MCQ

একটি curve কতটুকু bend curvature.
একটি " এর direction tangent line.

tangent এর rate of change = ঐ corresponding curvature.

circular helix

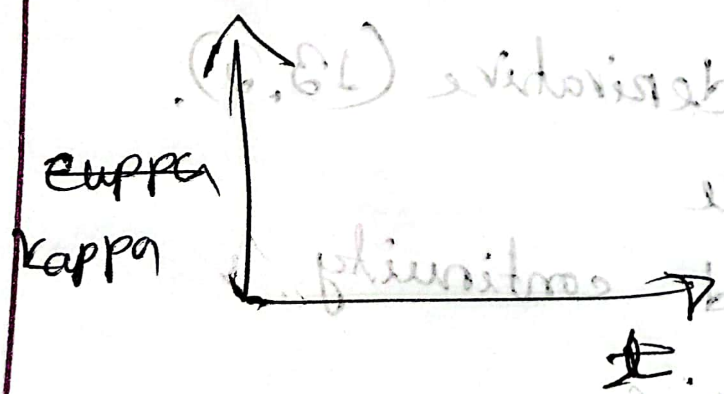
$$\boxed{x = a\cos t;\ y = a\sin t;\ z = ct}$$

helix

$z = c$

cylinder

$$\text{curvature} = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$



$$\frac{df}{dt} = \lim_{n \to 0} f(t+h) -$$

$z = f(x, y) \Rightarrow$ multivaraiable function

y এর rate of change বের করতে হলে x hold and vise versa.

↳ partial change

↳ derivative partial derivative.

$z = f(x, y_0)$

f_x $\frac{\delta f}{\delta x} = \lim_{\substack{h \to 0 \\ \Delta x \to 0}} \frac{f(x + \overset{\Delta x}{h}) - f(x, y)}{\underset{\Delta x}{h}}$

partial derivative

f_y $\frac{\delta f}{\delta y} = \lim_{\substack{h \to 0 \\ \Delta y \to 0}} \frac{f(x, y + \overset{\Delta y}{h}) - f(x, y)}{\underset{\Delta y}{h}}$

(x_0, y_0) point এর respect এ।

x_0, y_0 এর respect এ:

x, y এর

বসায়ে দিব।

Find $f_x(1,3)$ and $f_y($

Example 3:

Find $\frac{\delta z}{\delta x}$ and $\frac{\delta z}{\delta y}$ of

$$z = x^4 \sin(xy^3)$$

$$\frac{\delta z}{\delta x} = y^3 x^4 \frac{d}{dx} \cos(xy^3) + 4x^3 \sin(xy^3)$$

$$= 4x^3 \sin(xy^3) + y^3 x^4 \cos(xy^3)$$

$$\frac{\delta z}{\delta y} = x^4 \cos(xy^3)\, 3y^2$$

$$= 3x^4 y^2 \cos(xy^3)$$

Example 11 If $f(\rho, \theta, \phi) = \rho^2 \cos\phi \sin\theta$.

$$\frac{\delta^2 f}{\delta x \delta y} = \frac{\delta}{\delta x}\left(\frac{\delta f}{\delta y}\right)$$

$$\frac{\delta^2 f}{\delta y \delta x}$$

$$\frac{\delta^2 f}{\delta x \delta y} = \frac{\delta^2 f}{\delta y \delta x}.$$

Continuous and f differentiable

f_{xx}, f_{yy}.

$$f_{xyy} = \frac{\delta^3 f}{\delta y \delta y \delta x}.$$

$f(x, y) = y^2 e^x + y$.

$$\frac{\delta f}{\delta x} = y^2 e^x$$

$$\frac{\delta f}{\delta y} = 2ye^x.$$

$$\frac{\delta f}{\delta y} = 2e^x.$$

Determine if $f(x,y) = \frac{x^2}{y^3}$ is e increa

$f'(x) > 0$ increase

$f'(x) < 0$ decrease

f'

$(1, 2)$

2D

excluded → open disk.

$$\boxed{f_{xy} = f_{yx}}$$

I2.6 $\longrightarrow$ Motion along path.

$$v = \frac{ds}{dt}.$$

$$a = \frac{dv}{dt}.$$

$$= \frac{d^2s}{dt^2}$$

chain rule (1.3.5)

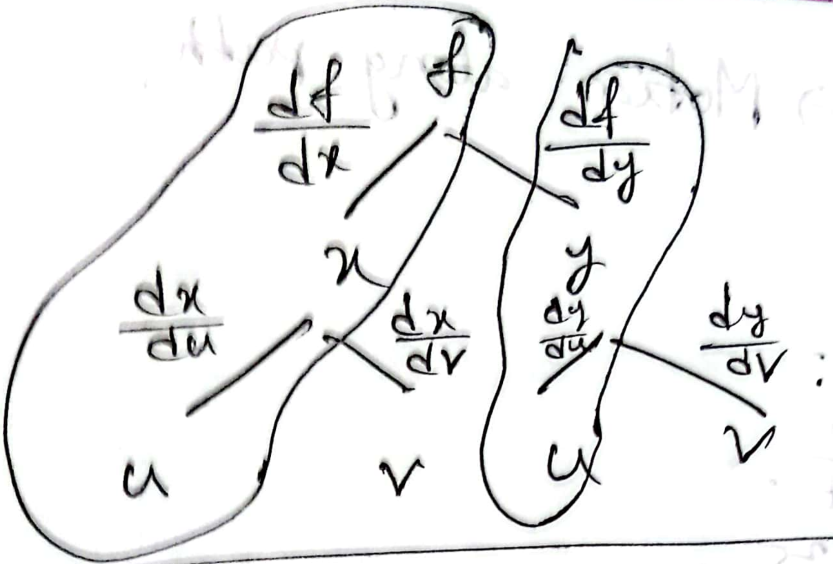
$x = x(t)$ $\quad \frac{dx}{dt} \longrightarrow$ (2)

$y = y(t)$ $\quad \frac{dy}{dt} \longrightarrow$ (1)

$$\frac{dy}{dt} = \frac{\frac{dy}{dt}}{dx/dt} = \frac{dy}{dx}.$$

$$\begin{tikzcd}
 & f \arrow[dl, "\frac{\delta f}{\delta x}"'] \arrow[dr, "\frac{\delta f}{\delta y}"] & \\
x \arrow[d, "\frac{dx}{dt}"'] & & y \arrow[d, "\frac{dy}{dt}"] \\
t & & t
\end{tikzcd}$$

$$\frac{\delta f}{\delta t} = \frac{\delta f}{\delta x} \cdot \frac{dx}{dt} + \frac{\delta f}{\delta y} \cdot \frac{dy}{dt}.$$

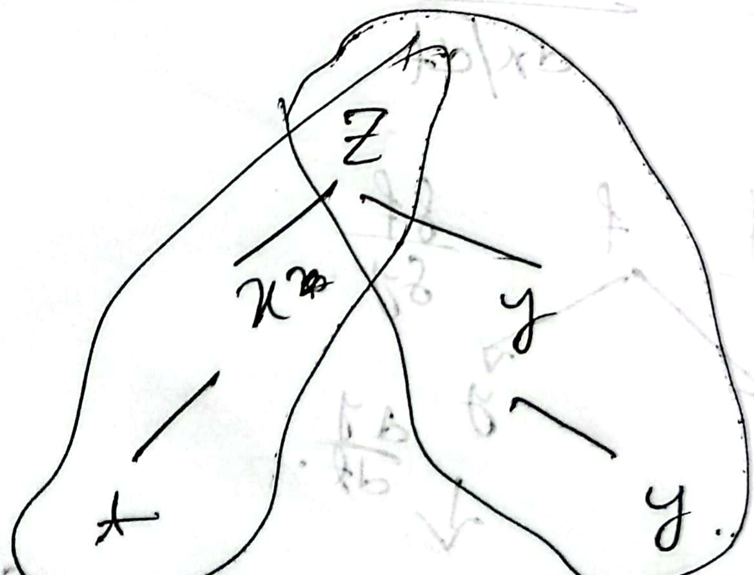


$$\boxed{\frac{\delta f}{\delta u} = \frac{df}{dx} \cdot \frac{dx}{du} + \frac{df}{dy} \cdot \frac{dy}{du}}$$

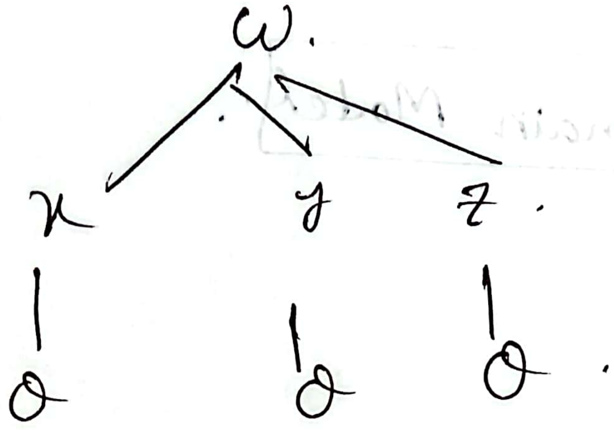
Example 1 ;

$$z = x^2 y, \quad x = t^2, \quad y = t^3.$$

$$\frac{dz}{dz}.$$



Example $w=\sqrt{x^2+y^2+z^2}$, $x=\cos\theta$, $y=\sin\theta$, $z=\tan\theta$.



$$\boxed{\frac{dw}{d\theta} = \cancel{\delta x}\,\frac{\delta w}{\delta x}\cdot\frac{dx}{d\theta} + \frac{\delta w}{\delta y}\cdot\frac{\delta y}{\delta \theta} + \frac{\delta w}{\delta z}\cdot\frac{\delta z}{\delta \theta}}$$

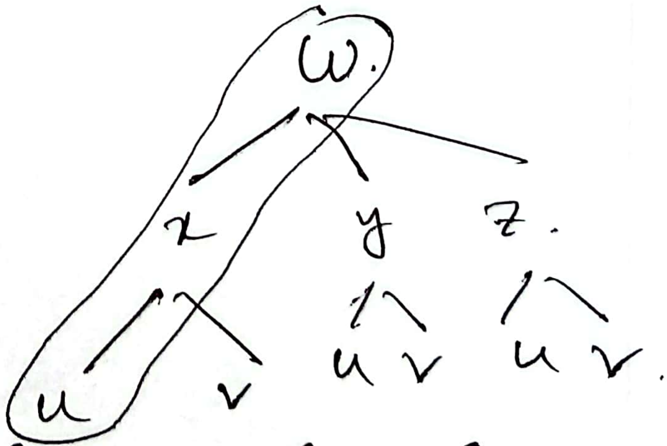
definitely [illegible]

$w = e^{xyz}$

$x = 3u+v$

$y = 3u-v$

$z = u^2+v$



$$\frac{\delta w}{\delta u} = \frac{\delta w}{\delta x}\cdot\frac{\delta x}{\delta u} + \frac{\delta w}{\delta y}\cdot\frac{\delta y}{\delta u} + \frac{\delta w}{\delta z}\,\frac{\delta z}{\delta u}$$

$$\frac{\delta w}{\delta v} = \frac{\delta w}{\delta x}\cdot\frac{\delta x}{\delta v} + \frac{\delta w}{\delta y}\cdot\frac{\delta y}{\delta v} + \frac{\delta w}{\delta z}\cdot\frac{\delta z}{\delta v}$$

$\boxed{\text{MCQ}}$ $\boxed{12.10.25.}$

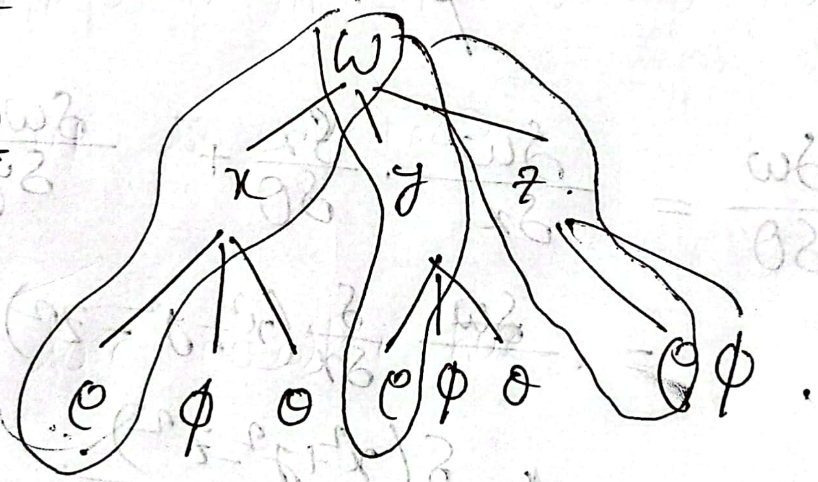
Suppose that,

$$\omega = x^2 + y^2 - z^2,$$

and
$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

use chain rule to find $\frac{\delta w}{\delta \rho}$ and $\frac{\delta w}{\delta \theta}$.

$\cancel{\delta w}$



$$\frac{\delta w}{\delta \rho} = \frac{\delta w}{\delta x} \cdot \frac{\delta x}{\delta \rho} + \frac{\delta w}{\delta y} \cdot \frac{\delta y}{\delta \rho} + \frac{\delta w}{\delta z} \cdot \frac{\delta z}{\delta \rho}.$$

$$= \frac{\delta(x^2+y^2-z^2)}{\delta x} \cdot \frac{\delta(\rho \sin\phi \cos\theta)}{\delta \rho} +$$

$$\frac{\delta(x^2+y^2-z^2)}{\delta y} \cdot \frac{\delta(\rho \sin\phi \sin\theta)}{\delta \rho} + \cancel{\frac{\delta(x^2+y^2-z^2)}{\delta y}}$$

$$\cdot \cancel{\frac{\delta(\rho \sin\phi \sin\theta)}{\delta \rho}} + \frac{\delta(x^2+y^2-z^2)}{\delta z} \cdot \frac{\delta(\rho \cos\phi)}{\delta \rho}$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$.
$-\cos 2\theta = \sin^2\theta - \cos^2\theta$.

$2x \cdot \sin\phi\cos\theta + 2y\sin\phi\sin\theta - 2z\cos\phi$.

$= 2\cdot(\rho\sin\phi\cos\theta)\cdot\sin\phi\cos\theta + 2(\rho\sin\phi\sin\theta)$

$\sin\phi\sin\theta - 2(\rho\cos\phi)\cos\phi$.

$= 2\rho\sin^2\phi(\cos^2\theta + \sin^2\theta) - 2\rho\cos^2\phi$.

$= 2\rho\sin^2\phi - 2\rho\cos^2\phi$.

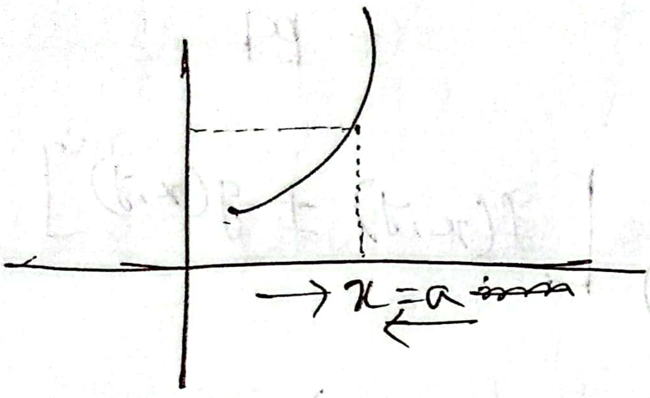
$= -2\rho\cos 2\phi$.

$$\frac{\delta w}{\delta\theta} = \frac{\delta w}{\delta x}\cdot\frac{\delta x}{\delta\theta} + \frac{\delta w}{\delta y}\cdot\frac{\delta y}{\delta\theta}.$$

$$= \frac{\delta w}{\quad}\ \frac{\delta}{\delta x}(x^2+y^2-z^2)\cdot\frac{\delta}{\delta\theta}(\rho\sin\phi\cos\theta)$$

$$+ \frac{\delta(x^2+y^2-z^2)}{\delta y}$$

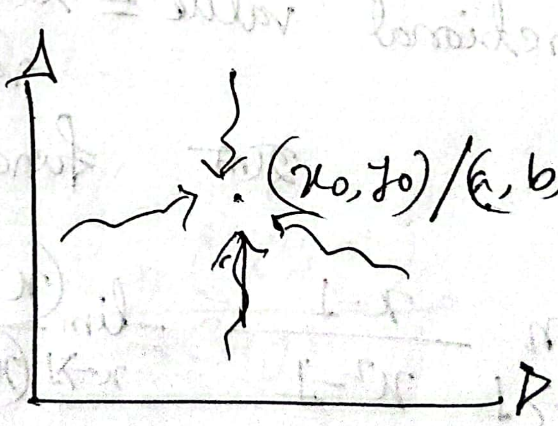
$$\boxed{y = f(x)}$$



$\lim\limits_{x \to a} f(x) = L$. (limit অবস্থা বের করি যে সমস্ত point এ function টা defined না।)

$\boxed{L.H.L = R.H.L}$: Limit exists.

$z = f(x+y)$.



$$\lim_{\substack{x \to x_0 \\ y \to y_0}} f(x+y) = M.$$

$$\boxed{\begin{array}{l} x \to a \\ y \to b \end{array}}$$

1) $x = a$

2) $y = b$.

3) $y - b = m(x-a)$ মূলবিন্দুগামী

$y = mx + b - ma$.

4) $y = x^2$

Same value limit exists.

$$\lim_{x \to a} \{f(x) \cdot g(x)\} = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

$$= pq$$

$$\lim_{(x,y) \to (x_0, y_0)} \left[f(x,y) \pm g(x,y) \right]$$

let $\lim_{(x,y) \to (x_0,y_0)} f(x,y) = L.$

$$\lim_{(x,y) \to (x_0,y_0)} g(x,y) = M.$$

so, $\lim_{(x \to y) \to (x_0,y_0)} \left[f(x,y) \pm g(x,y) \right] = L \pm M.$

functional value = ~~limit~~ function এর limit value.

হলে function টি continuous.

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

$$= \frac{1}{2}.$$

$$f(x,y) = \frac{x^2 - y^2}{x^2+y^2}$$

Consider the point $(0,0)$.

13.2 Limit & continuity.

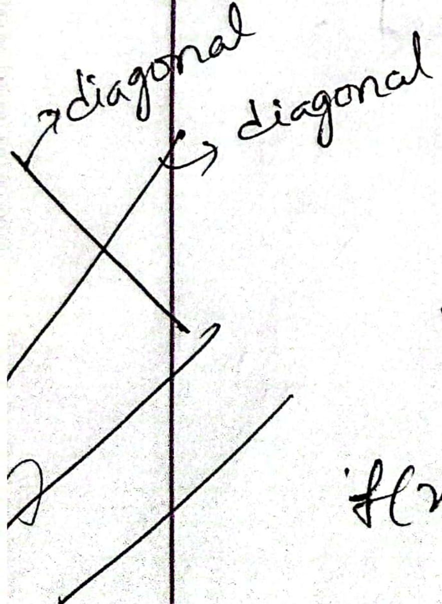
$x = a$.

$y = b$.

$y - b = m(x-a)$. ; $y = mx + b - mx$.

$y = x^2$.

৩ ভাবে লিখতে পারি.



$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

$$\lim_{\substack{x\to a \\ y\to b}} f(x,y) = L.$$

$$f(x,y) \to (0,0) \quad \frac{\sin(x^2+y^2)}{x^2+y^2}$$

$$\lim_{(x,y)\to(a,b)} c = c.$$

consider $f(x,y) = \dfrac{y}{x+y-1}$:

find $\displaystyle\lim_{(x,y)\to(1,0)} \frac{xy}{x+y-1}$:

$x \to 1$
$y \to 0$

$x = 1$.

$f(1,y) = \dfrac{y}{1+y-1} = \dfrac{y}{y} = 1$.

$\displaystyle\lim_{y\to 0} f(1,y) = 1$...

$y = 0$

$\displaystyle\lim_{x\to 1} f(x,0) = \frac{0}{x+0-1} = 0$

so limit doesn't exists.

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}.$$

$x=0$

$$f(0,y)= \frac{-y^2}{y^2} = -1.$$

$y \to 0$.

$$f(x,0) = \frac{x^2}{x^2} = 1 \therefore$$

$x \to 0$

limit doesn't exists.

Example: 3.2.16

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2},$$

$y=x$

$$= \lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+x^2}$$

$$= \frac{x^2}{2x^2} = \frac{1}{2}.$$

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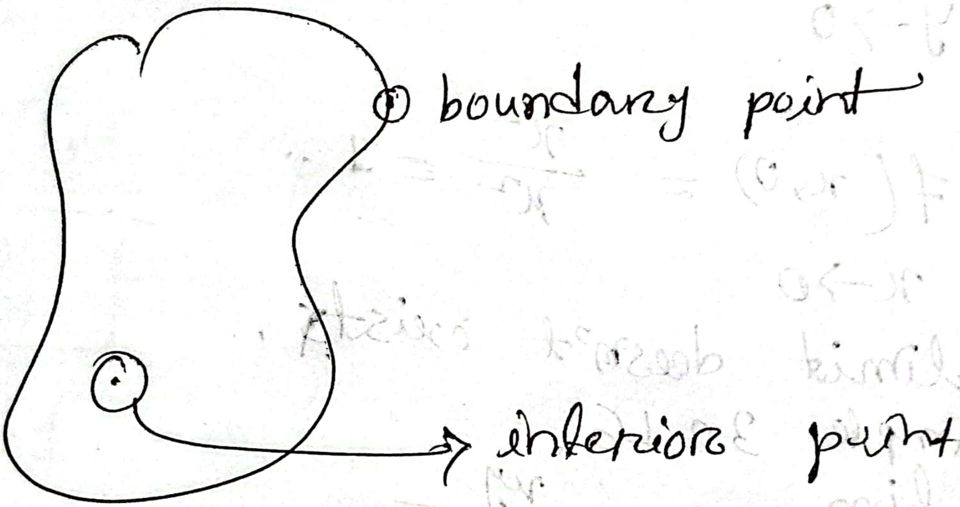
$\hookrightarrow$ op close disk,

[ ]

$\hookrightarrow$ close disk.

( ) $\rightarrow$ open disk.



$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0).$$

$f(x) = \sqrt{x}$ [3, 10].

$\cancel{\lim_{x\to 15}} f(x) \rightarrow$ can't calculate.

4. Polinomial func is always continuous.

$$\cdot f(x,y) = \frac{2x-y}{x^2+y^2}$$

Where is $f(x,y) = \frac{2x-y}{y=x^2}$.

Example 7: Find $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$

$\boxed{13.1, 13.2, 13.3, 13.5}$