

Project 1

Consider a porous catalytic 2D rod ($\rho = 1500 \text{ kg/m}^3$ and $C_p = 1000 \text{ J/kg.K}$) with 5 cm length. The rod is initially at 400 K and suddenly the outside fluid changes to 500 K. The heat transfer coefficient is 50 W/m²K on the left side and 5 W/m²K in the right side. The heat source is given by the following empirical equation

$S = a - b(T - 400)^2$, with $a = 100 \text{ kW/m}^3$ and $b = 0.002 \text{ kW/m}^3\text{K}^2$ $400\text{K} < T < 600\text{K}$, and $S = 0$, for $T > 600$ and $S = 1000 \text{ kW/m}^3$ for $T < 400 \text{ K}$

The thermal conductivity is given by

$k = k_0 + k_1(T - 400\text{K})$, where $k_0 = 2.0 \text{ W/mK}$ and $k_1 = 0.002 \text{ W/mK}^2$.

- 1) Assuming steady state, determine the temperature profile for 6, control volumes. Write all the canonical equations using scientific notation with 3 significant digits and solve this system using the tri-diagonal matrix algorithm.
- 2) Repeat 1) for 12 and 24 nodes and use the Richardson extrapolation to compute the exact value of the temperature in the center.
- 3) Simulate the system in transient conditions (starting from a uniform temperature of 400 K) and determine the time-scale (in seconds) to reach the steady-state, using 12 nodes. Use the implicit Euler scheme and find, by trial-and-error, the appropriate time-step of the simulation.
- 4) Estimate the time required to reach steady-state.
- 5) Prepare a concise report (pdf format) describing the computations and results. Additional files with data analysis should be submitted together with the report.

The report should include the following Table.

Simulation Summary

Matrix of the linear system of equations for 6 nodes.
Relative error of the temperature in the center for 6 nodes
Recommended time-step for unsteady state simulation (in seconds).
Time-scale (in seconds) to reach the steady-state.
