Department of Computer Science

California State University, Channel Islands

MATHCOMPPH-546: Pattern Recognition

Lesson 1 phys546 Review of The Fourier Domain RE_1B

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7.5 Consider an image that is black except for a single pixel wide stripe from the top left to the bottom right (Fig. E7.1, top left). Can you explain its Fourier transform (Fig. E7.1, bottom left)? Also, consider an image of noise (Fig. E7.1, top right), i.e. every pixel has a random value, independent of all other pixels. Can you explain its Fourier transform (Fig. E7.1, bottom right)? What does the bright spot in the middle of the noise Fourier transform image represent? Why does the Fourier transform of the noise appear dark gray?

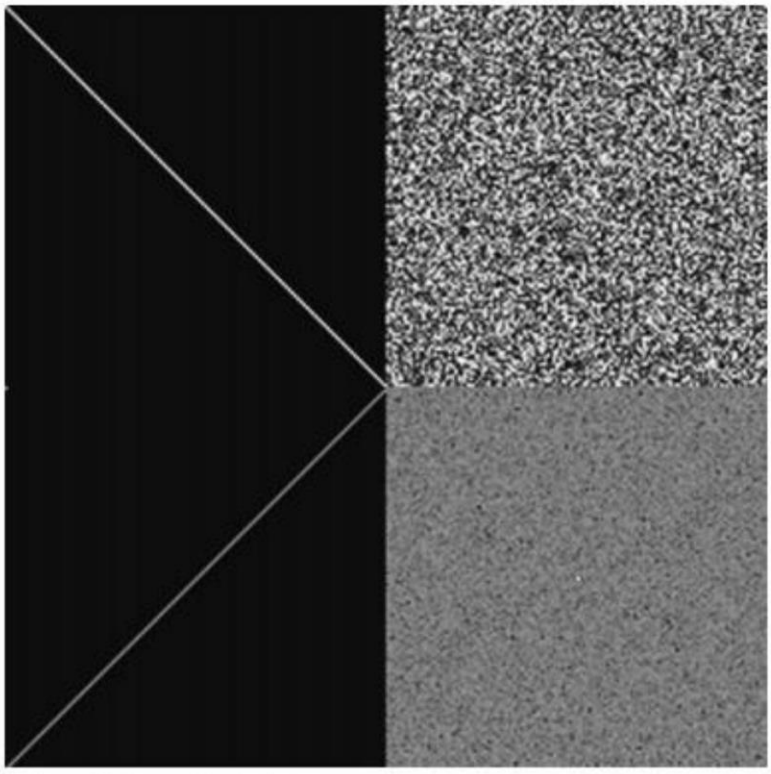
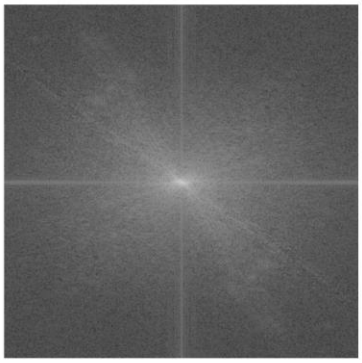
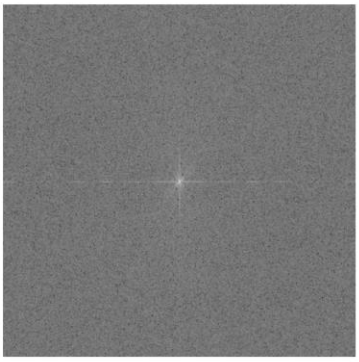


Figure E7.1 Stripe and noise, and their Fourier transforms.

Ans. The line in the image is an impulse function, and its fourier transform is constant from -Inf to +Inf. It's a solid line rather than a dotted line since we need to do an impulse function, which necessitates all frequencies.

7.6 What is the result of performing a Fourier transform on the Fourier transform of an image? Try it out. Can you explain the result?

Ans.

A double Fourier transformation to an image has few mathematical applications, but it does have many computing ones. It aids in addressing the order of magnitude of mistake due to the size of the signal manipulated in terms of calculation.

7.10 What sampling frequency should be used to avoid aliasing if the full width at half maximum, FWHM, of the point spread function is 5 mm?

Ans. It's a function defined by the difference of two extreme values of the independent variable, at which the dependent variable equals half of its maximum value, according to WHM.

$$FWHM = {f_{max}}/{2} \quad \therefore \quad {1}/{2} = e^{5|x|} \quad \rightarrow \quad \ln({1}/{2}) = 5|x|$$

$$|x| = 0.139, \qquad -0.139 \leq x \leq 0.139$$

$$FWHM = \mathbf{0.2772529}$$

7.11 A computed tomography imaging system has a point spread function with a full width at half maximum, FWHM, of 2 mm. How small do the image pixels have to be to avoid aliasing problems? For a field of view of 50 cm how many pixels are required along each side of the image?

Ans. We need to keep the pixel size under 1mm for the point spread function with an FWHM of 2mm and to avoid aliasing. With a view field of 50 cm and pixel size=1mm (due to FWHM), we need at least 500mm (50 cm) x 1mm, or 500 pixels on each side of the image, which implies we need a 512x512 image.

7.12 How can an analog image be recovered from its sampled (digitized) version? Describe the operations required in (i) the spatial domain and (ii) the frequency domain. Comment on the conditions to be met for accurate recovery of the analog image.

Ans. We'll need some interpolation with Smoothing and Sharpening for an analog image recovery. Linear or non-linear smoothing in the spatial domain is possible. Non-Linear filters include Min, Max, Mid, and Median. Linear filters include Box and Weight average filters. First order derivative (Gradient) and Second order derivative sharpening can be used in the spatial domain (Laplacian). Smoothing (ideal low passes, etc.) is done in the frequency domain.

Low pass filters (Butterworth, Gaussian) and sharpening (Ideal high pass, Butterworth high pass, Gaussian high pass). My suggestion for accurate analog picture recovery is to employ perfect interpolation, which is tough to achieve and not a simple operation.