Consider $\Omega =]0,1[\times]0,1[$ and the boundary value problem

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} & \text{in } \Omega \\ u(x,0) = u_0(x) & x \in [0,1] \\ u(a,t) = u_a(t) & t \in]0,1] \\ u(b,t) = u_b(t) & t \in]0,1], \end{cases} \tag{41}$$

where u_0, u_a and u_b shall be determined in such a way that the solution of the problem (42) is $u(x,t) = \sin(\pi x)e^{-\pi^2 t}$.

1. Write a Matlab routine that receives the value of N_x and N_t, the number of sub-intervals in space and time, resp. and calculates the numerical solution of the problem by using the explicit method.
2. Test the routine by taking several choices of N_x and N_t, keeping the ratio $\frac{h_t}{h_x^2}$ constant and estimate the order of convergence.
3. Write a Matlab routine that solves the problem using the implicit method.