Problem 1

a)

Starting from RNAP balance:

(1)

From JDV’s class notes: Deterministic Mathematical Models of Transcription:

(2)

(3)

Closed complex for gene *j*:

(4)

Open complex for gene *j*:

(5)

(6)

Closed complex for gene *i*:

(7)

Open complex for gene *i*:

(8)

(9)

New RNAP balance:

(10)

Solving for free RNAP concentration:

(11)

Divide top and bottom by τ-1X,j K-1X,j :

(12)

(13)

Open complex of gene *j* in terms of total RNAP:

(14)

Kinetic rate of transcription:

(15)

(16)

b) For an N-gene system to be approximately equivalent to the 1-gene system, the εj term needs to be minimized. The quantity (1 + τ) will never be smaller than 1, so trying to minimize it would serve little purpose. A very small Gi to Gj ratiomeans that there is a relatively low concentration of other genes besides j in the system; however, this case seems like the trivial answer, since the system could be approximated as a 1-gene system, especially if Gj is orders of magnitude larger than Gi. If τ for all other genes except j is much larger than it is for j, transcription of all those other genes is initiation limited, or inactivated.