

Math notation, statistics measure central tendency

μ = Population mean ($\frac{\text{divides}}{\text{Total number}}, n$) \rightarrow Average of entire group

\bar{x} = Sample mean ($\frac{\text{divides}}{\text{Total number}}, n-1$) \rightarrow Average of subset of the population

σ = standard deviation (s.d)

σ^2 = variance

$$SD = \sqrt{\text{Var}} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

$$\text{variance} = \frac{\sum (x_i - \bar{x})^2}{n}$$

Sample s.d

$$\sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

sample variance

$$\frac{\sum (x_i - \bar{x})^2}{n}$$

Population s.d

$$\sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

population variance

$$\frac{\sum (x_i - \bar{x})^2}{n}$$

\Rightarrow class interval = upper limit - lower limit

Frequency \rightarrow count of occurrence (how often a value appears)

Range \rightarrow diff b/w largest and smallest value
= max - min

~~Measure of~~

Measure of central tendency

$$1 \quad \text{Mean}(\mu) = \frac{\text{sum of values}}{\text{no. of values}}$$

2 Median \rightarrow arrange descending / ascending order, for odd no: of data point - middle value (median) else even - average of 2 - middle values

3) mode - most frequently occurring values in the same population

Measure of dispersion

1. Range = max - min

2. Variance (σ^2) = $\frac{\sum (x_i - \mu)^2}{n}$

3. S.D (σ) = $\sqrt{(\sigma^2)}$

4. Interquartile Range (IQR)

$$IQR = Q_3 - Q_1$$

5. Quartiles (Q) - divides the dataset into 4 equal parts

Q_1 - Median of the lower 25%

Q_2 - median (50%)

Q_3 - Median of the upper 25%

Outlier - extreme values (far from the actual values)

6. Coefficient of Variance (CV)

$$CV = \frac{\sigma}{\mu} \times 100$$

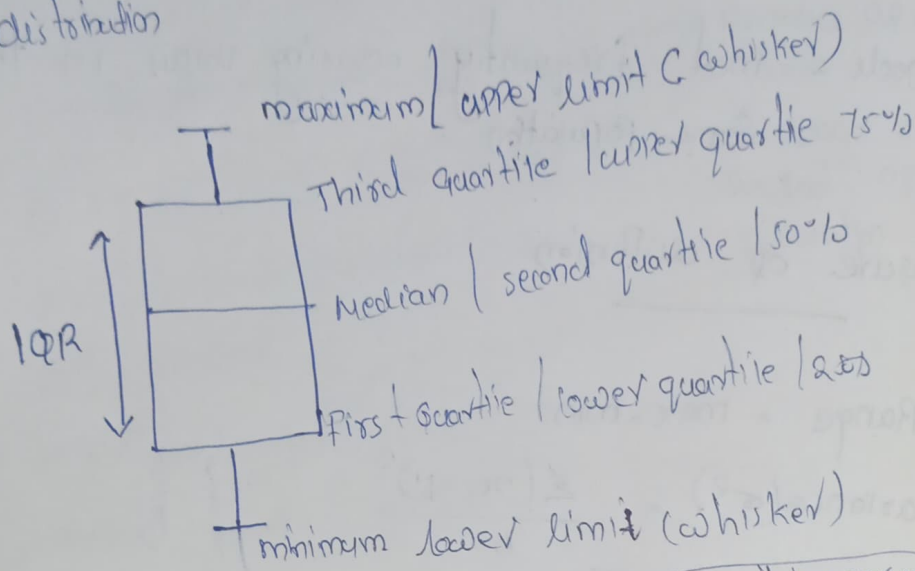
σ = S.D
 μ = mean

7. Mean Absolute Deviation

$$MAD = \frac{\sum_{i=1}^n |x_i - \mu|}{n}$$

Box Plot

data visualization that summarizes a dataset distribution



$$\text{upper limit} = Q_3 + 1.5 \times \text{IQR}$$

$$\text{lower limit} = Q_1 - 1.5 \times \text{IQR}$$

Condition for outliers
outlier < lower limit
or
outlier > upper limit

Covariance

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

Correlation

$$\rho(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \sigma_y}$$

range $\rightarrow -1$ to 1

$0 \rightarrow$ light

$-1 \rightarrow$ highly -ve correlated

$+1 \rightarrow$ highly +ve correlated

Z-Score

$$Z\text{-score} = \frac{x_i - \mu}{\sigma}$$

x_i → data value

μ → mean

$\sigma = SD$

Hypothesis Testing

Null hypothesis (H_0)

There is no significant difference or effect

Alternative hypothesis (H_1)

There is a significant effect i.e. the given statement can be false.

Level of significance (α)

It is a threshold used to determine statistical significance (value common - 0.05, 0.01, or 0.10)
↓
conclusion

Z-test

used to compare means when sample size is large
 $n > 30$ and population variance - known.

Steps

1. State null hypothesis
2. State Alternative hypothesis.
3. Choose your significance level (α)
4. Calculate your Z-test statistics

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

\bar{x} = sample mean

μ = population mean

σ = population SD

n = sample size.

5. Find P-value using z-table and z -test statistic computed if ~~P~~

6. If P -value $> \alpha$

Then we fail to reject null hypothesis
(Accept Alternative hypothesis)

7. There are one tailed and two tailed tests.

Two tailed, Alternative hypothesis look like
this

$$H_0: \mu = \mu_0 \text{ v/s } H_1: \mu \neq \mu_0$$

one tailed, look like (A.H)

$$\text{Right tailed, } H_0: \mu \leq \mu_0 \text{ v/s } H_1: \mu > \mu_0$$

$$\text{Left tailed, } H_0: \mu \geq \mu_0 \text{ v/s } H_1: \mu < \mu_0$$

Ex: A particular companies chocolate bars are supposed to have an average weight of 50 ~~amps~~ grams according to the manufacturer we want to test if a sample of chocolate bars deviates significantly from this weight data:

50.8, 49.5, 50.2, 51, 49.7, 50.3, 49.8, 50.5, 49.6

50.1 population standard deviation 1.5 grams?

$$H_0: \mu = 50$$

$$H_1: \mu \neq 50$$

$$\alpha = 0.05$$

$$\bar{x} = 50.15$$

$$SD = \frac{s}{\sqrt{n}} = \frac{1.5}{\sqrt{10}} = \underline{\underline{0.47434}}$$

$$z = \frac{\bar{x} - \mu_0}{SD} = \frac{50.15 - 50}{0.47434}$$

$$z = 0.8162$$

p-value - 2 tailed

$$p = 0.7518$$

Here $\alpha = 0.05$, $p = 0.7518$, $z = 0.8162$

$$0.05 < 0.7518$$

So fail to reject H_0

\therefore No significant deviation from 50g.

T-test

• we need T-test because population standard deviation σ is not always available.

• we will compute T-test statistic (based on Problem statement and type of t-test)

Type of T Test	Null hypothesis	Alternative hypothesis	DF
• one sample T-test	Sample mean = Reference mean	Sample mean \neq Reference mean	df = n-1
• Independent sample T-test	mean value in both group are same.	mean value in both group not same	df = $n_1 + n_2 - 2$
• Paired sample t-test	mean value of the difference b/w the pairs is zero	mean value of difference b/w the pairs is not zero	df = n-1

one sample T-test steps

compare the sample mean to a known population mean

steps

1. state the null hypothesis
2. state the alternative hypothesis
3. choose your significance level (α)
4. calculate your T-test statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

\bar{x} = sample mean

μ = population mean

s = sample SD

$$s = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

5. critical t-value find from T-table using α
6. if t statistic $<$ critical t-value, we fail to reject null hypothesis
7. same eq as z test

$$H_0: \mu = 50$$

$$H_1: \mu \neq 50$$

$$\bar{x} = 50.15, \quad s = 0.5104, \quad n = 10$$

$$SE = \frac{s}{\sqrt{n}} = 0.1614$$

T-test statistic $t = \frac{\bar{x} - \mu_0}{SE}$

$$= \frac{50.15 - 50}{0.1614}$$

$$= 0.929$$

$$df = 10 - 1 = 9$$

Value (2-tailed) T-Value = 2.262

t-test statistic P-value

$$0.929 < 0.262$$

\therefore we fail to reject null hypothesis.

Independent Sample T-test

Compare means of 2 independent groups

Steps

~~compare means of 2 independent groups~~

1. state null hypothesis
2. state alternative hypothesis
3. Choose your α
4. Calculate your T-test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

For one tailed.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

For 2 tailed

$$\text{(Pooled SD)} \quad s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

3. Find the critical t-value from T-table using 2
 6. If t-statistics < critical t value, we fail to reject Null hypothesis.

Ex: Board A = 49.5, 50.1, 49.8, 50.3, 50

Board B = 50.4, 49.9, 50.6, 50.2, 50.1

$$H_0: \mu_A \neq \mu_B$$

$$H_1: \mu_A \neq \mu_B$$

$$\bar{x}_A = 49.94$$

$$n_A = 5$$

$$\bar{x}_B = 50.24$$

$$n_B = 5$$

$$\bar{x}_A - \bar{x}_B = -0.30$$

$$s_A = 0.816$$

$$s_B = 0.265$$

$$s_p = \sqrt{\frac{(5-1)(0.816)^2 + (5-1)(0.265)^2}{5+5-2}}$$

$$= 0.292$$

$$t = \frac{49.94 - 50.84}{0.892 \sqrt{1/5 + 1/5}}$$

$$= \frac{0.3}{0.847} = -1.684$$

$$df = n_1 + n_2 - 2 = 8$$

$$\alpha = 0.05$$

$$\text{critical } t\text{-value} = 2.306$$

t statistic

critical t-value

-1.684

< 2.306

Here we fail to reject the null hypothesis

z-test

z-test statistic

z-table

p-value vs α

T-test

T-test statistic

vs

Alpha (α) df tailness

↓

critical t-value

Matrix

• $8 \times c = m \times n$ (order of a matrix)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

• $a_{ij} = a_{ji} \rightarrow$ Symmetric matrix

• Identity matrix $= I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Eg. $A = \begin{bmatrix} 6 & 3 \\ 9 & 4 \\ 0 & 7 \end{bmatrix}_{3 \times 2}$ $C = \begin{bmatrix} 1 & 0 \\ 8 & 4 \\ 11 & 31 \end{bmatrix}_{3 \times 2}$

$$\Rightarrow A + C = \begin{bmatrix} 7 & 3 \\ 12 & 8 \\ 11 & 38 \end{bmatrix}_{3 \times 2}$$

~~Singular~~

• Single Value \times matrix \rightarrow scalar matrix

• $A_{m \times 2} \times B_{2 \times n} \rightarrow \text{Result}_{m \times n}$

[1st matrix column = second matrix row]

• A Transpose (A^T)

If $A = A^T$ Symmetric Matrix (only for square matrix)

Identity matrix is also symmetric

• Determinant

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad |M| = (ad - bc)$$

• Inverse Matrix (M^{-1})

$$M \cdot M^{-1} = I$$

• M^{-1} does not exist when $|M| = 0$

~~Eigen~~

• Eigen values / vectors

- Eigen values are unique scalar values associated with a square matrix or linear transformation.
- They indicate how much an eigen vector is stretched or compressed during the transformation.

• Represented by $AV = \lambda V$

A = Matrix

V = eigenvector

λ = eigen value.

• $2 \times 2 = 2$ eigenvalues

$3 \times 3 \rightarrow 3$ eigenvalues.

Probability - chance of happen an event

• event - get an outcome from sample space.

•
$$\text{Probability} = \frac{\text{event count}}{(\text{S}) \text{ count}}$$

Event \rightarrow independent
 \rightarrow dependent.

~~Joint Probability~~

~~$P(c) = \frac{15}{32}$ $P(cT) = \frac{17}{32}$~~

~~$P(m) = \frac{2}{32}$ $P(m) = \frac{30}{32}$~~