NUMERICAL SIMULATION OF THE SIR MODEL AND ITS PROPERTIES

Modelization and simulation of biosystems

SANDRA ALONSO PAZ
Computational Biology Master, Universidad Politécnica de Madrid

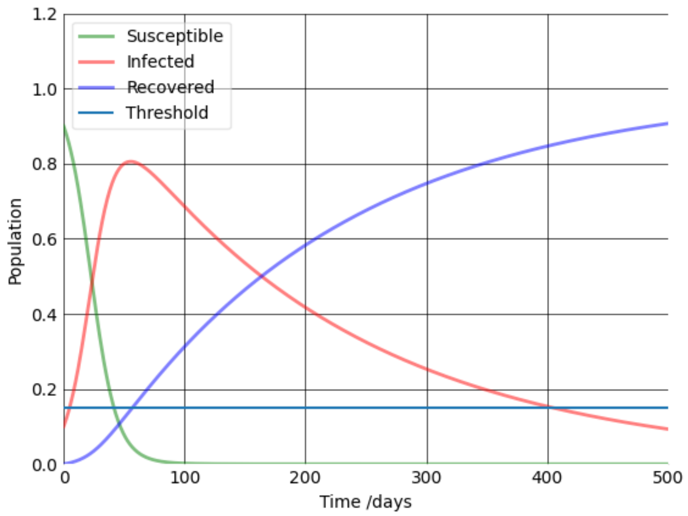
Exercise 1:

Implement and simulate the differential equations (use numerical methods for ODE)

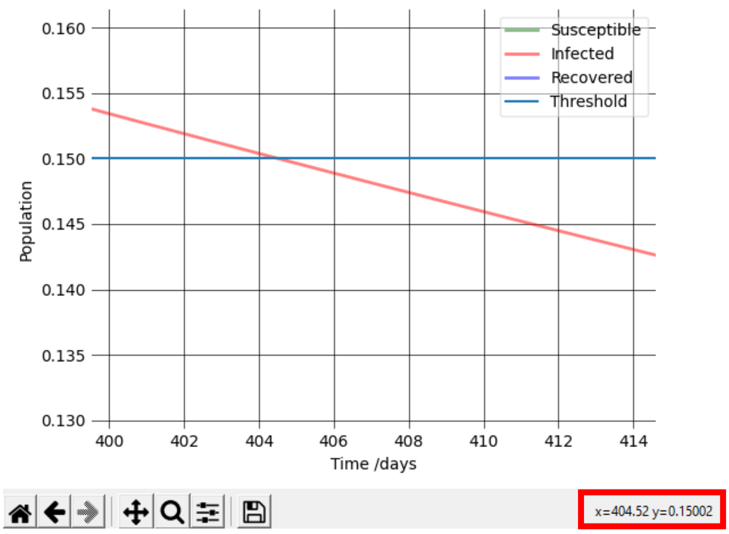
1. **Which parameters define the time scale in the system?**

   For this research I have set initial values for S0, I0 and R0:

   - S0 = 0.9
   - I0 = 0.1
   - R0 = 0.0

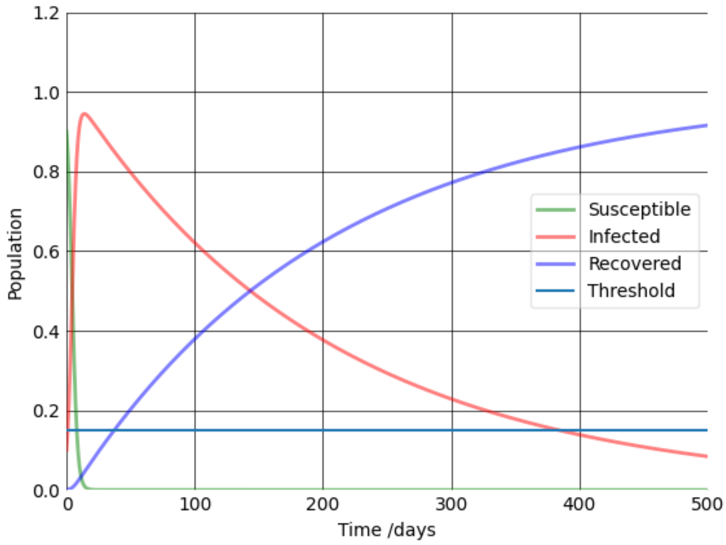


In addition, I have set a threshold at 0.15. This means that diseases will be considered eradicated when less of the 15% of the population is infected. Therefore, as we can observe in the plot below, the end of the infectious disease is set at day 405.

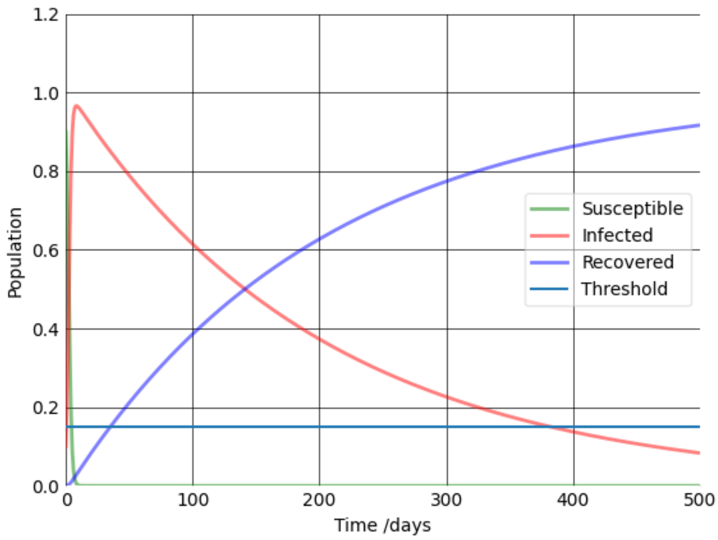


β (infection rate) and μ (infection rate) are the parameters which define the time scale in the system. By a handmade python script, I will plot some cases where those parameters change in order to see its importance:

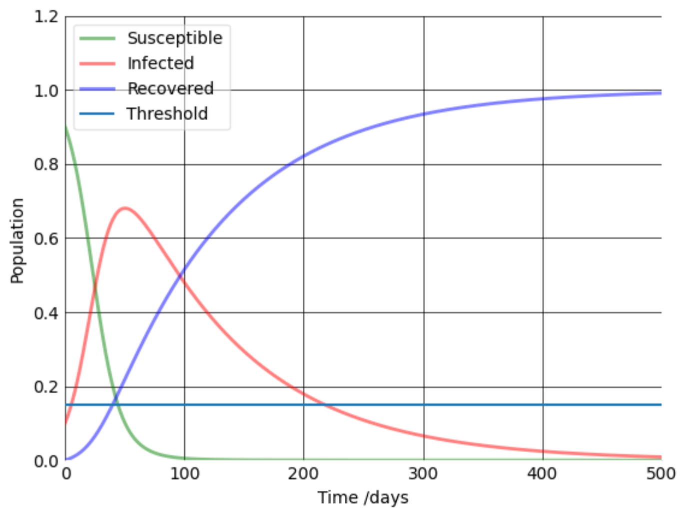
a. β = 0.5 and μ = 0.005



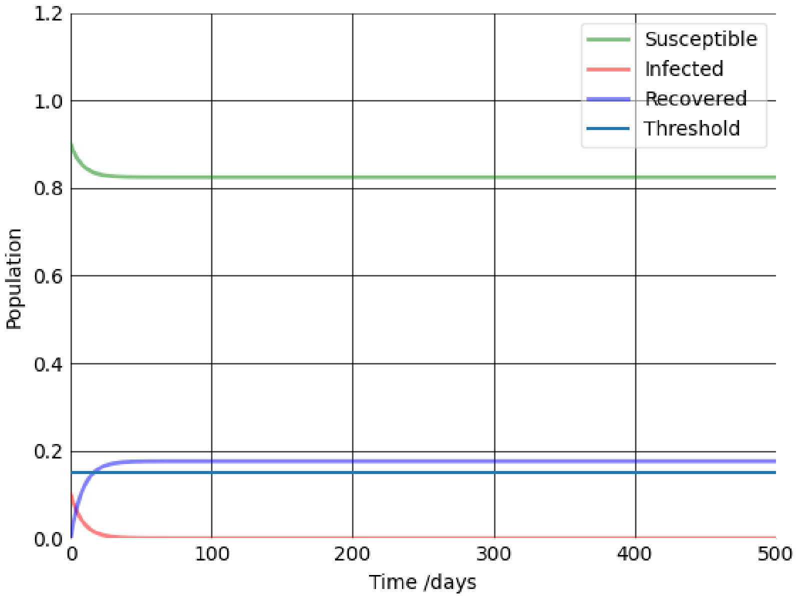
b. β = 0.9 and μ = 0.005



c. β = 0.1 and μ = 0.01



d. $\beta = 0.1$ and $\mu = 0.2$



Conclusions:

If β value increases (plots a and b), susceptible population drops quickly and, in consequence, infected population highly increase. As recovery population changed a bit, the end of the infectious disease becomes proportionally earlier.

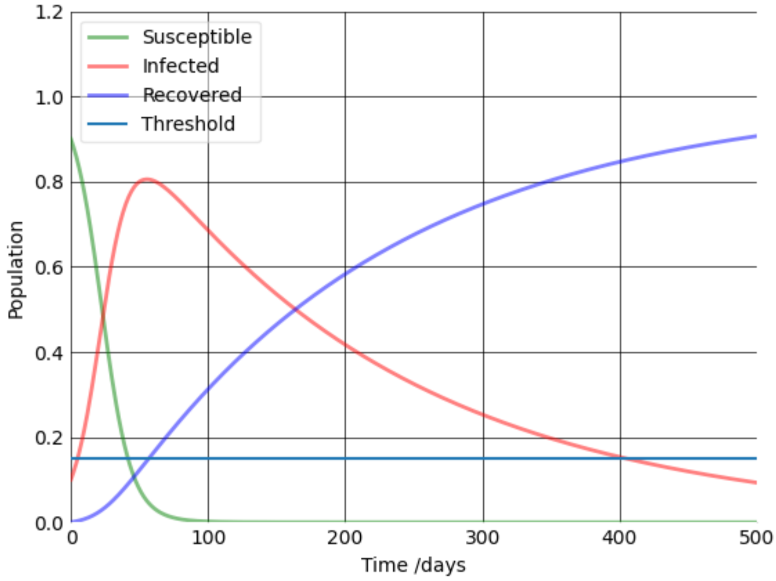
If μ increases (plots c and d), people who get infected become recovered in a faster rate. Therefore, there are les people infected and the infectious disease becomes extinct much earlier. It is remarkable the case that as μ increases, it makes more difficult for the disease to happen. For example, in last case (μ=0.5) the recovery rate is very big so not all the population get infected and therefore, the epidemic never happens.

Finally, higher values of β and μ, accelerate the infectious disease process.

2. **Fix parameters above the epidemic threshold and change the initial conditions. Can you find endemic states? Plot the phase space of the variables (S,I) for several initial conditions. Are these results consistent with the stability analysis?**

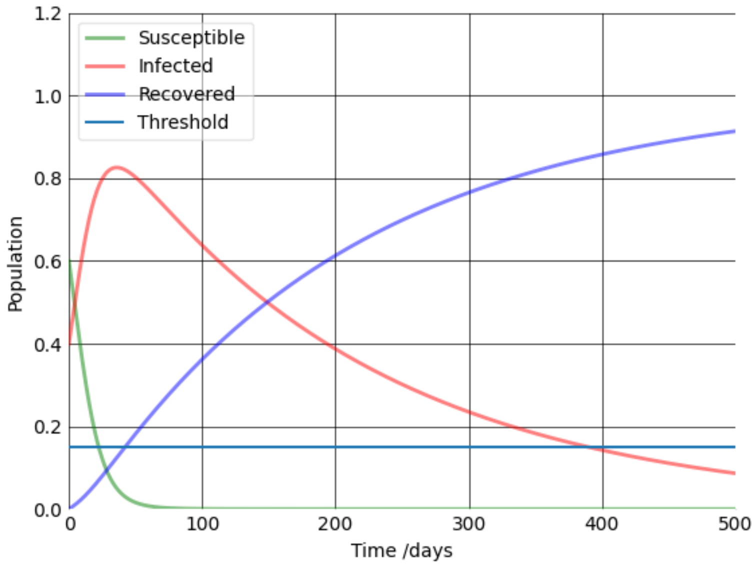
Talking about epidemics, endemic states are produced when an infection is constantly maintained at a baseline level in a geographic area without external inputs. Therefore, we cannot set μ value because if the recovery rate is constant, disease will tend to be eradicated. For this reason, in our model, obtaining an endemic state is impossible.

- **Initial state (S0=0.9, I0=0.1, R0=0, β =0.1, μ=0.005)**

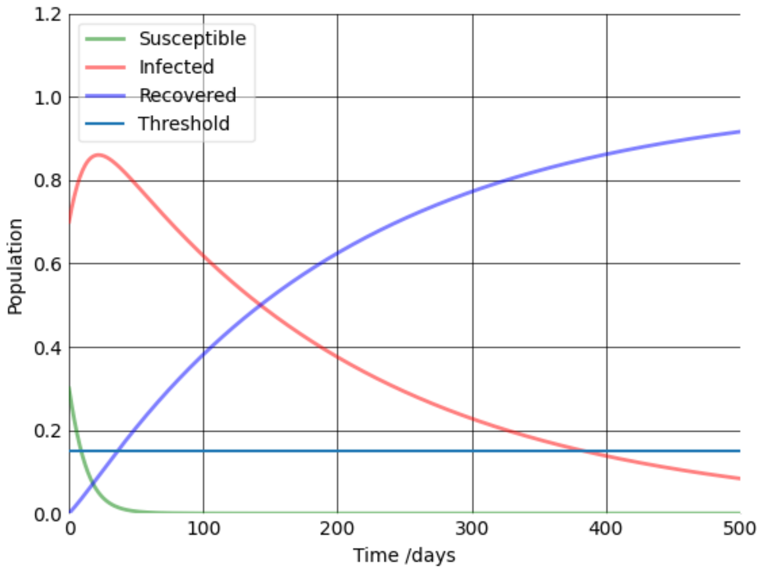


- **Modifying S0 and I0:**
At the end of the execution, it is sure that all the population will be infected and then recovery, so S+I=0 and R=population size. For this reason, a stable solution is obtained and not an endemic one.

  - **New state (S0=0.6, I0=0.4, R0=0, β =0.1, μ=0.005)**

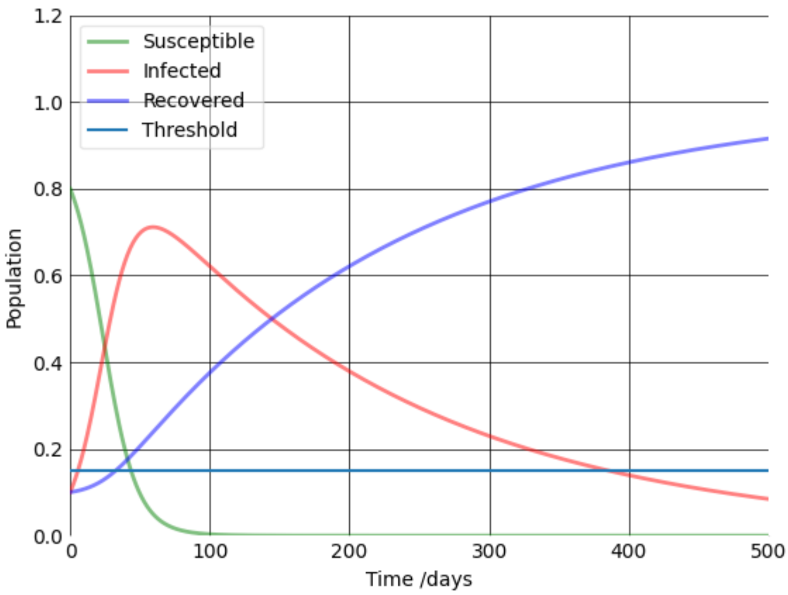


- **New state (S0=0.3, I0=0.7, R0=0, β =0.1, μ=0.005)**

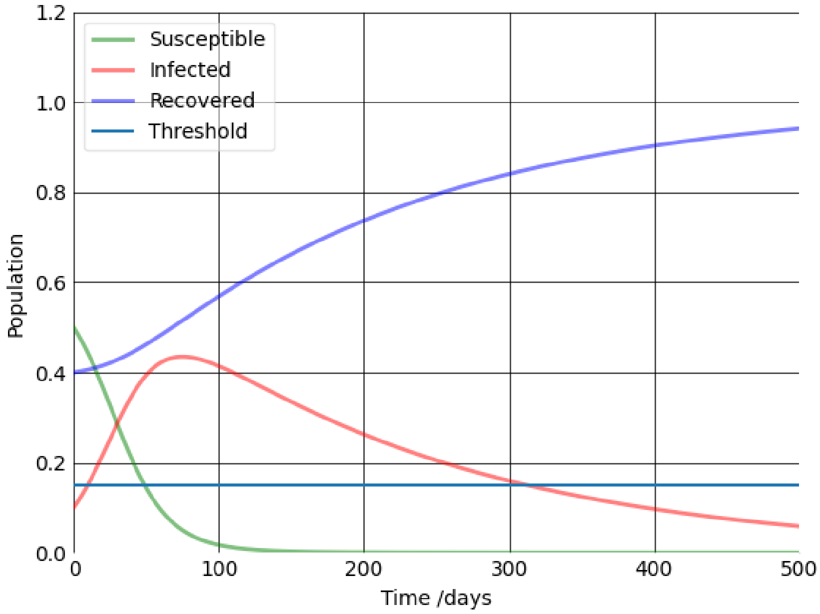


- **Modifying S0, I0 and R0:**
  In this example not all the population get infected due to R0>0. As a result, the rest of population will become infected and then recovered, obtaining then, a stable state.

  - **New state (S0=0.8, I0=0.1, R0=0.1, β =0.1, μ=0.005)**



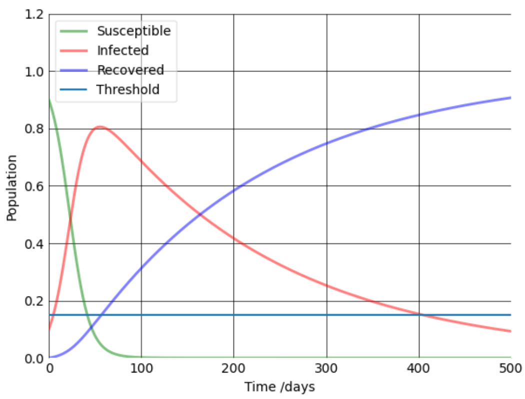
- **New state ($S0=0.5$, $I0=0.1$, $R0=0.4$, $\beta=0.1$, $\mu=0.005$)**



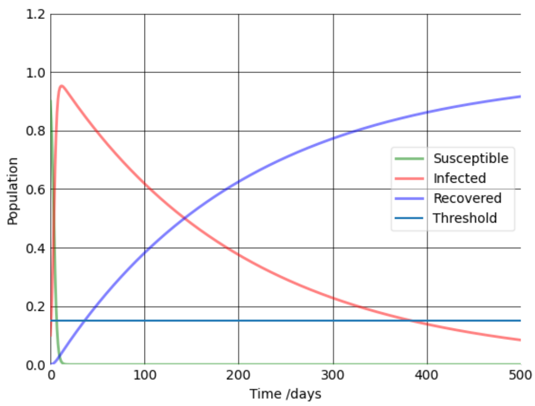
3. **Fix initial conditions and change parameters staying above the epidemic threshold. What is the maximum incidence of the disease? How does the maximum incidence and its time of occurrence change with the parameters?**

As the analysis done before, the mean values to change the evolution of the epidemic are **β** and **μ.** So, this are the values we have to modify for obtaining the maximum incidence. In **μ** case, we will need to decrease it but, as it already to low we will focus in **β** value. I have plotted the evolution of the disease for higher values of this parameter. As we can observe, as higher is **β,** as higher is the maximum value and as earlier we can obtain it.

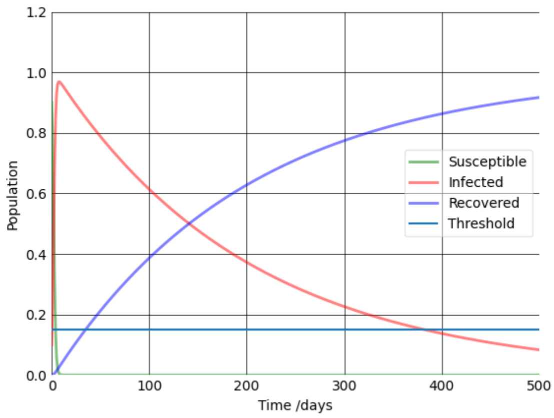
Initial conditions: $S0=0.9$, $I0=0.1$ and $R0=0.0$



β = 0.1

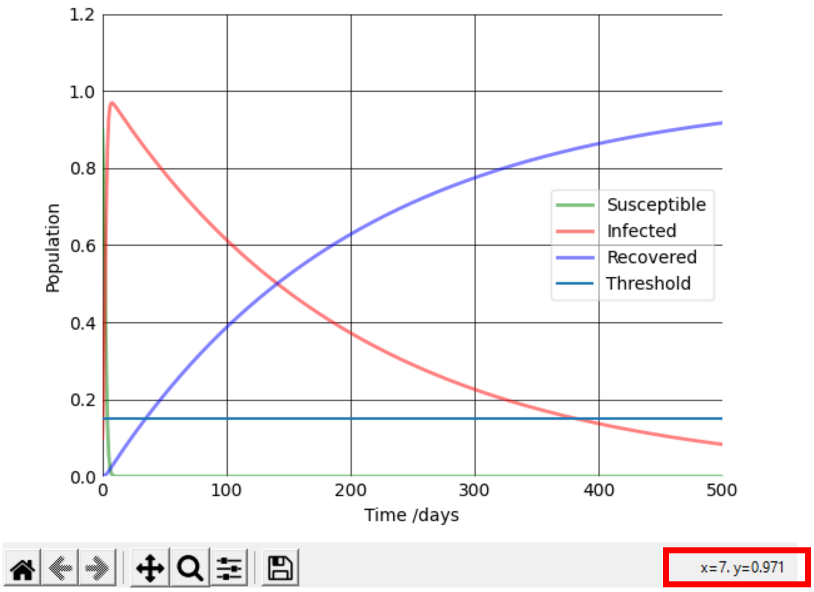


β = 0.6

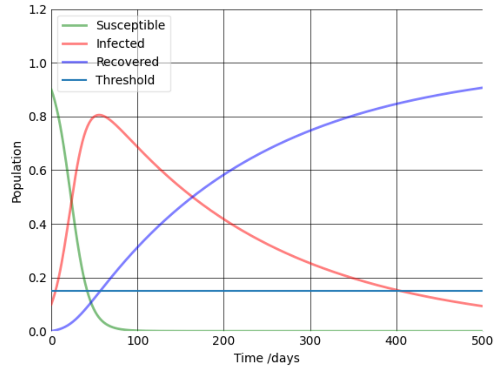


β = 0.99

In this scenario we obtain the maximum value (97,1%) at day 7 with **β** value of 0.99

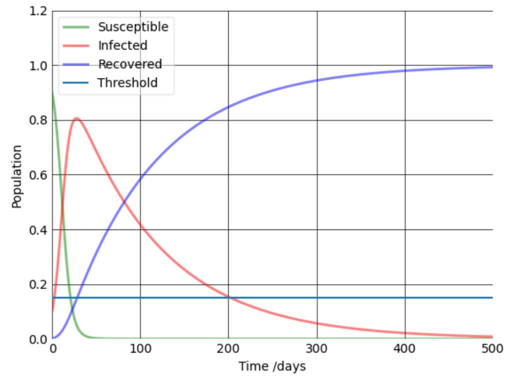


4. **Fix initial conditions and the value of the epidemic threshold, and change b and m with this condition. Do you see differences in the trajectories?**
As R0 = β / µ, and the threshold has been fixed at 0.15 (constant), the relationship between µ and β must be proportional. Let's see how the is their behavior. The initial condition is:



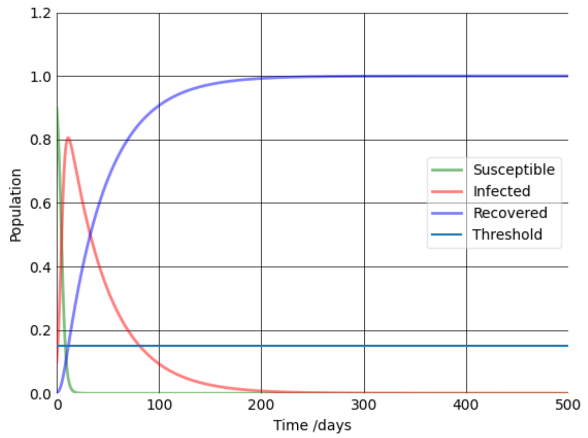
S0=0.9, I0=0.1, R0=0.0, β =0.1, µ=0.005

Now, I have duplicated β =0.1, µ=0.005, therefore:



S0=0.9, I0=0.1, R0=0.0, β =0.2, µ=0.01

Now, I have multiplied β =0.1, µ=0.005 by five, therefore:



S0=0.9, I0=0.1, R0=0.0, β =0.5, µ=0.025

Conclusions:

If the threshold is fixed, the variance of β and μ value must be proportional (same disease with higher or lower rates). As we can observe in the presented plots, changes in β and μ values modify the speed of the model but not the incidence (population infected). Therefore, it is sure that as higher values of β and μ, as early the diseased will be eradicated but not with less infected cases.