Unit 11.2.

Two vectors are equivalent if and only if their corresponding components are equal.

$\langle a, b, c \rangle$ and $\langle 1, -4, 2 \rangle$ is equal if $a = 1, b = -4, c = 2$.

$V = \langle V_1, V_2 \rangle$ and $W = \langle W_1, W_2 \rangle$ are vectors in 2-space and k is a scalar

$$V + W = \langle V_1 + W_1, V_2 + W_2 \rangle$$

$$V - W = \langle V_1 - W_1, V_2 - W_2 \rangle$$

$$kV = k \langle V_1, V_2 \rangle$$
$$= \langle kV_1, kV_2 \rangle.$$

$V = \langle V_1, V_2, V_3 \rangle$ and $W = \langle W_1, W_2, W_3 \rangle$ are vectors in 3-space and k is a scalar

$$V + W = \langle V_1 + W_1, V_2 + W_2, V_3 + W_3 \rangle$$

$$V - W = \langle V_1 - W_1, V_2 - W_2, V_3 - W_3 \rangle$$

$$kV = k \langle V_1, V_2, V_3 \rangle$$
$$= \langle kV_1, kV_2, kV_3 \rangle$$

If $\overrightarrow{P_1P_2}$ is a vector in 2-space with initial point $P_1(x_1, y_1)$ and terminal point $P_2(x_2, y_2)$, then

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle .$$

in 3-space $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle .$$

The <u>norm</u> of a vector $v = \langle v_1, v_2 \rangle$ in 2-space is given by $\|v\| = \sqrt{v_1^2 + v_2^2}$.

The <u>norm</u> of a vector $v = \langle v_1, v_2, v_3 \rangle$ in 3-space is given by $\|v\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Unit vector

$i = \langle 1, 0 \rangle$, $j = \langle 0, 1 \rangle$ in $\boxed{\text{2-space}}$

$i = \langle 1, 0, 0 \rangle$, $j = \langle 0, 1, 0 \rangle$, $k = \langle 0, 0, 1 \rangle$ in $\boxed{\text{3-space}}$

[F] Every vector in 2-space is expressable uniquely in terms of i and j.

$v = \langle v_1, v_2 \rangle = \langle v_1, 0 \rangle + \langle v_2, 0 \rangle = v_1 \langle 1,0 \rangle + v_2 \langle 0,1 \rangle = v_1 i + v_2 j$

and every vector in 3-space is expressable uniquely in terms of i, j and k.

$$v = \langle v_1, v_2, v_3 \rangle$$
$$= \langle v_1, 0, 0 \rangle + \langle 0, v_2, 0 \rangle + \langle 0, 0, v_3 \rangle$$
$$= v_1 \langle 1,0,0 \rangle + v_2 \langle 0,1,0 \rangle + v_3 \langle 0,0,1 \rangle.$$
$$= v_1 i + v_2 j + v_3 k$$

Example 5: Find the uni vector that has the same direction as $v = 2i + 2j - k$.

The vector v's norm $= \sqrt{(2)^2 + (2)^2 + (1)^2}$

$$= \sqrt{4+4+1} = \sqrt{9} = 3.$$

~~in~~ unit vector $u = \frac{v}{\|v\|}$

$$= \frac{2i + 2j - k}{3} = \frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k$$

Example: 6

a) Find the vectore of length 2 that makes an angle of $\pi/4$ with the positive x-axis.

$$V = v\cos\theta i + v\sin\theta j$$
$$= 2\cos\frac{\pi}{4} i + 2\sin\frac{\pi}{4} j$$
$$= 2 \cdot \frac{1}{\sqrt{2}} i + 2\frac{1}{\sqrt{2}} j$$
$$= \sqrt{2} i + \sqrt{2} j.$$

(b) Find the angle that the vector $V = \sqrt{3} i + j$ makes with the positive x-axis.

$$\frac{V}{\|V\|} = -\frac{\sqrt{3}}{2} i + \frac{1}{2} j.$$
$$= \cos\theta i + \sin\theta j$$

$$\|V\| = \sqrt{(-\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2.$$

$$\cos\theta = -\sqrt{3}/2$$
$$\theta = \cos^{-1}(-\sqrt{3}/2).$$
$$= 5\pi/6.$$

Example 7. A vector v of length $\sqrt{5}$ that extends along the line through A and B. Find the components of v.

$A(0,0,4)$, $B(2,5,0)$

$\vec{AB} = B\langle 2,5,0\rangle - A\langle 0,0,4\rangle$

$= \langle 2-0, 5-0, 0-4\rangle$ $\quad \|v\| = \sqrt{5}$.

$\vec{AB} = \langle 2, 5, -4\rangle$.

$\|\vec{AB}\| = \sqrt{(2)^2+(5)^2+(-4)^2} = \sqrt{4+25+16}$

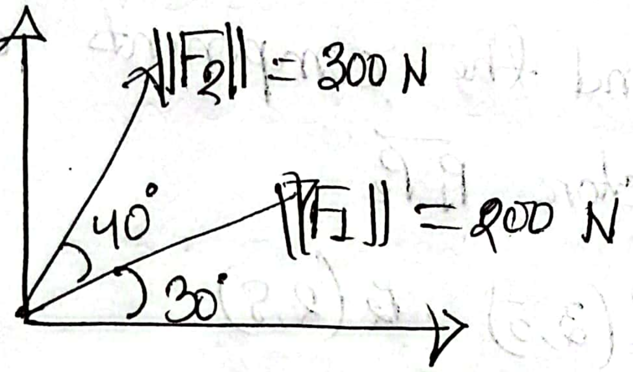
$= \sqrt{45} = 3\sqrt{5}$

$$v = \|v\| \left(\frac{\vec{AB}}{\|\vec{AB}\|} \right)$$

$$= \sqrt{5} \left\langle \frac{2}{3\sqrt{5}}, \frac{5}{3\sqrt{5}}, -\frac{4}{3\sqrt{5}} \right\rangle$$

$$= \left\langle \frac{2}{3}, \frac{5}{3}, -\frac{4}{3} \right\rangle.$$

Example 8. Suppose that two forces are applied to an eye bracket as
Find the magnitude of the resultant and the angle θ that it makes with the positive x axis.



resultant $F = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos 40°}$

$F_1 = 200 \cos 30 i + 200 \sin 30 j$
$= 100\sqrt{3} i + 100 j$

$F_2 = 300 \cos 70 i + 300 \sin 70° j$

$F = F_1 + F_2$

$F = (100\sqrt{3} + 300\cos 70) i + (100 + 300 \sin 70) j$

$F = F\cos\theta i + F\sin\theta j$

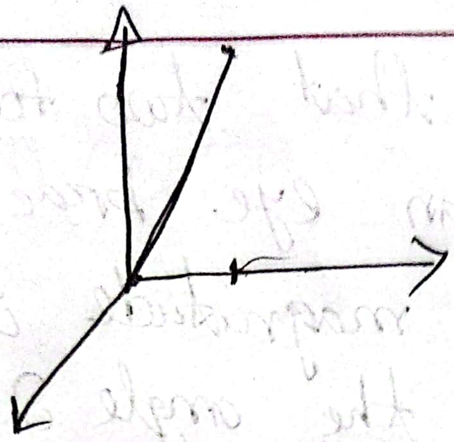
$||F|| = \sqrt{(100\sqrt{3} + 300\cos 70°)^2 + (100 + 300\sin 70°)^2}$

$= 471$ N.

$||F|| \cos\theta = 100\sqrt{3} + 300\cos 70°$

$\cos\theta = \dfrac{100\sqrt{3} + 300\cos 70°}{471}$

1)



7, Find the components of the vector $\overrightarrow{P_1P_2}$

(a) $P_1(3,5)$, $P_2(2,8)$.

$\overrightarrow{P_1P_2} = \langle 2-3, 8-5 \rangle$
$= \langle -1, 3 \rangle$.

~~9. (a) Find the~~

27. (b) Find unit vectors that satisfy ~~that~~ the stated conditions.

Oppositely directed to $6\hat{i} - 4\hat{j} + 2k$.

$\vec{V} = 6\hat{i} - 4\hat{j} + 2k$.

$\|\vec{V}\| = \sqrt{6^2 + (-4)^2 + (2)^2} = \sqrt{36+16+4} = 2\sqrt{14}$.

$$\vec{w} = \frac{-\vec{v}}{\|v\|} = \frac{-6i + 4j - 2k}{2\sqrt{14}}$$

$$= \cancel{\frac{6}{}} -\frac{3}{\sqrt{14}} i + \frac{2}{\sqrt{14}} j - \frac{1}{\sqrt{14}} k.$$

$$= \frac{1}{\sqrt{14}}(-3i + 2j - k).$$

29. In each part, find two unit vectors in 2-space that satisfy the stated condition.

(a) Parallel to the line $y = 3x + 2$.

$x = 0, y = 2$

$x = 1, 5.$

$A_1 \langle 0, 2 \rangle, A_2 \langle 1, 5 \rangle.$

$\cancel{\vec{B_1B_2}}$ $\vec{A_1A_2} = \cancel{\langle 0-1, 2-5 \rangle} \langle 5-2, 1-0 \rangle$

$= \cancel{\langle -1, -3 \rangle}$ $\langle 3, 1 \rangle$

$\langle 1-0, 5-2 \rangle \; (1, 3)$

$\|\vec{A_1A_2}\| = \sqrt{3^2 + 1^2} = \sqrt{10}$

so parallel $\langle 1/\sqrt{10}, 3/\sqrt{10} \rangle$ and $\langle -1/\sqrt{10}, -3/\sqrt{10} \rangle$.

ⓒ Perpendicular to the line

$y = -5x + 1.$

~~x=0, y=1.~~

$y = \frac{1}{5}x$

$x = 0, y = 0$

$x = 1 \quad y = \frac{1}{5}.$

$A_1 \langle 0, 0 \rangle, A_2 \langle 1, \frac{1}{5} \rangle.$

$A_1A_2 \langle 1 - 0, \frac{1}{5} - 0 \rangle$

$\vec{A_1A_2} \langle 1, \frac{1}{5} \rangle.$

$\|\vec{A_1A_2}\| = \sqrt{(1)^2 + (\frac{1}{5})^2} = \sqrt{1 + \frac{1}{25}} = \sqrt{\frac{26}{25}} = \pm\frac{\sqrt{26}}{5}$

$$\boxed{\pm \frac{\sqrt{26}}{5} \langle 1, \frac{1}{5} \rangle.}$$

Perpendicular.

40. In each part, find two unit vectors in 3-space that satisfy the stated condition.

(a) Perpendicular to the xy plane $= \pm k$.

(b) Perpendicular to the yz plane $= \pm i$

(c) Perpendicular to the zx plane $= \pm j$.

57. A vector w is said to be a linear combination of the vectors v_1 and v_2 if w can be expressed as $w = c_1 v_1 + c_2 v_2$, where c_1 and c_2 are scalars.

(a) Find scalars c_1 and c_2 to express the vector $4j$ as a linear combination of the vectors $v_1 = 2i - j$ and $v_2 = 4i + 2j$

$$4j = c_1(2i - j) + c_2(4i + 2j)$$

$$4j = (2c_1 + 4c_2)i + (-c_1 + 2c_2)j$$

which gives $c_1 = -2$ and $c_2 = 1$.

(b)

58. A vector w is a linear combination of vectors v_1, v_2 and v_3 if w can be expressed as $w = c_1v_1 + c_2v_2 + c_3v_3$ where c_1, c_2, c_3 are scalars.

(a) Find scalars c_1, c_2 and c_3 to express $\langle -1, 1, 5\rangle$ as a linear combination of $v_1 = \langle 1, 0, 1\rangle$, $v_2 = \langle 3, 2, 0\rangle$ and $v_3 = \langle 0, 1, 1\rangle$.

$$-1i + 1j + 5k = c_1v_1 + c_2v_2 + c_3v_3.$$