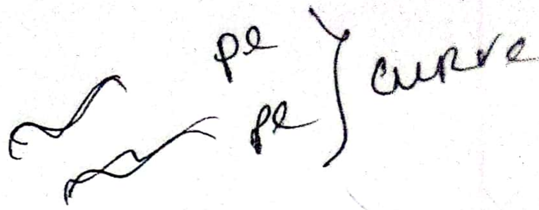


Unit 12.1.

If f and g are well-behaved functions, then the pair of parametric eqns

$$x = f(t),\ y = g(t).$$

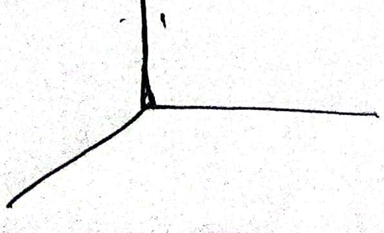
generates a curve in 2 space that is traced in a specific direction as t increases.

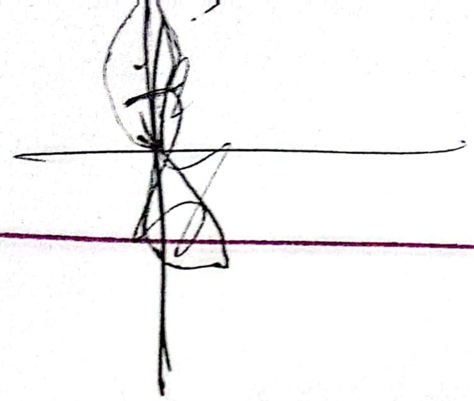
We call the $\boxed{\text{curve}}$ together with its orientation the graph of the parametric equations or the parametric curve represented by the eqns.

Example 1. The parametric eqns

$$x = 1 - t,\ y = 3t,\ z = 2t.$$

represents a line in 3-space that passes through (1, 0, 0) and is parallel to the vector $\langle -1, 3, 2 \rangle$. Since x decreases as t increases, the line has the orientation.





In particular, if we choose $x=t$ as the parameter and substitute this into the eqns. $z=x^3$ and $y=x^2$, we obtain the parametric eqns,

$$x=t,\ y=t^2,\ z=t^3 \text{ ——— } (3).$$

The twisted cubic defined by the eqns in (3) is the set of points of the form (t, t^2, t^3) for real values of t, if we view each of these points as a terminal point for a vector r, then

$$r = \langle x, y, z\rangle = \langle t, t^2, t^3\rangle = ti + t^2 j + t^3 k.$$

$\downarrow$

vector valued function of a real variable / vector valued function.

$a = x(t),\ b = y(t),\ c = z(t) \Rightarrow$ parametric eqn.

$$r = r(t) = \langle x(t), y(t), z(t)\rangle$$
$$= x(t)i + y(t)j + z(t)k.$$

$x(t), y(t), z(t)$ are called the component functions or the components of $r(t)$.

Example 3

The component functions of

$r(t) = \langle t, t^2, t^3 \rangle = t\,i + t^2 j + t^3 k$

The domain of a vector-valued function $r(t)$ is the set of allowable values for t.

If $r(t)$ is defined in terms of component functions and the domain is not specified explicitly, then ~~it will b~~ the domain in the intersection of the natural domains of the component funcs, this is called the natural domain of $r(t)$

Example 4:

Find the natural domain of

$r(t) = \langle \ln|t-1|, e^t, \sqrt{t} \rangle$

$= \ln|t-1|\, i + e^t j + \sqrt{t}\, K.$

The natural domains of the component functions

$x(t) = \ln|t-1|$, $y(t) = e^t$, $z(t) = \sqrt{t}$.

are $(-\infty, 1) \cup (1, \infty)$, $(+\infty, -\infty)$ $[0, \infty)$.

respectively.

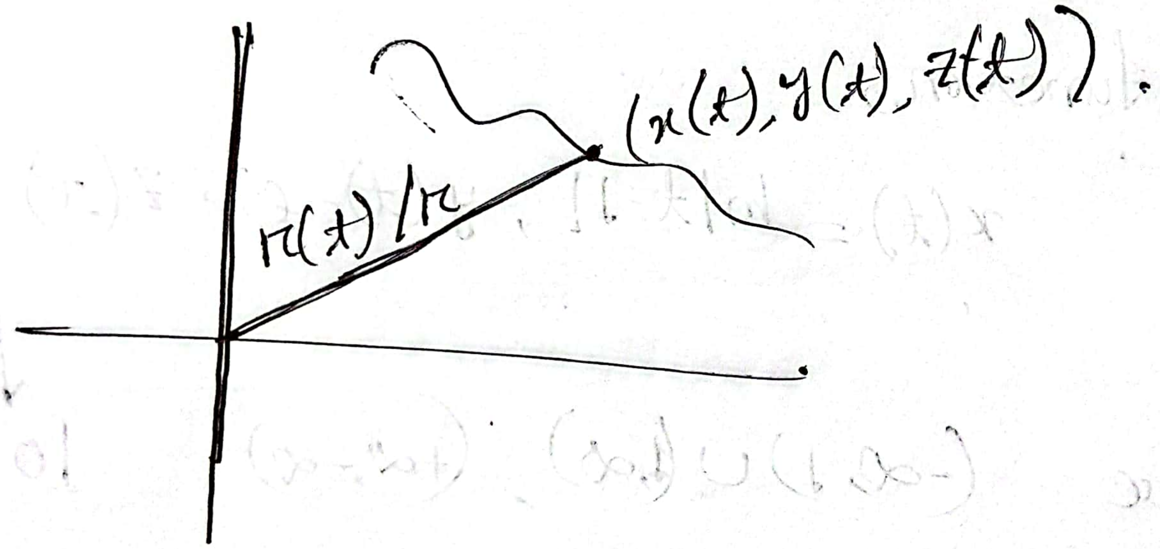
The intersections of this sets is

~~$-\infty, 1 \cup \infty$~~ $(1, \infty) \cup [0, 1)$.

$\boxed{1}$ The graph of the vector valued function is the graph of the parametric equations.

4. Describe the graph of the vector valued function.

$$r(t) = \langle \cos t, \sin t, t \rangle$$
$$= \cos t\, i + \sin t\, j + t\, k.$$



Example 6: Sketch the graph and a radius vector of

(a) $r(t) = \cos t\, i + \sin t\, j$

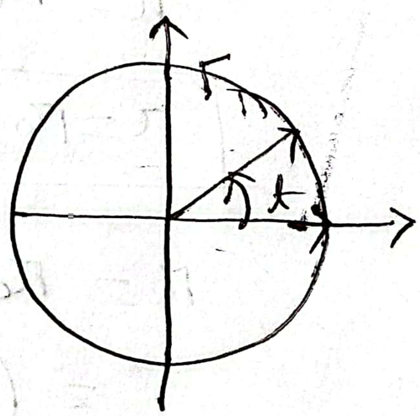
(b) $r(t) = \cos t\, i + \sin t\, j + 2k.$

ⓐ $r(t) = \cos t\, i + \sin t\, j.$

The parametric eqns are

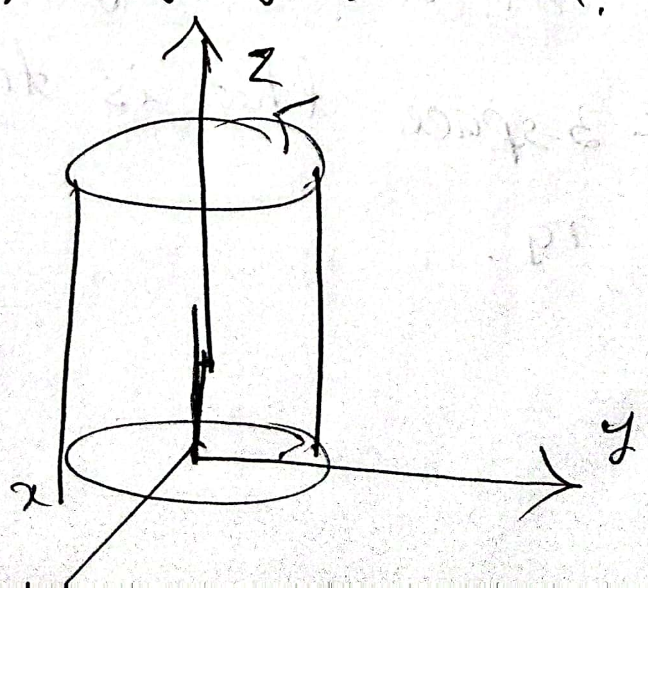
$x = \cos t,\ y = \sin t.$

So the graph is a circle of radius 1, centered at the origin, and oriented counterclockwise.

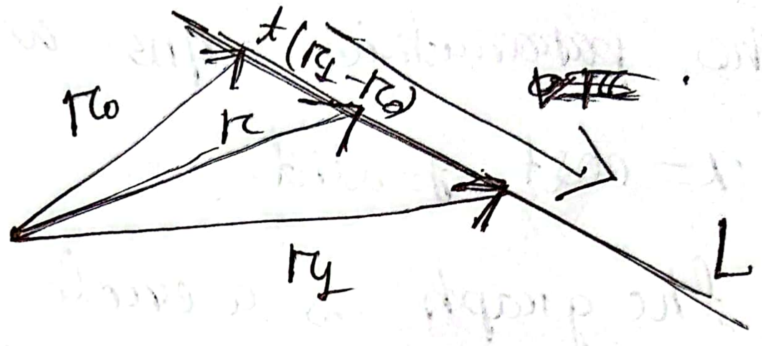


ⓑ The corresponding parametric eqns are

$x = \cos t,\ y = \sin t,\ z = 2.$



Vector form of a line segement



$r = r_0 + tv.$ $\quad\quad v = r_1 - r_0$

$$\boxed{r = r_0 + t(r_1 - r_0)}$$

$r = r_0 + r_1 t - r_0 t.$

$= (1-t)r_0 + r_1 t.$

Thus the eqn represent vector form of a line segment in 2-space or 3-space that is traced from r_0 to r_1.

Quick check exercise:

(i) (a) Express the parametric equations

$x = \frac{1}{t}$, $y = \sqrt{t}$, $z = \sin^{-1} t$.

as a single equation of the form

$r = x(t)\,i + y(t)\,j + z(t)\,k.$

$= \frac{1}{t}\,i + \sqrt{t}\,j + \sin^{-1} t\,k.$

(1-4): Find the domain of $r(t)$ and the value of $r(t_0)$.

$r(t) = \cos t\,i - 3t\,j.$

domain of $\cos t$ $(-\infty, +\infty)$

domain of $-3t$ $(-\infty, +\infty)$

intersection of $\cos t, -3t$'s domain

$(-\infty, +\infty)$.

5-6: Express the parametric equations as a single vector equation of the form

$r = x(t)i + y(t)j$

or $r = x(t)i + y(t)j + z(t)k$.

(5). $x = 3\cos t$, $y = t + \sin t$.

$r = 3\cos t\, i + (t + \sin t) j$.

(6) $x = 2t$, $y = 2\sin 3t$, $z = 5\cos 3t$.

$r = 2t\, i + 2\sin 3t\, j + 5\cos 3t\, k$.

(7-8) Find the parametric equation that correspond to the given vector eqn.

7. $r = 3t^2 i - 2j$.

parametric eqn: $x = 3t^2$ $y = -2$.

(8) $r = (2t - 1)i - 3\sqrt{t}\, j + \sin 3t\, k$.

parametric eqn: $x = 2t - 1$, $y = -3\sqrt{t}$, $z = \sin 3t$.

9-14 ; Describe the graph of the eqn.

9. $r = (3-2t)i + 5t\,j$.

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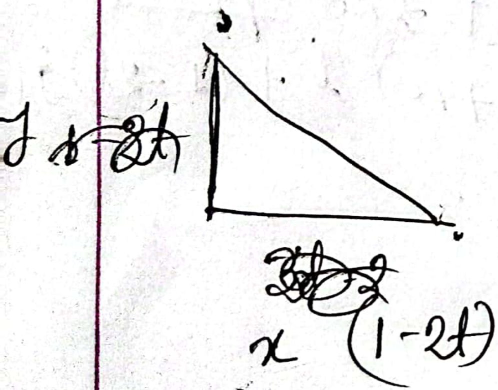
15. a) Find the slope of the line in 2-space that is represented by the vector equation $r = (1-2t)\,i - (2-3t)\,j$.

$r = \left(1 - 2\cdot\frac{1}{2}\right)i - 2 - 3$.



$(1-2t)\cdot(3t-2) = 0$.

$2t = 1$, $t = \frac{1}{2}$ $3t = 2$, $t = \frac{2}{3}$

$x = 1-2t$,

$$\boxed{t = \frac{1-x}{2}}$$

$y = -2 + 3t$ $y = -2 + 3\left(\frac{1-x}{2}\right)$.

$y = -\frac{1}{2} - \frac{3}{2}x$.

$m = -\frac{3}{2}$.

parametric eqn

$x = 1 - 2t$.

$y = -(2 - 3t)$.

$= -2 + 3t$.

so the parallel vector of the line $\langle -2, 3 \rangle$.

$= -2i + 3j$.

(b) Find the coordites of the point where the line $r = (2+t) i + (1-2t) j + 3tk$ intersects the xz plane.

$1 - 2t = 0$.

$2t = 1$

$t = \frac{1}{2}$.

$r = (2+t) i + (1-2t) j + 3tk$.

$= \left(2 + \frac{1}{2}\right) i + \left(1 - 2 \cdot \frac{1}{2}\right) j + 3 \cdot \frac{1}{2} k$.

$= \frac{5}{2} i + 0 j + \frac{3}{2} k$.

coordinates $\left(\frac{5}{2}, 0, \frac{3}{2}\right)$

16 Q) Find the y-intercept of the line in 2-space that is represented by the vector eqn $r = (3+2t)\,i + 5t\,j$.

$$3+2t = 0.$$
$$2t = -3$$
$$t = -\frac{3}{2}.$$

$$y = 5t = 5\left(\frac{3}{2}\right) = -\frac{15}{2}.$$

(8) Find the coordinates of the point where the line
$$r = t\,i + (1+2t)\,j - 3t\,k.$$
intersects the plane $3x - y - z = 2$.

$$x = t.$$
$$y = 1+2t.$$
$$z = -3t.$$

$$3t - (1+2t) - (-3t) = 2.$$
$$t = 3/4.$$

the points of ~~coordinat~~ intersection.
$$\left(\frac{3}{4}, \frac{5}{2}, -\frac{9}{4}\right)$$

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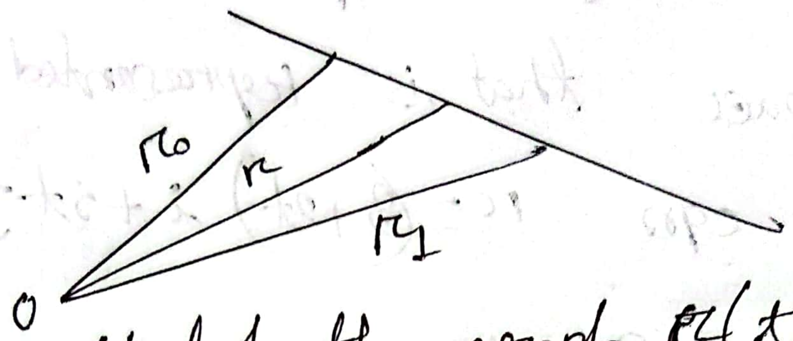

Sketch the graph $r(t)$ and show the direction.

(22) $r(t) = 2i + tj$.

$x = 3t - 4$

$y = 6t + 2$;

$3t - 4 = x$.

$3t = x + 4$.

$t = \frac{x+4}{3}$

$y = 6 \cdot \left(\frac{x+4}{3}\right) + 2$.

$y = 2(x+4) + 2$.

$y = 2x + 8 + 2 = 2x + 10$.

23. $r(t) = \langle 3t-4, 6t+2 \rangle$

$x = 3t-4$

$y = 6t+2$

$r(t) = (1+\cos t)\,i + (3-\sin t)\,j$

$0 \le t \le 2\pi$

$x = 1+\cos t$

$y = (3-\sin t)$

$x-1 = \cos t$

$t = \cos^{-1}(x-1)$

if $t = 0^\circ$

$r(t) = (1+\cos 0^\circ)\,i + (3-\sin 0^\circ)\,j$

$= 1 \cdot i + 3j$

$= i+3j$

x

$x = 1+\cos t$

$y = 3-\sin t$

$(x-1) = \cos t$

$(y-3) = \sin t$

$(x-1)^2 + (y-3)^2 = 1$

24. $r(t) = \langle 2\cos t, 5\sin t \rangle$; $0 \le t \le 2\pi$.

$x = 2\cos t$.

$y = 5\sin t$.

$\frac{x}{2} = \cos t$ ——— (i)

$\frac{y}{5} = \sin t$ ——— (ii)

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{5}\right)^2 = \sin^2 t + \cos^2 t.$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{5}\right)^2 = 1.$$

25. $r(t) = \cosh t\, i + \sinh t\, j$.

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$x = \cosh t$.

$y = \sinh t$

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(26) $r(t) = \sqrt{t}\,i + (2t+4)\,j$.

$x = \sqrt{t}$.

$y = 2t+4$.

$t = x^2$.

$y = 2x^2+4$.

(27) $r(t) = 2\cos t\,i + 2\sin t\,j + tk$.

$x = 2\cos t$.

$y = 2\sin t$

$z = t$.

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33. If r_0 and r_1 are vectors in 3-space, then the graph of the vector-valued function

$r(t)$ is the straight line segment joining the terminal points of r_0 and r_1.

$$r(t) = (1-t)r_0 + t r_1.$$

$r(t) =$

$r = r_0 + vt$. $\qquad v = r_1 - r_0$.

$= r_0 +$

37-38: Sketch the curve of intersection of the surface, and find a vector eqn for the curve in terms of the parameter $x = t$.

37. $9x^2 + y^2 + 9z^2 = 81$, $y = x^2$.

$y = t^2$.

$9t^2 + t^4 + 9z^2 = 81$.

$9z^2 = 81 - 9t^2 - t^4$

$z^2 = \frac{81}{9} - \frac{9t^2}{9} - \frac{t^4}{9}$.

$z = 9 - t^2 - \frac{t^4}{9}$.

$z = \sqrt{9 - t^2 - \frac{t^4}{9}}$.

$r(t) = x i + y j + z k$

$= t i + t^2 j + \sqrt{9 - t^2 - \frac{t^4}{9}}\, k$.

39. Show that the graph of

$$r = t\sin t\, i + t\cos t\, j + t^2 k.$$

lies on the paraboloid $z = x^2 + y^2$

~~let~~ compare the two eqns

$x = t\sin t$

$y = t\cos t.$

$z = t^2.$

$x^2 + y^2 = t^2\sin^2 t + t^2\cos^2 t.$

$= t^2(\sin^2 t + \cos^2 t).$

$= t^2$

40. Show that the graph of

$$r = t i + \frac{1+t}{t} j + \frac{1-t^2}{t} k.$$

lies on the plane $x - y + z + 1 = 0.$

$x = t$

$y = \frac{1+t}{t}.$

$z = \frac{1-t^2}{t}$

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$$L.H.S = t - \frac{1+t}{t} + \frac{1-t^2}{t} + 1.$$

$$= \frac{t^2 - (1+t) + (1-t^2) + t}{t}$$

$$= \frac{t^2 - 1 - t + 1 - t^2 + t}{t}$$

$$= 0$$

$$= R.H.S.$$

49. a) Find parametric eqns for the curve of intersection of the circular cylinder $x^2+y^2=9$ and the parabolic cylinder $z=x^2$ in terms of a parameter t for which

$x = 3\cos t.$

$z = (3\cos t)^2 = 9\cos^2 t.$

$x^2+y^2=9$

$y^2 = 9 - x^2 = 9 - 9\cos^2 t.$

$y^2 = 9(1-\cos^2 t) = 9\sin^2 t.$

$y = 3\sin t.$