~ N (2 u,4 σ^2) and

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer -**D. (48.5, 151.5)**

z = (X̅ -μ)/σ **where** μ = 100, σ = 20

X̅ = z\*σ+μ

P (outside area) = 1-0.99

P (towards left from a) = 0.01/2 = -0.005

P (towards left from b) = 0.01/2 = 0.005

z (0.005) = -2.57 (using z-score table)

X̅ (point a) = (-2.57\*(20) +100 = 48.6

X̅ (point b) = -(-2.57\*(20) +100 = 151.4

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer-**

z = (X̅ -μ)/σ

X̅1 = 5

X̅2 = 7

Mean Profit = (5+7) \*45 = 540

SD = (√32+√42) \* 45 = 225

1. **Range –**

z95 **=** (X̅ ± μ)/σ

X̅ = μ ± (z95\* σ)

= 540 ± (1.96\*225)

**= (99,981)**

1. **5th percentile of profit –**

X̅ = μ + (z5\* σ)

z5 = -1.64

= 540 + (-1.64\*225)

**= 169.875**

1. **larger probability of making a loss –**

**Division 1 –**