Phys/Comp 510 Advanced Image Analysis

HW#5B

Read through ch.6 of the textbook 'Digital Image Processing for Medical Applications', and do questions 7.5, 7.6, 7.10, 7.11 and 7.12.

7.5 Consider an image that is black except for a single pixel wide stripe from the top left to the bottom right (Fig. E7.1, top left). Can you explain its Fourier transform (Fig. E7.1, bottom left)? Also, consider an image of noise (Fig. E7.1, top right), i.e. every pixel has a random value, independent of all other pixels. Can you explain its Fourier transform (Fig. E7.1, bottom right)? What does the bright spot in the middle of the noise Fourier transform image represent? Why does the Fourier transform of the noise appear dark gray?

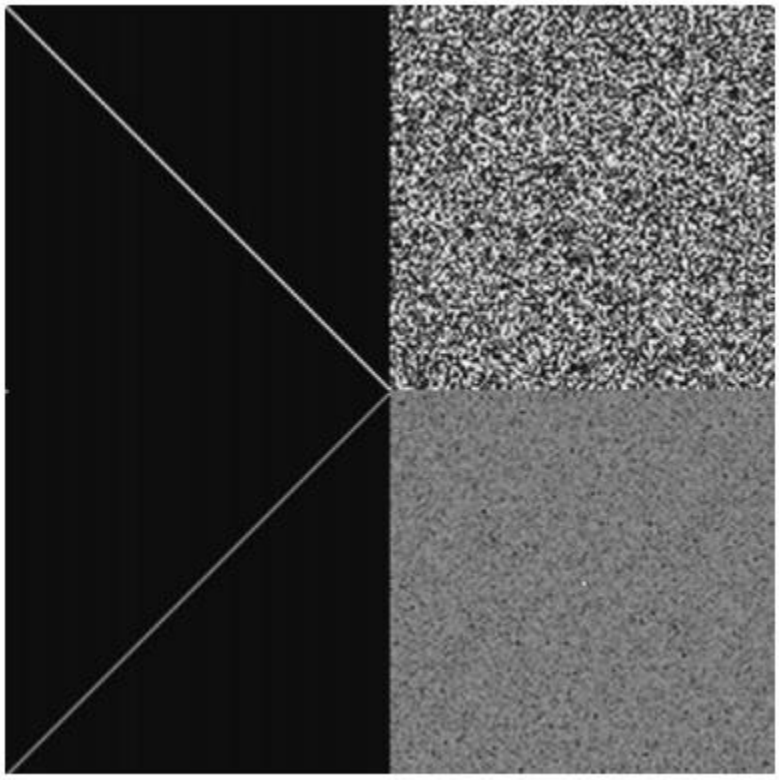
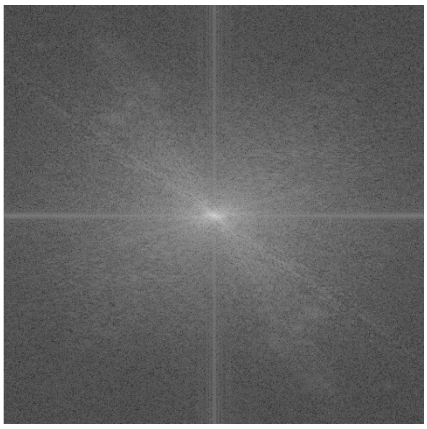
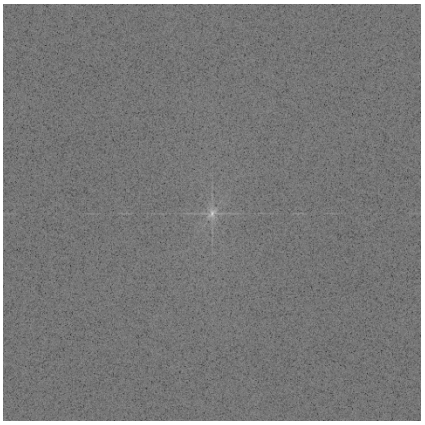


Figure E7.1 Stripe and noise, and their Fourier transforms.

The line given in the image is a impulse function and its fourier transform is constant over all frequencies from -Inf to +Inf. It is a solid line instead of dotted line because we have to impulse function and for it we require all the frequencies.

7.6 What is the result of performing a Fourier transform on the Fourier transform of an image? Try it out. Can you explain the result?

Mathematically there are not many applications of performing a double Fourier transformation to an image, but computationally it does. In terms of computation, it helps handling the order of magnitude of the error due to the size of the signal manipulated.

7.10 What sampling frequency should be used to avoid aliasing if the full width at half maximum, FWHM, of the point spread function is 5 mm?

For WHM, it is a function given by the difference of two extreme values of the independent variable, at which the dependent variable is equal to half of its max value.

$$FWHM = {f_{max}}/{2} \qquad \therefore \qquad {1}/{2} = e^{5|x|} \qquad \rightarrow \qquad \ln({1}/{2}) = 5|x|$$

$$|x| = 0.139, \qquad -0.139 \leq x \leq 0.139$$

$$FWHM = \mathbf{0.2772529}$$

7.11 A computed tomography imaging system has a point spread function with a full width at half maximum, FWHM, of 2 mm. How small do the image pixels have to be to avoid aliasing problems? For a field of view of 50 cm how many pixels are required along each side of the image?

For the point spread function with FWHM of 2mm and to avoid aliasing problem we need to keep the pixel size less than 1mm. For a view field of 50 cm, and considering pixel size=1mm (due to FWHM), it requires at least 500mm (50 cm) x 1mm, i.e. 500 pixels on each side of the image, which means we need a 512x512 image to fit the image.

7.12 How can an analog image be recovered from its sampled (digitized) version? Describe the operations required in (i) the spatial domain and (ii) the frequency domain. Comment on the conditions to be met for accurate recovery of the analog image.

For an analog image recovery, we need some suitable interpolation with operations such as Smoothing and Sharpening. Smoothing in **spatial domain** can be Linear or Non-Linear. Linear can be either Box filter or Weight average filter; Non-Linear can be any of Min, Max, Mid or Median. Sharpening in spatial domain can be First order derivative (Gradient) and Second order derivative (Laplacian). While the operations in **frequency domain** are smoothing (Ideal low passes, Butterworth low pass, Gaussian low pass) and sharpening (Ideal high pass, Butterworth high pass, Gaussian high pass).

My comment on the **condition for accurate recovery of analog image** would be that we can use ideal interpolation -which is difficult accomplish and not a trivial task.