Unit 11.3.

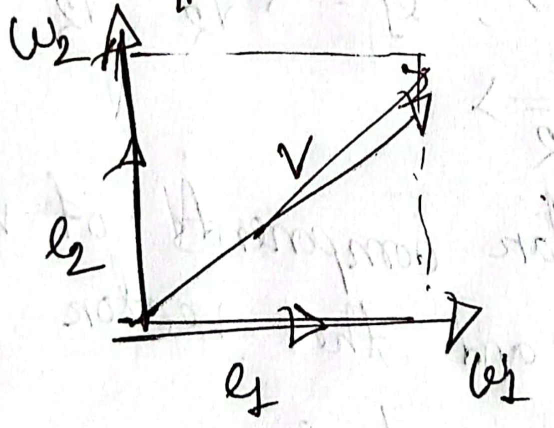
If $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$ are vectors in 2-space, then the dot product of u and v is $u \cdot v = u_1 v_1 + u_2 v_2$.

for 3-space $u = \langle u_1, u_2, u_3 \rangle$ and $v = \langle v_1, v_2, v_3 \rangle$

so the dot product $= u_1 v_1 + u_2 v_2 + u_3 v_3$.

The direction cosines of a non-zero vector $v = v_1 i + v_2 j + v_3 k$. are

$$\cos\alpha = \frac{v_1}{\|v\|}, \quad \cos\beta = \frac{v_2}{\|v\|}, \quad \cos\gamma = \frac{v_3}{\|v\|}.$$



$v = w_1 + w_2$

$$\boxed{v = k_1 e_1 + k_2 e_2.}$$

We can find k_1 by taking the dot product of v and e_1.

$$\boxed{v \cdot e_2 = k_2}$$

$$v \cdot e_1 = (k_1 e_1 + k_2 e_2)\, e_1$$

$$v \cdot e_1 = k_1 \|e_1\|^2 + k_2 e_2 e_1 = k_1 \|e_1\|^2 = k_1.$$

$k_1 = v.e_1$.

$k_2 = v.e_2$

$v = k_1 e_1 + k_2 e_2$.

$= (v.e_1) e_1 + \cancel{(k_2 \cdot v_2)} (v.e_2) e_2$.

$\|v\| \cos\theta$ $\quad\quad$ $\|v\| \sin\theta$.

$= (\|v\| \cos\theta) e_1 + (\|v\| \sin\theta) e_2$

Example 5:

Let $v = \langle 2, 3 \rangle$, $e_1 = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$. and

$e_2 = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

Find the scalar components of v along e_1 and e_2 and the vector compon~~ts~~ents of v along e_1 and e_2.

$v = k_1 e_1 + k_2 e_2.$

$k_1 = v \cdot e_1 = \langle 2,3 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle.$

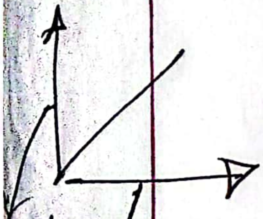
$= \frac{2}{\sqrt{2}} + \frac{3}{\sqrt{2}} = \sqrt{2} + \frac{3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

$k_2 = v \cdot e_2 = \langle 2,3 \rangle \cdot \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle.$

$= -\frac{2}{\sqrt{2}} + \frac{3}{\sqrt{2}}$

$= -\sqrt{2} + \frac{3}{\sqrt{2}} = \frac{1}{\sqrt{2}}.$

vector components v $k_1 e_1 = \frac{5}{\sqrt{2}} \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = \langle \frac{5}{2}, \frac{5}{2} \rangle$



$k_2 e_2 = \frac{5}{\sqrt{2}} \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = \langle -\frac{1}{2}, \frac{1}{2} \rangle.$

Orthogonal projection

ORthogonal projections.

$Proj_{e_1} v = (v \cdot e_1) e_1 = k_1 e_1.$

$Proj_{e_2} v = (v \cdot e_2) e_2 = k_2 e_2.$

In general, if e is a unit vector, then we define the orthogonal projection of v on e to be

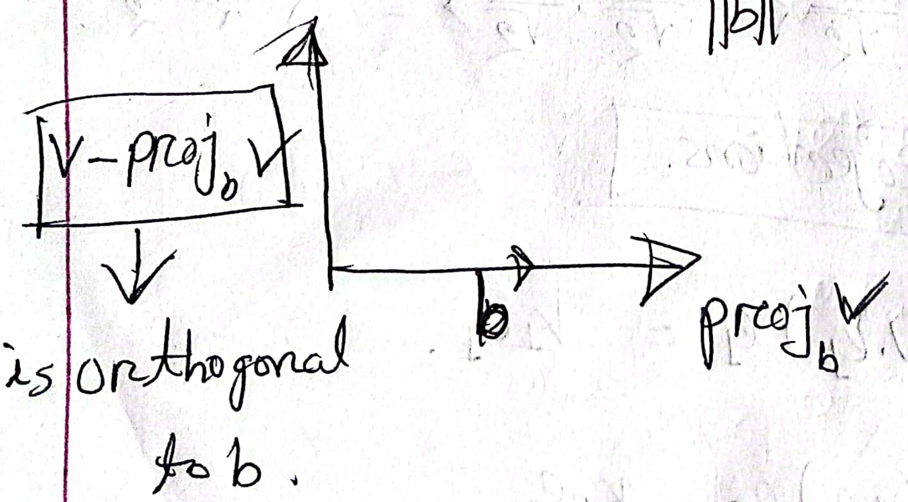
$$\mathrm{Proj}_e v = (v \cdot e)\, e.$$

For arbitrary nonzero vector b.

$$\mathrm{Proj}_b v = (v e_1) e_1$$

$$= \left(v \cdot \frac{b}{\|b\|}\right)\left(\frac{b}{\|b\|}\right)$$

$$= \frac{v \cdot b}{\|b\|^2}\, b.$$



Example 7. Find the orthogonal projection of $v = i+j+k$ on $b = 2i+2j$ and then find the vector component of v orthogonal to b.

$$\text{proj}_b v = (v \cdot e_1) e_1$$

$$= \cancel{i+j+k} \left(v \cdot \frac{\vec{b}}{\|b\|} \right) \frac{\vec{b}}{\|b\|}$$

$$= \frac{\vec{v} \cdot \vec{b}}{\|b\|^2} \vec{b}$$

$$= \left\{ (i+j+k) \cdot \frac{2i+2j}{\sqrt{(2)^2+(2)^2}} \right\} \cdot \frac{2i+2j}{\sqrt{2^2+2^2}}$$

$$= \left\{ (i+j+k) \cdot \frac{2i+2j}{2\sqrt{2}} \right\} \frac{2i+2j}{2\sqrt{2}}$$

$$= \left\{ (i+j+k) \left(\frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j \right) \right\} \cdot \left(\frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j \right)$$

$$= \frac{(i+j+k)(2i+2j)}{8} (2i+2j)$$

$$= \frac{2+2}{8} (2i+2j)$$

$$= \frac{4}{8} (2i+2j) = \frac{1}{2} (2i+2j) = i+j.$$

orthogonal to $b = v - (i+j) = k$.

1. (a)-(d)

$$\cos\theta = \frac{u.v}{\|u\|\ \|v\|}$$

২: (a) acute angle = সূক্ষ্মকোণ

obtuse angle = স্থূলকোণ

orthogonal = সমকোণ.

right angle = $90°$.

৪ (a) $v.v_1 = \|v\|\ \|v_1\| \cos\alpha$.

$$\cos\alpha = \frac{v.v_1}{\|v\|.\|v_1\|}$$

$$= \frac{(ai+bj)(-bi+aj)}{\sqrt{a^2+b^2}.\sqrt{a^2+b^2}}$$

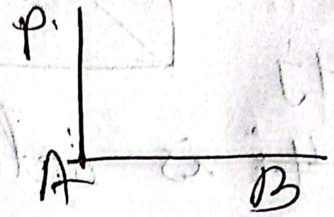
$$= \frac{-ab+ab}{a^2+b^2}$$

$$\cos\alpha = 0$$

$$\alpha = 90°.$$

~~20.5.24~~

13. Find r so that the vector from the point $A(1,-1,3)$ to the point $B(3,0,5)$ is orthogonal to the vector from A to the point $P(r,r,r)$.



$$\vec{AB} = (3i+0\cdot j+5k)-(i-j+3k)$$

$$= 3i-i+0j+j+5k-3k$$

$$= 2i+j+2\hat{k}$$

$$\vec{AP} = (r-1)i+(r+1)j+(r-3)k$$

$$\vec{AB}\cdot\vec{AP} = 0$$

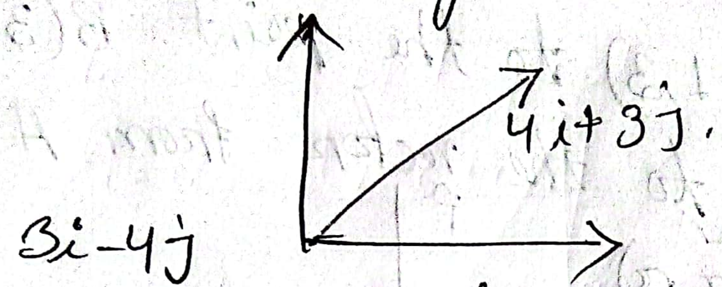
$$2(r-1)+1(r+1)+2(r-3)=0$$

$$2r-2+r+1+2r-6=0$$

$$5r-7=0$$

$$r=\frac{7}{5}$$

14. Find two unit vectors in 2-space that make an angle of $45°$ with $4i+3j$.



~~3j-4i~~ is orthogonal to and has the same length as $4i+3j$.

so $u_1 = (4i+3j) + (3i-4j)$

$= 7i - j$

and $u_2 = (4i+3j) + (-1)(3i-4j)$

$= i+7j$ each make an of $45°$ with $4i+3j$.

so unit vectors

$$\frac{7i-j}{\sqrt{7^2+(1)^2}} = \frac{7i-j}{\sqrt{50}}$$

$$\frac{i+7j}{\sqrt{7^2+1^2}} = \frac{i+7j}{\sqrt{50}}$$

15-16. Find the direction cosines of v and confirm that they satisfy eqn 5. Use the direction cosines to approximate the direction angles

a) $\cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$

$\cos\beta = \frac{1}{\sqrt{3}}$

$\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$

$\cos\gamma = -\frac{1}{\sqrt{3}}.$

$\gamma = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) =$

(b) $\cos\alpha = \frac{2}{3}$

$\alpha = \cos^{-1}\left(\frac{2}{3}\right).$

$\cos\beta = -\frac{2}{3}.$

$\beta = \cos^{-1}\left(-\frac{2}{3}\right).$

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20. Show that 2 non-zero vectors v_1 and v_2 are orthogonal if and only if their direction cosines satisfy

$$\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_2\cos\gamma_1 = 0.$$

$$u = \|u\| \langle \cos\alpha_1, \cos\beta_1, \cos\gamma_1 \rangle.$$

$$v = \|v\| \langle \cos\alpha_2, \cos\beta_2, \cos\gamma_2 \rangle$$

u and v are perpendicular if and only if $u \cdot v = 0$.

$$\|u\| \|v\| \langle \cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2 = 0$$

$$\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2 = 0.$$

22. Find, to the nearest degree, the acute angle formed by two diagonals of a cube.

$d_1 = ai + aj + ak$.

$d_2 = ai + aj - ak$.

$$\cos\alpha = \frac{d_1 \cdot d_2}{\|d_1\| \, \|d_2\|} = \cancel{\cos 71^\circ}$$

$$= 71^\circ.$$

23. vector $\vec{u} =$

unit vector $\frac{\vec{u}}{\|u\|}$ $(i + j + k)$

$$(\quad)i + (\quad)j + (\quad)k.$$

$\cos\alpha$ $\quad$ $\cos\beta$ $\quad$ $\cos\gamma$.