Computational Methods in Finance
Computational project 1

18 March 2025

Instructions

This project shall be solved in groups of three or four people. The solution of the project shall be sent by email to prsantunes@tecnico.ulisboa.pt by 21 April 2025. The submission must include a pdf file with a short report and a zip file contained all the Matlab files that were developed to solve the computational part of the project. Please include a Matlab script allowing to reproduce all the numerical results and figures that are presented in the report.

1. Consider the region

$$\mathcal{R}_V^T = \{(S,t),\ 0 < S < S^*,\ 0 \le t \le T\}\,, \tag{1}$$

for a sufficiently large S^* and the terminal/boundary value problem for Black-Scholes equation defining $V(S,t)$ to be the value of an option at the point (S,t). In order to replace the terminal value problem associated with Black-Scholes equation by an initial value problem we perform a change of variables $U(S,t) := V(S,T-t)$ and consider the problem

$$\begin{cases} \frac{\partial U}{\partial t} = \frac{\sigma^2}{2} S^2 \frac{\partial^2 U}{\partial S^2} + rS\frac{\partial U}{\partial S} - rU & \text{in } \mathcal{R}_V^T \\ U(S,0) = u_0(S) & S \in [0,S^*] \\ U(0,t) = u_a(t) & t \in [0,T] \\ U(S^*,t) = u_b(t) & t \in [0,T] \end{cases} \tag{2}$$

for some functions u_0, u_a and u_b that depend on the type of option and are assumed to be known.

   (a) Write a Matlab routine for solving problem (2) using the method of lines using fourth order Runge-Kutta method for solving the system of ODEs.

   (b) Calculate and plot the solution of (2) obtained by the numerical method in the case of a European put option with the parameters $r = 0.06$, $\sigma = 0.3$, $T = 1$, $K = 10$ and taking $S^* = 15$.

2. Write a Matlab routine for pricing an American option, using Crank-Nicolson method and the Projected Successive Over-relaxation (PSOR) method. Apply it to calculate and plot the solution in the continuation region of an American put option with the parameters $r = 0.06$, $\sigma = 0.3$, $T = 1$, $K = 10$ and taking $S^* = 15$.