Unit II.6

Planes parallel to the coordinate planes.

The graph of the eqn $x = a$ in an xyz - consists of all points of the form (a, y, z), where y and z are arbitrary.

☐ $x = a$ is parallel to yz plane. $(a, 0, 0)$ $(a, \text{arbi}, \text{arbi})$

☐ $y = b$ is parallel to the zx plane.

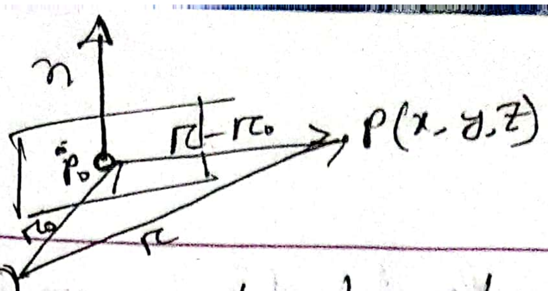
(arbitrary point, b, arbitrary point).

☐ $z = c$ is parallel to the xy plane

(arbitrary point, arbitrary point, c).

☐ Planes determined by a point and a normal vector.

A Plane will be determined by a point and a normal vector.



here
normal vector = perpendicular to the plane

a point = a point on the plane.

suppose that we want to find an eqn of the equation of the plane passing through $P_0(x_0, y_0, z_0)$ and perpendicular to the vector

$n = \langle a, b, c \rangle$

Define the vectors r_0 and r as

$r_0 = \langle x_0, y_0, z_0 \rangle$ and $r = \langle x, y, z \rangle$

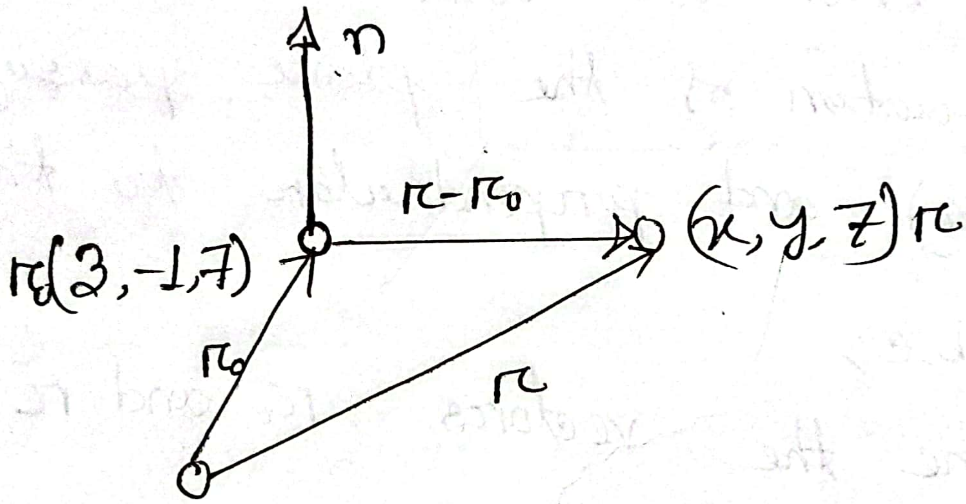
$$n \cdot (r - r_0) = 0.$$

$$\langle a, b, c \rangle \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

This is called the point-normal-form of the eqn of a plane.

Example 1 Find an eqn of the plane passing through the point $(3,-1,7)$ and perpendicular to the vector $n = \langle 4, 2, -5\rangle$



$r - r_0$

$$\langle x-3,\ y+1,\ z-7\rangle \cdot \langle 4, 2, -5\rangle = 0.$$

$$4(x-3) + (2)(y+1) + (-5)(z-7) = 0.$$

$$4x - 12 + 2y + 2 - 5z + 35 = 0.$$

$$\boxed{4x + 2y - 5z + 25 = 0.}$$

$\boxed{4}$ $ax+by+cz+d=0$.
is a plane that has the vector
$n=\langle a,b,c\rangle$ as a normal.

Example 2. Determine whether the planes

$3x-4y+5z=0$ and $-6x+8y-10z-4=0$.
are parallel.

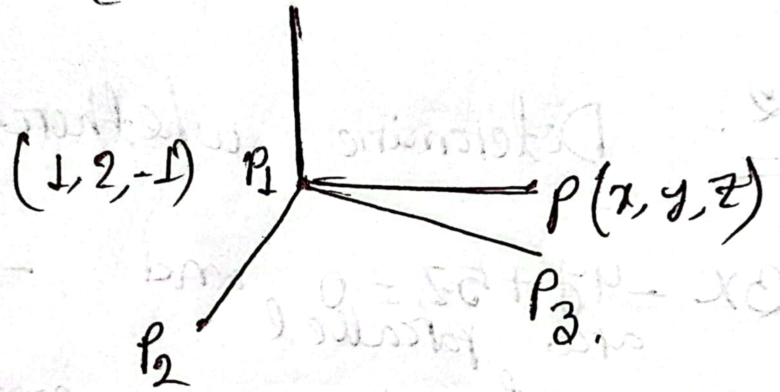
For first eqn normal of the plane $n_1=\langle 3,-4,5\rangle$

For second eqn normal of the plane $n_2=\langle -6,8,-10\rangle$

$$\boxed{n_1=-2n_1.}$$

so the noromals are parallel, and hence so are the planes.

Example 3: Find an eqn of the plane through the points $P_1(1,2,-1)$, $P_2(2,3,1)$ and $P_3(3,-1,2)$.



calculate $\vec{P_1P_2}$, $\vec{P_1P_3}$

$\vec{P_1P_2} \times \vec{P_1P_3}$ = normal.

$P_1P \cdot n = 0$

which is the equation of the plane.

Example 4.

Determine whether the line

, $x = 3+8t,\ y = 4+5t,\ z = -3-t.$

is parallel to the plane $x - 3y + 5z = 12.$

$L: x = 3+8t,\ y = 4+5t,\ z = -3-t.$

vector $v = \langle 8, 5, -1 \rangle$

and from the plane's eqn $x - 3y + 5z - 12 = 0$

normal, $n = \langle 1, -3, 5 \rangle.$

~~v.n~~

For the line and plane to be parallel, the vectors v and n must be orthogonal (right angle)

$v.n = 8 - 15 - 5$

$= -12$

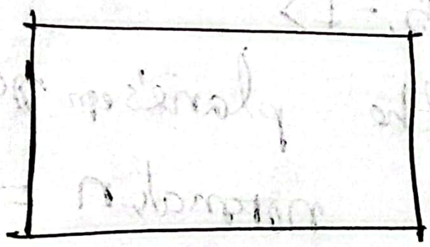
Thus this line and plane are not parallel.

~~Example 4: Find the acute an~~

Example 5: Find the intersection of the line and the plane

L: $x = 3+8t$, $y = 4+5t$, $z = -3-t$.

and the plane: $x - 3y + 5z - 12 = 0$.



Let (x_0, y_0, z_0) be the point of intersection, then the coordinates of this point will both satisfy the plane's eqn and parametric eqn of the line.

$$x_0 - 3y_0 + 5z_0 = 12.$$

$$x_0 = 3+8t_0, \quad y_0 = 4+5t_0, \quad z_0 = -3-t_0$$

$$3(3+8t_0) - 3(4+5t_0) + 5(-3-t_0) = 12.$$

$$t_0 = -3.$$

$(x_0, y_0, z_0) = (-21, -11, 0)$.

Example 6 Find the acute angle of intersection between the two planes.

$$2x - 4y + 4z = 6 \quad \text{and} \quad 6x + 2y - 3z = 4.$$

normal $n_1 = \langle 2, -4, 4 \rangle$ normal $n_2 = \langle 6, 2, -3 \rangle$.

$$n_1 \cdot n_2 = \|n_1\| \, \|n_2\| \cos\theta.$$

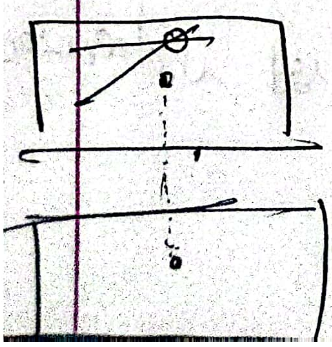
$$\cos\theta = \frac{n_1 n_2}{\|n_1\| \, \|n_2\|}$$

$$\theta = 79^\circ.$$

Example 7: Find an equation for the line $\boxed{L \text{ of intersection}}$ of the planes

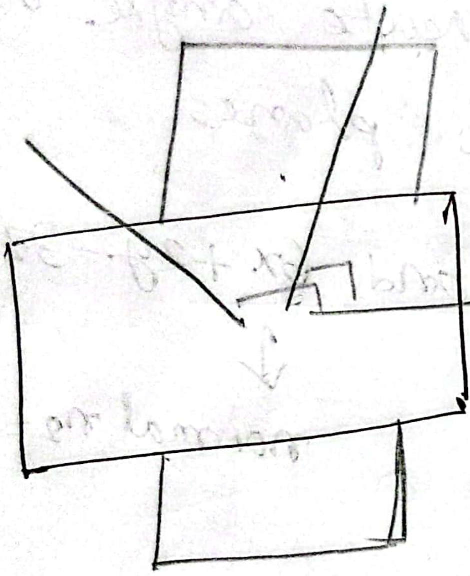
~~in~~ $2x - 4y + 4z = 6$ and

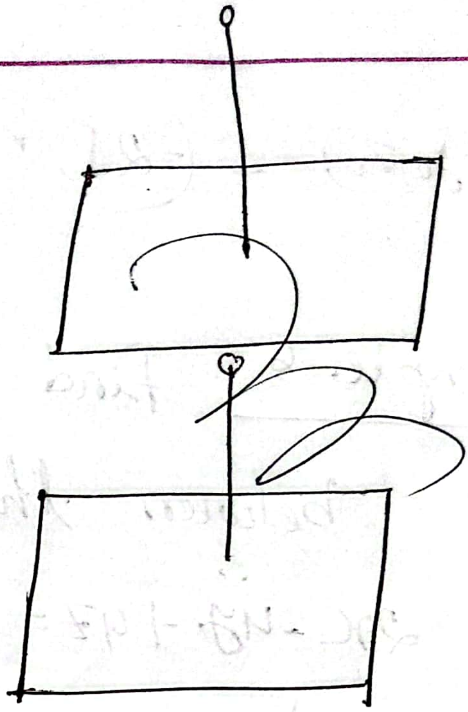
$6x + 2y - 3z = 4$.



$n_1 = \langle 2, -4, 4 \rangle$.

$n_2 = \langle 6, 2, -3 \rangle$.





$$v = n_1 \times n_2$$
$$= \langle 2, -4, 4 \rangle \times \langle 6, 2, -3 \rangle$$
$$= \langle 4, 30, 28 \rangle$$

v is orthogonal to n_1, n_2 of the first and second plane.

That is v is parallel to L, the intersections of the two planes.

To find a point on L must intersect the xy-plane, $z=0$,

$2x - 4y = 6.$

$6x + 2z = 4.$

with solution $x = 1, y = -1.$

Thus $P(+1, -1, 0)$ is a point L.

A vector equation for L.

$$\langle x, y, z \rangle = (1, -1, 0) + t(4, 30, 28).$$

Example 9.

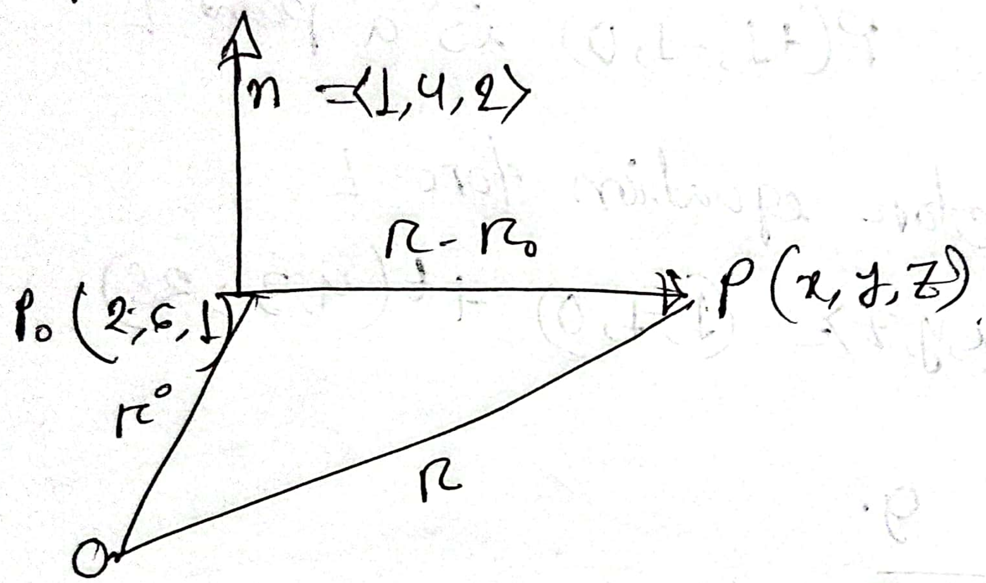
The planes

$x + 2y - 2z = 3$ and $2x + 4y - 4z = 7.$

are parallel since their normals, $\langle 1, 2, -2 \rangle$ and $\langle 2, 4, -4 \rangle$ are parallel vectors. Find the distance between these planes

3-6.

Find an eqn of the plane that passes through.



$r - r_0 = \langle x-2,\ y-6,\ z-1 \rangle$

$n \cdot (r - r_0) = 0.$

$1(x-2) + 4(y-6) + 2(z-1) = 0$

$x - 2 + 4y - 24 + 2z - 2 = 0$

$$\boxed{x + 4y + 2z - 28 = 0.}$$

11.-12. Find an equation of the plane that passes through the given points

11. $P_1(-2,1,1)$ $P_2(0,2,3)$ and $P_3(1,0,-1)$.

calculate $\overrightarrow{P_1P_2}$

$\overrightarrow{P_1P_3}$.

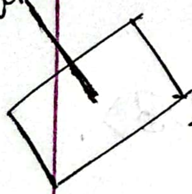
$$n = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}.$$

P_1P

$V.n=0$

parallel.

$\boxed{n \cdot P_1P = 0.}$ eqn.



13-14 Determine whether the planes are parallel, perpendicular or neither.

line

$\cos\theta$

$\cos 90^\circ = 0$

(a) $2x-8y-6z-2=0$. $n_1 = \langle 2,-8,-6\rangle$

$-x+4y+3z-5=0$ $n_2 = \langle -1,4,3\rangle$

$\cancel{n_1 \times}$ $n_1 \cdot n_2 = 0 \rightarrow$ perpendicular

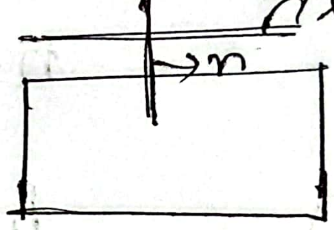
$n_1 \cdot n_2 = 1 \rightarrow$ orthogonal.

15-16: Determine whether the line and plane are parallel, perpendicular or neither.

(a) $x = 4+2t,\ y = -t,\ z = -1-4t \Rightarrow v = \langle 2,-1,-4 \rangle$

$3x+2y+z-7=0.\ n = \langle 3,2,1 \rangle.$

line



if parallel, $v \cdot n = 0$.

$\langle 2,-1,-4 \rangle \cdot \langle 3,2,1 \rangle$

$= 6-2-4 = 0$. so parallel.

(b) $x = t,\ y = 2t,\ z = 3t \Rightarrow \vec{v} = \langle 1,2,3 \rangle.$

$x - y + 2z = 5 \quad n \Rightarrow \langle 1,-1,2 \rangle.$

$v \cdot n = 0$ if parallel.

$\langle 1,2,3 \rangle \cdot \langle 1,-1,2 \rangle$

$= \cancel{1+}\ 1-2+6$

$=$ 5. neither parallel nor perpendicular

16.

(a) $x = 3 - t,\ y = 2 + t,\ z = 1 - 3t.\quad \vec{v} = \langle -1, 1, -3 \rangle.$

$2x + 2y - 5 = 0.\quad n = \langle 2, 2, 0 \rangle.$

$\vec{v} \cdot n = \langle -1, 1, -3 \rangle \langle 2, 2, 0 \rangle$

$-2 + 2 + 0 = 0$

(b) $x = 1 - 2t,\ y = t,\ z = -t.\quad \bar{u} = \langle -2, 1, -1 \rangle$

$6x - 3y + 3z = 1.$

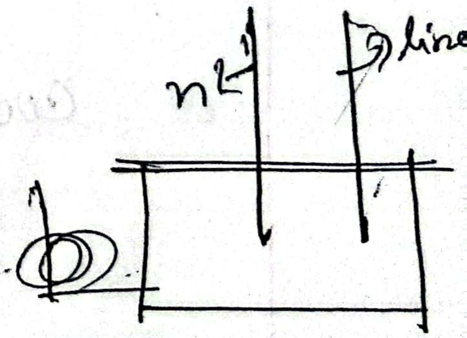
$\bar{n} = \langle 6, -3, 3 \rangle.$

$\bar{u} \cdot \bar{n} = \langle -2, 1, -1 \rangle \cdot \langle 6, -3, 3 \rangle.$

$= \cancel{-12 - 3 - 3}$

$=$

$\boxed{\bar{n} = -3\bar{u}}$



~~pa~~ ~~both~~ perpendicular.

$\boxed{17.-18}$. Determine whether the line and plane intersect. if so, find the coordinates of the intersection.

a) $x = t,\ y = t,\ z = t$

$3x - 2y + z - 5 = 0.$

$x_0 = t,\ y_0 = t,\ z_0 = t.$

$3x_0 - 2y_0 + z_0 - 5 = 0$

$3t - 2t + t - 5 = 0.$

$2t = 5.$

$t = \frac{5}{2}.$

coordinates of the intersection

$\left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right).$

Example 9.

The planes $x+2y-2z=3$

and $2x+4y-4z=7$.

are parallel since their normals

$\langle 1,2,-2\rangle$ and $\langle 2,4,-4\rangle$ are parallel.

vectors,

we select an arbitrary points in one of the planes and compute its distance to the ~~out~~ other plane.

By setting $y=z=0$ in the eqn

$x+2y-2z=3$.

we obtain point $P_0(3,0,0)$

from $P_0\,(3,0,0)$ to the plane $2x+4y-4z=7$

is ~~for~~ $D=\dfrac{|2\cdot 3+4\cdot 0-4\cdot 0-7|}{\sqrt{2^2+4^2+4^2}}$

$=\left|\dfrac{-1}{6}\right| = \dfrac{1}{6}$.

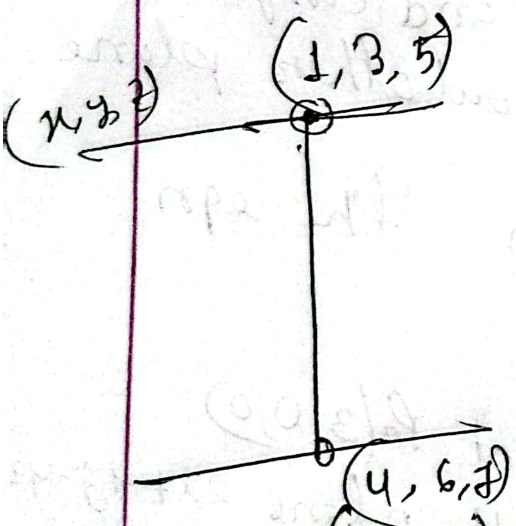
47: $x = 1+7t,\ y = 3+t,\ z = 5-3t.$

$x = 4-t,\ y = 6,\ z = 7+2t,$

for first line the vector $\vec{u} = \langle 7, 1, -3 \rangle$.

for 2nd line the vector $\vec{v} = \langle -1, 0, 2 \rangle$.

$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ 7 & 1 & -3 \\ -1 & 0 & 2 \end{vmatrix}$$



$$= \langle 2, -11, 1 \rangle.$$

$$2(x-1) + (y-3)(-11) + (z-5) = 0.$$

$\cancel{2x - 11y + z + 51 = 0}.$

$4\varphi\ \cancel{2.4 - 11.}$

$2x - 2 - 11y + 33 + z - 5 = 0$

$2x - 11y + z + 26 = 0.$

$$\frac{2\times 4 - 11\cdot 6 + 7 + 26}{\sqrt{4^2+6^2+7^2}}$$

$$= \frac{25}{\sqrt{101}}$$

$$= \frac{25}{\sqrt{101}}.$$