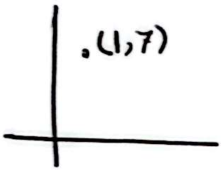
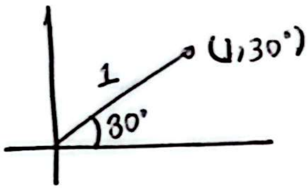
Math:
(01)

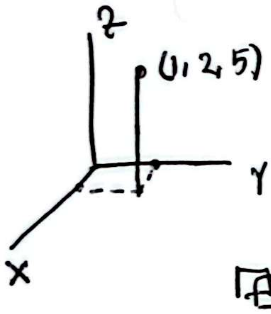
26/08/25

Syllabus: ch-11 to 15 of Anton book

ch→15 ⇒ important







$x = r\cos\theta, \; y = r\sin\theta$

$r = \sqrt{x^2+y^2}, \; \tan\theta = y/x$

$r = a$

↳ eqn of circle

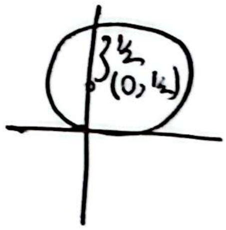
→ $\sqrt{x^2+y^2} = a$

→ $x^2+y^2 = a^2$

$c(0,0), \; r = a$

$r = \sin\theta$

$c(0,\frac{1}{2}), \; r = \frac{1}{2}$



$r = \cos\theta$ ↳ circle

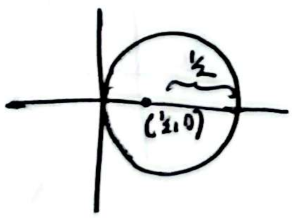
$r^2 = r\cos\theta$

$x^2+y^2 = x$

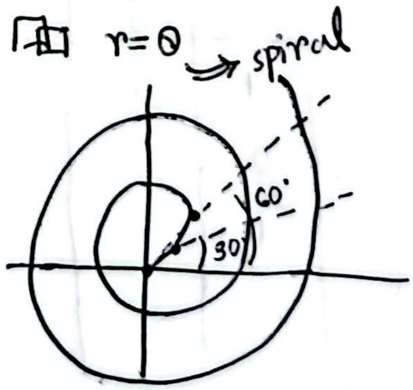
$x^2+y^2-x = 0$

$(x-\frac{1}{2})^2 + y^2 = (\frac{1}{2})^2$

$c(\frac{1}{2},0), \; r = \frac{1}{2}$



$r = \theta$ ⇒ spiral

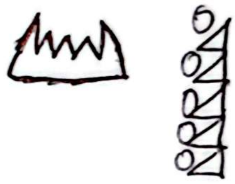


MCMP

r3h24kw~~c~~c

Math → Anton

(from 10th chapter)



→ Computer Networking

Top Down Approach

→ Kurose, Ross

$\boxed{\text{H}}$ $r = 1 + \cos\theta$

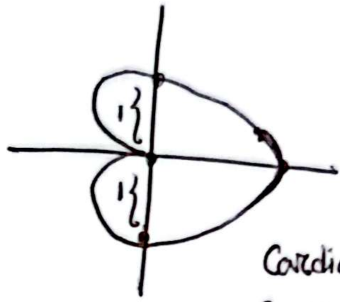
when, $\theta = 0$, $r = 1+1 = 2$

$\theta = \frac{\pi}{2}$, $r = 1+0 = 1$

$\theta = \pi$, $r = 1-1 = 0$

$\theta = \frac{3\pi}{2}$, $r = 1$

$\theta = 2\pi$, $r = 2$



Cardiac Curve

Ch-10

Parametric Eqns:

$y = f(x)$

$x = t \hookrightarrow$ Parameter

$y = f(t)$

Trajectory → সঞ্চারপথ

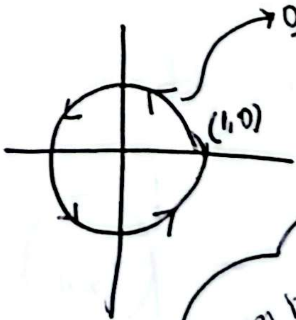
$(x, y) \to (t, f(t))$

Example-2 Find the graph of the eqn $x = \cos t$, $y = \sin t$, $(0 \le t \le 2\pi)$

$x^2 + y^2 = \cos^2 t + \sin^2 t = 1 \to x^2 + y^2 = 1$ ↳ circle $c(0,0), r = 1$

or,

t	x	y
0	1	0
$\pi/2$	0	1
π	-1	0
.		



orientation of the curve, anticlockwise

t এর increase এর সাথে অগ্রসর হয় graph

Example-3 $x = 2t-3,\ y = 6t-7$

$3x = 6t-9,$

$= 6t-7-2$

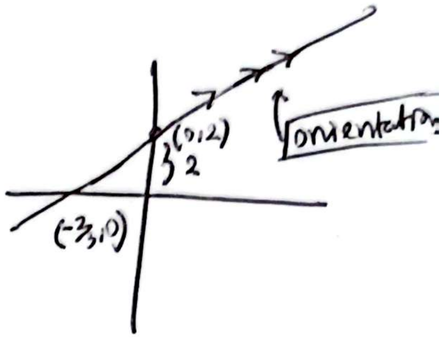
$= y-2$

$y = 3x+2$

$3x - y = -2$

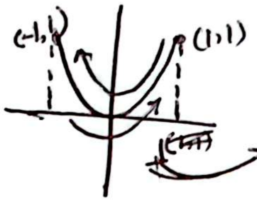
$\frac{x}{-2/3} + \frac{y}{2} = 1$

$(-2/3, 0),\ (0, 2)$



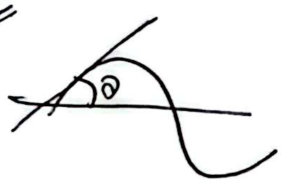
Example-4 $x = \sin t,\ y = \sin^2 t$

$y = x^2$



orientation in both directions

Tangent



$\tan\theta = \frac{dy}{dx}$

$\text{tangent} = \frac{dy}{dx}$

$x = x(t),\ y = y(t)$

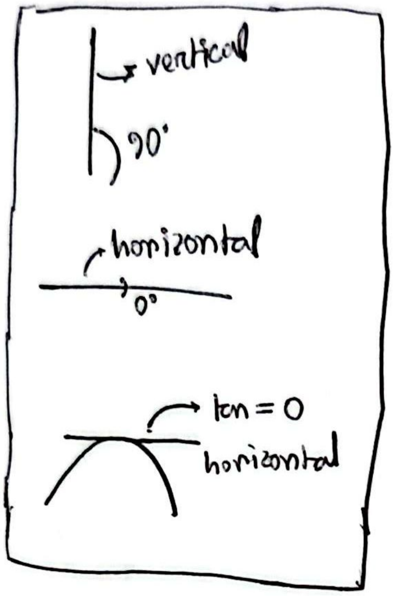
$\frac{dx}{dt},\ \frac{dy}{dt}$

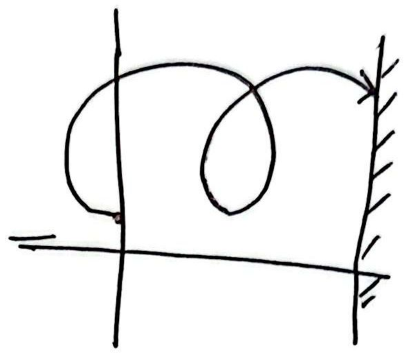
$$\frac{dy/dt}{dx/dt} = \frac{dy}{dx}$$

Example-4 $\frac{dy}{dx} = \frac{dy/dt}{dy/dt} =$

Example 5: $x = t - 3\sin t,\ y = 4 - 3\cos t \quad (t \geq 0)$

crashes at $t = 10$





$y = x^2$

$\frac{dy}{dx} = 2x$

$\frac{d^2y}{dx^2} = 2 \rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right)$

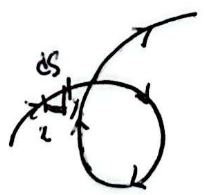
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(y')$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}(y'')$$

Arc length for parametric curves

বক্ররেখা



$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

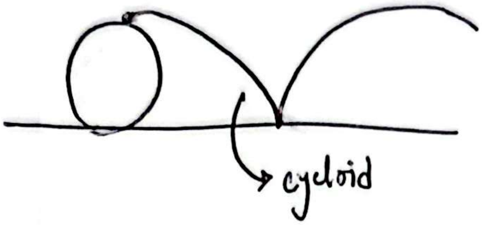
$ds^2 = x^2 + y^2$

$ds = \sqrt{x^2 + y^2}$

~~circle~~ line এখন circle এর Arc length

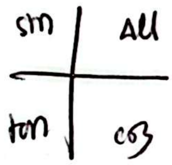
ভাগ radius = α

Cycloid



10.2 Polar Co-ordinates

(r, θ)



Example-6: $r = \cos 2\theta$

$r = \sin(n\theta)$ or, $r =$ ~~sin(n~~ $\cos(n\theta)$

number of loop

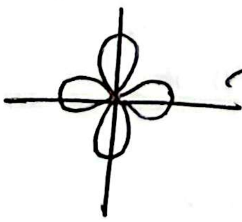
~~1) no. of loop θ = n, when~~ n = 00

① n = odd, no of loop = odd

⑪ n = even, " " " = 2n

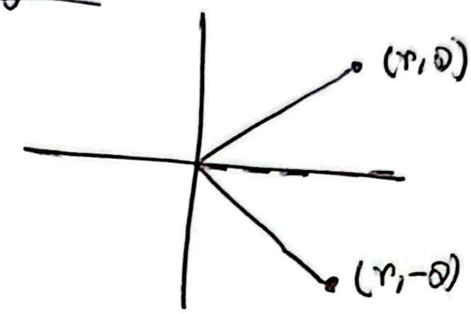
Ex $r = \cos 2\theta$

loop $= 2n$
$= 4$

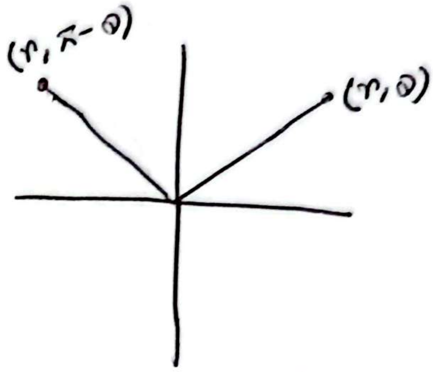


→ petal eqn
↳ looks like flower

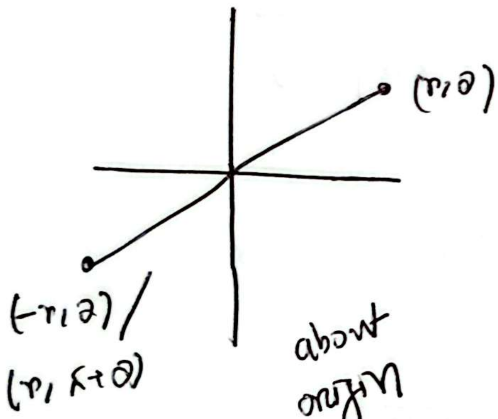
Symmetry Test



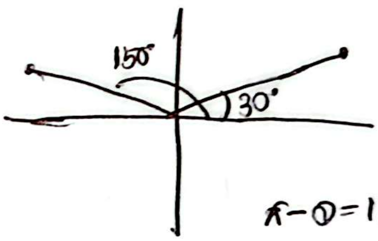
respect to x axis



respect to y axis



about origin



$\pi - \theta = 150^\circ$ ✓

$\frac{\pi}{2} + \theta = 120^\circ$ ✗

families of cardioids and limacons:

$r = a \pm b \sin\theta$ → limacons eqn

$r = 1 + \sin\theta$

$a/b = 1/1 = 1$, cardiac

10.4

Tangent lines to Polar Curves

$x = x(t)$, $\quad x = r\cos\theta,\ y = r\sin\theta$

$y = y(t)$ $\quad r = f(\theta)$

$x = f(\theta)\cos\theta \Rightarrow \frac{dx}{d\theta} = f(\theta)(-\sin\theta) + \cos\theta \frac{df}{d\theta}$

$y = f(\theta)\sin\theta \Rightarrow \frac{dy}{d\theta} = f(\theta)\cos\theta + \sin\theta \frac{df}{d\theta}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $\qquad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$L = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

arc. length

$L = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}$

$= \sqrt{(f(\theta))^2 + (f'(\theta))^2}$

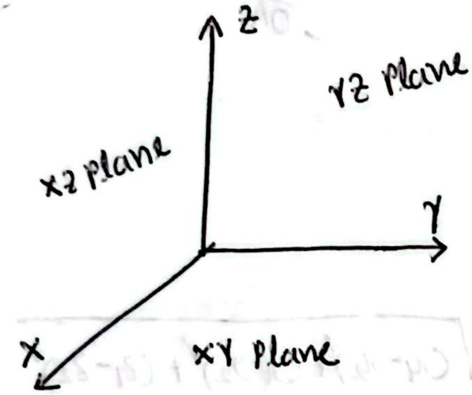
$= \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ \cancel{d}\ d\theta$

read at home
10.1 to 10.3

Math – Multivariable Calculus & Geometry

31/08/25 SUN

Three dimensional space; Vectors (Chapter 11)



$xz \to (x, 0, z)$

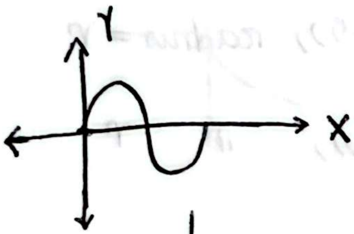
$yz \to (0, y, z)$

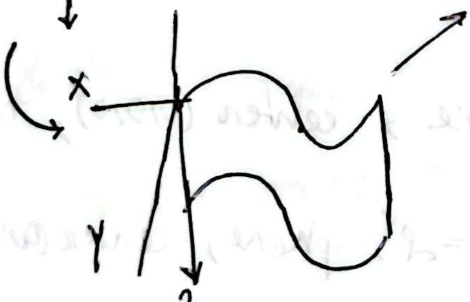
$xy \to (x, y, 0)$

$x \to (x, 0, 0)$

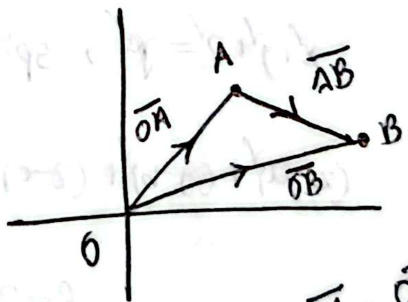
$y \to (0, y, 0)$

$z \to (0, 0, z)$



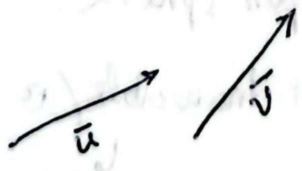


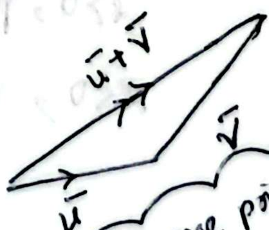
cylindrical co-ordinate system



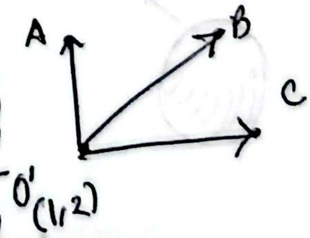
$\overline{OA}$, $\overline{OB}$

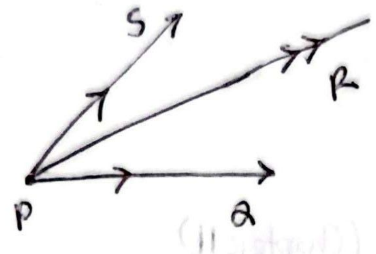
$$\boxed{\overline{AB} = \overline{OB} - \overline{OA}}$$

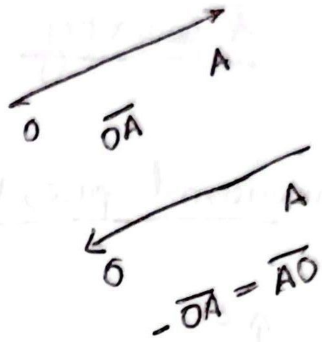




Reference point may not be at origin (0,0)







11.1

$(x_1, y_1), (x_2, y_2)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

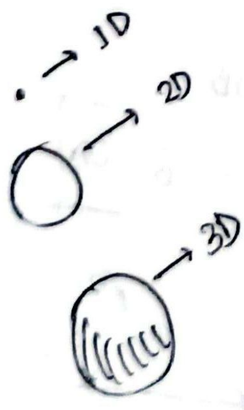
$(x_1, y_1, z_1), (x_2, y_2, z_2)$, $d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$

circle $\Rightarrow$ $x^2+y^2 = r^2$, center $(0,0)$, radius $= r$

$(x-p)^2 + (y-q)^2 = r^2$ $\quad$ (p,q), $\quad$ " $\quad$ r

$x^2+y^2+z^2 = \rho^2$, sphere, center $(0,0,0)$, radius $= \rho$

$(x-a)^2 + (y-b)^2 + (z-c)^2 = \rho^2$, sphere, center (a,b,c), radius $= \rho$



$\rho = 0$, point at 3D

$\rho > 0$, proper sphere

$\rho < 0$, not drawable / ~~it~~ not real

↳ no graph

$x^2+y^2+z^2-2x-4y+8z+17=0$

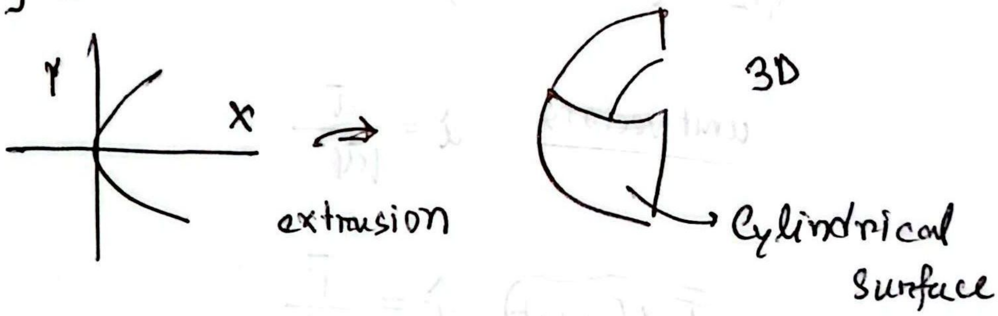
center, radius = ?

$\underline{x^2-2x+1} + \underline{y^2-4y+4} + \underline{z^2+8z+16} + 17 - 1 - 4 - 16 = 0$

$(x-1)^2 + (y-2)^2 + (z+4)^2 = 4 = 2^2$

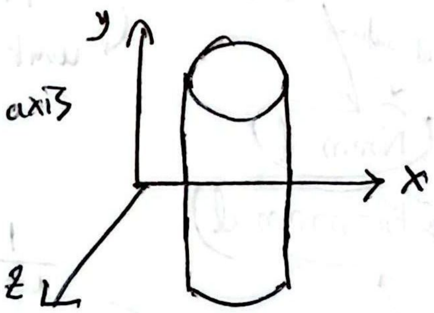
$C(1, 2, -4)$, $r = 2$

$y = x^2$

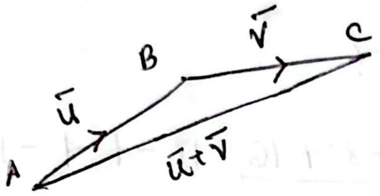


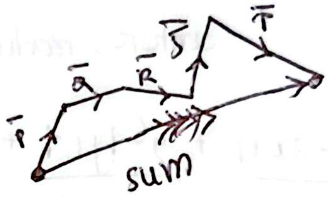
$x^2+z^2=1$ ⌣ circle

3D, extrusion by y axis



11.2 Vectors





$\bar{v} = 2\hat{i} + 3\hat{j} = \langle 2, 3 \rangle$

$P_1(x_1, y_1)$, $P_2(x_2, y_2)$

$\overline{P_1P_2} = (x_2 - x_1, y_2 - y_1)$

unit vectors $\quad \hat{a} = \dfrac{\bar{a}}{|\bar{a}|}$

$\bar{T} \rightarrow$ Tangent, $\quad \hat{T} = \dfrac{\bar{T}}{|\bar{T}|}$

unit tangent

parpendicular

Normal

Bi-normal

parpendicular upon both Tangent and Normal

$\dfrac{1}{x-2}$, $\quad x = 2 \rightarrow$ singular point

not defined at $x = 2$.

singular মানে ঝামেলা

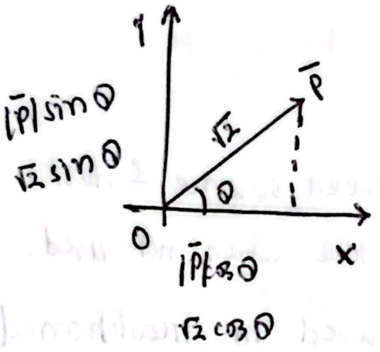
like singular matrix

where $|A| = 0$

so, A^{-1} not possible

$\Rightarrow$ standardize করা মানে → unit vector বের করা

normalize



$$\sqrt{2} = \langle \sqrt{2}\cos\theta, \sqrt{2}\sin\theta \rangle$$

$$= \sqrt{2}\cos\theta\,\hat{i} + \sqrt{2}\sin\theta\,\hat{j}$$

$$\bar{V} = |\bar{V}|\cos\theta\,\hat{i} + |\bar{V}|\sin\theta\,\hat{j}$$

example 6

(a)

$$v = \langle 2\cos\pi/4, 2\sin\pi/4 \rangle$$

$$= 2\cos\pi/4\,\hat{i} + 2\sin\pi/4\,\hat{j} = \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$$

(b) $v = -\sqrt{3}\hat{i} + \hat{j}$

$|v| = 2$

$\cancel{v=}$

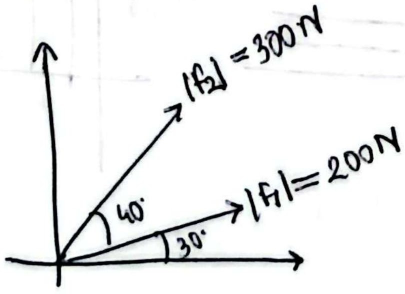
$\theta = 150^\circ = 5\pi/6$

Software

RVP আছে

next lab এর আগে

example 8



$$\bar{F_1} = 200\cos 30\,\hat{i} + 200\sin 30\,\hat{j}$$

$$\bar{F_2} = 300\cos 70\,\hat{i} + 300\sin 70\,\hat{j}$$

$$\bar{F_1} + \bar{F_2} = (200\cos 30 + 300\cos 70)\,\hat{i} + (200\sin 30 + 300\sin 70)\,\hat{j}$$

$|\bar{F_1} + \bar{F_2}| = 471N, \theta = 54.20$

Math- Multivariable Calc & Geometry

02/09/25 TUES

11.3 Dot product & Projections

$$\bar{u}_1 \cdot \bar{u}_2 = |\bar{u}_1||\bar{u}_2| \cos\theta$$

$$\bar{u}_1 = a_1\hat{i} + b_1\hat{j}, \quad \bar{u}_2 = a_2\hat{i} + b_2\hat{j}$$

$$\bar{u}_1 \cdot \bar{u}_2 = \cancel{a_1 b_2 +} \; a_1a_2 + b_1b_2$$

$$\bar{u}_1 \times \bar{u}_2 = |\bar{u}_1||\bar{u}_2| \sin\theta \cdot \hat{\eta}$$

$$\bar{u}_1 \times \bar{u}_2 = \begin{vmatrix} \hat{i} & \hat{j} \\ a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

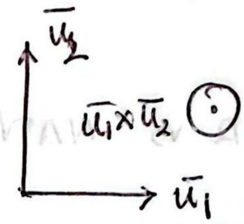
$$\hat{i}\cdot\hat{i} = 1\cdot1\cdot\cos 0 = 1, \quad \hat{j}\cdot\hat{j} = 1, \quad \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = 1\cdot1\cdot\cos 90 = 0, \quad \hat{j}\cdot\hat{k} = 0, \quad \hat{k}\cdot\hat{i} = 0$$

if two vectors are $\perp$

$$\bar{u}_1 \cdot \bar{u}_2 = 0$$

if two vectors are $\parallel$, $\bar{u}_1 \times \bar{u}_2 = 0$



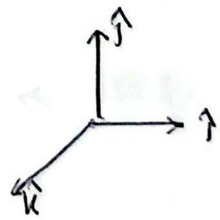
$$\bar{u}_1 \times \bar{u}_2 \perp \bar{u}_1$$

$$\bar{u}_1 \times \bar{u}_2 \perp \bar{u}_2$$

so, $(\bar{u}_1 \times \bar{u}_2)\cdot \bar{u}_1 = 0$

$$(\bar{u}_1 \times \bar{u}_2)\cdot \bar{u}_2 = 0$$

$$\hat{i}\times\hat{i} = 1\cdot1\cdot\sin 0 = 0, \quad \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$



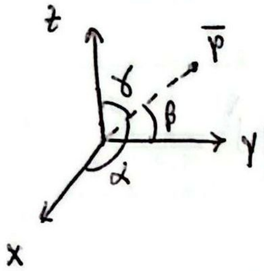
$$\hat{i}\times\hat{j} = 1\cdot1\cdot\sin 90\cdot\hat{k} = \hat{k}$$

$$\hat{j}\times\hat{i} = -\hat{k}, \quad \hat{j}\times\hat{k} = \hat{i}, \quad \hat{k}\times\hat{i} = \hat{j}$$

$$\hat{i}\times\hat{k} = -\hat{j}$$

angle between vectors

$$\cos\theta = \frac{\bar{u}_1 \cdot \bar{u}_2}{|\bar{u}_1| \cdot |\bar{u}_2|}$$



$\bar{p}\cdot\hat{i} = |\bar{p}|\cdot 1\cdot\cos\alpha \Rightarrow \cos\alpha = \dfrac{\bar{p}\cdot\hat{i}}{|\bar{p}|}$

α, β, γ

$\uparrow$

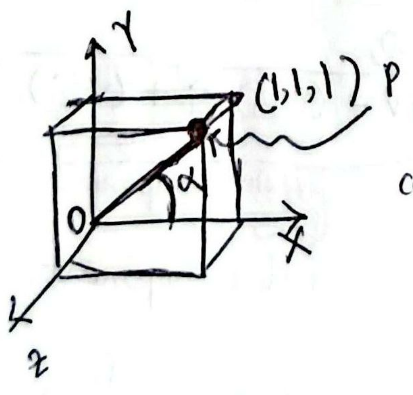
Direction angles

$$\cos\beta = \frac{\bar{p}\cdot\hat{j}}{|\bar{p}|}$$

$$\cos\gamma = \frac{\bar{p}\cdot\hat{k}}{|\bar{p}|}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

example-4: find the angle between diagonal of a cube and one of its edges.



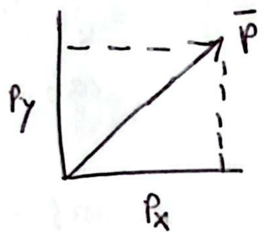
$$\cos\alpha = \frac{\bar{p}\cdot\hat{i}}{|\bar{p}|} = \frac{1}{\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}$$

$$\alpha = \cos^{-1} 1/\sqrt{3}.$$

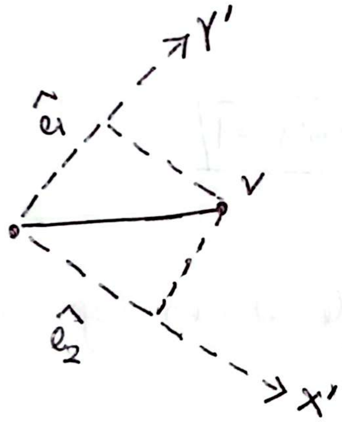
$\hat{e}_1, \hat{e}_2 \to$ if given as unit vectors

$\bar{u}_1 = a_1\hat{e}_1 + b_1\hat{e}_2$

<u>decomposing vectors into orthogonal components:</u>



$\bar{P} = P_x\hat{i} + P_y\hat{j}$



$v = k_1\hat{e}_1 + k_2\hat{e}_2$

$v\cdot\hat{e}_1 = k_1\cdot 1 + k_2\cdot 0 = k_1$

$v\cdot\hat{e}_2 = k_2$

so,

$$\boxed{V = (v\cdot\hat{e}_1)\,\hat{e}_1 + (v\cdot\hat{e}_2)\,\hat{e}_2}$$

vector component

scalar component

<u>Example-5:</u> $V = (2,3)$, $e_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $e_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$V = (v\cdot e_1)\,\hat{e}_1 + (v\cdot e_2)\,\hat{e}_2$$

$$= \left(\sqrt{2} + \frac{3}{\sqrt{2}}\right)\hat{e}_1 + \left(-\sqrt{2} + \frac{3}{\sqrt{2}}\right)\hat{e}_2 = \frac{5}{\sqrt{2}}\hat{e}_1 + \frac{1}{\sqrt{2}}\hat{e}_2$$

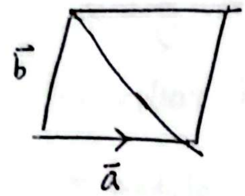
Box product,

$$\bar{u}\cdot(\bar{v}\times\bar{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \text{ or, } (\bar{u}\times\bar{v})\cdot\bar{w}$$

Indicates Volume

if $\bar{u}\cdot(\bar{v}\times\bar{w}) = 0$

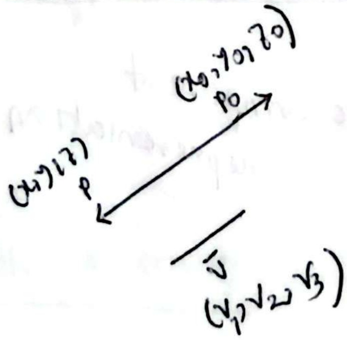
then, $\bar{u}, \bar{v}, \bar{w}$ lie in the same plane

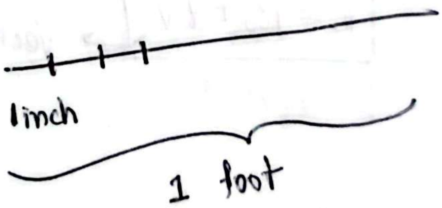


$\square$ Area $= |\bar{a}\times\bar{b}|$

Δ Area $= \frac{1}{2}|\bar{a}\times\bar{b}|$

11.5 Parametric Eqns of Lines





$\boxed{\overline{PP_0} = t\,\bar{v}}$ $\rightsquigarrow$ $\langle x-x_0, y-y_0, z-z_0\rangle = t\langle v_1, v_2, v_3\rangle$

P any point on line → variable

P_0 → a point through which the line passes.

Vectors equal when component equal

parametric eq: $x - x_0 = tv_1$, $y = y_0 + tv_2$, $z = z_0 + tv_3$

$\Rightarrow$ so, we need a point (P_0) that the line passes through and a parallel vector ($\bar{V}$) to the line

example 3

$L_1 \rightarrow$ passing point $(1, 5, -1)$

parallel vector $\langle 4, -4, 5 \rangle$

$L_2 \rightarrow$ passing point $(2, 4, 5)$

parallel vector $\langle 8, -3, 1 \rangle$

$r = (x, y, z)$, $r_0 = (x_0, y_0, z_0)$, $v = (a, b, c)$

$\boxed{\bar{r} = \bar{r_0} + t\bar{V}}$ $\rightarrow$ Vector component representation

11.5 examples & exercise homework

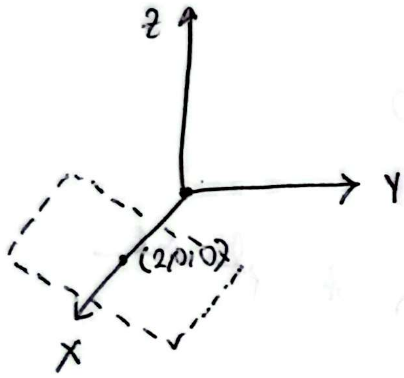
imiki T13

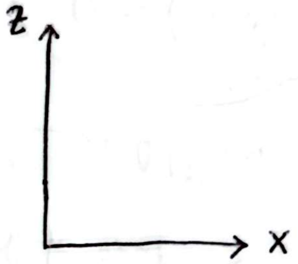
Multivariable Calculus & Geometry

11/09/25 THURS

$$\bar{r} = \bar{r}_0 + t\bar{v}$$

(2,0,0)





$$\bar{n}(\bar{r}-\bar{r}_0) = 0$$

Q# find an eqn of the plane passing through the point $(3,-1,7)$ and perpendicular to the vector $\bar{n} = \langle 4,2,-5 \rangle$

02# Determine whether the planes $3x-4y+5z=0$ and $-6x+8y-10z-4=0$ are parallel.

03# Find an equation of the plane through the points $P_1(1,2,-1)$, $P_2(2,3,1)$ and $P_3(3,-1,2)$.

$\bar{r} \longrightarrow \bar{r}_0$ $\quad \overline{r r_0}$

এর সাথে normal or perpendicular

$$\bar{n} \cdot \underline{(\bar{r} - \bar{r}_0)} = 0$$

line

DA হলো 0 হবে

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

↳ general eqn of plane

plane এর সাথে perpendicular vector থাকবে

↳ $\bar{n}$

↳ orthogonal vector

<u>Ans. to Q</u> $\quad \bar{n} \Rightarrow \langle 4, 2, -5 \rangle$, $r_0 \Rightarrow \langle 3, -1, 7 \rangle$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$4(x - 3) + 2(y + 1) - 5(z - 7) = 0$$

$$4x - 12 + 2y + 2 - 5z + 35 = 0$$

$$4x + 2y - 5z + 25 = 0$$

plane eqn, $\boxed{ax + by + cz + d = 0}$

↳ $\langle a, b, c \rangle \Rightarrow$ corresponding normal vector

⇒ two lines are parallel if there normals are scalar multiples of each other

Ans. to 2

normal of ম) plane $\overline{n_1} = \langle 3, -4, 5 \rangle$

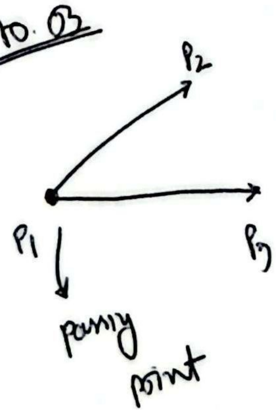
normal " 2nd " $\overline{n_2} = \langle -6, 8, -10 \rangle$

here, $\overline{n_2} = -2\overline{n_1}$

linear algebra অনুযায়ী linear dependence property full fill করলে parallel

যদি n dimensions এ ভেক্টর থাকে check করা যায়

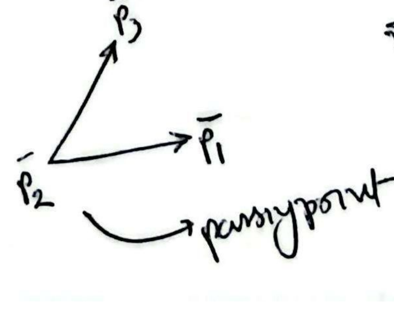
Ans. to. Q3



$\overline{n} = \overline{P_1P_2} \times \overline{P_1P_3}$

can do by other points like P_2, P_3 also,

$\overline{n} = \overline{P_2P_3} \times \overline{P_2P_1}$



$P_1 = \langle 1, 2, -1 \rangle$, $P_2 = \langle 2, 3, 1 \rangle$, $P_3 = \langle 3, -1, 2 \rangle$

$\overline{P_1P_2} = \langle 2-1, 3-2, 1+1 \rangle = \langle 1, 1, 2 \rangle$

$\overline{P_1P_3} = \langle 3-1, -1-2, 2+1 \rangle = \langle 2, -3, 3 \rangle$

$$\overline{P_1P_2} \times \overline{P_1P_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = \hat{i}(3+6) - \hat{j}(3-4) + \hat{k}(-3-2)$$

$$= 9\hat{i} + \hat{j} - 5\hat{k}$$

$\bar{n} = 9\hat{i} + \hat{j} - 5\hat{k}$

$\hat{n} = \langle 9, 1, -5 \rangle$

$P(1, 2, -1)$

$9(x-1) + 1(y-2) + (-5)(z+1) = 0$

CamScanner

Determine whether the line, $x = 3+8t$, $y = 4+5t$, $z = -3-t$ is parallel to the plane $x - 3y + 5z = 12$.

normal vector $= \langle 1, -3, 5 \rangle$
$\vec{n}$

parallel vector

$\vec{p} = \langle 8, 5, -1 \rangle$

$\vec{p} \cdot \vec{n} = 8 - 15 - 5$

$\neq 0$

not parallel

will intersect somewhere

finding the intersection point,

$3+8t - 3(4+5t) + 5(-3-t) = 12$

$3 + 8t - 12 - 15t - 15 - 5t = 12$

$-12t = 36$

$t = -3$

$x = 3 - 24 = -21$, $y = 4 - 15 = -11$, $z = -3 + 3 = 0$

$\boxed{(-21, -11, 0)}$

(x_1, y_1), (x_2, y_2)

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

distance → point to plane

→ plane to plane (parallel)

→ plane to skew ~~line plane~~ line on other parallel plane

line on a plane to skew → means not parallel ~~line~~ lines.

$(x_0, y_0, z_0) \rightarrow ax + by + cz + d = 0$

$$\text{distance} = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

angle between two planes

↳ find angle between normals

$$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{\|\vec{n_1}\| \, \|\vec{n_2}\|}$$

The planes $x+2y-2z=3$ and $2x+4y-4z=7$ are parallel. Find the distance between these planes.

$x+2y-2z=3$ — ① $x+2y-2z=7/2$ — ②

~~(3,0,0)~~ $y=z=0$, from ① $\Rightarrow x=3$

So, point $(3,0,0)$ upon plane ①

so distance from point $(3,0,0)$ to plane ② would be the answer

The lines

$L_1: x = 1+4t,\ y = 5-4t,\ z = -1+5t$

$L_2: x = 2+8t,\ y = 4-3t,\ z = 5+t$

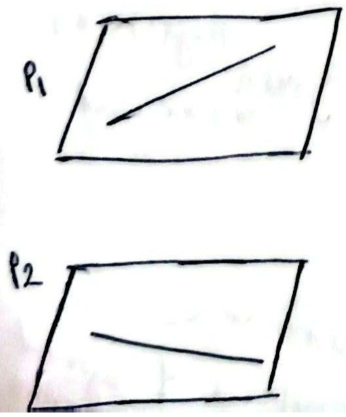
are skew. $\longrightarrow$ means not parallel lines.

But the planes containing them $\Rightarrow$ parallel.

Find the distance between them

P1

P2



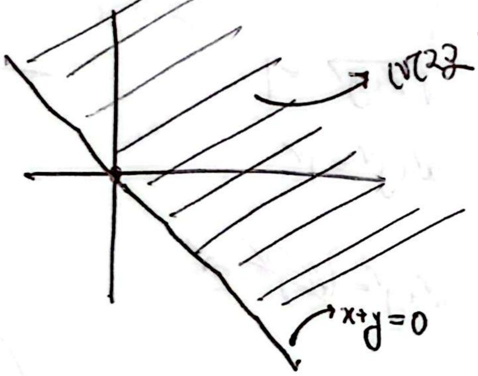
Multivariable Calculus & Geometry 15/09/25 | MON

Determine the domain of each of the following & present them:— graphically

(i) $f(x,y) = \sqrt{x+y}$

(ii) $f(x,y) = \sqrt{x} + \sqrt{y}$

(iii) $f(x,y) = \ln(9 - x^2 - 9y^2)$



মানে ≥ 0

(i) $x+y \geq 0$

$x+y=0$

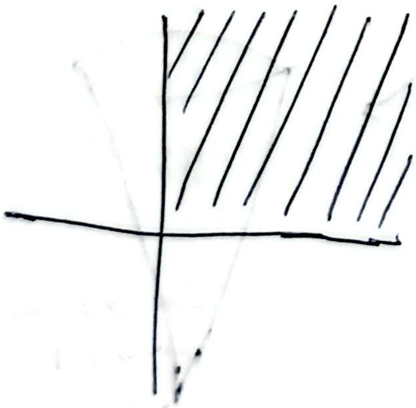
$y=-x$

ln এর ভিতর (+)ve হতে হয় defined

$\ln 0 =$ undefined

(iii)

(ii) $x \geq 0$ and $y \geq 0$



$9 - x^2 - 9y^2 > 0$

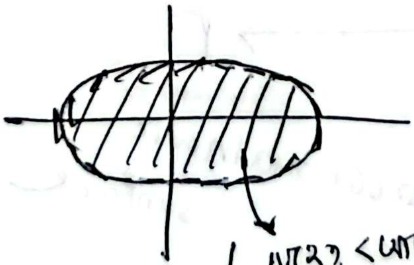
$x^2 + 9y^2 < 9$

$x^2 + 9y^2 = 9$

$\frac{x^2}{9} + y^2 = 1$

$\frac{x^2}{3^2} + \frac{y^2}{1^2} = 1$

center $(0,0)$, $a = 3, 1$



মানে < আছে

এখানে Boundary accept করবে না, কারণ equal নাই

3D figure কে 2D তে convert করলে
আমরা দেখব 2D

convert করার পর রেখাগুলো পাবো contour বলা হয় যা, 2D তে

Identify the level curves of $f(x,y) = \sqrt{x^2+y^2}$ and sketch few of them.

let, $z = \sqrt{x^2+y^2}$ → 3D curve

let, $z = 0$, $\sqrt{x^2+y^2} = 0$

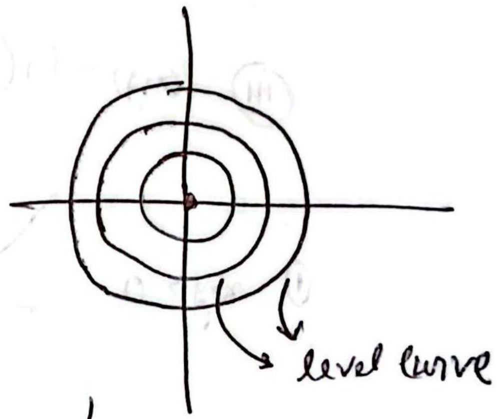
$x^2+y^2 = 0$

$z = 1$, $\sqrt{x^2+y^2} = 1$

$x^2+y^2 = 1^2$

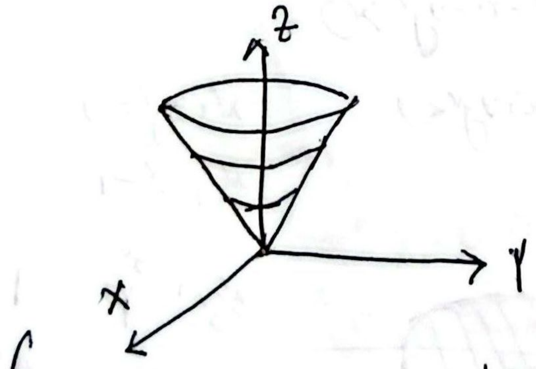
$z = 2$ $\quad x^2+y^2 = \cancel{(1)^2}\ 2^2$

z এর মান radius হবে

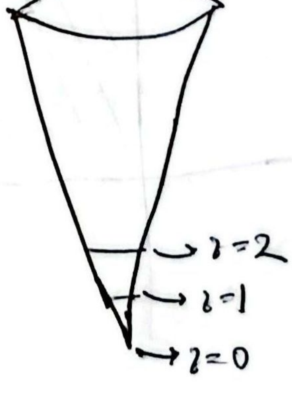


Contour plot

multiple level curve মিলে যা form হয়



3D তে figure কে বলে quadratic surface



এই 3D তে $x=1$ plane এর উপরে ও $y=2$ এর উপরে দেখতে কেমন হবে

☐ Sketch the trace of,

$$f(x,y) = 10 - 4x^2 - y^2$$

for the plane $x=1$ and $y=2$.

$(1,0,0)$ ~~(0,1,0)~~ $(0,2,0)$

↳ যেটা মানে হলো plane টা y ও z axis এর parallel

↳ যেটা হবে x ও z axis এর parallel

$y=2$, $f(x,2) = 10-4x^2-4 = 6-4x^2$

$z = 6-4x^2 \longrightarrow$ Parabola

$x=1 \longrightarrow$ parabola

12.1/ Introduction to Vector valued functions

$f(t) = t \longrightarrow$ t parametric function

$g(t) = t^2$

$h(t) = 3t$

$$\bar{r}(t) = \underline{f(t)}\hat{i} + \underline{g(t)}\hat{j} + \underline{h(t)}\hat{k}$$

component function

যে parametric function দ্বারা ভিন্ন vector valued function তৈরি হয়

↳ scalar, direction নেই

↳ value ও আছে direction ও আছে

↳ vector valued function.

| t এর value change এর সাথে $\bar{r}$ পরিবর্তন হবে

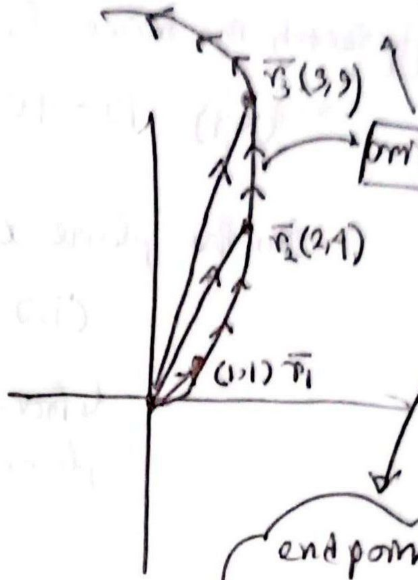
suppose, $\vec{r}(t) = t\hat{i} + t^2\hat{j}$

~~t(0)~~ $t=0,\ \vec{r} = 0\hat{i} + 0\hat{j}$

$t=1,\ \vec{r} = \hat{i} + \hat{j}$

$t=2,\ \vec{r} = 2\hat{i} + 4\hat{j}$

$t=3,\ \vec{r} = 3\hat{i} + 9\hat{j}$



বা | direction of increasing parameter

$\boxed{\text{orientation}}$

endpoint গুলোর দিয়ে curve পাবো

vector valued function এর curve

vector valued function এর domain

↳ প্রত্যেকটি component function এর domain এর intersection

সেই natural domain

vector valued function এর limit

exist করবে ⇒ যদি প্রত্যেকটি component funct এর limit থাকে

" " " continuous " " " continuous

" " " differentiable " " " differentiable example 1

parametric eqns

$x = 1-t, \; y = 3t, \; z = 2t$

(1,0,0) parallel vector

passing point (-1,3,2)

example 2 describe the parametric curve represented by the eqns,

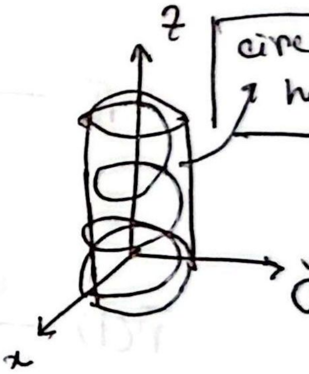
$x = a\cos t, \; y = a\sin t, \; z = ct$

$x^2 + y^2 = a^2$

xy plane a circle form

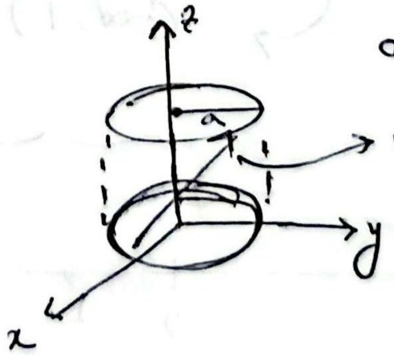
यह a radius का

fixed



$x = a\cos t, \; y = a\sin t, \; z = 2$

$x^2 + y^2 = a^2$



circular cylinder

orientation

दिशा बिटा

$\bar{r}(t) = (\ln|t-1|, e^{t}, \sqrt{t})$

$= \ln|t-1|\,\hat{i} + e^t\,\hat{j} + \sqrt{t}\,\hat{k}$

find domain

$\ln(t-1) \rightarrow$ domain $\rightarrow t > 1$

$e^t \longrightarrow \mathbb{R}$

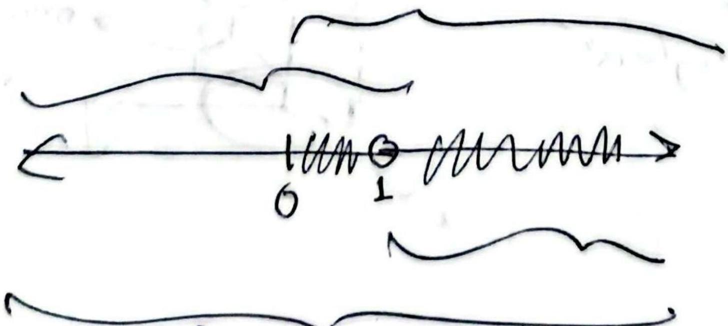
$\sqrt{t} \longrightarrow t \geq 0$

$\bar{r}(t) \longrightarrow$ Domain: $\{t \in \mathbb{R} : t > 1\}$

★★★ Domain, Range, Sketch এর ধরনের problem মনে রাখবে

But $\ln|t-1| \rightarrow$ modulus থাকায়

$\hookrightarrow (-\infty, 1) \cup (1, +\infty)$



so, $\bar{r}(t)$ Domain

$\{t \in \mathbb{R} : t \geq 0$ and $t \neq 1\}$

or ω,

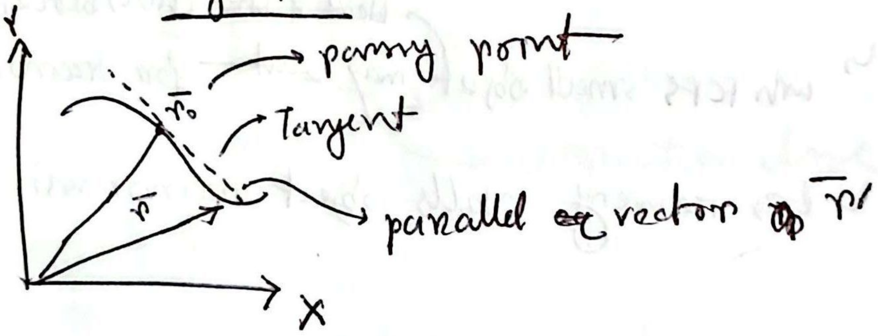
$$[0,1) \cup (1, +\infty)$$

⇒ 12.2 এর limit, differentiation এর math শিখে রাখবো

⇒ 12.2.5 Theorem proof দেখতে হবে

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

<u>Tangent</u>



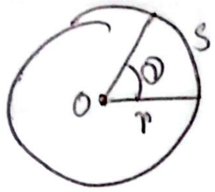
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

⇒ 12.2.8 Theorem খুবই important,
ভালোভাবে বুঝে দেখবো

12.3 | Change of Parameter; Arc length

$s = r\theta$

Arc length ← s, radius ← r, θ → angle



→ প্রত্যেকটা straight line infinite radius এর কোনো না কোনো circle এর Arc.

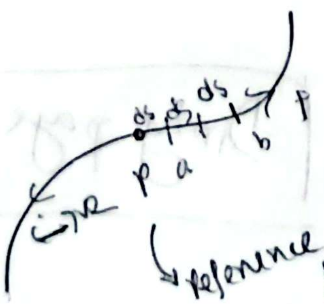
$\bar{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j}$ → circle

radius a, angle → t

$s = at$

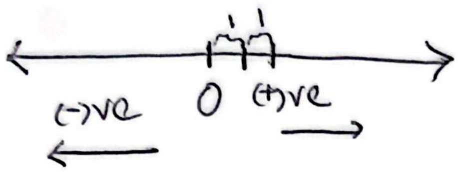
$t = s/a$

$\bar{r}(s) = a\cos(s/a)\,\hat{i} + a\sin(s/a)\,\hat{j}$ → Arc length parameterization



→ reference point ধরে নাও যেকোনো

→ তোমার যদি direction কে +ve, opposite কে (-)ve ধরে নাও।



$\Rightarrow$ এরপর ds থেকে Total length পেতে Integrate করবো।

smooth vector valued function

↳ 1st derivative exists and first derivative always non zero.

$\bar{r}(t) = t^2\hat{i} + 2t^3\hat{j}$; $0 \le t \le 1$

$\bar{r}'(t) = 2t\hat{i} + 6t^2\hat{j}$

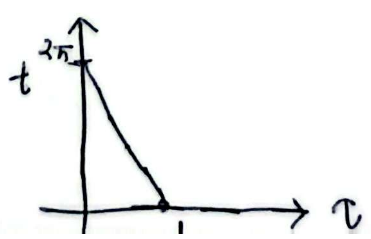
$\bar{r}'(0) = \langle 0, 0 \rangle$

↳ so, not smooth vector valued func.

$\Rightarrow$ Theorem 12.3.1

Example 4 $\quad t = g(\tau)$ $\quad r(t) = \cos t\,\hat{i} + \sin t\,\hat{j}$ $(0 \le t \le 2\pi)$ ← circle এর eqn

$0 \le \tau \le 1$

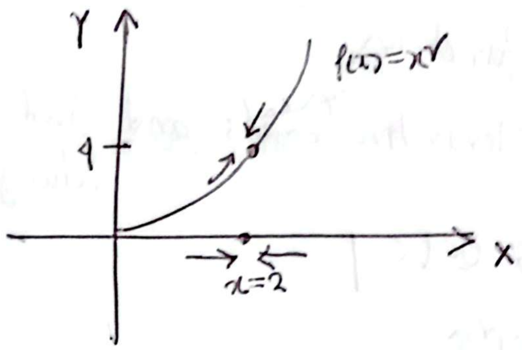


Limits and Continuity

$$\lim_{x \to a} f(x) = L$$

$$f(x) = x^2$$

$$\lim_{x \to 2} f(x) = 4$$



Limit exist করতে হলে, $LHL = RHL$

continuous হতে হলে, $LHL = RHL = f(x)$

⇒ multivariable এ বিভিন্ন দিক থেকে approach করি যদি একটি common point এ converge করে তাহলে বলি limit exist করে

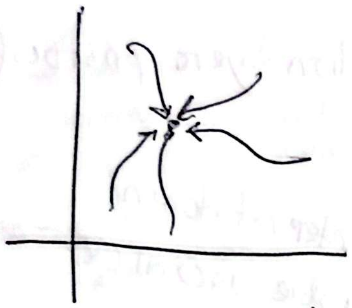
$$\lim_{(x,y) \to (x_0, y_0)} f(x,y) = L$$

3D তে আমরা কোনো curve এর along limit বের করতে পারি

limits along curves

→ সবচেয়ে easy curve হলো straight line

☆☆☆ limit, continuity, differentiability থেকে একটা না একটা problem final এ definitely থাকবে → 7/8 mark এর



in 3D finding limit

example 3.2.5 $\quad f(x,y)=\dfrac{x^2-y^2}{x^2+y^2}$

$$\lim_{(x,y)\to(0,0)} f(x,y) = ?$$

$x=0$ $\quad$ $y=0$

→ এই দুই line এর respect এ আমরা limit বের করবো along

→ দুটো converge করলে limit exist করে।

example 3.2.10 $\quad f(x,y)=\dfrac{x^2 y}{x^4+y^2}$