

# **Accurate implementation of DeepMD-v2 Potential calculation in HALMD for single species, extension to multi-species and Automatic-Differentiation-Based Force Computation**

by

**Sandip Kumar Sah**

Master Thesis in Computational Science

Submission: 09 January 2026

Supervisor: Prof. Dr. Felix Höfling

## **Statutory Declaration**

|                               |                   |
|-------------------------------|-------------------|
| Family Name, Given/First Name | Sah, Sandip Kumar |
| Matriculation number          | 5589263           |
| Kind of thesis submitted      | Master Thesis     |

### **English: Declaration of Authorship**

I hereby declare that the thesis submitted was created and written solely by myself without any external support. Any sources, direct or indirect, are marked as such. I am aware of the fact that the contents of the thesis in digital form may be revised with regard to usage of unauthorized aid as well as whether the whole or parts of it may be identified as plagiarism. I do agree my work to be entered into a database for it to be compared with existing sources, where it will remain in order to enable further comparisons with future theses. This does not grant any rights of reproduction and usage, however.

This document was neither presented to any other examination board nor has it been published.

### **German: Erklärung der Autorenschaft (Urheberschaft)**

Ich erkläre hiermit, dass die vorliegende Arbeit ohne fremde Hilfe ausschließlich von mir erstellt und geschrieben worden ist. Jedwede verwendeten Quellen, direkter oder indirekter Art, sind als solche kenntlich gemacht worden. Mir ist die Tatsache bewusst, dass der Inhalt der Thesis in digitaler Form geprüft werden kann im Hinblick darauf, ob es sich ganz oder in Teilen um ein Plagiat handelt. Ich bin damit einverstanden, dass meine Arbeit in einer Datenbank eingegeben werden kann, um mit bereits bestehenden Quellen verglichen zu werden und dort auch verbleibt, um mit zukünftigen Arbeiten verglichen werden zu können. Dies berechtigt jedoch nicht zur Verwendung oder Vervielfältigung.

Diese Arbeit wurde noch keiner anderen Prüfungsbehörde vorgelegt noch wurde sie bisher veröffentlicht.

.....  
Date, Signature

## **Abstract**

# Contents

|          |  |          |
|----------|--|----------|
| <b>1</b> | <b>Introduction</b>  | <b>1</b> |
| 1.1      | Motivation . . . . .   | 1        |
| 1.2      | Objectives and Scope . . . . .                                   | 2        |
| <b>2</b> | <b>Background</b>  | <b>3</b> |
| 2.1      | HALMD Software . . . . .   | 3        |
| 2.2      | Neural Network Potentials . . . . .                              | 4        |
| 2.3      | Deep Potential Molecular Dynamics (DeepMD) . . . . .             | 5        |
| <b>3</b> | <b>Methodology</b>   | <b>6</b> |
| 3.1      | Overview of Implementation . . . . .                             | 6        |
| 3.2      | Model Parameter Extraction . . . . .                             | 6        |
| 3.2.1    | Frozen Model Structure . . . . .                                 | 6        |
| 3.2.2    | Extraction Procedure . . . . .                                   | 8        |
| 3.3      | Coordinate System Extension . . . . .                            | 10       |
| 3.3.1    | Explicit coordinate wrapping . . . . .                           | 11       |
| 3.3.2    | Ghost-cell periodic extension . . . . .                          | 11       |
| 3.3.3    | Environment-matrix construction interface . . . . .              | 12       |
| 3.3.4    | Normalization using $t_{\text{avg}}, t_{\text{std}}$ . . . . .   | 12       |
| 3.3.5    | Summary of improvements . . . . .                                | 12       |
| 3.3.6    | Species-aware neighbour grouping . . . . .                       | 13       |
| 3.3.7    | Applying the <code>sel</code> constraint . . . . .               | 13       |
| 3.4      | Construction of the Geometric Matrix $R$ . . . . .               | 14       |
| 3.4.1    | Relative displacement and inverse-distance quantities . . . . .  | 15       |
| 3.4.2    | Raw geometric row . . . . .                                      | 15       |
| 3.4.3    | Cutoff behaviour and switching function . . . . .                | 15       |
| 3.4.4    | Species-aware neighbour ordering . . . . .                       | 15       |
| 3.4.5    | Normalization of geometric features . . . . .                    | 16       |
| 3.4.6    | Summary . . . . .  | 16       |
| 3.5      | Construction of the Embedding Matrix $G$ . . . . .               | 16       |
| 3.5.1    | Input and Output Dimensionality . . . . .                        | 16       |
| 3.5.2    | Structure of the Filter Network . . . . .                        | 17       |
| 3.5.3    | SpeciesPair-Dependent Embedding Networks . . . . .               | 17       |
| 3.5.4    | Runtime Network Selection and Evaluation . . . . .               | 17       |
| 3.5.5    | Summary . . . . .  | 18       |
| 3.6      | Descriptor Computation . . . . .                                 | 18       |
| 3.6.1    | Normalized geometric matrix . . . . .                            | 18       |
| 3.6.2    | Embedding matrix . . . . .                                       | 19       |
| 3.6.3    | Quadratic descriptor construction . . . . .                      | 19       |
| 3.6.4    | Summary . . . . .  | 20       |
| 3.7      | Potential Energy Calculation . . . . .                           | 20       |
| 3.7.1    | Baseline Implementation Prior to This Work . . . . .             | 20       |
| 3.7.2    | Species-Dependent Fitting Networks . . . . .                     | 21       |
| 3.7.3    | Per-Species Energy Offsets . . . . .                             | 21       |
| 3.7.4    | Integration with the Multi-Species Descriptor Pipeline . . . . . | 21       |
| 3.7.5    | Reproduction of DeepMD-v2 Energies . . . . .                     | 22       |
| 3.7.6    | Final Formulation . . . . .                                      | 22       |

|   |           |
|---|-----------|
| <b>4 Force Computation</b>  | <b>22</b> |
| 4.1 Automatic Differentiation (AD) . . . . .                                      | 23        |
| 4.1.1 Forward-mode AD (current HALMD implementation) . . . . .                    | 23        |
| 4.1.2 Reverse-mode AD (not yet implemented) . . . . .                             | 23        |
| 4.1.3 Hybrid analytic–AD differentiation . . . . .                                | 24        |
| 4.2 Derivatives of the Geometry Matrix $R$ . . . . .                              | 24        |
| 4.2.1 Step 1: Rewrite the Components of $R$ . . . . .                             | 25        |
| 4.2.2 Step 2: Derivative of the Distance $r_{ij}$ . . . . .                       | 25        |
| 4.2.3 Step 3: Derivative of the Switching Function . . . . .                      | 25        |
| 4.2.4 Step 4: Derivative of the Switched Inverse Distance . . . . .               | 26        |
| 4.2.5 Step 5: Derivative of the Directional Components . . . . .                  | 26        |
| 4.2.6 Step 6: Normalized Geometry Derivative . . . . .                            | 27        |
| 4.2.7 Step 7: Derivative with Respect to the Neighbour Coordinate $r_j$ . . . . . | 27        |
| 4.3 Descriptor Derivative . . . . .   | 27        |
| 4.4 Fitting Network Derivative . . . . .  | 29        |
| 4.4.1 Forward-mode automatic differentiation of the fitting network . . . . .     | 29        |
| 4.4.2 Relation to reverse-mode AD and future improvements . . . . .               | 30        |
| 4.5 Final Force Assembly . . . . .  | 30        |
| 4.5.1 Contribution from the fitting network . . . . .                             | 30        |
| 4.5.2 Contribution from the descriptor . . . . .                                  | 31        |
| 4.5.3 Contribution from geometric derivatives . . . . .                           | 31        |
| 4.5.4 Final force expression . . . . .  | 31        |
| <b>5 Results</b>  | <b>32</b> |
| 5.1 Energy Agreement Between DeepMD and HALMD . . . . .                           | 32        |
| 5.2 Force Comparison and Remaining Discrepancy . . . . .                          | 33        |
| 5.3 GPU Profiling and Performance Analysis . . . . .                              | 33        |
| <b>6 Discussion</b>   | <b>35</b> |
| <b>7 Conclusions and Future Work</b>  | <b>35</b> |

# 1 Introduction

## 1.1 Motivation

Molecular dynamics (MD) simulations play a central role in materials science by providing atomistic insight into the behavior of complex systems. Their predictive power depends on the interatomic potential used to approximate the underlying potential energy surface (PES). Classical empirical potentials are efficient but often too rigid to describe complex bonding environments, whereas *ab initio* methods offer high accuracy at prohibitive computational cost. Machine-learned interatomic potentials—particularly the Deep Potential Molecular Dynamics (DeepMD) framework—bridge this gap by achieving near-quantum mechanical accuracy with classical-MD performance.

The work by Andrés Cruz, titled “*Deep Neural Networks Potentials for Scalable Molecular Dynamics Simulations on Accelerator Hardware*” [1], represents an important step toward integrating Deep Potential models into the high-performance GPU-accelerated simulation engine HALMD. His work reconstructs the DeePMD-kit inference pipeline inside HALMD, extracts network weights from a trained TensorFlow model, and validates energy and force predictions for a *single-species Copper system*. In particular, his implementation focuses on the *two-body embedding smooth edition*, *DeepPot-SE descriptor* and reproduces the filter and fitting networks *only for a monoatomic system*, making it suitable for single metal.

However, the implementation in [1] remains limited to the simplest case of DeepMD-v2 architectures. It does not include multi-species support.

Additionally, several intermediate steps present in the full DeepMD-v2 computational graph—such as normalization layers, ghost body extension for periodicity in boundary condition, species-wise descriptor partitioning, and certain derivative pathways—were simplified or omitted in the previous implementation. Cruz’s work successfully established a working single-species DeepMD integration within HALMD.

This work builds directly on the work of Cruz [1] and advances it substantially. The primary contributions of the present work are:

1. Generalizing the HALMD Deep Potential integration to *multi-species, multi-body DeepMD-v2 models*, enabling simulations of binary and multicomponent alloy systems.
2. Reconstructing descriptor and fitting networks for the *full multi-species DeepPot-SE architecture*, including multi-body descriptor terms.
3. Improving the *accuracy* of the HALMD implementation by adding several computational steps missing in the previous work, such as proper cutoff normalization, multi-species descriptor algebra and so on.

Through these extensions, the present thesis transforms HALMD from supporting a prototype single-component DeepMD potential into a *high-accuracy, multi-species DeepMD-v2 engine*. This significantly broadens HALMD’s applicability, enabling large-scale molecular dynamics simulations of technologically relevant multicomponent materials at near-quantum mechanical accuracy.

## 1.2 Objectives and Scope

The objective of this thesis is to extend and generalize the Deep Potential (DeepPot) implementation in HALMD beyond the single-species, two-body descriptor developed by Cruz [1]. While Cruz's work successfully demonstrated that HALMD can evaluate a DeePMD-v2 model for a monoatomic copper system, several components required for full multi-species DeepMD-v2 inference were missing. Most notably, the previous implementation did not include (i) the periodic coordinate extension and neighbor-list construction used in DeepMD [2], (ii) normalization and scaling layers defined in the DP-v2 framework [3], (iii) species-dependent descriptors, or (iv) descriptor and filter weights that depend simultaneously on the central and neighbor species, as introduced in the multi-species DeepPot-SE descriptor [3].

This thesis addresses these limitations by implementing the complete multi-species DeepMD-v2 inference pipeline, including accurate periodic handling of atomic environments. Specifically, the thesis pursues the following objectives:

1. **Implement coordinate normalization, ghost-cell extension, and multi-type neighbor list construction.** The previous implementation did not include the canonical DeePMD preprocessing steps for periodic systems—wrapping coordinates into the primary simulation cell, generating ghost atoms to cover the cutoff radius, and constructing species-grouped neighbor lists—as described in the DeePMD methodology [2, 3]. Implementing these steps ensures that HALMD reproduces the correct local environments required by the DeepPot-SE descriptors under periodic boundary conditions.
2. **Generalize the descriptor pipeline to multi-species systems.** This includes implementing species-dependent neighbor counts  $N_c(a)$ , species-aware embedding matrices  $G_i$ , and the full multi-species DeepPot-SE (se\_e2\_a) descriptor introduced in DP-v2 [3].
3. **Implement species-dependent filter networks.** Extend the single-species embedding and filter networks used in [1] to support arbitrary numbers of atomic types, following the species-indexed filter network formulation defined in the DP-v2 architecture [3].
4. **Reproduce the full DeepMD-v2 inference procedure with higher accuracy.** Incorporate several computational steps omitted in previous work, including normalization layers, descriptor scaling operations, smooth cutoff functions, and full force backpropagation through descriptor derivatives, consistent with the DeepMD-v2 formulation [3, 2].
5. **Validate energy and force predictions for multi-component systems.** Compare results from HALMD against the DeePMD-kit reference implementation for multi-species models and quantify numerical deviations, following standard validation procedures established in DeepMD literature [2, 3].

**Scope.** This thesis focuses exclusively on the inference stage of DeepMD-v2, i.e., the computation of energies and forces from pre-trained models. Model training is outside the scope of this work. The implementation targets the multi-species DeepPot-SE descriptor and does not cover other descriptor families such as end-to-end or message-passing potentials. The work provides support for multi-species atomic systems relevant to alloys and chemically complex materials, while the underlying molecular dynamics algorithms

(integrators, thermostats, barostats) rely on HALMD’s existing infrastructure.

## 2 Background

### 2.1 HALMD Software

The High-Accuracy Large-scale Molecular Dynamics (HALMD) package is a high-performance, open-source molecular dynamics (MD) simulation framework designed to study the microscopic dynamics of liquids, glasses, and other condensed matter systems. HALMD is built around a modular C++ architecture with a strong emphasis on numerical precision, extensibility, and efficient parallel computation on graphics processing units (GPUs) [4]. It provides a versatile platform for conducting large-scale classical molecular dynamics simulations involving millions of particles.

HALMD employs the *classical molecular dynamics* approach, in which atomic motion is governed by Newton’s equations of motion,

$$m_i \frac{d^2 \mathbf{r}_i}{dt^2} = -\nabla_i U(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N),$$

where  $U$  is a predefined analytical potential describing interatomic interactions. The accuracy of a classical MD simulation depends on the accuracy of this potential. HALMD supports several built-in empirical pair potentials, such as Lennard–Jones, Gaussian Core, and Yukawa models, and additionally allows users to supply tabulated pair potentials. These empirical models do not involve quantum-mechanical calculations and thus differ from *ab initio* molecular dynamics (AIMD), where forces are computed directly from electronic structure methods.

One of HALMD’s defining strengths is its high-performance GPU backend. The software employs NVIDIA’s CUDA to accelerate computationally intensive tasks such as force evaluation, neighbor-list construction, and time integration [4], and more recent versions extend support to heterogeneous computing platforms through SYCL [5]. Spatial domain decomposition and cell binning enable efficient scaling on multi-GPU systems, achieving orders-of-magnitude speedups compared to CPU-only implementations. HALMD also supports mixed-precision arithmetic, selectively applying double precision to critical operations to ensure long-term numerical stability without compromising performance.

HALMD follows a modular design philosophy. Core components—integrators, interaction potentials, neighbor-list builders, and observables—are implemented as interchangeable modules configurable through a Lua scripting interface [6]. This modularity makes it straightforward to extend HALMD with new functionality, including custom potentials, analysis routines, and data exporters. Simulation data are stored in the H5MD format [7], an HDF5-based standard widely supported by analysis tools in computational physics and chemistry.

These characteristics make HALMD an ideal platform for integrating machine-learned interatomic potentials. Its GPU-accelerated, modular architecture provides the necessary infrastructure to replace conventional analytical potentials with neural-network-based force fields. In this thesis, HALMD serves as the host MD engine into which the Deep Potential Molecular Dynamics (DeepMD) model is embedded. The goal is to achieve *ab initio*-level accuracy within HALMD’s efficient large-scale simulation framework by implementing a full multi-species DeepMD-v2 inference pipeline.

## 2.2 Neural Network Potentials

In recent years, machine learning techniques—particularly deep neural networks (DNNs) have revolutionized the construction of interatomic potentials for molecular dynamics simulations. Traditional analytical potentials, while computationally efficient, are often limited in their ability to accurately capture many-body interactions and chemical complexities across diverse atomic environments. Neural Network Potentials (NNPs) overcome these limitations by learning the potential energy surface (PES) directly from *ab initio* reference data [8, 9].

In the NNP framework, a neural network is trained to approximate the mapping between an atomic configuration and its corresponding total energy and atomic forces. The total potential energy of a system is commonly expressed as a sum of atomic contributions [8]:

$$E = \sum_i E_i(\mathcal{R}_i),$$

where  $E_i$  denotes the atomic energy associated with atom  $i$ , and  $\mathcal{R}_i$  represents its local environment within a cutoff radius. This decomposition ensures extensivity and locality and enables efficient scaling with system size.

Training a neural network potential involves minimizing a loss function that combines the errors in energies, forces, and optionally virials. Following the formulation used in DeePMD-kit [2, 3], the loss is written as

$$\mathcal{L} = p_E L_E + p_F L_F + p_V L_V, \quad (1)$$

where the terms are defined as

$$L_E = \frac{1}{N_E} \sum_{n=1}^{N_E} (E_n^{\text{NN}} - E_n^{\text{DFT}})^2, \quad (2)$$

$$L_F = \frac{1}{3N_F} \sum_{n=1}^{N_F} \sum_{i=1}^{N_{\text{atoms}}} \|\mathbf{F}_{i,n}^{\text{NN}} - \mathbf{F}_{i,n}^{\text{DFT}}\|^2, \quad (3)$$

$$L_V = \frac{1}{9N_V} \sum_{n=1}^{N_V} \|\mathbf{V}_n^{\text{NN}} - \mathbf{V}_n^{\text{DFT}}\|^2. \quad (4)$$

The factors 1/3 and 1/9 arise from averaging the force and virial errors over their three and nine Cartesian components, respectively, ensuring consistent normalization across all contributions to the loss.

During inference, atomic coordinates are transformed into symmetry-preserving descriptors that are invariant under translation, rotation, and permutation of atoms of the same type [10, 11]. These descriptors serve as input to the neural network, which predicts atomic energies and corresponding forces in real time, achieving *ab initio*-level accuracy at computational cost comparable to classical MD.

A variety of neural-network-based interatomic potentials have been proposed, including Behler–Parrinello networks [8], Gaussian Approximation Potentials (GAP) [12], SchNet [13], and the E(3)-equivariant NequIP model [14]. Among these, the Deep Potential Molecular Dynamics (DeepMD) framework has emerged as one of the most widely adopted and computationally efficient implementations due to its smooth descriptor formulation and native GPU acceleration. The following subsection focuses specifically on **DeepMD-kit version 2**, which forms the theoretical and computational foundation for the present work.

## 2.3 Deep Potential Molecular Dynamics (DeepMD)

This work focuses on **Deep Potential Molecular Dynamics version 2 (DeepMD-kit v2)**, the second-generation implementation of the Deep Potential framework. DeepMD-kit v2 represents a major advancement over the original DeepMD formulation [2], introducing improved descriptor smoothness, enhanced multi-species handling, and a refined software architecture that enables highly efficient GPU execution [3]. These features make DeepMD-kit v2 particularly suitable for modeling chemically complex systems such as binary alloys within HALMD.

Deep Potential Molecular Dynamics is a machine-learning approach that approximates interatomic interactions with near *ab initio* accuracy while retaining computational efficiency comparable to classical MD. A neural network is trained on quantum-mechanical reference data (typically from DFT or AIMD) to map atomic configurations to energies and forces. The total potential energy of the system is decomposed into atomic energy contributions [2, 3]:

$$E = \sum_i E_i(\mathcal{R}_i),$$

where  $E_i$  is the atomic energy of atom  $i$ , and  $\mathcal{R}_i$  denotes its local environment within a cutoff radius  $r_c$ . This locality assumption yields linear scaling in the number of atoms and enables efficient parallelization on GPUs.

A defining element of DeepMD is the use of symmetry-preserving descriptors that are invariant under translations, rotations, and permutations of atoms of the same type. DeepMD-kit v2 employs the *Smooth Edition* (SE) descriptor family, provided in angular (se\_e2\_a) variants [3]. It is constructed from two matrices:

- the ***R*-matrix**, containing relative position vectors  $R_{ij} = \mathbf{r}_j - \mathbf{r}_i$  for neighbors  $j$  within the cutoff radius;
- the ***G*-matrix**, a normalized representation incorporating angular information and many-body geometric correlations.

Distances  $r_{ij} = |R_{ij}|$  are modulated by a smooth cutoff function to ensure continuity of energies and forces at the cutoff boundary [2].

The Deep Potential model in both DP-v1 and DP-v2 consists of two neural networks: an *embedding (filter) network* and a *fitting network* [2, 3].

1. **Embedding (Filter) Network.** This smaller feed-forward network transforms raw neighbor information into high-dimensional filter features. In DP-v2, a distinct embedding network is assigned to each atomic species, allowing the model to capture chemically specific interactions (e.g., Cu–Ag, Ni–Al) while preserving permutation invariance within each species [3]. The embedding network processes each neighbor independently.
2. **Fitting Network.** The outputs of the embedding networks are aggregated into a descriptor vector  $\mathbf{D}_i$ , which is passed to a larger fully connected fitting network that predicts the atomic energy  $E_i$ . Each species has its own fitting network parameters, enabling accurate modeling of multi-component materials.

Forces are obtained as the negative gradients of the total energy:

$$\mathbf{F}_i = -\frac{\partial E}{\partial \mathbf{r}_i}.$$

During training, these derivatives are computed automatically through TensorFlow’s back-propagation engine. During inference—which is the focus of this thesis—the gradients are obtained by applying the chain rule explicitly through the descriptor and neural-network layers, following the DeePot-v2 computational graph [3].

DeepMD-kit v2 introduces several enhancements that are essential for accurate multi-species modeling. These include species-dependent embedding networks, species-specific filter weights, descriptor normalization layers, and refined smooth cutoff schemes [3]. Together, these improvements enable DeePot-v2 to handle chemically diverse systems with greater smoothness, transferability, and numerical stability compared to its predecessor.

A trained DeepMD model is stored as a TensorFlow `frozen_model.pb` file trained with config file `input.json` file which specifies descriptor types, cutoff radii, hyperparameters, and precision settings. During inference, atomic coordinates are transformed into descriptors, processed by species-specific embedding networks, and fed into the fitting network to produce atomic energies and forces.

In this thesis, DeepMD-kit v2 serves as the machine-learned potential that is fully integrated into HALMD. All parameter extraction, descriptor reconstruction, and neural-network inference follow the DP-v2 specification precisely. By embedding this inference process into HALMD’s GPU-accelerated architecture, the present work enables large-scale simulations of multi-species systems, such as binary alloys, with near *ab initio* accuracy.

## 3 Methodology

### 3.1 Overview of Implementation

The integration of DeepMD-v2 into HALMD follows a modular workflow in which each stage provides the necessary inputs for the next. The process begins with extracting all descriptor and neural-network parameters from the `frozen_model.pb` and `input.json` files and converting them into a compact HDF5 format that HALMD can load efficiently. Using these parameters, HALMD reconstructs the local atomic environment in the exact DeepMD-v2 convention, including periodic wrapping, ghost-cell expansion, and species-ordered neighbour selection. From this environment, HALMD computes the descriptor by forming the geometric matrix  $R$ , evaluating species-dependent embedding networks to obtain  $G$ , and assembling the final descriptor vector  $D_i$  that characterises the environment of each atom. The descriptor is then passed through the species-specific fitting network to compute atomic energies, while forces are obtained by propagating derivatives through the descriptor and networks using a combination of analytic expressions and automatic differentiation. Together, these steps enable HALMD to perform full DeepMD-v2 inference—energies and forces—with its native C++/CUDA simulation loop.

### 3.2 Model Parameter Extraction

#### 3.2.1 Frozen Model Structure

The trained DeepMD-v2 potential is distributed as a frozen TensorFlow graph (`frozen_model.pb`). To perform manual inference inside HALMD, all relevant model parameters must be re-

constructed from the computation graph. The node list extracted from the model (see Appendix X) contains all information required for this reconstruction.

**Descriptor attributes.** The descriptor section of the graph exposes the physical and structural parameters needed to build the environment matrix:

- cutoff radii (`descrpt_attr/rcut`, `descrpt_attr/rcut_smth`),
- neighbour selection size (`descrpt_attr/sel`, `descrpt_attr/original_sel`),
- type mapping and normalization tensors (`descrpt_attr/t_avg`, `descrpt_attr/t_std`, `descrpt_attr/ntypes`).

These values uniquely determine the construction of  $\hat{R}_i$  and ensure that HALMD reproduces the same geometric preprocessing as DeepMD.

**Embedding network.** The embedding (filter) network parameters appear under `filter_type_0/*`. The suffix `_0` indicates that these parameters correspond to a central atom of species  $a = 0$ . DeepMD-v2 stores a separate filter (embedding) network for each central-atom species, so in a multi-species system one would observe additional blocks such as `filter_type_1/*`, `filter_type_2/*`, and so on.

Within each `filter_type_a` group, DeepMD-v2 uses node names that follow the pattern

$$\text{matrix}_{\ell,b}, \quad \text{bias}_{\ell,b},$$

where:

- $\ell$  is the layer index of the embedding MLP,
- $b$  is the *neighbour-species index*.

Thus, nodes such as `matrix_1_0`, `matrix_2_0`, `matrix_3_0` and their corresponding `bias_1_0`, `bias_2_0`, `bias_3_0` represent the three-layer embedding network used when:

$$\text{central species } a = 0, \quad \text{neighbour species } b = 0.$$

In a multi-species model, additional blocks would appear, for example:

- `matrix_1_1`, `matrix_2_1`, ... for neighbours of species  $b = 1$ ,
- `matrix_1_2`, `matrix_2_2`, ... for neighbours of species  $b = 2$ , etc.

Each embedding layer provides:

- a weight matrix (`matrix_{\ell,b}`),
- a bias vector (`bias_{\ell,b}`),
- a nonlinear activation function node (typically Tanh).

Collectively, these nodes define the mapping

$$\hat{s}_{ij} \mapsto G_{ij},$$

which transforms the normalized inverse distance into an embedding vector for the descriptor.

**Descriptor production.** The node `ProdEnvMatA` encapsulates the internal DeepMD operation that produces the environment matrix and its derivatives:

- relative positions and angular components (`o_rmat`),
- pairwise distances (`o_rij`),
- neighbour list (`o_nlist`),
- geometric derivatives (`o_rmat_deriv`).

These outputs provide the ground truth for validating HALMD’s geometric derivative implementation.

**Fitting network.** The atomic-energy fitting network appears under the node groups `layer_0_type_0/*`, `layer_1_type_0/*`, `layer_2_type_0/*`, and `final_layer_type_0/*`. Each layer contributes:

- a weight matrix,
- a bias vector,
- optional residual-timestep coefficients (`idt` nodes),
- a nonlinear activation function (`Tanh`).

Together, these nodes define the mapping from the descriptor  $D_i$  to the atomic energy  $E_i$ .

**Energy and force outputs.** The frozen graph also includes nodes providing:

- total energy (`o_energy`),
- per-atom energy (`o_atom_energy`),
- forces (`o_force`),
- virials (`o_virial`, `o_atom_virial`),

as well as all intermediate gradient nodes under `gradients/*`, used by DeepMD’s automatic differentiation engine.

Together, these extracted components provide a complete specification of the trained potential: descriptor parameters, embedding transformations, environment-matrix construction, fitting-network architecture, and the final energy/force outputs. Using these elements, HALMD can reproduce DeepMD-v2 inference exactly, without relying on TensorFlow.

### 3.2.2 Extraction Procedure

DeepMD-kit stores trained neural network potentials as a TensorFlow `frozen_model.pb` file, accompanied by an `input.json` file containing model hyperparameters. The `.pb` file encodes all numerical weights, biases, and auxiliary tensors in the form of TensorFlow computation graph nodes, while the JSON file specifies the descriptor type, cutoff radius, number of neurons per layer, activation functions, and species layout. Following the methodology of DeepMD-kit v1 and v2 [2, 3], this thesis extracts all necessary model

parameters from these files and converts them into an HDF5 representation suitable for efficient inference inside HALMD.

The extraction process begins by loading the TensorFlow graph definition from the `.pb` file. All tensors of type `Const` are scanned, and those whose node names match descriptor or fitting-network layers are decoded using TensorFlow’s low-level `tensor_util.MakeNdarray`. The DeepMD model contains one set of parameters per atomic species, and the species ordering in the output strictly follows the ordering defined in the DeepMD input configuration. For the work presented here, the descriptor type is always the angular Smooth Edition descriptor `se_e2_a`, which DeepMD-kit v2 uses for multi-species models requiring both radial and angular correlation encoding.

**Descriptor Parameters.** For each species, the descriptor section consists of a stack of fully connected layers with user-specified neuron counts, activation functions, and optional residual time-step connections. The extraction script identifies each descriptor layer via a naming pattern of the form

```
filter_type_{s}/matrix_{k}_{t_n}, filter_type_{s}/bias_{k}_{t_n},
```

where  $s$  indexes the species,  $k$  is the layer index, and  $t_n$  enumerates neighbor-type channels. For each layer, the script records:

- weight matrices,
- bias vectors,
- number of neurons,
- activation function (typically `tanh` or `linear` in the present work),
- the presence of a residual network branch.

DeepMD-v2 allows descriptor layers to use residual updates when `resnet_dt=true`. In that case, a per-layer time-step tensor is extracted and marked in the output structure, though the use of residual updates depends on the model’s training configuration.

**Embedding (Filter) Network.** In the DeepMD architecture, the descriptor network is conceptually equivalent to the “filter” or embedding network described in [2, 3]. Each species has its own filter-network parameters to account for species-dependent geometric correlations. The extracted filter-network weights are reshaped into a row-major layout compatible with HALMD’s GPU evaluation kernels.

**Fitting Network.** For each species, the fitting network (also called the main network) predicts the atomic energy contribution  $E_i$ . The extraction process retrieves:

- all intermediate fitting layers,
- species-dependent activation functions,
- weight and bias tensors for each layer,
- the species-specific atomic energy bias term `bias_atom_e`,
- the parameters of the final output layer.

The fitting-network layers are identified using a template of the form

```
layer_LL_type_<k><s>/matrix,    layer_LL_type_<k><s>/bias,
```

and similarly for the final layer `final_layer_type_<s>`. The final layer uses a fixed activation function, typically linear, consistent with the DeepMD energy formulation.

**Normalization Parameters.** DeepMD-v2 introduces descriptor normalization tensors `t_avg` and `t_std`, which are required to maintain numerical stability and descriptor smoothness. These tensors are extracted from the nodes

```
descript_attr/t_avg,    descript_attr/t_std,
```

and stored in the HDF5 output so that HALMD can apply the same normalization as the original DeepMD model.

**Organization and Output Format.** All extracted parameters are written into a structured HDF5 file using a hierarchical layout:

- global descriptor constants (cutoff radii, smoothing radii, normalization tensors),
- per-species descriptor network parameters,
- per-species fitting network parameters.

This structure mirrors the multi-species design of DeepMD-kit v2 and makes the extracted parameters directly usable by HALMD for inference. Unlike the single-species extraction used in the previous HALMD implementation [1], the present work extracts and stores fully species-resolved descriptor and fitting-network data, which are required for accurate multi-species DeepMD-v2 inference.

The resulting HDF5 file serves as the unified model representation for HALMD. During the simulation, HALMD loads this file, reconstructs the descriptor and neural networks using GPU-optimized data structures, and performs inference without relying on TensorFlow. This enables the Deep Potential model to be evaluated natively within HALMD’s simulation loop with high performance and full multi-species support.

### 3.3 Coordinate System Extension

A central contribution of this thesis is the redesign of HALMD’s environment–construction pipeline so that it follows the exact conventions required by DeepMD-v2. The earlier implementation by Cruz [1] relied on HALMD’s built-in periodic boundary handling and neighbour list, which correctly satisfy the minimum-image convention used in classical MD [4]. However, this approach provides only the minimal displacement between atoms and does not reproduce the full periodic environment expected by the Deep Potential (DP) descriptor pipeline. DeepMD-v2 requires explicit coordinate wrapping, ghost-cell expansion, species-aware neighbour grouping, and descriptor normalization [2, 3]. These steps are essential for reproducing the descriptor inputs used during training.

The new DeePMD-style environment constructor introduced in this work consists of the following components.

### 3.3.1 Explicit coordinate wrapping

DeepMD requires that all atomic coordinates are expressed inside the primary simulation cell before any descriptor quantities are computed. This is stricter than the minimum-image convention normally used in classical molecular dynamics, where only the \*differences\* between coordinates need to be mapped back into the simulation box. DeepMD, however, expects the \*absolute coordinates\* of every atom to lie within the interval

$$[0, L_\alpha) \quad \text{for each Cartesian direction } \alpha \in \{x, y, z\}.$$

To ensure consistency, each coordinate component is mapped explicitly into the primary simulation box using

$$\tilde{r}_{i\alpha} = r_{i\alpha} - L_\alpha \left\lfloor \frac{r_{i\alpha}}{L_\alpha} \right\rfloor.$$

This operation performs the following steps:

- Compute the integer number of box lengths by which the particle has moved:

$$n_\alpha = \left\lfloor \frac{r_{i\alpha}}{L_\alpha} \right\rfloor.$$

This may be positive (particle drifted to the right), negative (particle drifted to the left), or zero.

- Subtract exactly  $n_\alpha L_\alpha$  from the coordinate so that the result lies in the interval  $[0, L_\alpha)$ :

$$\tilde{r}_{i\alpha} = r_{i\alpha} - n_\alpha L_\alpha.$$

- The wrapped coordinates  $\tilde{r}_i$  now correspond to the representation DeepMD expects as input.

This differs from the minimum-image convention, which only wraps *relative displacements*. DeepMD's descriptor depends on absolute coordinates (through the ghost-cell extension and species grouping), so the wrapping step is necessary to guarantee that the computed descriptor matches the TensorFlow implementation exactly.

### 3.3.2 Ghost-cell periodic extension

DeepMD constructs atomic environments by explicitly replicating the simulation cell in all spatial directions before evaluating descriptor quantities. This ensures that every atom, including those close to a periodic boundary, retains a complete neighbourhood within the cutoff radius  $r_c$ . In contrast, HALMD's native neighbour list returns only the nearest periodic image, which is sufficient for classical MD but does not supply the full set of geometric relations required by the DeepMD descriptor pipeline.

To achieve DeepMD-compatible behaviour, the present work implements full ghost-cell tiling. For each wrapped coordinate  $\tilde{r}_i$ , the periodic images are generated as

$$\mathbf{r}_i^{(s)} = \tilde{\mathbf{r}}_i + s_x L_x \mathbf{e}_x + s_y L_y \mathbf{e}_y + s_z L_z \mathbf{e}_z, \quad s_\alpha \in [-n_{\text{buff},\alpha}, n_{\text{buff},\alpha}],$$

where  $L_\alpha$  are the box lengths and

$$n_{\text{buff},\alpha} = \left\lceil \frac{r_c}{L_\alpha} \right\rceil$$

ensures full spatial coverage of the cutoff sphere. For a cubic cell with  $n_{\text{buff}} = 1$ , this produces  $3^3 = 27$  images (the central box and its neighbouring tiles). Neighbour search is then performed over these images, and only the atoms whose replicas fall within the cutoff are retained.

This explicit tiling is necessary for descriptor correctness: it guarantees that all physically relevant neighbours are included, that the distances  $r_{ij}$  and direction vectors  $\mathbf{r}_{ij}$  match those used by DeepMD-kit, and that atoms near cell boundaries obtain descriptor rows identical to those of atoms in the interior. By reproducing DeepMD’s environment construction exactly, the HALMD implementation achieves full compatibility with the DeepMD-v2 descriptor generation process.

### 3.3.3 Environment-matrix construction interface

Previously, HALMD computed environment quantities directly from the neighbour list using minimum-image displacements. The new implementation builds environments from:

- wrapped coordinates,
- ghost-expanded coordinates,
- species-ordered neighbour lists.

The construction of the descriptor matrices ( $R$ ,  $G$ ) occurs later and is documented in Section 3.4.

### 3.3.4 Normalization using $t_{\text{avg}}$ , $t_{\text{std}}$

DeepMD-v2 normalizes descriptor features using training-time statistics:

$$X' = \frac{X - t_{\text{avg}}}{t_{\text{std}}}.$$

The previous HALMD implementation lacked this step; the new pipeline integrates normalization directly, ensuring full compatibility with DeepMD-v2.

### 3.3.5 Summary of improvements

The updated environment-construction step introduces:

- explicit DeePMD-style coordinate wrapping,
- full periodic ghost-cell expansion,
- species-aware neighbour grouping based on the `sel` configuration,
- a redesigned environment interface for descriptor construction,
- descriptor normalization matching DeepMD-v2.

These extensions form the necessary foundation for computing the  $R$  and  $G$  descriptor matrices (Section 3.4) and enable HALMD to accurately evaluate multi-species DeepMD-v2 models.

### 3.3.6 Species-aware neighbour grouping

DeepMD-v2 requires that neighbour atoms be organised in a very specific species-dependent format before any descriptor quantities are computed. This is fundamentally different from HALMD’s native neighbour list, which treats all neighbours uniformly and returns a single distance-sorted list independent of species. However, in DeepMD-v2 the descriptor is constructed under the assumption that each central atom has a fixed, preallocated number of neighbour slots for *each species type*, as defined by the model parameter `sel`:

$$\text{sel} = [N_c^{(1)}, N_c^{(2)}, \dots, N_c^{(B)}],$$

where  $N_c^{(b)}$  is the number of neighbours of species  $b$  that the model expects for each central atom.

This fixed layout is not optional: every block of neighbours associated with a neighbour species is passed through a species-specific embedding network  $N_\theta^{(a,b)}$ , and thus the descriptor depends critically on *which neighbour occupies which row* of the  $R$  and  $G$  matrices.

**Constructing species-specific neighbour lists.** To reproduce this behaviour, the new implementation replaces HALMD’s species-blind neighbour structure with species-partitioned lists. For a central atom  $i$  and each species label  $a$ , HALMD now constructs

$$\text{neighbors\_by\_type}[a] = \{ j \mid t_j = a, r_{ij} < r_c \},$$

where  $t_j$  is the species of neighbour  $j$ . Each such list contains only the neighbours belonging to species  $a$ .

**Distance sorting within each species group.** DeepMD requires each species block to be internally sorted by distance. Thus, for every species  $a$ , the list `neighbors_by_type[a]` is sorted according to  $r_{ij}$ :

$$\text{sort}(\text{neighbors\_by\_type}[a], \text{key} = r_{ij}).$$

This ensures that:

- the nearest neighbours of each species appear first,
- rows of the descriptor matrices corresponding to species  $a$  match the exact order used by DeepMD-kit,
- neighbour-dependent neural networks receive inputs in the correct, deterministic order.

### 3.3.7 Applying the `sel` constraint

For each central atom, DeepMD-v2 requires that the descriptor contains a fixed number of neighbour entries for each species, as prescribed by the vector

$$\text{sel} = [N_c^{(0)}, N_c^{(1)}, \dots].$$

After grouping neighbours by species and sorting them by distance, each species block is adjusted to have exactly  $N_c^{(a)}$  rows:

$$\widehat{\text{neighbors\_by\_type}}[a] = \begin{cases} \text{first } N_c^{(a)} \text{ neighbours,} & \text{if more than } N_c^{(a)} \text{ are available,} \\ \text{zero-padded rows,} & \text{if fewer than } N_c^{(a)} \text{ neighbours exist.} \end{cases}$$

This behaviour matches the DeepMD-v2 implementation, where missing neighbours are represented by zero-valued geometric entries in the environment matrix. Invalid neighbour indices are masked, resulting in

$$R_{ij} = 0, \quad \hat{s}_{ij} = 0, \quad G_{ij} = N_\theta^{(a,b)}(0),$$

for all padded rows. No duplication of the last valid neighbour is performed.

Zero-padding ensures that the assembled  $R$  and  $G$  matrices have the fixed, model-dependent shape required by DeepMD-v2 and that the fitting network receives inputs consistent with the TensorFlow/JAX implementation.

**Constructing the full neighbour ordering.** The final neighbour list for descriptor construction is obtained by concatenating species blocks in the fixed species order used by the model:

$$\text{neighbors\_ordered} = [\widehat{\text{neighbors\_by\_type}}[1], \widehat{\text{neighbors\_by\_type}}[2], \dots, \widehat{\text{neighbors\_by\_type}}[B]].$$

Every descriptor row in the matrices  $R$  and  $G$  now corresponds to a specific species and a specific position within that species block, exactly as expected by the DeepMD-v2 architecture.

**Importance of species grouping.** This organisation is not merely a convenience; it is required because:

- each species pair  $(a, b)$  uses a different embedding network  $N_\theta^{(a,b)}$ ,
- descriptor rows must map one-to-one to embedding-network evaluations,
- the quadratic descriptor  $D_i$  implicitly assumes this grouping when computing  $G^T \hat{R}$ ,
- the fitting network is trained on descriptors in this exact ordering.

A deviation from this layout would result in incorrect  $R$  and  $G$  matrices, leading to mismatched descriptors  $D_i$ , incorrect predicted energies, and incorrect forces.

Thus, the introduction of species-aware neighbour grouping is a key requirement for enabling HALMD to evaluate multi-species DeepMD-v2 models correctly.

### 3.4 Construction of the Geometric Matrix $R$

Once wrapped coordinates, ghost-cell expansions, and species-ordered neighbour lists have been obtained (Section 3.3), the next task is to construct the geometric matrix  $R$ . This matrix encodes the local atomic environment of a central atom  $i$  in a way that is translationally, rotationally, and permutation invariant [2, 3]. Each of the  $N_c^{(\text{total})}$  neighbour slots contributes one row of four geometric quantities.

### 3.4.1 Relative displacement and inverse-distance quantities

For each neighbour selected into the fixed-capacity list, the relative displacement is computed using the *wrapped and ghost-extended coordinates*:

$$\mathbf{r}_{ij} = \mathbf{r}_j^{(s)} - \tilde{\mathbf{r}}_i \in \mathbb{R}^3, \quad r_{ij} = \|\mathbf{r}_{ij}\| \in \mathbb{R}.$$

### 3.4.2 Raw geometric row

DeepMD defines the raw geometric features of neighbour  $j$  (row index  $k$ ) as

$$R_k^{\text{raw}} = \left[ s_{ij}, \frac{x_{ij}}{r_{ij}^2}, \frac{y_{ij}}{r_{ij}^2}, \frac{z_{ij}}{r_{ij}^2} \right] \in \mathbb{R}^4, \quad s_{ij} = \frac{1}{r_{ij}}.$$

Collecting all rows yields the matrix

$$R^{\text{raw}} \in \mathbb{R}^{N_c^{\text{(total)}} \times 4}.$$

### 3.4.3 Cutoff behaviour and switching function

To ensure smooth decay near the cutoff radius, DeepMD applies a quintic switching function:

$$f_{\text{sw}}(r) = \begin{cases} 1, & r \leq r_{\text{sm}}, \\ x^3(-6x^2 + 15x - 10) + 1, & r_{\text{sm}} < r < r_c, \\ 0, & r \geq r_c, \end{cases} \quad x = \frac{r - r_{\text{sm}}}{r_c - r_{\text{sm}}}.$$

The resulting weight vector is

$$w_k = f_{\text{sw}}(r_{ij(k)}), \quad w \in \mathbb{R}^{N_c^{\text{(total)}}}.$$

Applying this weight yields the weighted geometric matrix

$$R_{k,\alpha}^{\text{w}} = R_{k,\alpha}^{\text{raw}} w_k, \quad R^{\text{w}} \in \mathbb{R}^{N_c^{\text{(total)}} \times 4}.$$

### 3.4.4 Species-aware neighbour ordering

DeepMD-v2 requires neighbours to be arranged according to:

1. neighbour species,
2. increasing neighbour distance within each species,
3. a fixed per-species capacity determined by

$$\text{sel} = [N_c^{(0)}, N_c^{(1)}, \dots].$$

After this procedure, every central atom receives a geometric matrix with the same shape:

$$R^{\text{w}} \in \mathbb{R}^{N_c^{\text{(total)}} \times 4}.$$

### 3.4.5 Normalization of geometric features

DeepMD-v2 applies feature-wise normalization using tensors

$$t_{\text{avg}}, t_{\text{std}} \in \mathbb{R}^{N_c^{\text{(total)}} \times 4}.$$

The normalized geometric matrix is

$$\hat{R}_{k,\alpha} = \frac{R_{k,\alpha}^{\text{w}} - (t_{\text{avg}})_{k,\alpha}}{(t_{\text{std}})_{k,\alpha}}, \quad \hat{R} \in \mathbb{R}^{N_c^{\text{(total)}} \times 4}.$$

The first column,

$$\hat{s}_{ij} = \hat{R}_{ij,0},$$

serves as the scalar input to the embedding network.

### 3.4.6 Summary

The constructed matrix

$$\hat{R} \in \mathbb{R}^{N_c^{\text{(total)}} \times 4}$$

is bitwise compatible with DeepMD-v2 and provides the geometric inputs for the next stage of the descriptor.

## 3.5 Construction of the Embedding Matrix $G$

The second stage of the DeepPot-SE descriptor transforms each normalized inverse distance  $\hat{s}_{ij} = \hat{R}_{ij,0}$  into a high-dimensional embedding vector through a species-pair-specific *filter network*. Collecting the embeddings for all neighbour slots produces the matrix

$$G \in \mathbb{R}^{N_c^{\text{(total)}} \times M},$$

where  $M$  is the embedding dimension, and  $N_c^{\text{(total)}} = \sum_b N_c^{(b)}$  is the total neighbour capacity specified by the model's `sel` vector.

### 3.5.1 Input and Output Dimensionality

Each neighbour slot contributes a single scalar input

$$\hat{s}_{ij} \in \mathbb{R},$$

obtained from the first column of the normalized geometric matrix  $\hat{R}$ .

The embedding network maps this scalar to an  $M$ -dimensional output:

$$G_{ij} \in \mathbb{R}^M.$$

Stacking the embedding vectors for all neighbour rows yields

$$G = \begin{bmatrix} G_{i1}^T \\ G_{i2}^T \\ \vdots \\ G_{iN_c^{\text{(total)}}}^T \end{bmatrix} \in \mathbb{R}^{N_c^{\text{(total)}} \times M}.$$

The ordering of the rows is identical to the ordering of the rows of  $\hat{R}$ : neighbours are grouped by species and sorted by distance within each species block.

### 3.5.2 Structure of the Filter Network

For each central–neighbour species pair  $(a, b)$ , DeepMD-v2 defines a dedicated multi-layer perceptron (MLP)

$$N_{\theta}^{(a,b)} : \mathbb{R} \rightarrow \mathbb{R}^M.$$

Each of these MLPs consists of:

- an input layer of width 1 (the scalar  $\hat{s}_{ij}$ ),
- $L$  hidden layers with species-pair–dependent widths,
- a final output layer of dimension  $M$ ,
- Tanh activations between layers (except the output layer).

For a filter network with layer widths

$$1 \rightarrow H_1 \rightarrow H_2 \rightarrow \cdots \rightarrow H_L \rightarrow M,$$

the forward pass is

$$\begin{aligned} h^{(1)} &= \tanh\left(W_{a,b}^{(1)} \hat{s}_{ij} + b_{a,b}^{(1)}\right), \\ h^{(\ell)} &= \tanh\left(W_{a,b}^{(\ell)} h^{(\ell-1)} + b_{a,b}^{(\ell)}\right), \quad \ell = 2, \dots, L, \\ G_{ij} &= W_{a,b}^{(L+1)} h^{(L)} + b_{a,b}^{(L+1)}. \end{aligned}$$

These weights  $W_{a,b}^{(\ell)}$  and biases  $b_{a,b}^{(\ell)}$  are extracted from the Frozen Model under node names of the form ‘filter\_type\_a/matrix\_l\_b’ and ‘filter\_type\_a/bias\_l\_b’ (see Section 3.2). The subscript  $b$  corresponds to the neighbour species, confirming explicitly that the network architecture depends on both the central and neighbour species.

### 3.5.3 SpeciesPair–Dependent Embedding Networks

For a central atom of species  $a$  and a neighbour of species  $b$ , the embedding is computed using the matching filter network:

$$G_{ij} = N_{\theta}^{(a,b)}(\hat{s}_{ij}).$$

Different neighbour species therefore produce embeddings through different MLPs even when the geometric distances are identical. This mechanism is essential for modelling alloy systems, where cross-species interactions must be distinguishable in the descriptor.

HALMD stores the entire set of such networks in a two-dimensional structure:

$$\text{neural\_networks}[a][b] \equiv N_{\theta}^{(a,b)}.$$

### 3.5.4 Runtime Network Selection and Evaluation

After species-grouped neighbour lists are constructed (Section 3.3), each neighbour row  $j$  is associated with a neighbour-species label  $b_{(j)}$ . The embedding vector for that row is obtained via

$$G_{ij} = N_{\theta}^{(a,b_{(j)})}(\hat{s}_{ij}).$$

Thus:

- row  $j$  of  $\hat{R}$  determines  $\hat{s}_{ij}$ ,
- the species list determines  $b_{(j)}$ ,
- the corresponding network  $N_\theta^{(a,b_{(j)})}$  is applied,
- the resulting  $G_{ij}$  is stored in row  $j$  of  $G$ .

This ensures strict alignment between rows of  $\hat{R}$  and rows of  $G$ , exactly as in the DeepMD-v2 computational graph.

### 3.5.5 Summary

The embedding matrix  $G$  is constructed through the following steps:

- extract the scalar geometric input  $\hat{s}_{ij}$  from the normalized  $R$ -matrix,
- select the appropriate species-pair filter network  $N_\theta^{(a,b)}$ ,
- evaluate its multi-layer structure using the trained weights and biases,
- assemble all embedding vectors into the matrix  $G \in \mathbb{R}^{N_c^{(\text{total})} \times M}$ .

The resulting matrix provides the learned many-body representation that, together with  $\hat{R}$ , forms the complete input to the quadratic descriptor described in the next section.

## 3.6 Descriptor Computation

The DeepPot-SE (se\_e2\_a) descriptor encodes the environment of a central atom  $i$  through a combination of (i) normalized geometric features collected in the matrix  $\hat{R}_i$  and (ii) learned nonlinear embeddings collected in the matrix  $G_i$ . These two matrices are then contracted into a quadratic descriptor  $D_i$ , which forms the input to the species-dependent fitting network predicting the atomic energy  $E_i$ .

DeepMD-v2 assigns neighbour capacity on a per-species basis. Given species set  $\mathcal{S}$  and model parameters  $N_c^{(b)}$  for each species  $b \in \mathcal{S}$ , the total neighbour capacity is

$$N_c^{(\text{total})} = \sum_{b \in \mathcal{S}} N_c^{(b)}. \quad (4.1)$$

All matrices involved in descriptor computation are dimensioned using this total capacity.

### 3.6.1 Normalized geometric matrix

Once the wrapped coordinates, periodic images, and species-ordered neighbour lists have been constructed (Section 3.3), the geometric features for atom  $i$  are assembled into the matrix

$$\hat{R}_i \in \mathbb{R}^{N_c^{(\text{total})} \times 4}. \quad (4.2)$$

Each row corresponds to a specific neighbour slot and contains four quantities:

1. the normalized inverse distance  $\hat{s}_{ij}$ ,
2. the three normalized angular components  $\hat{x}_{ij}/r_{ij}^2$ ,  $\hat{y}_{ij}/r_{ij}^2$ ,  $\hat{z}_{ij}/r_{ij}^2$ .

Normalization follows the statistics extracted from the trained model:

$$\hat{R}_{ij,\alpha} = \frac{R_{ij,\alpha} - (t_{\text{avg}})_{\alpha}}{(t_{\text{std}})_{\alpha}}, \quad \alpha = 0, \dots, 3. \quad (4.3)$$

The first column is used as the scalar geometric input for the nonlinear embedding:

$$\hat{s}_{ij} = \hat{R}_{ij,0}. \quad (4.4)$$

This matrix therefore represents a geometrically structured, normalized description of the neighbourhood of atom  $i$ , with rows aligned to the species-block ordering of DeepMD-v2.

### 3.6.2 Embedding matrix

Each neighbour slot  $j$  has a known neighbour species  $b(j)$ . DeepMD-v2 assigns a distinct filter (embedding) network for every central–neighbour pair  $(a, b)$ , where  $a$  is the central species:

$$G_{ij} = N_{\theta}^{(a,b(j))}(\hat{s}_{ij}), \quad (4.5)$$

with  $N_{\theta}^{(a,b)} : \mathbb{R} \rightarrow \mathbb{R}^M$ .

Evaluating all such networks yields

$$G_i \in \mathbb{R}^{N_c^{(\text{total})} \times M}. \quad (4.6)$$

Only the first  $M'$  embedding channels are required for the descriptor:

$$G_i^{\text{trunc}} = G_i[:, 1:M'] \in \mathbb{R}^{N_c^{(\text{total})} \times M'}. \quad (4.7)$$

This embedding stage transforms the purely geometric scalar inputs  $\hat{s}_{ij}$  into learned, species-dependent representations, with rows aligned exactly to the rows of  $\hat{R}_i$ .

### 3.6.3 Quadratic descriptor construction

The DeepPot-SE descriptor is obtained by combining the geometric and embedding matrices into a quadratic tensor:

$$D_i = \frac{1}{(N_c^{(\text{total})})^2} (G_i^T \hat{R}_i) (\hat{R}_i^T G_i^{\text{trunc}}). \quad (4.8)$$

The intermediate products have the following shapes:

$$\begin{aligned} G_i^T \hat{R}_i &\in \mathbb{R}^{M \times 4}, \\ \hat{R}_i^T G_i^{\text{trunc}} &\in \mathbb{R}^{4 \times M'}, \\ D_i &\in \mathbb{R}^{M \times M'}. \end{aligned} \quad (4.9)$$

DeepMD-v2 flattens this matrix in row-major order to obtain the final descriptor vector:

$$\mathbf{D}_i \in \mathbb{R}^{MM'}. \quad (4.10)$$

This vector is the complete learned representation of the neighbourhood of atom  $i$ , structured and normalized exactly as required by the DeepMD fitting network.

### 3.6.4 Summary

The descriptor pipeline implemented in HALMD reproduces the DeepMD-v2 formulation without modification. In particular:

- the neighbour axis matches the species-resolved capacities  $N_c^{(b)}$ ,
- the geometric matrix  $\hat{R}_i$  implements all preprocessing steps required by DeepMD-v2,
- the embedding matrix  $G_i$  is produced by the full set of species-pair networks  $N_\theta^{(a,b)}$ ,
- the quadratic descriptor  $D_i$  and its scaling follow the DeepMD-v2 definition,
- the final descriptor vector  $\mathbf{D}_i$  matches TensorFlow output to floating-point precision.

This yields a complete, dimensionally consistent descriptor pipeline suitable for accurate, high-performance DeepMD-v2 inference inside HALMD.

## 3.7 Potential Energy Calculation

In the Deep Potential (DeePot) formalism, the total potential energy of a system of  $N$  atoms is written as a sum of atomic contributions,

$$E = \sum_{i=1}^N E_i, \quad (5)$$

where the atomic energy  $E_i$  is obtained by evaluating a *species-dependent* fitting neural network on the descriptor vector  $\mathbf{D}_i$ . For an atom of species  $s_i$ , the energy is computed as

$$E_i = \text{NN}_{s_i}(\mathbf{D}_i), \quad (6)$$

where  $\text{NN}_{s_i}$  denotes the DeepMD-v2 fitting network associated with species  $s_i$ . HALMD reconstructs these networks directly from the TensorFlow frozen graph (`frozen_model.pb`), including all linear layers, activation functions, residual branches, and output transformations.

### 3.7.1 Baseline Implementation Prior to This Work

The earlier HALMD implementation [1] supported inference only for *single-species* DeepMD models. It successfully reproduced DeepMD energies for monoatomic systems but lacked the structural components required for DeepMD-v2 multi-species inference:

- only one fitting network for all atoms,
- no integration with multi-species descriptors ( $\hat{R}, G, D$ ),
- no per-species energy shift (`bias_atom_e`),
- no mechanism for mapping descriptor vectors to the appropriate network.

Therefore, multi-component DeepMD-v2 models could not be evaluated within HALMD.

### 3.7.2 Species-Dependent Fitting Networks

DeepMD-v2 defines a separate fitting neural network for each atomic species. In the TensorFlow graph, these networks appear under node groups of the form:

`layer_ℓ_type_a/matrix`,    `layer_ℓ_type_a/bias`,    `final_layer_type_a/*`,

where:

- $a$  indexes the central-atom species,
- $\ell$  is the layer index,
- each pair of `matrix` and `bias` nodes defines an affine layer.

HALMD reconstructs the complete set

$$\{ \text{NN}_0, \text{NN}_1, \dots, \text{NN}_{S-1} \},$$

and evaluates the appropriate network according to

$$E_i = \text{NN}_{s_i}(\mathbf{D}_i).$$

The names of the corresponding TensorFlow nodes make the species mapping explicit:

- for species  $a = 0$ : `layer_0_type_0/matrix`, `layer_1_type_0/matrix`, `final_layer_type_0/matrix`, etc.
- for species  $a = 1$ : `layer_0_type_1/matrix`, `layer_1_type_1/matrix`, `final_layer_type_1/matrix`, etc.

These node groups fully specify all weights and biases of each species-dependent fitting network.

### 3.7.3 Per-Species Energy Offsets

DeepMD-v2 optionally includes constant energy offsets for each species, stored under:

`bias_atom_e`.

HALMD extracts these tensors and evaluates

$$E_i = \text{NN}_{s_i}(\mathbf{D}_i) + b_{s_i}. \quad (7)$$

This ensures that the final energy scale matches the one used during DeepMD model training.

### 3.7.4 Integration with the Multi-Species Descriptor Pipeline

The descriptor vector  $\mathbf{D}_i$  supplied to the fitting network has the exact structure expected by the DeepMD-v2 TensorFlow implementation. Its construction incorporates:

- the normalized geometric matrix  $\hat{R}_i \in \mathbb{R}^{N_c^{(\text{total})} \times 4}$ ,
- neighbour-slot ordering based on the model's `sel` vector,
- embedding vectors from species-pair networks  $N_\theta^{(a,b)}(\hat{s}_{ij})$ ,

- truncation to the first  $M'$  embedding channels,
- the quadratic descriptor construction  $D_i \in \mathbb{R}^{M \times M'}$ ,
- flattening into  $\mathbf{D}_i \in \mathbb{R}^{MM'}$ .

The ordering and dimensionality of  $\mathbf{D}_i$  must match the structure expected by the fitting networks defined in `layer_type_a/*` and `final_layer_type_a/*`. The revised HALMD implementation enforces this consistency.

### 3.7.5 Reproduction of DeepMD-v2 Energies

With the full multi-species pipeline integrated, HALMD reproduces DeepMD-v2 energy predictions to floating-point precision across all tested systems:

- monoatomic models (Cu),
- multi-species alloy models (HEA),
- alternative Copper models with different normalization statistics.

Both total energies and per-atom energies match DeepMD's TensorFlow-based outputs.

### 3.7.6 Final Formulation

HALMD now evaluates the energy of each atom using the DeepMD-v2 expression

$$E_i = \text{NN}_{s_i}(\mathbf{D}_i) + b_{s_i}, \quad (8)$$

and accumulates the total energy as

$$E_{\text{tot}} = \sum_{i=1}^N E_i. \quad (9)$$

This formulation is bitwise compatible with the DeepMD-v2 computational graph and is suitable for high-performance molecular dynamics simulations of multi-species systems within HALMD.

## 4 Force Computation

In the Deep Potential (DeeP) framework, forces are obtained as the negative gradient of the total potential energy with respect to atomic coordinates,

$$\mathbf{F}_i = -\frac{\partial E}{\partial \mathbf{r}_i}.$$

Because the energy is produced through a multi-stage mapping involving geometric quantities ( $R$ ), species-dependent embedding networks ( $G$ ), and a species-dependent fitting network, the force computation requires evaluating a sequence of nested derivatives.

DeepMD expresses this as a structured chain rule passing through:

$$\mathbf{r} \xrightarrow{\text{geometry}} R \xrightarrow{\text{embedding}} G \xrightarrow{\text{fitting network}} E,$$

so that each force component involves contributions from all atoms appearing in the local environment of the corresponding descriptor.

The remainder of this section describes the derivative computation in HALMD. We begin with an overview of automatic differentiation (AD), which is used to evaluate all neural-network derivatives. We then detail the analytic derivatives of the geometry matrix  $R$ , the descriptor  $D$ , and finally show how these quantities are combined with the fitting-network derivatives to assemble the total atomic forces.

## 4.1 Automatic Differentiation (AD)

The computation of atomic forces in DeepMD requires propagating derivatives through a sequence of transformations involving geometric preprocessing, species-pair embedding networks, quadratic descriptor construction, and finally a species-dependent fitting network. While the geometric derivatives admit closed-form expressions, the neural networks involve nonlinear activations and residual connections that make analytic differentiation impractical. For these components, HALMD employs *automatic differentiation* (AD).

Automatic differentiation evaluates derivatives by decomposing a computation into elementary operations and applying the chain rule locally [15, 16]. Two modes of AD are commonly used:

### 4.1.1 Forward-mode AD (current HALMD implementation)

Forward-mode AD propagates derivatives *together with the value* of every intermediate variable. Intuitively, each quantity in the computation carries a “derivative tag” that is updated whenever the variable participates in an operation.

This mode is efficient when:

- the number of inputs is small, and
- each function has a scalar or low-dimensional output.

These conditions hold for the DeepMD *embedding networks*  $N_\theta^{(a,b)}$ , each of which maps a single scalar input  $\hat{s}_{ij}$  to an  $M$ -dimensional output. HALMD therefore evaluates both the value

$$G_{ij}, \quad \frac{\partial G_{ij}}{\partial \hat{s}_{ij}}$$

using forward-mode AD provided by Boost.Autodiff [17].

Forward-mode AD is also used in the fitting network to compute  $\partial E_i / \partial D_{i,\alpha}$ , since each descriptor component  $D_{i,\alpha}$  can be marked as a differentiable input.

### 4.1.2 Reverse-mode AD (not yet implemented)

Reverse-mode AD performs the computation in two sweeps:

1. a forward sweep computes all intermediate values,
2. a backward sweep accumulates derivatives from the output back through the computational graph.

Intuitively, reverse-mode AD works like backpropagation in neural networks: one starts from the scalar loss (or, in this case, the atomic energy  $E_i$ ) and distributes sensitivities backward across all intermediate variables.

Reverse-mode AD is most efficient when:

- the function has a *single scalar output*, and
- the number of inputs is large.

This matches the structure of the *fitting network*:

$$E_i = \text{NN}_{s_i}(\mathbf{D}_i),$$

where the input dimension  $\dim(\mathbf{D}_i) = MM'$  may be several hundred. In such a setting, reverse-mode AD would greatly reduce computational cost, as the full Jacobian  $\partial E_i / \partial D_{i,\alpha}$  could be computed in a *single* backward pass.

At present, HALMD implements only forward-mode AD. Although correct, this results in one forward-mode evaluation per descriptor entry, which is more expensive than necessary for large multi-species models. Incorporating reverse-mode AD in the fitting network is therefore a promising future improvement for performance-critical simulations.

#### 4.1.3 Hybrid analytic–AD differentiation

All geometric derivatives—including those of the wrapped coordinates, ghost-cell transformations, inverse-distance expressions, and switching functions—are evaluated analytically. AD is used exclusively inside the neural-network components. This hybrid strategy offers:

- exact agreement with the DeepMD-v2 TensorFlow implementation,
- efficient evaluation of nonlinear network derivatives,
- clear separation between analytic geometric contributions and AD-driven neural contributions.

The following sections detail the analytic components required to complete the force calculation: derivatives of the geometric mapping, descriptor derivatives, and the assembly of the total force.

## 4.2 Derivatives of the Geometry Matrix $R$

To compute forces, we require the derivative of each row of the geometry matrix  $R$  with respect to the coordinates of the central atom  $i$ . Each row depends on the neighbour displacement

$$\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i = (d_x, d_y, d_z), \quad r_{ij} = \|\mathbf{r}_{ij}\|,$$

and on the switched inverse distance

$$\tilde{s}_{ij} = \frac{1}{r_{ij}} f_{\text{sw}}(r_{ij}).$$

We now derive all components of  $\partial R_{ij} / \partial \mathbf{r}_i$  using only the multivariate chain rule.

### 4.2.1 Step 1: Rewrite the Components of $R$

For neighbour  $j$ , the geometry row is

$$R_{ij} = [\tilde{s}_{ij}, \tilde{s}_{ij} \frac{d_x}{r_{ij}}, \tilde{s}_{ij} \frac{d_y}{r_{ij}}, \tilde{s}_{ij} \frac{d_z}{r_{ij}}].$$

We denote the three directional components compactly as

$$S_\alpha = \tilde{s}_{ij} \frac{d_\alpha}{r_{ij}}, \quad \alpha \in \{x, y, z\}.$$

### 4.2.2 Step 2: Derivative of the Distance $r_{ij}$

Following the derivation in Section 3.4, the distance can be written as

$$r_{ij} = (d_x^2 + d_y^2 + d_z^2)^{1/2}.$$

Using the chain rule,

$$\frac{\partial r_{ij}}{\partial d_\alpha} = \frac{d_\alpha}{r_{ij}}, \quad \alpha \in \{x, y, z\}.$$

Since

$$d_\alpha = r_{j,\alpha} - r_{i,\alpha},$$

we also have

$$\frac{\partial d_\alpha}{\partial \mathbf{r}_i} = -\mathbf{e}_\alpha,$$

and combining these yields

$$\frac{\partial r_{ij}}{\partial \mathbf{r}_i} = \sum_\alpha \frac{d_\alpha}{r_{ij}} (-\mathbf{e}_\alpha) = -\frac{1}{r_{ij}} (d_x, d_y, d_z) = -\hat{\mathbf{r}}_{ij}.$$

### 4.2.3 Step 3: Derivative of the Switching Function

The switching function used in DeepMD is

$$f_{sw}(r) = \begin{cases} 1, & r \leq r_{sm}, \\ x^3(-6x^2 + 15x - 10) + 1, & r_{sm} < r < r_c, \\ 0, & r \geq r_c, \end{cases} \quad x = \frac{r - r_{sm}}{r_c - r_{sm}}.$$

Its derivative with respect to the distance  $r$  is obtained using the chain rule,

$$\frac{df_{sw}}{dr} = \begin{cases} 0, & r \leq r_{sm}, \\ \frac{df_{sw}}{dx} \cdot \frac{1}{r_c - r_{sm}}, & r_{sm} < r < r_c, \\ 0, & r \geq r_c, \end{cases}$$

where

$$\frac{df_{sw}}{dx} = x^2 (-30x^2 + 60x - 30).$$

This expression enters the derivative of the switched inverse distance  $\tilde{s}_{ij} = r_{ij}^{-1} f_{\text{sw}}(r_{ij})$ , via

$$\frac{\partial \tilde{s}_{ij}}{\partial r_{ij}} = -\frac{1}{r_{ij}^2} f_{\text{sw}}(r_{ij}) + \frac{1}{r_{ij}} \frac{df_{\text{sw}}}{dr}(r_{ij}),$$

which is subsequently used in the derivatives of all components of the geometry matrix  $R$ .

#### 4.2.4 Step 4: Derivative of the Switched Inverse Distance

The first component of the geometry row is

$$R_{ij,0} = \tilde{s}_{ij} = \frac{1}{r_{ij}} f_{\text{sw}}(r_{ij}).$$

Differentiating with respect to  $r_{ij}$ ,

$$\frac{\partial \tilde{s}_{ij}}{\partial r_{ij}} = -\frac{1}{r_{ij}^2} f_{\text{sw}}(r_{ij}) + \frac{1}{r_{ij}} \frac{df_{\text{sw}}}{dr}(r_{ij}).$$

Applying the chain rule with  $\partial r_{ij}/\partial \mathbf{r}_i = -\hat{\mathbf{r}}_{ij}$ ,

$$\frac{\partial R_{ij,0}}{\partial \mathbf{r}_i} = \frac{\partial \tilde{s}_{ij}}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \mathbf{r}_i} = -\frac{\partial \tilde{s}_{ij}}{\partial r_{ij}} \hat{\mathbf{r}}_{ij}.$$

#### 4.2.5 Step 5: Derivative of the Directional Components

Consider a single directional term

$$S_\alpha = \tilde{s}_{ij} \frac{d_\alpha}{r_{ij}}.$$

We differentiate it with respect to  $\mathbf{r}_i$  using the product rule. First differentiate with respect to distance:

$$\frac{\partial}{\partial r_{ij}} \left( \frac{d_\alpha}{r_{ij}} \right) = d_\alpha \left( -\frac{1}{r_{ij}^2} \right) = -\frac{d_\alpha}{r_{ij}^2}.$$

Then

$$\frac{\partial S_\alpha}{\partial r_{ij}} = \frac{\partial \tilde{s}_{ij}}{\partial r_{ij}} \frac{d_\alpha}{r_{ij}} - \tilde{s}_{ij} \frac{d_\alpha}{r_{ij}^2}.$$

Next we differentiate  $S_\alpha$  with respect to the displacement component  $d_\alpha$ :

$$\frac{\partial S_\alpha}{\partial d_\alpha} = \tilde{s}_{ij} \frac{1}{r_{ij}}.$$

Using  $\partial d_\alpha/\partial \mathbf{r}_i = -\mathbf{e}_\alpha$ , the chain rule gives

$$\frac{\partial S_\alpha}{\partial \mathbf{r}_i} = \frac{\partial S_\alpha}{\partial r_{ij}} \frac{\partial r_{ij}}{\partial \mathbf{r}_i} + \frac{\partial S_\alpha}{\partial d_\alpha} \frac{\partial d_\alpha}{\partial \mathbf{r}_i}.$$

Substituting all expressions:

$$\frac{\partial S_\alpha}{\partial \mathbf{r}_i} = - \left( \frac{\partial \tilde{s}_{ij}}{\partial r_{ij}} \frac{d_\alpha}{r_{ij}} - \tilde{s}_{ij} \frac{d_\alpha}{r_{ij}^2} \right) \hat{\mathbf{r}}_{ij} - \frac{\tilde{s}_{ij}}{r_{ij}} \mathbf{e}_\alpha.$$

This formula directly produces the derivative of  $R_{ij,1}, R_{ij,2}, R_{ij,3}$ .

#### 4.2.6 Step 6: Normalized Geometry Derivative

DeepMD-v2 uses component-wise normalization:

$$\hat{R}_{ij,\alpha} = \frac{R_{ij,\alpha} - (t_{\text{avg}})_{\alpha}}{(t_{\text{std}})_{\alpha}}.$$

Since the normalization parameters are constants,

$$\frac{\partial \hat{R}_{ij,\alpha}}{\partial \mathbf{r}_i} = \frac{1}{(t_{\text{std}})_{\alpha}} \frac{\partial R_{ij,\alpha}}{\partial \mathbf{r}_i},$$

#### 4.2.7 Step 7: Derivative with Respect to the Neighbour Coordinate $\mathbf{r}_j$

All formulas above were derived for the derivative with respect to the *central* atom  $i$ . The derivative with respect to the *neighbour* atom  $j$  is obtained immediately from the identity

$$\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i,$$

which implies

$$\frac{\partial d_{\alpha}}{\partial \mathbf{r}_j} = +\mathbf{e}_{\alpha}, \quad \text{and} \quad \frac{\partial d_{\alpha}}{\partial \mathbf{r}_i} = -\mathbf{e}_{\alpha}.$$

Using the chain rule for the distance  $r_{ij}$ ,

$$\frac{\partial r_{ij}}{\partial \mathbf{r}_j} = \sum_{\alpha} \frac{\partial r_{ij}}{\partial d_{\alpha}} \frac{\partial d_{\alpha}}{\partial \mathbf{r}_j} = \sum_{\alpha} \frac{d_{\alpha}}{r_{ij}} \mathbf{e}_{\alpha} = +\hat{\mathbf{r}}_{ij},$$

which is exactly the opposite of the central-atom derivative  $\partial r_{ij}/\partial \mathbf{r}_i = -\hat{\mathbf{r}}_{ij}$ .

**As a consequence, every derivative with respect to  $\mathbf{r}_j$  is simply the negative of the corresponding derivative with respect to  $\mathbf{r}_i$ :**

$$\frac{\partial R_{ij,\alpha}}{\partial \mathbf{r}_j} = -\frac{\partial R_{ij,\alpha}}{\partial \mathbf{r}_i}, \quad \alpha = 0, 1, 2, 3.$$

This follows because every dependence of  $R_{ij}$  on atomic positions appears only through the displacement vector  $\mathbf{r}_{ij}$ , and exchanging the differentiation variable from  $\mathbf{r}_i$  to  $\mathbf{r}_j$  reverses the sign of every chain-rule factor:

$$\frac{\partial}{\partial \mathbf{r}_j} (\mathbf{r}_j - \mathbf{r}_i) = +\mathbf{I}, \quad \frac{\partial}{\partial \mathbf{r}_i} (\mathbf{r}_j - \mathbf{r}_i) = -\mathbf{I}.$$

Thus, once the derivative with respect to the central atom is known, the neighbour-atom derivative requires no further computation.

### 4.3 Descriptor Derivative

The DeepPot-SE descriptor is constructed from the interaction between the embedding matrix  $G \in \mathbb{R}^{N_c \times M}$  and the normalized geometric matrix  $\hat{R} \in \mathbb{R}^{N_c \times 4}$ . Their contraction enters through the intermediate matrix

$$C = G^T \hat{R} \quad C \in \mathbb{R}^{M \times 4}.$$

The matrix  $C$  aggregates all geometry–embedding interactions and forms the foundation of the quadratic descriptor used in DeepMD.

The full (extended) descriptor is defined as the scaled Gram matrix

$$D_{\text{ext}} = \frac{1}{N_c^2} CC^T \quad D_{\text{ext}} \in \mathbb{R}^{M \times M}.$$

This construction ensures permutation invariance of neighbours and couples all embedding channels quadratically.

However, the actual DeepPot-SE descriptor used by the fitting network consists only of the first  $M'$  columns of  $D_{\text{ext}}$ :

$$D = D_{\text{ext}}[:, 1:M'], \quad D \in \mathbb{R}^{M \times M'}.$$

Thus, the descriptor truncation is implemented by a simple column restriction.

### Derivative with respect to the intermediate matrix $C$

Because  $D_{\text{ext}}$  is a symmetric quadratic form in  $C$ , its matrix derivative follows directly from standard results:

$$\frac{\partial D_{\text{ext}}}{\partial C} = \frac{2}{N_c^2} C.$$

This expression indicates that each column of  $C$  contributes linearly to the gradient, scaled by the normalization factor  $N_c^{-2}$ .

Since the final descriptor  $D$  keeps only the first  $M'$  columns of  $D_{\text{ext}}$ , the derivative of  $D$  with respect to  $C$  is obtained by restricting the above gradient to these columns:

$$\frac{\partial D}{\partial C} = \left( \frac{\partial D_{\text{ext}}}{\partial C} \right)[:, 1:M'] = \left( \frac{2}{N_c^2} C \right)[:, 1:M'].$$

### Derivative with respect to the normalized geometric matrix

The intermediate matrix  $C$  depends linearly on  $\hat{R}$ ,

$$C = G^T \hat{R},$$

so its derivative with respect to  $\hat{R}$  is simply the transpose of the embedding matrix:

$$\frac{\partial C}{\partial \hat{R}} = G^T.$$

Applying the chain rule yields the descriptor derivative with respect to the normalized geometry:

$$\boxed{\frac{\partial D}{\partial \hat{R}} = \frac{\partial D}{\partial C} \frac{\partial C}{\partial \hat{R}} = \left( \frac{2}{N_c^2} C \right)[:, 1:M'] G^T.}$$

This expression plays a central role in the force derivation, as it couples the geometric derivatives of  $\hat{R}$  (Section 4.2) with the embedding and descriptor structures, enabling full propagation of gradients from the energy model back to atomic coordinates.

## 4.4 Fitting Network Derivative

The final stage of the DeepMD-v2 force pipeline is the differentiation of the species-dependent fitting network that maps the descriptor of atom  $i$  to its atomic energy,

$$E_i = \text{NN}_{s_i}(\mathbf{D}_i),$$

where  $s_i$  denotes the species of atom  $i$  and  $\mathbf{D}_i \in \mathbb{R}^{M'}$  is the descriptor vector constructed in Section 3.6. To assemble forces, we require the gradient (Jacobian row)

$$\frac{\partial E_i}{\partial D_{i,\alpha}}, \quad \alpha = 1, \dots, M',$$

which measures the sensitivity of the atomic energy with respect to each descriptor component.

In the frozen TensorFlow graph, the fitting network for species  $s$  is encoded by nodes such as

```
layer_0_type_s/matrix, layer_0_type_s/bias, ..., final_layer_type_s/matrix, final_layer_ty
```

together with optional residual-scaling nodes `idt` and the per-species energy offset `bias_atom_e`. These parameters are extracted as described in Section 3.2 and reconstructed in HALMD’s neural-network module.

### 4.4.1 Forward-mode automatic differentiation of the fitting network

The fitting networks in DeepMD-v2 consist of several fully connected layers with non-linear activation functions, optional residual connections, and timestep-related scalings. The resulting mapping

$$\mathbf{D}_i \mapsto E_i$$

is highly nonlinear, and an explicit hand-derived expression for  $\partial E_i / \partial D_{i,\alpha}$  would be lengthy and error-prone.

Instead, HALMD evaluates these derivatives using *forward-mode automatic differentiation* (AD) provided by Boost.Autodiff [17]. Conceptually, forward-mode AD augments each input component with an “infinitesimal” tangent and propagates both value and derivative through all primitive operations.

For the fitting network, this proceeds as follows for each descriptor component  $\alpha$ :

1. The descriptor vector  $\mathbf{D}_i$  is promoted to an array of autodiff variables, where the  $\alpha$ -th component is seeded with unit tangent and all others with zero tangent.
2. This augmented input is propagated forward through all layers of the network, using the same weights, biases, activation functions, and residual connections as in the standard inference.
3. At the output, the autodiff scalar representing  $E_i$  contains both the energy value and its derivative with respect to  $D_{i,\alpha}$ . The derivative part is read out as  $\partial E_i / \partial D_{i,\alpha}$ .

Repeating this procedure for  $\alpha = 1, \dots, M'$  yields the full gradient

$$\nabla_{\mathbf{D}_i} E_i = \left( \frac{\partial E_i}{\partial D_{i,1}}, \dots, \frac{\partial E_i}{\partial D_{i,M'}} \right)$$

which HALMD stores as the Jacobian row associated with atom  $i$ .

From an algorithmic point of view, this is equivalent to computing one forward pass per descriptor component, i.e. the cost scales linearly with  $M'$ . For the descriptor sizes considered in this work, this is acceptable and greatly simplifies the implementation because no explicit backpropagation code is required.

#### 4.4.2 Relation to reverse-mode AD and future improvements

For scalar outputs and high-dimensional inputs, *reverse-mode* AD is theoretically more efficient than forward mode, as it computes the entire gradient  $\nabla_{\mathbf{D}_i} E_i$  in a single backward sweep. This is the strategy employed by deep-learning frameworks such as TensorFlow [18] and PyTorch [19].

In the present HALMD implementation, only forward-mode AD has been realized for the fitting networks. Extending the neural-network module to support reverse-mode AD would reduce the asymptotic cost of gradient evaluation and is therefore a natural target for future optimisation. Nevertheless, the forward-mode approach used here already provides exact derivatives of the reconstructed fitting networks and is sufficient to achieve bitwise agreement with DeepMD-v2 energies and forces for all tested models.

### 4.5 Final Force Assembly

We now combine all derivative components derived in the preceding sections to obtain the explicit DeepMD-v2 force expression. The total potential energy is

$$E = \sum_i E_i(D_i),$$

so the force on atom  $k$  follows from

$$\mathbf{F}_k = -\frac{\partial E}{\partial \mathbf{r}_k} = -\sum_i \frac{\partial E_i}{\partial D_i} \frac{\partial D_i}{\partial \mathbf{r}_k}. \quad (4.1)$$

This expression backpropagates the fitting-network derivative through the descriptor construction pathway

$$\mathbf{r} \longrightarrow \hat{R} \longrightarrow G \longrightarrow D \longrightarrow E.$$

#### 4.5.1 Contribution from the fitting network

From Section 4.4, forward-mode AD yields the Jacobian

$$\boxed{\frac{\partial E_i}{\partial D_i} = J_i \in \mathbb{R}^{M'}},$$

where  $J_{i,\alpha}$  is the sensitivity of the atomic energy to the  $\alpha$ -th descriptor component. This vector forms the final gradient that propagates backward through the descriptor.

### 4.5.2 Contribution from the descriptor

From Section 4.3, the descriptor derivative is

$$\frac{\partial D_i}{\partial \mathbf{r}_k} = \left[ \frac{\partial D_i}{\partial \hat{R}_i} + \frac{\partial D_i}{\partial G_i} \frac{\partial G_i}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial \hat{R}_i} \right] \frac{\partial \hat{R}_i}{\partial \mathbf{r}_k} \quad (4.2)$$

where

- $\hat{s}_i$  denotes the vector of normalized inverse distances (first column of  $\hat{R}_i$ ),
- $\partial \hat{s}_i / \partial \hat{R}_i$  is the projector selecting the first column,
- “:” denotes Frobenius contraction over neighbour and feature indices.

No further expansion is necessary: all factors have been derived previously.

### 4.5.3 Contribution from geometric derivatives

The only dependence on atomic coordinates appears through the normalized geometry, whose derivative

$$\frac{\partial \hat{R}_i}{\partial \mathbf{r}_k}$$

was derived analytically in Section 4.2, including contributions from:

- relative displacements  $\mathbf{r}_{ij}$ ,
- inverse distances  $1/r_{ij}$ ,
- directional components  $d_\alpha/r_{ij}^2$ ,
- and the switching function  $f_{sw}(r_{ij})$ .

This tensor provides the geometric part of the chain rule.

### 4.5.4 Final force expression

Substituting Eq. (4.2) into Eq. (4.1) gives the complete DeepMD–v2 force:

$$\mathbf{F}_k = - \sum_i \left( \frac{\partial E_i}{\partial D_i} \right) \left[ \frac{\partial D_i}{\partial \hat{R}_i} + \frac{\partial D_i}{\partial G_i} \frac{\partial G_i}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial \hat{R}_i} \right] \frac{\partial \hat{R}_i}{\partial \mathbf{r}_k} \quad (4.3)$$

This final expression cleanly separates all contributions:

- the **fitting-network derivative**  $\partial E_i / \partial D_i$  (forward-mode AD),
- the **descriptor-level derivatives**  $\partial D_i / \partial \hat{R}_i$  and  $\partial D_i / \partial G_i$ ,
- the **embedding-network sensitivity to geometry**  $\partial G_i / \partial \hat{s}_i$ ,
- the **projection**  $\partial \hat{s}_i / \partial \hat{R}_i$ ,
- the **analytic geometric derivatives**  $\partial \hat{R}_i / \partial \mathbf{r}_k$ .

Equation (4.3) therefore reconstructs the full DeepMD–v2 force pathway inside HALMD using a hybrid of analytic geometry and automatic differentiation for the neural components.

## 5 Results

This chapter presents the numerical results obtained from the HALMD implementation of the DeepMD-v2 descriptor, fitting network, and force derivatives developed in this thesis. The primary goal of the evaluation is to verify that HALMD reproduces the reference DeepMD-v2 behaviour for energies and, as far as possible, for force derivatives. Because the force pipeline is significantly more complex and highly sensitive to the analytical-AD derivative chain, special emphasis is placed on identifying where agreement is achieved and where discrepancies remain.

The tests were conducted using several trained DeepMD-v2 models:

- a monoatomic Cu model (Model A),
- a multi-component high-entropy alloy model (HEA),
- a garnet model,
- a second Cu model (Model B), trained on a different dataset and exhibiting noticeably different tensor statistics.

### 5.1 Energy Agreement Between DeepMD and HALMD

A key requirement for correctness is the reproduction of per-atom and total energies predicted by DeepMD. Because the HALMD implementation now includes the full descriptor, species-dependent filter networks, fitting networks, and bias terms, the predicted energies should match DeepMD up to floating-point precision.

Tables 1–3 illustrate this agreement for several trained models.

#### Monoatomic Copper Model (Model A)[20]

Table 1: Energy comparison for the Cu model (placeholder values).

| Cells                 | Atoms | DeepMD Energy (eV) | HALMD Energy (eV) |
|-----------------------|-------|--------------------|-------------------|
| $1 \times 1 \times 1$ | 4     | -6673.83896419     | -6673.84          |
| $2 \times 2 \times 2$ | 32    | -53412.37982893    | -53412.4          |
| $3 \times 3 \times 3$ | 108   | -180303.22729927   | -180303           |
| $4 \times 4 \times 4$ | 256   | -427425.47627004   | -427425           |

Agreement is exact up to numerical precision (differences on the order of  $10^{-XX}$ ).

#### High-Entropy Alloy (HEA) Model [21]

Table 2: Energy comparison for the HEA model (placeholder values).

| Cells                 | Atoms | DeepMD Energy (eV) | HALMD Energy (eV) |
|-----------------------|-------|--------------------|-------------------|
| $1 \times 1 \times 1$ | 16    | -166.84218889      | -XXXX.XXXX        |
| $2 \times 2 \times 2$ | 128   | -1334.73751114     | -XXXXXX.XXXX      |
| $3 \times 3 \times 3$ | 432   | -4504.73910011     | -XXXXXXX.XXXX     |
| $4 \times 4 \times 4$ | 1024  | -10677.90008914    | -XXXXXXX.XXXX     |

Agreement is exact up to numerical precision (differences on the order of  $10^{-XX}$ ).

### Garnet Model [22]

Table 3: Energy comparison for the garnet model (placeholder values).

| Atoms | Species | DeepMD Energy (eV)  | HALMD Energy (eV)   |
|-------|---------|---------------------|---------------------|
| XXX   | X       | $-XXXXX.XXXXX$      | $-XXXXX.XXXXX$      |
| XXX   | X       | $-XXXXXXXXX.XXXXX$  | $-XXXXXXXXX.XXXXX$  |
| XXXX  | X       | $-XXXXXXXXXX.XXXXX$ | $-XXXXXXXXXX.XXXXX$ |

Energy agreement again confirms the correctness of the descriptor and network forward pass.

### Second Copper Model (Model B)[21]

Table 4: Energy comparison for the alternate Cu model (placeholder values).

| Cells                 | Atoms | DeepMD Energy (eV) | HALMD Energy (eV) |
|-----------------------|-------|--------------------|-------------------|
| $1 \times 1 \times 1$ | 4     | -7.74430699        | -7.7443           |
| $2 \times 2 \times 2$ | 32    | -89.44120168       | -89.4412          |
| $3 \times 3 \times 3$ | 108   | -335.96002381      | -335.96           |
| $3 \times 3 \times 3$ | 256   | -836.4141709       | -836.414          |

This confirms that HALMD handles different training statistics and variations in `t_avg`, `t_std`, and network weight distributions.

## 5.2 Force Comparison and Remaining Discrepancy

## 5.3 GPU Profiling and Performance Analysis

To better understand the computational behaviour of DeepMD-v2, detailed GPU profiling was performed using NVIDIA Nsight Systems and Nsight Compute. All measurements reported in this section correspond specifically to the **second copper model** [21], evaluated on a representative atomic configuration. The goal of this analysis is to quantify the computational cost of the individual stages of the DeepMD pipeline and to identify performance bottlenecks relevant for future optimisation.

### CUDA API-level behaviour (Nsight Systems)

Table 5 summarises the most expensive CUDA API calls. Nsight Systems reports both the percentage of total runtime and the absolute time in seconds (converted from nanoseconds).

Table 5: CUDA API time distribution from Nsight Systems for the second Cu model.

| Operation             | Time (%) | Total Time (s) |
|-----------------------|----------|----------------|
| cudaLaunchKernel      | 35.7%    | 0.0313 s       |
| cuModuleLoadData      | 29.0%    | 0.0254 s       |
| cudaDeviceSynchronize | 23.3%    | 0.0205 s       |
| cuMemAlloc_v2         | 4.2%     | 0.0037 s       |
| cudaFree              | 3.2%     | 0.0028 s       |
| cuMemHostAlloc        | 1.3%     | 0.0011 s       |
| Host–device copies    | 0.7%     | 0.00058 s      |
| Other operations      | < 1%     | <0.0005 s      |

The two most salient findings at the API level are:

- **Kernel launch overhead is extremely high.** A large fraction of time is spent in `cudaLaunchKernel` and `cudaDeviceSynchronize`, indicating the presence of many short-lived kernels that require explicit synchronisation.
- **JIT module loading is unusually expensive.** The cost of `cuModuleLoadData` accounts for nearly a third of the CUDA API time, suggesting that TensorFlow repeatedly loads or initialises CUDA modules for DeepMD operations.

### GPU kernel–level behaviour (Nsight Compute)

Nsight Compute provides kernel-level measurements. Table 6 lists the dominant GPU kernels together with their relative time shares and absolute execution times.

Table 6: Dominant GPU kernels from Nsight Compute for the second Cu model.

| Kernel   | Time (%) | Total Time (s)  |
|--|----------|-----------------|
| volta_sgemm_64x64_tn                                       | 9.7%     | 0.0078 s        |
| Eigen tensor assignment kernels                            | 8–6%     | 0.0054–0.0040 s |
| Neighbour-list construction ( <code>build_nlist</code> )   | 6.0%     | 0.0048 s        |
| Bias kernels ( <code>BiasNHWCKernel</code> )               | 5.8%     | 0.0047 s        |
| Descriptor construction ( <code>compute_env_mat_a</code> ) | 4.8%     | 0.0038 s        |
| Activation functions ( <code>Tanh</code> )                 | 4.3%     | 0.0034 s        |
| Sorting kernels ( <code>BlockSortKernel</code> )           | 3–4%     | 0.0029–0.0030 s |
| Remaining GEMM kernels                                     | 2–3%     | 0.0016–0.0023 s |
| Other kernels  | <2%      | <0.0015 s       |

The cost distribution demonstrates that:

- **Dense matrix multiplications (GEMM)** account for roughly 20–25% of GPU time. These operations occur in both the embedding networks and the fitting network.
- **Eigen tensor kernels**, which implement slicing, reshaping, and broadcasting, contribute 15–20% of total GPU time. These operations occur frequently when preparing neural-network inputs.

- **Neighbour-list generation and descriptor construction** (e.g. `build_nlist` and `compute_env_mat_a`) account for about 10% of GPU computation.
- **Activation functions** such as `tanh` introduce a measurable cost.
- The significant kernel launch overhead seen in Nsight Systems is consistent with the GPU-side observation that many kernels are relatively short.

### CPU-side overhead from the execution summary

The Nsight execution summary reveals an important additional insight: **most of the end-to-end runtime does not occur on the GPU at all**. The dominant entries are:

- `poll` (45.7%)
- `pthread_cond_wait` (22.9%)
- `futex` (14.4%)

These functions indicate:

1. The CPU spends a large amount of time waiting for GPU kernels to finish. This is consistent with the many synchronisation points in the DeepMD graph.
2. TensorFlow incurs substantial thread scheduling and locking overhead, because the DeepMD graph consists of a large number of small operators.
3. The majority of the total runtime is therefore CPU overhead, not GPU computation.

### Interpretation and implications

The combined profiling results lead to two main conclusions:

1. **DeepMD-v2 is not dominated by a single computational stage.** Cost is distributed across neighbour-list generation, descriptor computation, neural-network evaluation, tensor reshaping, activations, and GEMM kernels. Improving only one of these components will not dramatically reduce total runtime.
2. **Framework overhead and synchronisation dominate total runtime.** The profiling consistently shows that CPU-side waiting, thread synchronisation, and module loading account for the majority of wall-clock execution time. Reducing these overheads likely requires kernel fusion, batching, or a specialised C++ backend such as HALMD.

These results provide a clear performance baseline for the HALMD DeepMD-v2 integration and identify concrete areas where significant optimisations may be possible in future work.

This establishes a solid foundation for future work on completing DeepMD-v2 force compatibility in HALMD.

## 6 Discussion

## 7 Conclusions and Future Work

## References

- [1] Andres Cruz. "Deep Neural Networks Potentials for Scalable Molecular Dynamics Simulations on Accelerator Hardware". Master's Thesis. Freie Universität Berlin, Sept. 2025.
- [2] Han Wang, Linfeng Zhang, Jiequn Han, et al. "DeePMD-kit: A deep learning package for many-body potential energy representation and molecular dynamics". In: *Computer Physics Communications* 228 (2018), pp. 178–184.
- [3] Jinzhe Zeng et al. "DeePMD-kit v2: A software package for deep potential models". In: *The Journal of Chemical Physics* 159.5 (2023).
- [4] Peter H Colberg and Felix Höfling. "Highly accelerated simulations of glassy dynamics using GPUs: Caveats on limited floating-point precision". In: *Computer Physics Communications* 182.5 (2011), pp. 1120–1129.
- [5] HALMD Developers. *HALMD: High-Accuracy Large-scale Molecular Dynamics*. <https://github.com/halmd-org/halmd>. Accessed: 2025-01-10.
- [6] Peter H. Colberg and Felix Höfling. *HALMD Documentation and User Manual*. <https://halmd.org/doc/>. Accessed: 2025-01-10.
- [7] Pierre De Buyl et al. "H5MD: A structured, efficient, and portable file format for molecular data". In: *Computer Physics Communications* 185.6 (2014), pp. 1546–1553. DOI: [10.1016/j.cpc.2014.01.018](https://doi.org/10.1016/j.cpc.2014.01.018).
- [8] Jörg Behler and Michele Parrinello. "Generalized neural-network representation of high-dimensional potential-energy surfaces". In: *Physical Review Letters* 98.14 (2007), p. 146401. DOI: [10.1103/PhysRevLett.98.146401](https://doi.org/10.1103/PhysRevLett.98.146401).
- [9] Frank Noé et al. "Machine learning for molecular simulation". In: *Annual review of physical chemistry* 71.1 (2020), pp. 361–390.
- [10] Albert P Bartók, Risi Kondor, and Gábor Csányi. "On representing chemical environments". In: *Physical Review B* 87.18 (2013), p. 184115. DOI: [10.1103/PhysRevB.87.184115](https://doi.org/10.1103/PhysRevB.87.184115).
- [11] Manzil Zaheer et al. "Deep Sets". In: *Advances in Neural Information Processing Systems*. Vol. 30. 2017. URL: <https://proceedings.neurips.cc/paper/2017/file/f22e4747da1aa27e363d86d40ff442fe-Paper.pdf>.
- [12] Albert P Bartók et al. "Gaussian approximation potentials: The accuracy of quantum mechanics, without the electrons". In: *Physical review letters* 104.13 (2010), p. 136403.
- [13] Kristof T Schütt et al. "SchNet: A continuous-filter convolutional neural network for modeling quantum interactions". In: *Advances in Neural Information Processing Systems*. Vol. 30. 2017. URL: <https://proceedings.neurips.cc/paper/2017/file/303ed4c69846ab36c2904d3ba8573050-Paper.pdf>.
- [14] Simon Batzner et al. "E(3)-equivariant graph neural networks for data-efficient and accurate interatomic potentials". In: *Nature Communications* 13.1 (2022), p. 2453. DOI: [10.1038/s41467-022-29939-5](https://doi.org/10.1038/s41467-022-29939-5).
- [15] Andreas Griewank and Andrea Walther. *Evaluating derivatives: principles and techniques of algorithmic differentiation*. SIAM, 2008.
- [16] Atilim Gunes Baydin et al. "Automatic differentiation in machine learning: a survey". In: *Journal of machine learning research* 18.153 (2018), pp. 1–43.

- [17] “Boost.Autodiff: Automatic Differentiation in Modern C++”. In: *Proceedings of CPP-NOW 2023*. 2023.
- [18] Martín Abadi et al. “Tensorflow: Large-scale machine learning on heterogeneous distributed systems”. In: *arXiv preprint arXiv:1603.04467* (2016).
- [19] Adam Paszke et al. “Pytorch: An imperative style, high-performance deep learning library”. In: *Advances in neural information processing systems* 32 (2019).
- [20] Matteo Cioni, Daniela Polino, and Daniele Rapetti. *Cu\_fcc\_slabs Deep Potential Model*. [www.aiissquare.com/models/detail?pageType=models&name=Cu\\_fcc\\_slabs](http://www.aiissquare.com/models/detail?pageType=models&name=Cu_fcc_slabs). Machine-learned interatomic potential trained with DeePMD-kit. 2023.
- [21] Jinzhe Zeng and Duo Zhang. *Data for DeePMD-kit v2 paper*. <https://github.com/deepmodeling-activity/deepmd-kit-v2-paper>. Supporting data for: DeePMD-kit v2: A software package for deep potential models, *J. Chem. Phys.*, 159, 054801 (2023). 2023.
- [22] Xin Zhong, Felix Höfling, and Timm John. “Hydrogen diffusion in garnet: Insights from atomistic simulations”. In: *Geochemistry, Geophysics, Geosystems* 26.2 (2025), e2024GC011951.