CELLULAR AUTOMATON

Modelization and simulation of biosystems

MSTER'S DEGREE IN COMPUTATIONAL BIOLOGY 2021-2022

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Exercise 4.1 -Study the dynamics of one of the following sets of rules for two arrays of 256 and 257 cells with periodic boundary conditions

A **cellular automaton** consists of a regular grid of cells, each in one of a finite number of states, such as on and off (in contrast to a coupled map lattice). For each cell, a set of cells called its neighborhood is defined relative to the specified cell. An initial state (time t = 0) is selected by assigning a state for each cell.

A new generation is created (advancing t by 1), according to some fixed rule (generally, a mathematical function) that determines the new state of each cell in terms of the current state of the cell and the states of the cells in its neighborhood. Typically, the rule for updating the state of cells is the same for each cell and does not change over time, and is applied to the whole grid simultaneously.

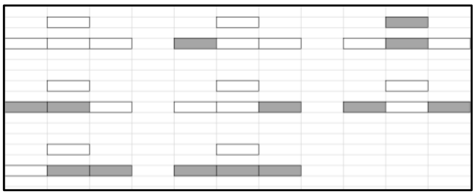
In order to simulate some cellular automata, I have chosen the second set proposed in this exercise which is composed of 3 rules: **4, 90, 137.**

For this task I have created a **Jupyter notebook** for plotting the result of these combinations. But firstly, lets convert those numbers into binary code.

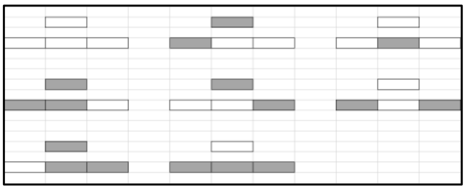
- 4 = 100(b)
- 90 = 1011010(b)
- 137 = 10001001(b)

These codes will be the bases of our cellular automaton and they look like this:

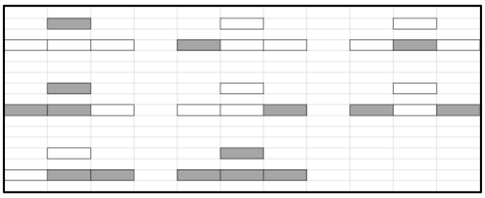
Rule 4



Rule 90:

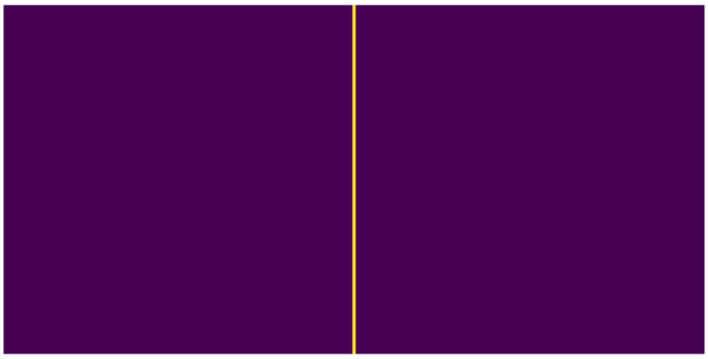


Rule 137:

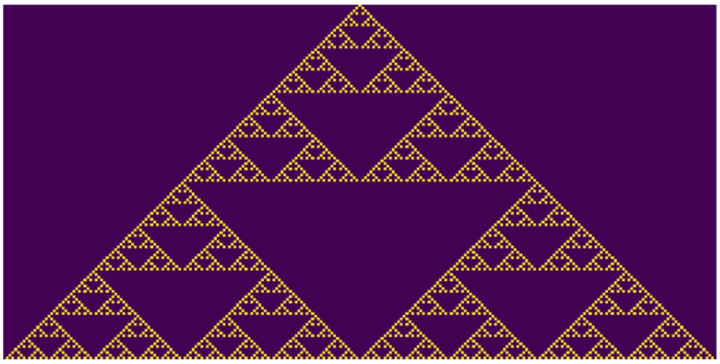


Let's represent them with different initial conditions.

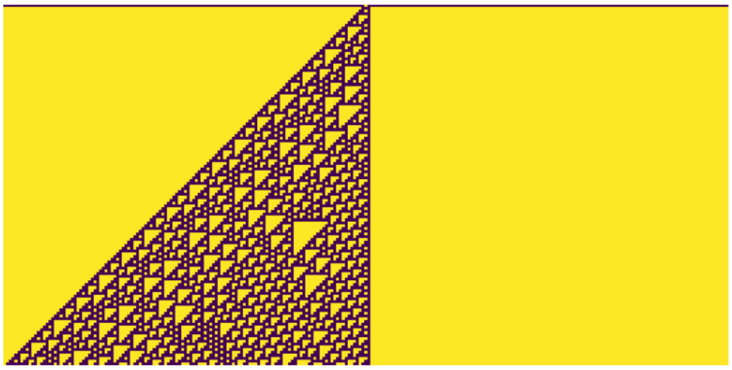
1. **A single black (1) cell in the middle of the array**
    a. **Rule 4:**



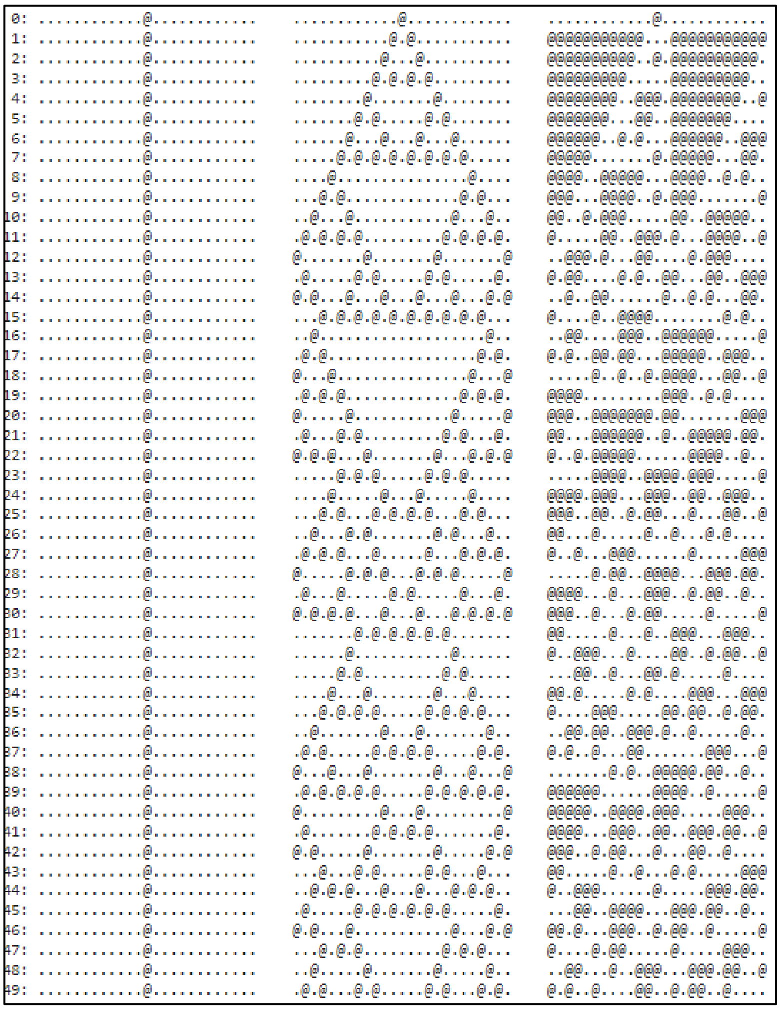
    b. **Rule 90**



    c. **Rule 137**



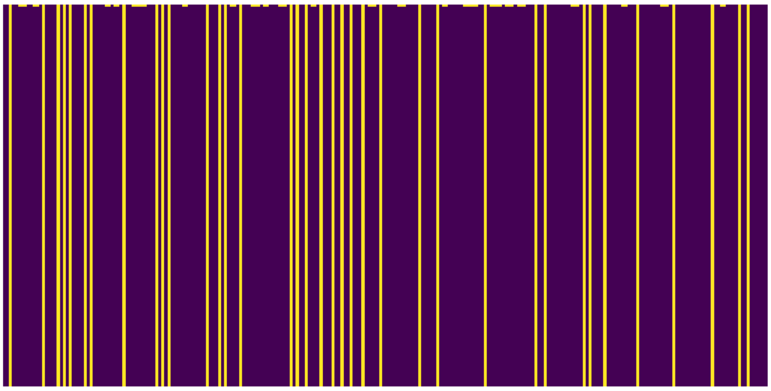
d. **Although this representation looks so nice, it is impossible to calculate repetitions, therefore I have used an algorithm which reproduces the rule and shows them with '.' (0) and @ (1): (Initial state: 0000000000001000000000000)**



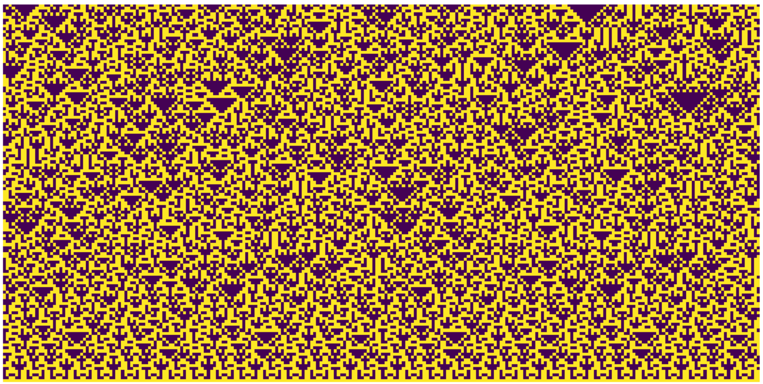
e. **Answer the following analysis questions:**

- **Can you see repeated patterns? How long does it take for a pattern to repeat?**
  Yes, rule 4 and 90 are clearly repeated. Otherwise, rule 137 does not seem a repeated pattern but their shapes are stables.
  - Rule 4: takes just one step to be repeated.
  - Rule 90: takes 48 steps to be repeated.
  - Rule 137: does not seems to be repeated.
- **Classify your rules into Wolfram classes.**
  - Rule 4: Class 1
  - Rule 90: Class 2
  - Rule 137: Class 3

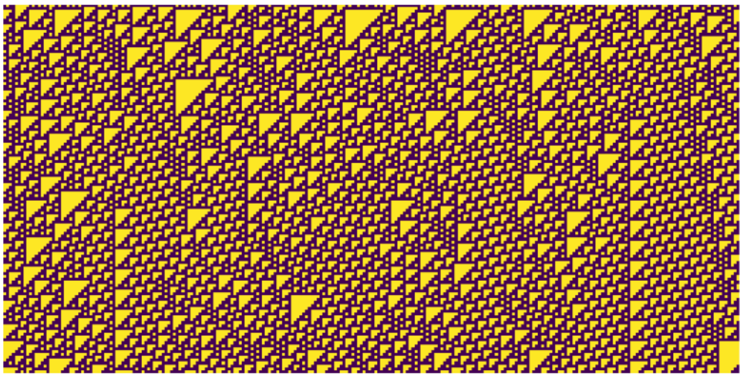
2. **Half black (1) cells and half white (0) cells at randomly chosen positions**
    a. **Rule 4**



    b. **Rule 90**



    c. **Rule 137**



d. Although this representation looks so nice, it is impossible to calculate repetitions, therefore I have used an algorithm which reproduces the rule and shows them with '.' (0) and @ (1): (Initial state: 11010011010010100100111010)

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e. Answer the following analysis questions:

- **Can you see repeated patterns? How long does it take for a pattern to repeat?**

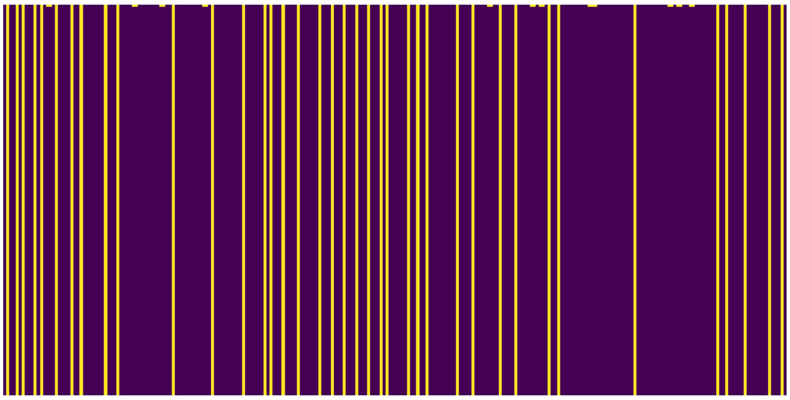
  Yes, rule 4 and 90 are repeated. Otherwise, rule 137 does not seem a repeated pattern but their shapes are stables.

  - Rule 4: takes just one step to be repeated.
  - Rule 90: takes 26 steps to be repeated.
  - Rule 137: does not seems to be repeated.

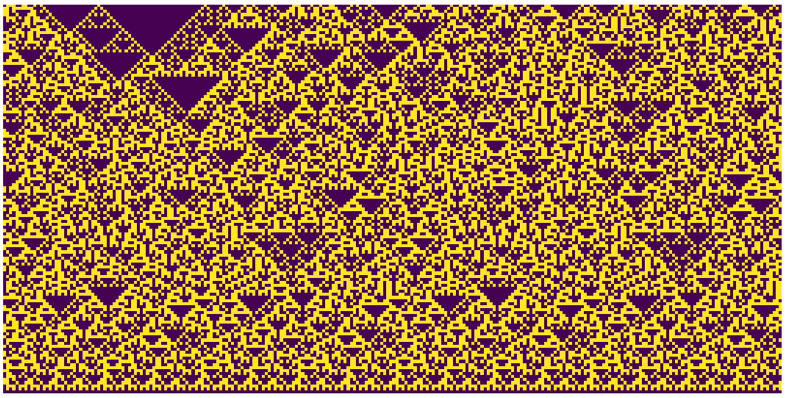
- **Classify your rules into Wolfram classes.**
  - Rule 4: Class 1
  - Rule 90: Class 3
  - Rule 137: Class 4

3. **25% of black (1) cells and 75% of white (0) cells (approximately)**

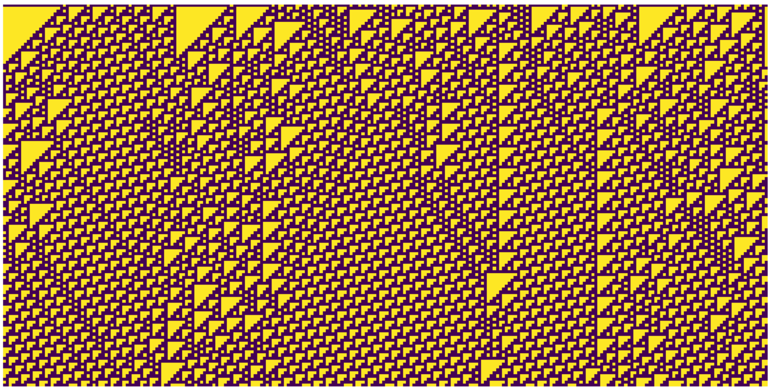
   a. **Rule 4:**



   b. **Rule 90:**



   c. **Rule 137:**



d. **Although this representation looks so nice, it is impossible to calculate repetitions, therefore I have used an algorithm which reproduces the rule and shows them with '.' (0) and @ (1): (Initial state: 10100100010001)**

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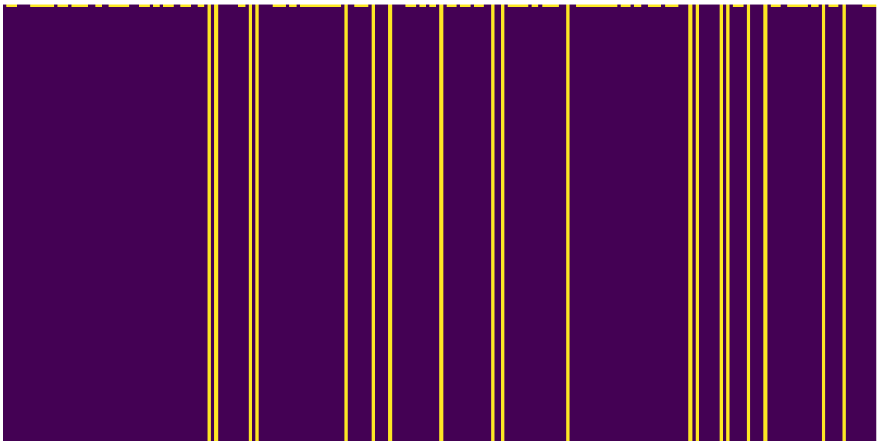
e. **Answer the following analysis questions:**

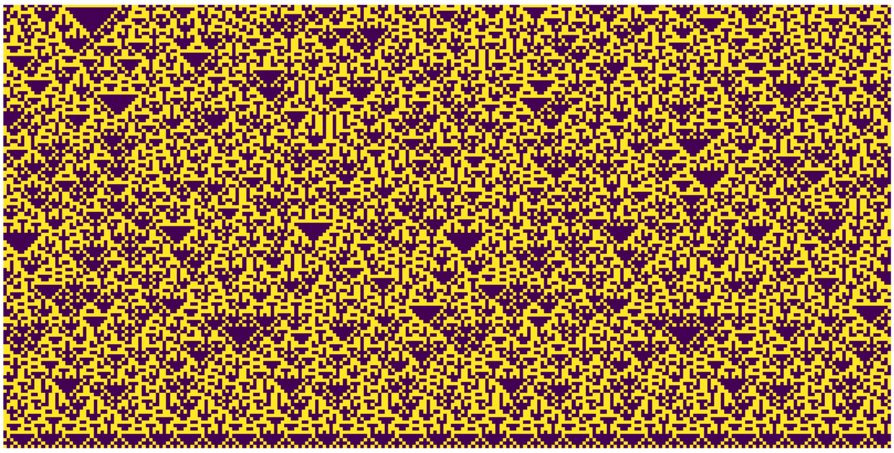
- **Can you see repeated patterns? How long does it take for a pattern to repeat?**
  Yes, rule 4, 90 and 137 are repeated.
  - Rule 4: takes just one step to be repeated.
  - Rule 90: takes 30 steps to be repeated.
  - Rule 137: takes 7 steps to be repeated.

- **Classify your rules into Wolfram classes.**
  - Rule 4: Class 1
  - Rule 90: Class 3
  - Rule 137: Class 4

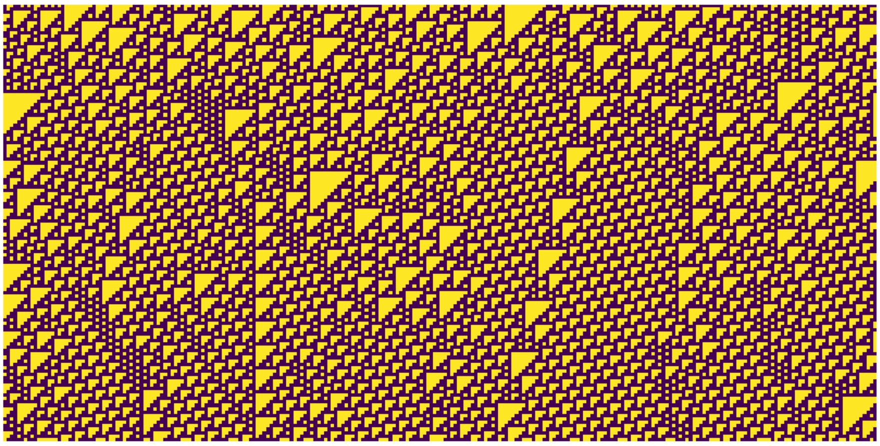
4. **90% of black (1) cells and 10% of white (0) cells (approximately)**
   (Initial state: 11111111011111111011)
   a. **Rule 4**



   b. **Rule 90**



   c. **Rule 137**



d. Although this representation looks so nice, it is impossible to calculate repetitions, therefore I have used an algorithm which reproduces the rule and shows them with '.' (0) and @ (1):

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39: ....................   @..@.@@.@@@.@.@@.@@.   ..@.....@@.@.....@..
40: ....................   .@@..@@.@.@...@@.@@@   @...@@@.@....@@@...@
41: ....................   @@@@@@@....@.@@@.@.@   ..@.@@....@@.@@..@..
42: ....................   @.....@@..@..@.@....   @...@..@@.@..@.....@
43: ....................   .@...@@@@@.@@...@...   ..@....@.......@@@..
44: ....................   @.@.@@...@.@@@.@.@..   @...@@...@@@@@.@@..@
45: ....................   ....@@@.@..@.@....@.   ..@.@..@.@@@@..@....
46: ....................   ...@@.@..@@...@..@.@   @........@@@.....@@@
47: ....................   ..@@@..@@@@@.@.@@...   ..@@@@@@.@@..@@@.@@.
48: ....................   .@@.@@@@...@...@@@..   @.@@@@@..@...@@..@..
49: ....................   @@@.@..@@.@.@.@@.@@.   ..@@@@.....@.@.....@
```

e. Answer the following analysis questions:

- **Can you see repeated patterns? How long does it take for a pattern to repeat?**
  Yes, rule 90 and 137 are repeated. Otherwise, rule 137 does not seem a repeated pattern but their shapes are stables.
  - Rule 4: keeps stable during all steps with all 0.
  - Rule 90: takes 24 steps to be repeated.
  - Rule 137: takes 23 steps to be repeated.

- **Classify your rules into Wolfram classes.**
  - Rule 4: Class 1
  - Rule 90: Class 3
  - Rule 137: Class 4

Pay especial attention to the length of the cycles. Can you see repeated patterns? How long does it take for a pattern to repeat? Does this time vary with the array length? In the light of the results obtained, try to classify your rules into Wolfram classes. Is it easy? Where do you find difficulties?

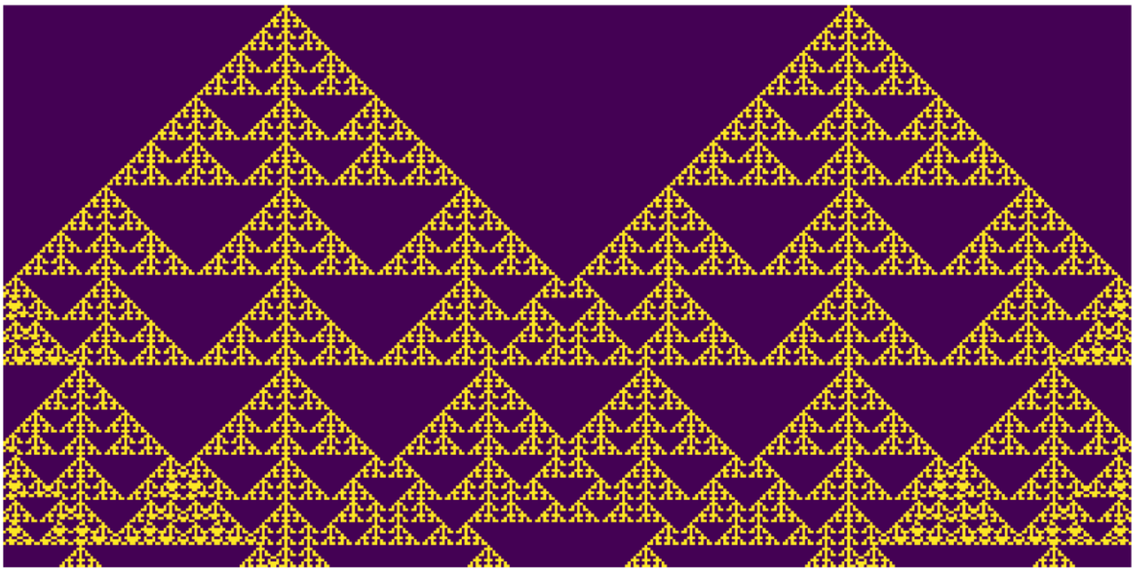
Conclusion:
It is common to see repeated patterns in most of the rules. As I have seen the repetition time depends on the initial state we chose. For instance, the initial state which takes less time for the patterns to be repeated is the tirth one (25 of 1 and 75% of 0). In rule 4 it only takes on step of time, in rule 90 it takes 30 steps and for rule 137 7 steps.

As I have developed all the rules with an initial array of 256 (yellow and purple examples) and 20 lengths (@ and . example), I was able to see that as shorter is the array, before they get repeated.

It has not been easy to classify each pattern. In my case, was difficult to separate class 2 from class 3 because the only difference is how quicky created figures are destroyed by the sounding noise. In addition, as it depends on the array length, it is easy for a rule to change its class.

Exercise 4.2 – Design (invent!) your own cellular automaton

For this task I have used a python code where I was able to introduce the rule characteristics. I have tried a lot to obtain a beautiful figure and this one is which I loved the most.



My new rule: