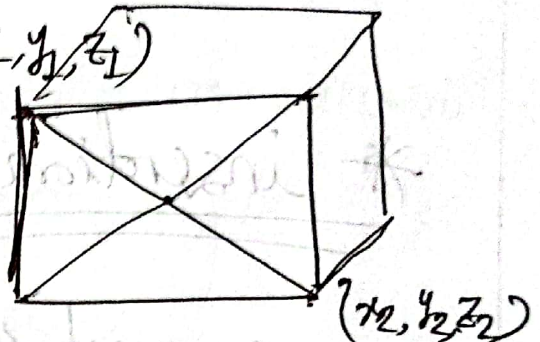
Unit 11.1

4. Suppose ~~a~~ that a box has its faces parallel to the coordinate planes and the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are endpoints of a diagonal.

(a) Find the coordinates of the remaining six corners.



(x_1, y_1, z_1) (x_1, y_1, z_2)

(x_2, y_1, z_1) (x_2, y_2, z_2)

1 1 1   2 2 2

1 1 2
1 2 1
1 2 2
2 1 1
2 1 2
2 2 1

(x_1, y_1, z_2)
(x_1, y_2, z_1)
(x_1, y_2, z_2)
(x_2, y_1, z_1)
x_2, y_1, z_2

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Chapter 11.1.

- xy plane, consists of all points of the form $(x, y, 0)$
- xz plane consists of all points of the form $(x, 0, z)$
- yz-plane consists of all points of the form $(0, y, z)$.
- x-axis consists of all points of the form $(x, 0, 0)$
- y-axis consists of all points of the form $(0, y, 0)$.
- z-axis, consists of all points of the form $(0, 0, z)$.

Recall that in 2 space the distance d between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

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$\boxplus$ The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ in 3-space is

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}.$$

$\boxplus$ Recall that the standard equation of the circle in 2-space that has center (x_0, y_0) and radius r is

$$(x-x_0)^2 + (y-y_0)^2 = r^2.$$

$\boxplus$ Standard equation of the sphere in 3-space that has center (x_0, y_0, z_0) and radius r is

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2.$$

$\boxplus$ $x^2 + y^2 + z^2 + Gx + Hy + Iz + J = 0.$
represents a sphere, a point or has no graph.

$\boxed{P}$ $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=k$,

if $k=0$ then the sphere has radius 0, so the graph is the single point (x_0, y_0, z_0).

if $k<0$, so the eqn is not satisfied by any value of x, y and z, so it has no graph.

do example -3,4.

$\boxed{P}$ sketch the graph of $x^2+z^2=1$ in 3 space

Ans: Since y doesn't appear in this eqn the graph is a cylindrical surface generated by extrusion parallel to the y-axis. In the xz-plane the

12. Find the distance from the point $(-5, 2, -3)$ to the

a) xy-plane
$d = \sqrt{(-3)^2} = 3.$

(b) xz-plane $d = \sqrt{(2)^2} = 2.$

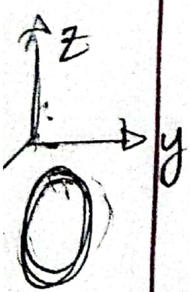
(c) yz-plane $d = \sqrt{(-5)^2} = 5.$

(d) x-axis $d = \sqrt{(2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}.$

(e) y-axis $d = \sqrt{(-5)^2 + (-3)^2} = \sqrt{15+9} = \sqrt{24}.$

(f) z-axis $d = \sqrt{(-5)^2 + (2)^2} = \sqrt{25+4} = \sqrt{29}$

15. In each part find an eqn of the sphere with centre $(2, -1, -3)$ and satisfying the given condition.

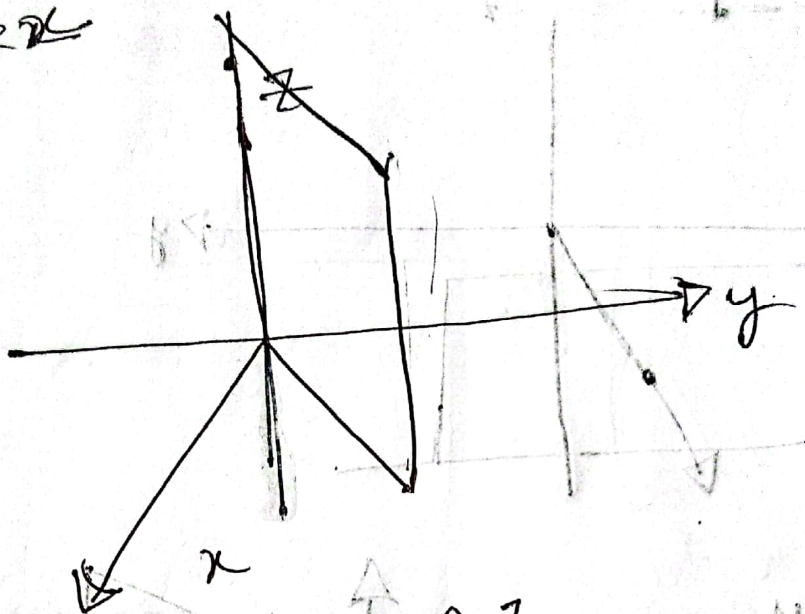


(a) Tangent to the xy plane $r = -3$

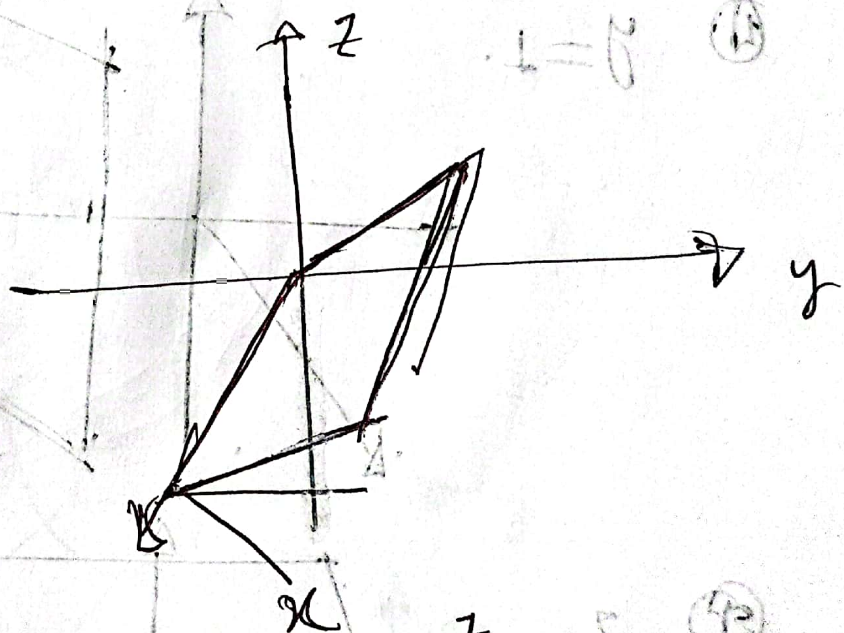
(b) Tangent to the xz plane $\Rightarrow r = -1$.

(c) Tangent to the yz plane. $\Rightarrow r = 2$.

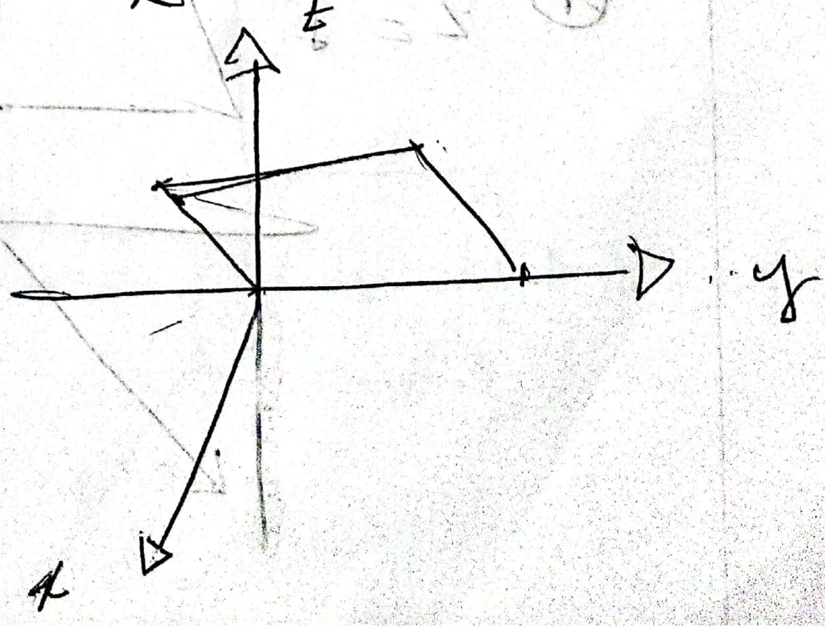
29. a) $y = x$



(b) $y = z$

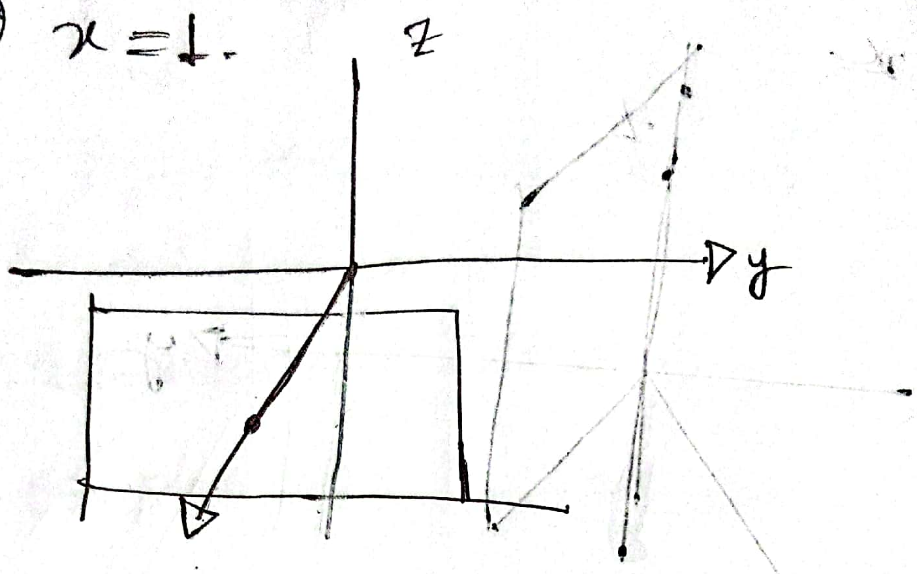


(c) $x = z$

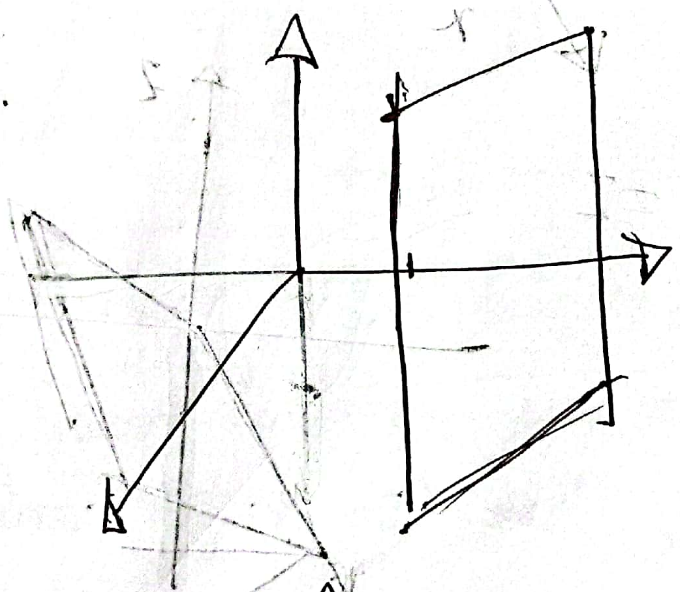


80.

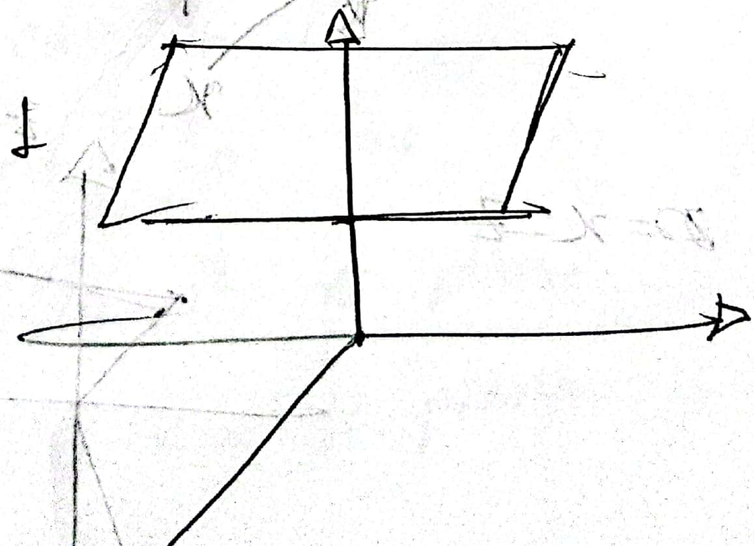
(2) $x = 1.$



(1) $y = 1.$



(3) $z = 1$



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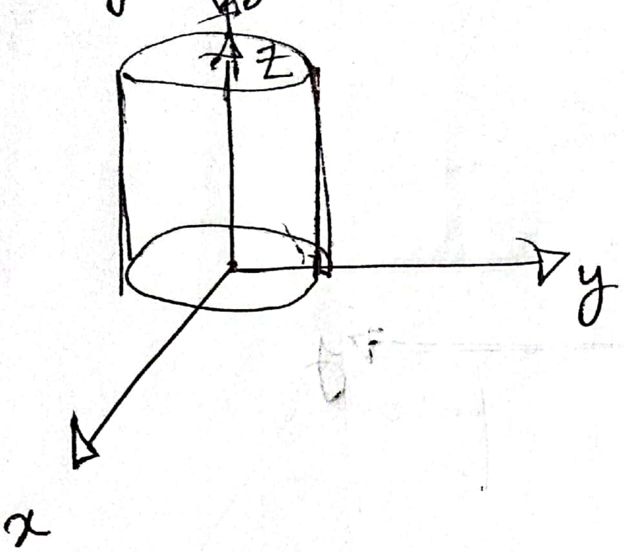
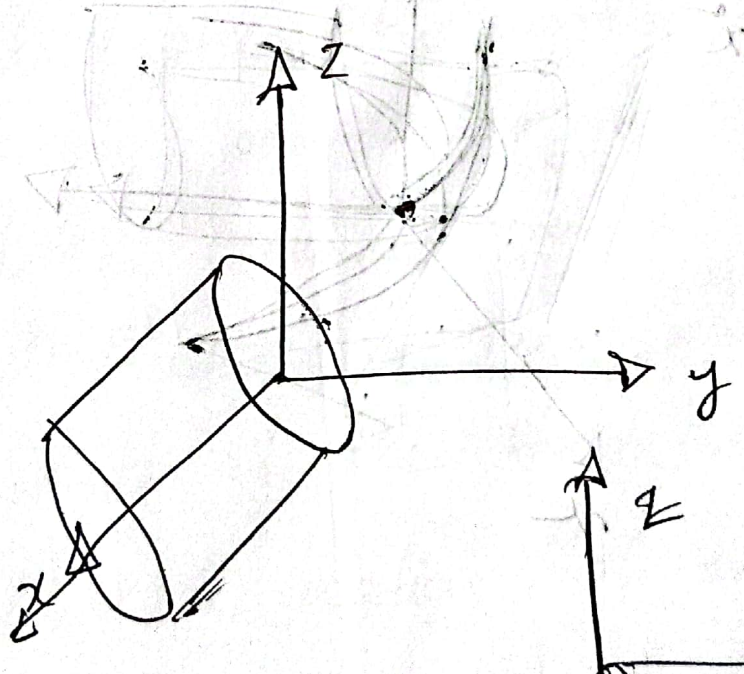
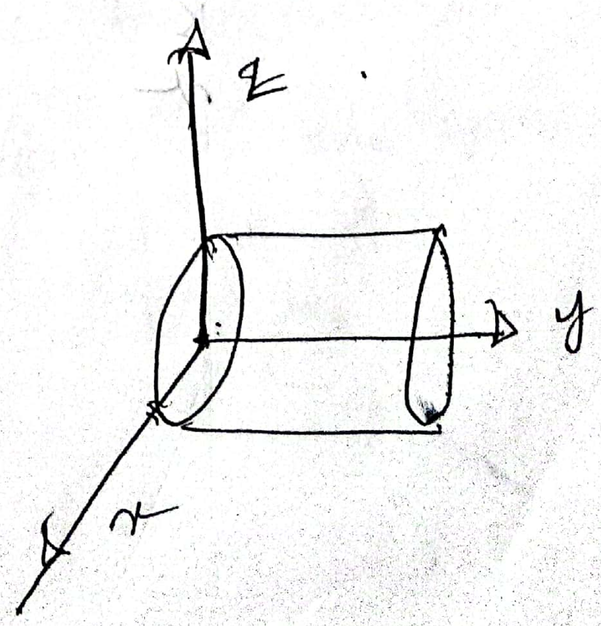
31. (2) $x^2+y^2=25$.



(4) $y^2+z^2=25$.

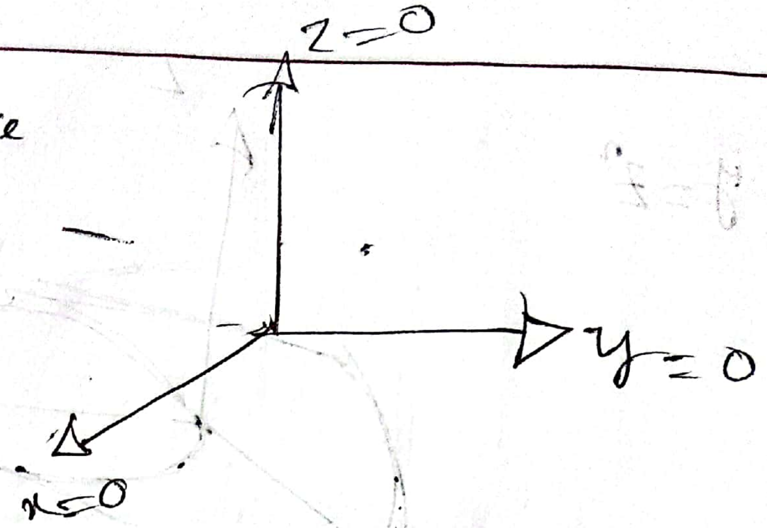


(5) $x^2+z^2=25$



1.7

Let sphere has center (a,b,c) and radius $r>0$



The three coordinate planes are $x=0$, $y=0$ and $z=0$

The distance from the center (a,b,c) to plane $x=0$ is $|a|$

$|a|=r$

The distance from the center (a,b,c) to plane $y=0$ is $|b|$.

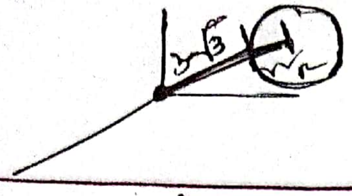
$|b|=r$

The distance from the center (a,b,c) to plane $z=0$ is $|c|$;

$|c|=r$

Therefore the center is (r,r,r)

The points of tangency are $(r,0,0)$, $(0,r,0)$, $(0,0,r)$

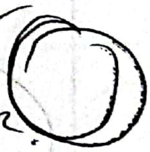


18. A sphere has center in the first octant and is tangent to each of the three coordinate planes. The distance from the origin to the sphere is $3-\sqrt{3}$ units. Find an equation for the sphere.

The distance from the origin to the sphere is $3-\sqrt{3}$ units.

The distance from the origin to the center is $r\sqrt{3}$

$\sqrt{r^2+r^2+r^2}$
$=\sqrt{3r^2}$
$=\sqrt{3}r$



$$r\sqrt{3}-r = 3-\sqrt{3}.$$

$$r(\sqrt{3}-1) = 3-\sqrt{3}.$$

$$r(\sqrt{3}-1) = (\sqrt{3})\cdot\sqrt{3}-\sqrt{3}.$$

$$r(\sqrt{3}-1) = \sqrt{3}(\sqrt{3}-1).$$

$$\boxed{r = \sqrt{3}}.$$

(1-4)

Thus the center is (r,r,r) and
the radius is also r.

center $(\sqrt{3},\sqrt{3},\sqrt{3})$.
radius is $\sqrt{3}$.

$$(x-\sqrt{3})^2+(y-\sqrt{3})^2+(z-\sqrt{3})^2=(\sqrt{3})^2.$$

$$\boxed{(x-\sqrt{3})^2+(y-\sqrt{3})^2+(z-\sqrt{3})^2=3}$$

Clarification

distance from $x=0$ and
center is r.

distance from $y=0$ and center is r
distance from $z=0$ and center is r

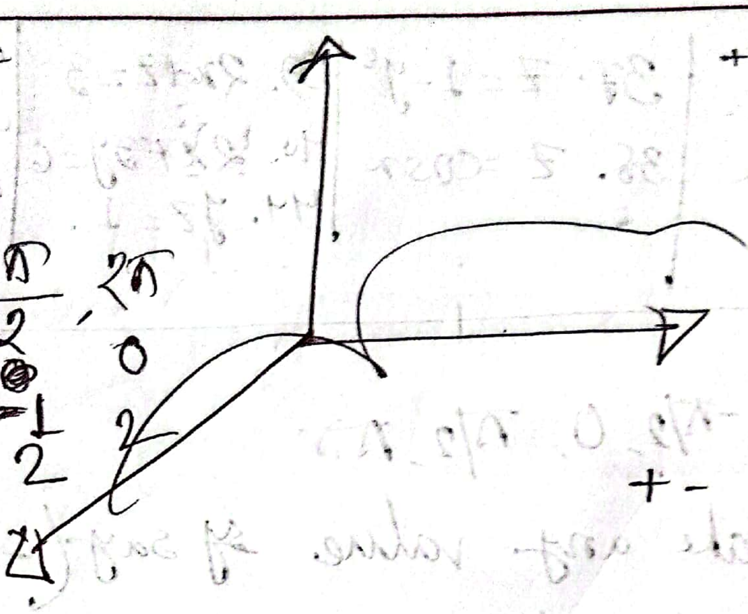
so center (r,r,r) and origin$(0,0,0)$

$$d=\sqrt{(r-0)^2+(r-0)^2+(r-0)^2}.$$

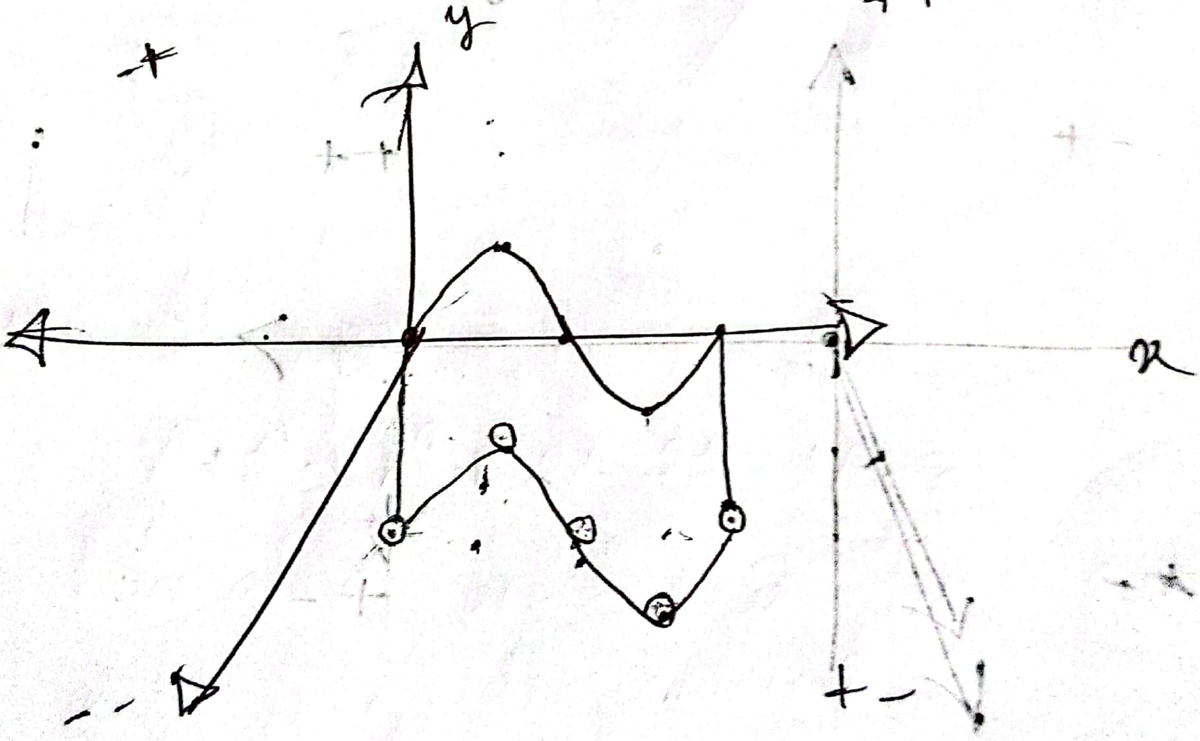
$$d=\sqrt{3r^2}=\sqrt{3}r.$$

$z = 2$

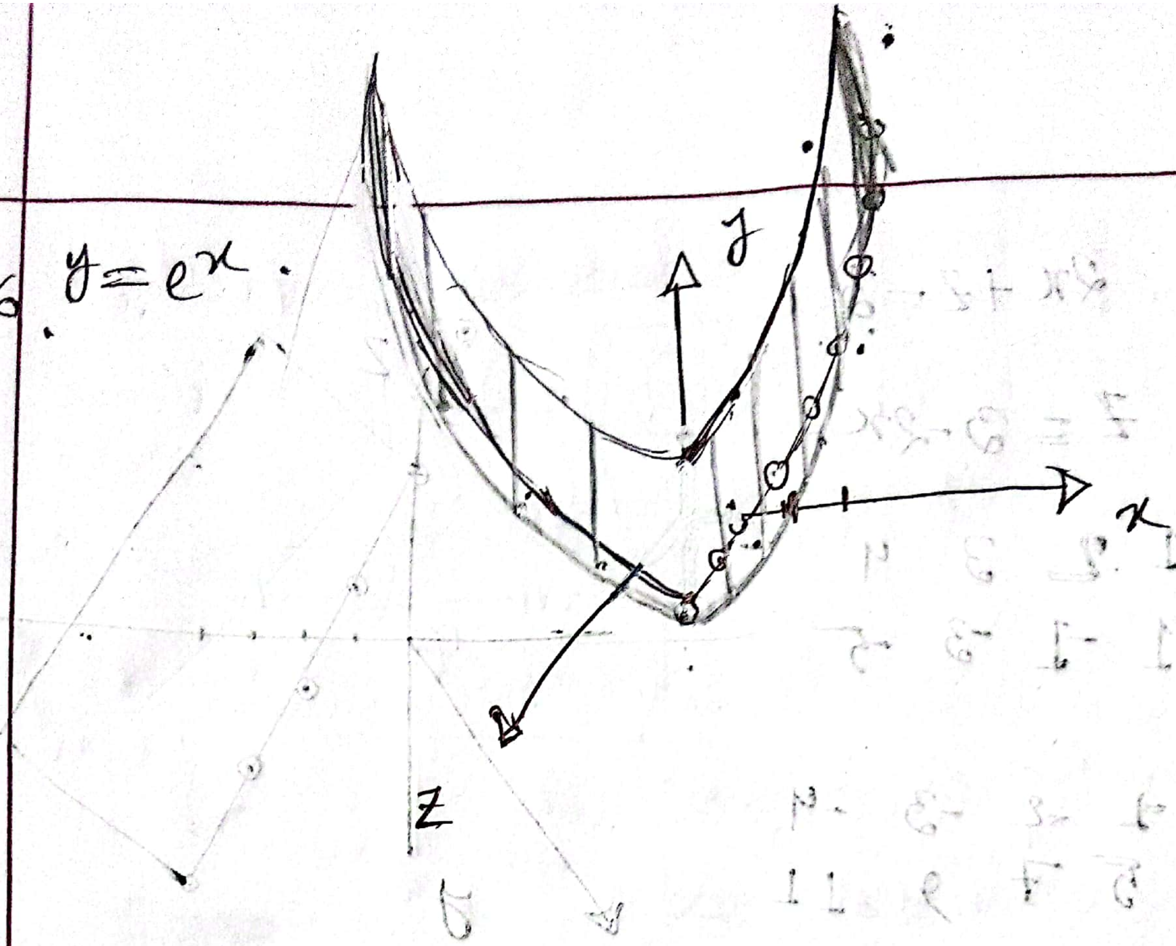
0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
0	1	0	-1	0
2	2	2	2	2



$y = \sin x$

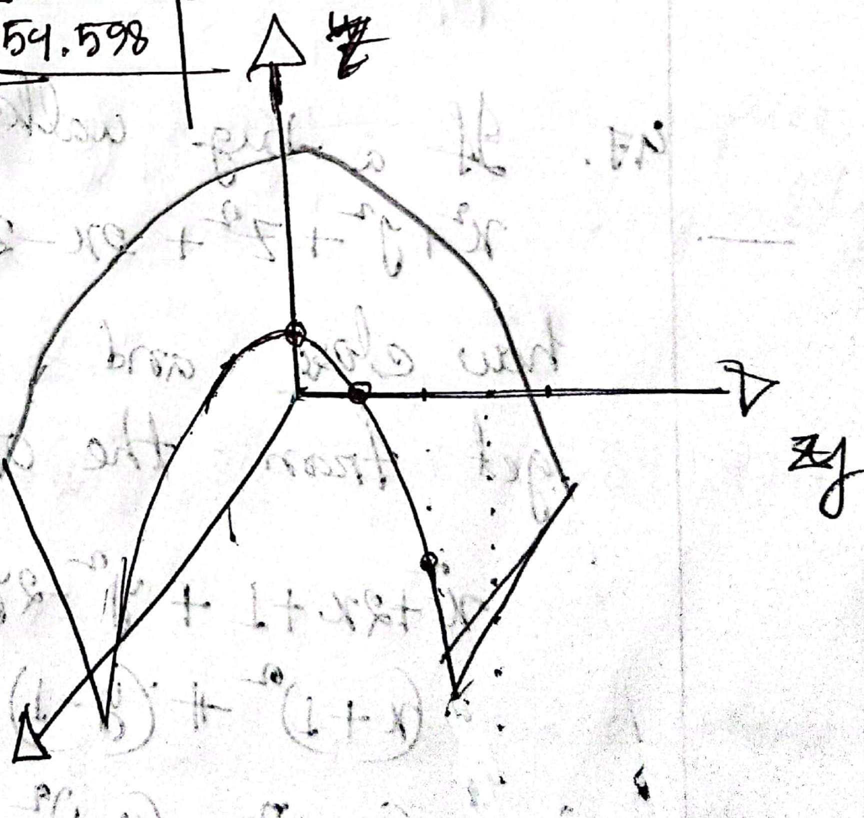


36. $y = e^x$



0	1	2	3	4
1	2.72	7.4	20.08	54.598

37. $z = 1 - y^2$



0 1 2 3 4

1 0 -3 -8 -15

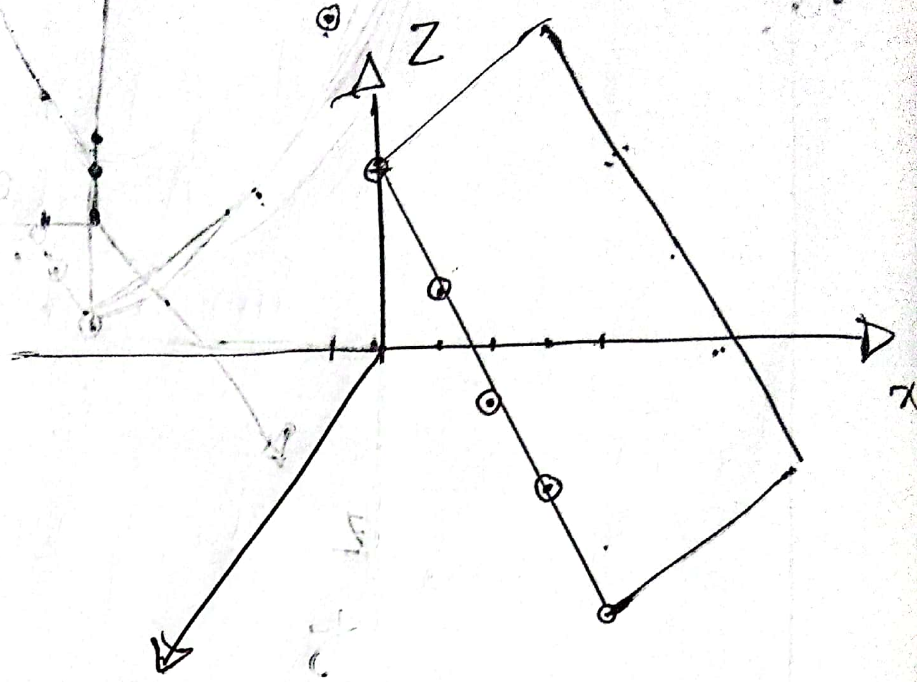
-1

39. $2x+z=3$

$z = 3-2x$

0	1	2	3	4
3	1	-1	-3	-5

0	-1	-2	-3	-4
3	5	7	9	11



47. If a bug walks on the sphere:

$x^2+y^2+z^2+2x-2y-4z-3=0$

how close and how far can it get from the origin?

$x^2+2x+1+y^2-2y+1+z^2-4z+4=3+1+1+4$

$(x+1)^2+(y-1)^2+(z-2)^2=9$

$(x+1)^2+(y-1)^2+(z-2)^2=3^2$

or center $(-1,1,2)$ and radius $=3$.

The distance between the center and origin is $d\langle = \sqrt{(0+1)^2 + (0-1)^2 + (0-2)^2}$.

$$d = \sqrt{1+1+4}$$

$$\boxed{d = \sqrt{6}.}$$

$d < 3$, so the origin is inside the sphere. The longest distance is $3+\sqrt{6}$. the shortest distance is $3-\sqrt{6}$.

$\boxed{48}$. Describe the set of all points in 3-space whose coordinates satisfy the inequality $x^2+y^2+z^2-2x+8z \le 8$.

$$x^2-2x+1+y^2-20.y+(0)^2+z^2+8z+16 \le 8+1+16$$

$$(x-1)^2+(y-0)^2+(z+4)^2 \le 25.$$

all points are on and inside the sphere which radius is 5 and the center is $(1,0,-4)$.

50. The distance between a point $P(x, y, z)$

$$\boxed{AP = 2BP}$$

and the point $A(1, -2, 0)$ is twice the distance between P and the point $B(0, 1, 1)$ show that the set of all such points is a sphere and find the center and radius of the sphere.

$$PA = \sqrt{(x-1)^2 + (y+2)^2 + (z-0)^2}$$

$$= \sqrt{(x-1)^2 + (y+2)^2 + z^2}$$

$$PB = \sqrt{(x-0)^2 + (y-1)^2 + (z-1)^2}$$

$$= \sqrt{x^2 + (y-1)^2 + (z-1)^2}$$

$$\left(\sqrt{(x-1)^2+(y+2)^2+z^2}\right)^2 = \left(2\sqrt{x^2+(y-1)^2+(z-1)^2}\right)^2$$

$$(x-1)^2+(y+2)^2+(z)^2 = 4\left(x^2+(y-1)^2+(z-1)^2\right).$$

$$\Rightarrow x^2-2x+1+y^2+4y+4+z^2 = 4\left(x^2+y^2-2y+1+z^2-2z+1\right)$$

$$\Rightarrow 4\left(x^2+y^2+z^2-2y-2z+2\right) = x^2+y^2+z^2-2x+4y+5.$$

we get,

$$3x^2+3y^2+3z^2+2x-12y-8z+3=0.$$

we get $\left(x+\frac{1}{3}\right)^2+(y-2)^2+\left(z-\frac{4}{3}\right)^2 = \frac{44}{9}.$

center $\left(-\frac{1}{3}, 2, \frac{4}{3}\right)$, radius $\sqrt{\frac{44}{9}} = \frac{2\sqrt{11}}{3}.$

52. Consider the eqn

$$x^2+y^2+z^2+Gx+Hy+Iz+J=0.$$

and let $K=G^2+H^2+I^2-4J$.

(a) Prove that the eqn represents a sphere if $K>0$, a point if $K=0$ and has no graph if $K<0$.

$$\left(x+\frac{G}{2}\right)^2+\left(y+\frac{H}{2}\right)^2+\left(z+\frac{I}{2}\right)^2=\frac{K}{4}.$$

So, the eqn represents a sphere when $K>0$, a point when $K=0$ and no graph when $K<0$

(b). Center $C\left(-\frac{G}{2},-\frac{H}{2},-\frac{I}{2}\right)$ and radius $\frac{\sqrt{K}}{2}$.

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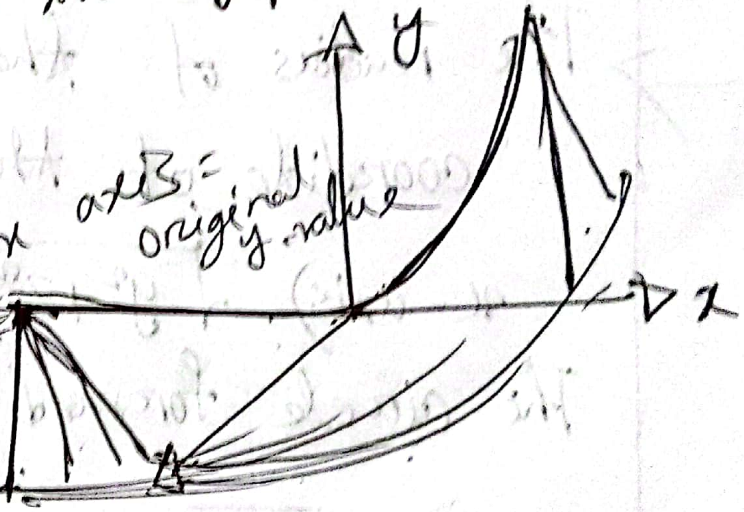
53 (a) At $x = c$ the trace of the surface is the circle $y^2 + z^2 = f(c)^2$. so the surface is given by $y^2 + z^2 = [f(x)]^2$.

$\to$ 3D surface formed when a curve is rotated around ...

(b) Find an eqn of the surface of revolution that is generated by revolving the curve $y = e^x$ in the xy-plane about the x-axis.



distance from the x axis = original y value

$\sqrt{y^2+z^2}$ = radius =

$$\sqrt{y^2 + z^2} = (e^x)$$

(c) $3x^2 + 4y^2 + 4z^2 = 16$.

$\frac{3}{4}x^2 + y^2 + z^2 = 4$.

$y^2 + z^2 = 4 - \frac{3}{4}x^2$.

52 (b) you are given, $y = e^x$

This is curve in the xy-plane (so z=0)

When you revolve a curve around ~~the~~ the x-axis, each point on the curve traces out a circle in 3D space.

The radius of that circle = the y-coordite of the point (distance from x-axis). $\sqrt{y^2+z^2}$

The circle formed has radius $r = e^x$.

$$\sqrt{y^2+z^2} = e^x$$

$$y^2+z^2 = (e^x)^2$$

$$y^2+z^2 = e^{2x}$$

~~54. a) The surface of~~

c) Show that, the ellipsoid $3x^2+4y^2+4z^2=16$ is a surface of revolution about the $\boxed{x\text{-axis}}$ by finding a curve $\boxed{y=f(x)}$ in the xy-plane that generates it.

$$\cancel{3x^2}\quad 3x^2+4y^2+4z^2=16$$

$$4y^2+4z^2=16-3x^2$$

$$4(y^2+z^2)=\frac{16}{4}-\frac{3}{4}x^2$$

$$y^2+z^2=\boxed{4-\frac{3}{4}x^2}$$

distance from x axis

~~radius~~ $\sqrt{y^2+z^2}$

$$\text{radius}=\sqrt{4-\frac{3}{4}x^2}$$

54'a) $x=f(y) \Rightarrow$ Radius.

distance from y axis $=\sqrt{x^2+z^2}$ $\qquad (\sqrt{x^2+z^2})^2=[f(y)]^2$