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# LOGISTIC EQUATION RESPRESENTATION

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Master in Computational Biology  
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## 1. Introduction

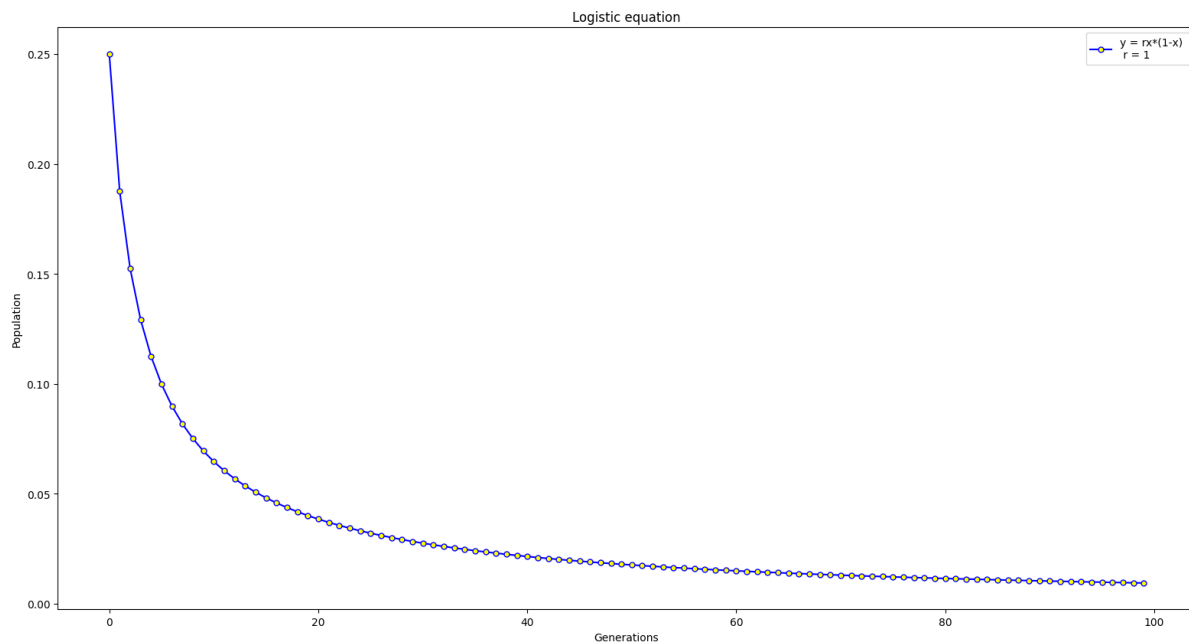
The logistic function or logistic curve is a mathematical function used in several fields including biology, biomathematics, demography, etc. Focusing on biology it is a commonly way to represent the population growth or diseases spread.

This function could be represented by a differential equation:  $\frac{dx}{dt} = rx(1 - x)$  where  $x$  is the population size and  $y$  is a period of time. Finally,  $r$  represents the rate between reproduction and mortality in the given population.

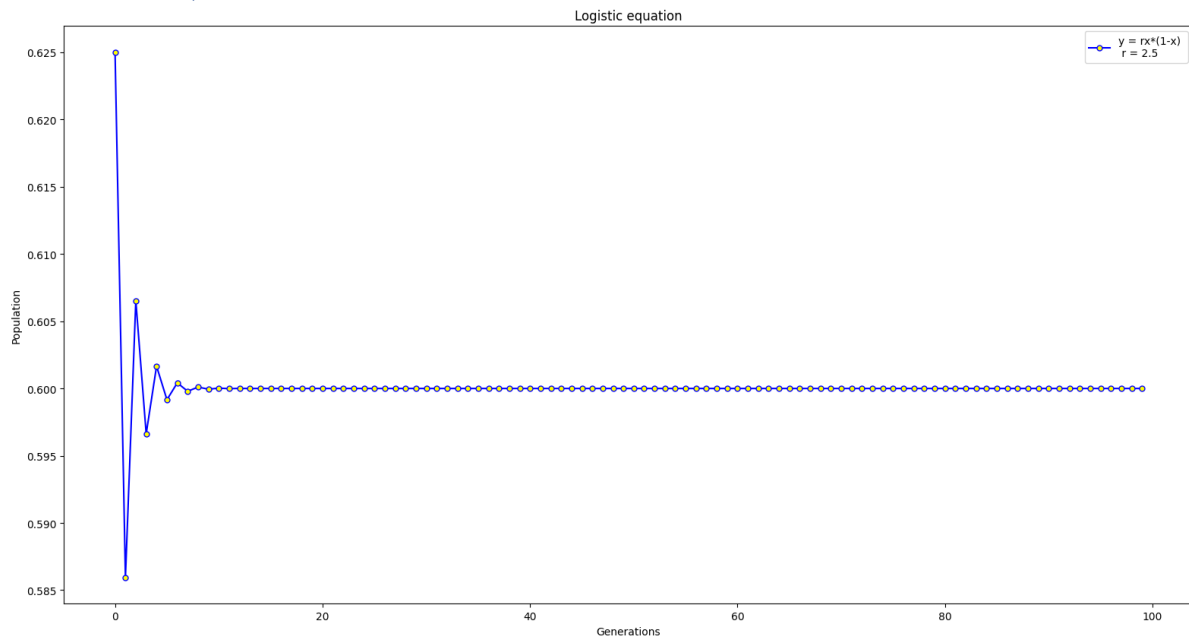
## 2. Test cases

Twelve test cases are represented below to see the differences when we vary the values of  $x$  and  $r$ .

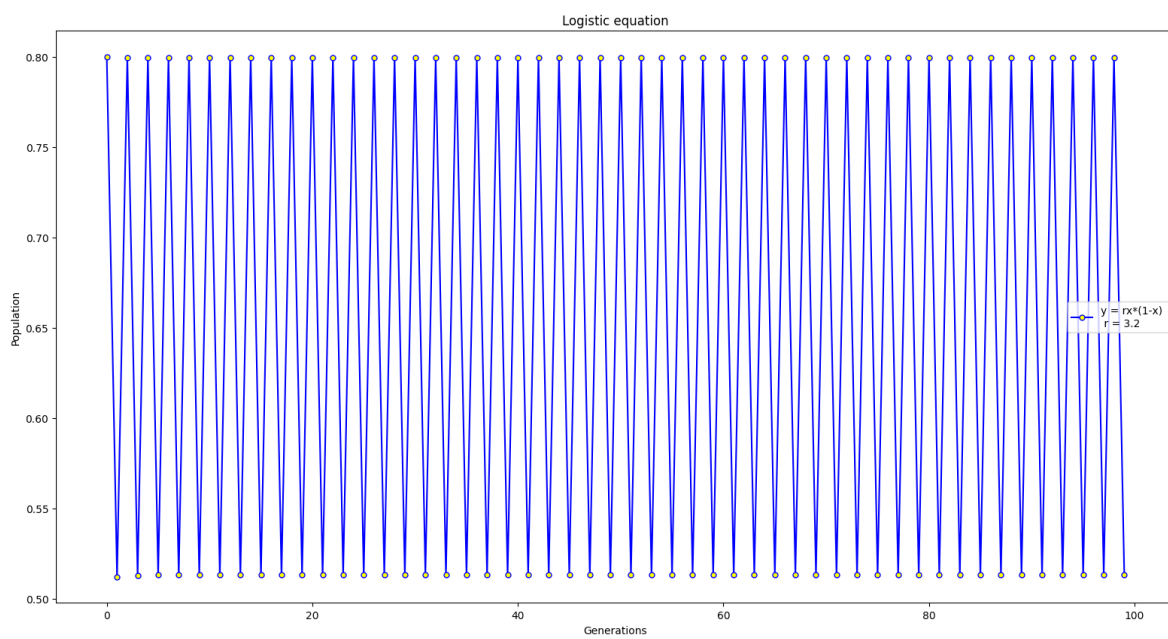
a.  $x = 0.5, r = 1$



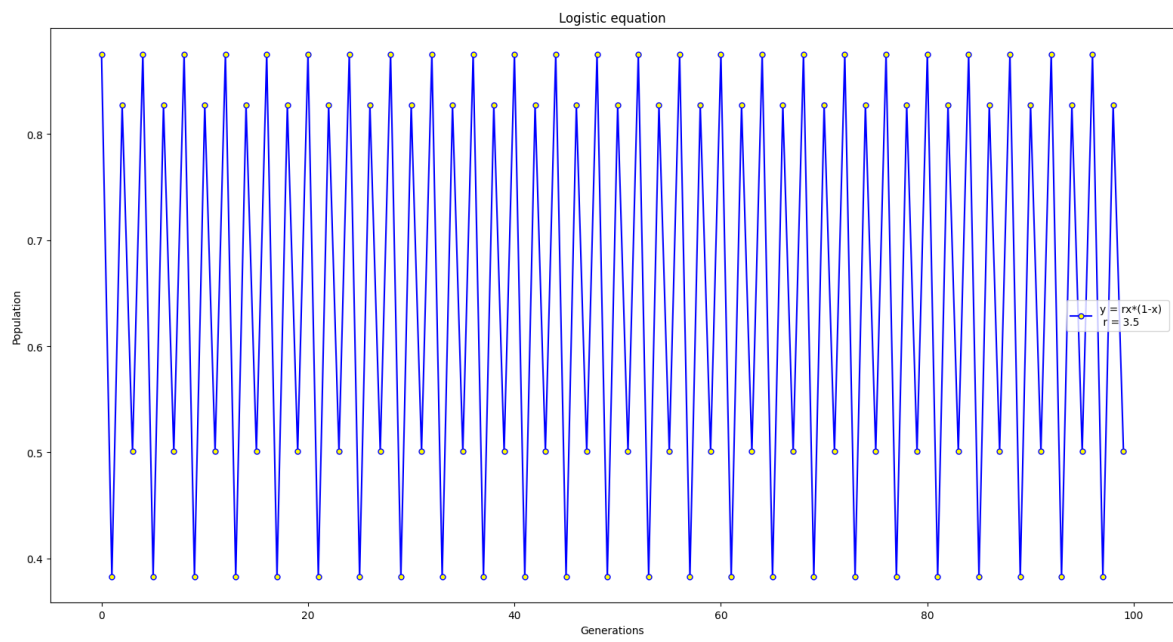
b.  $x = 0.5, r = 2.5$



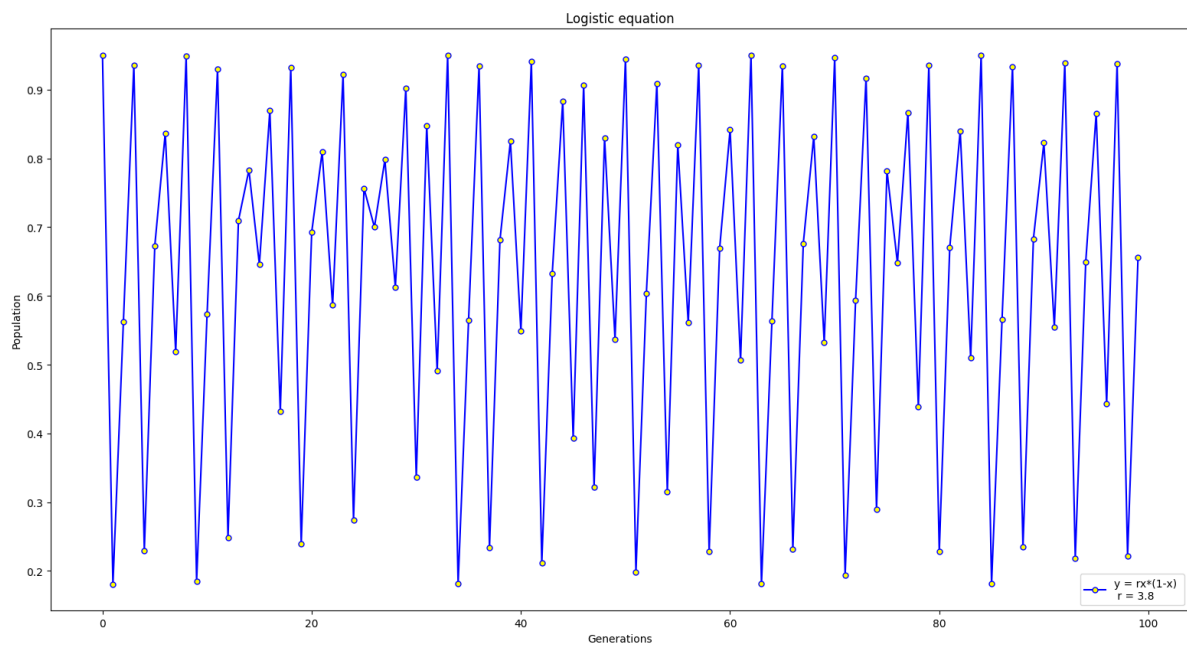
c.  $x = 0.5, r = 3.2$



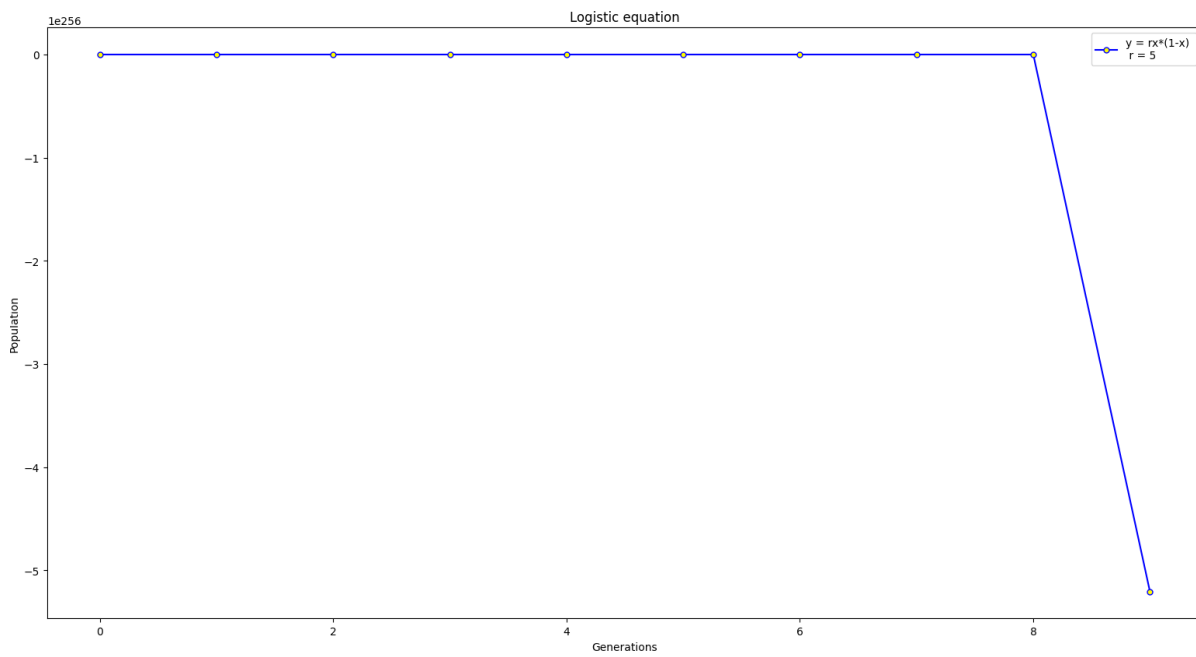
d.  $x = 0.5, r = 3.5$



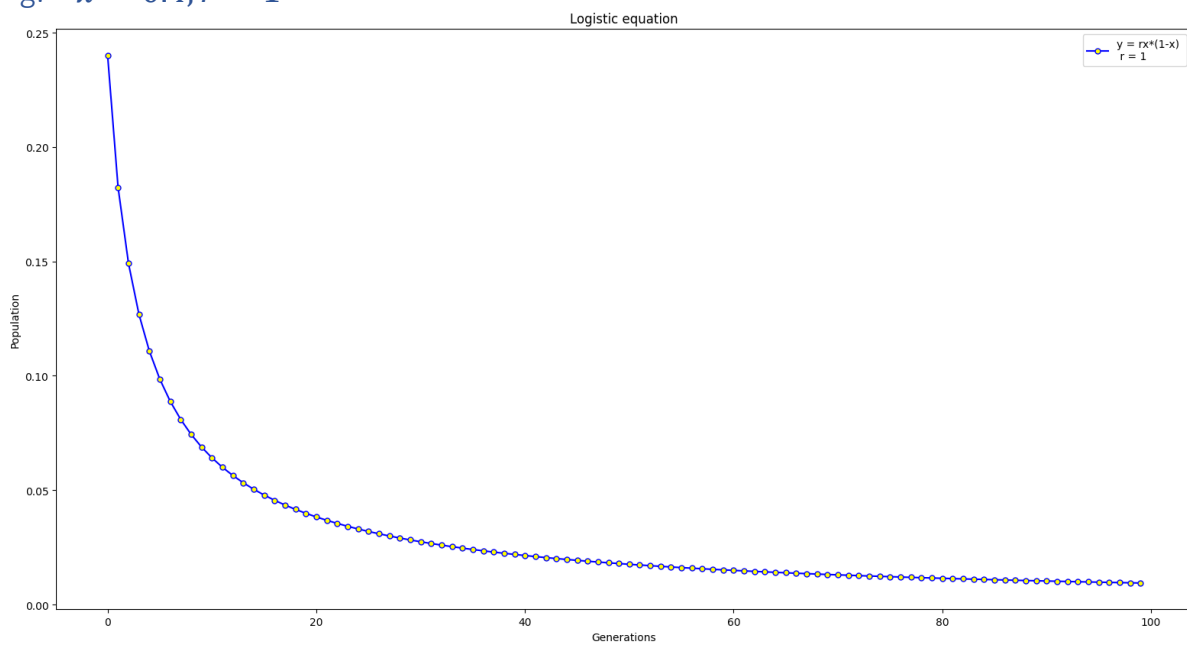
e.  $x = 0.5, r = 3.8$



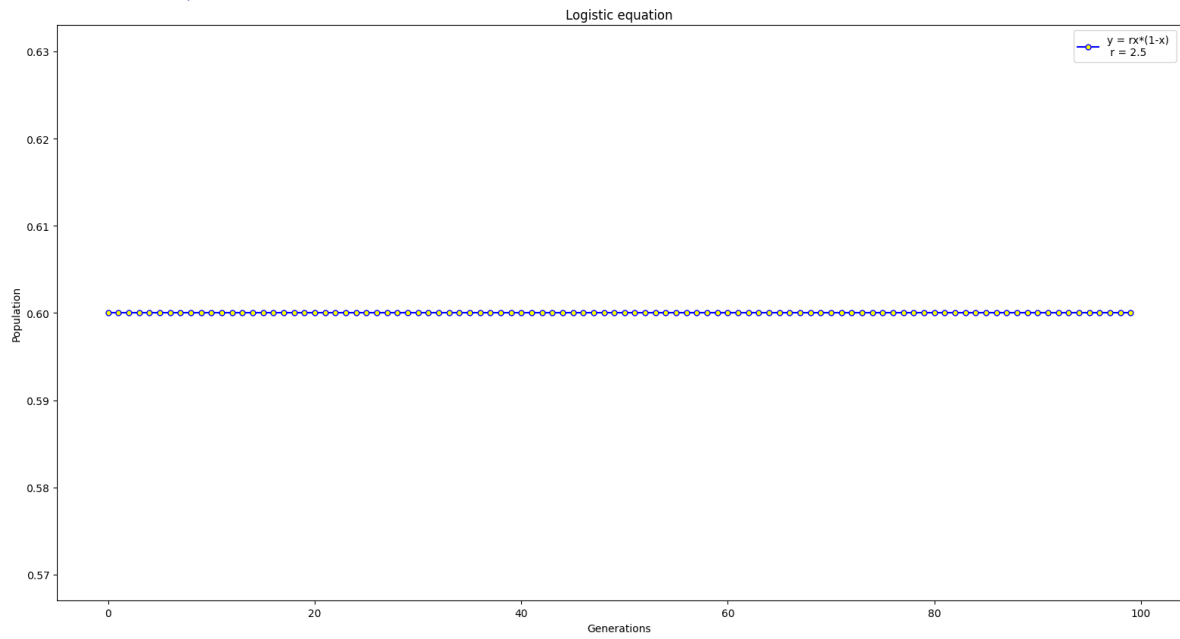
f.  $x = 0.5, r = 5$



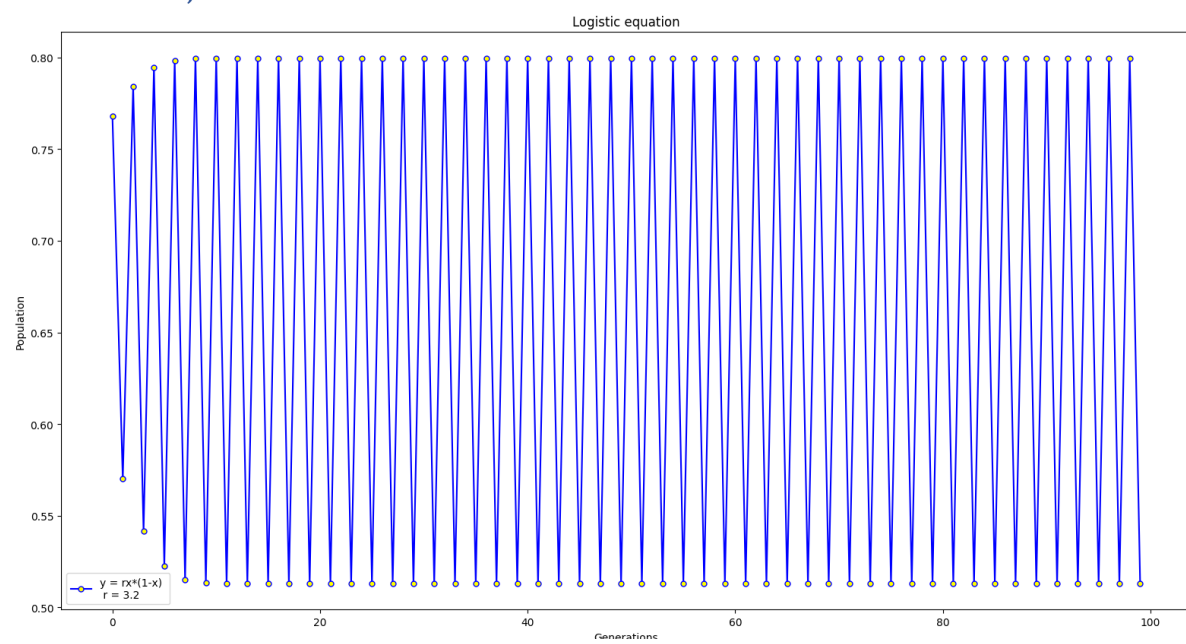
g.  $x = 0.4, r = 1$



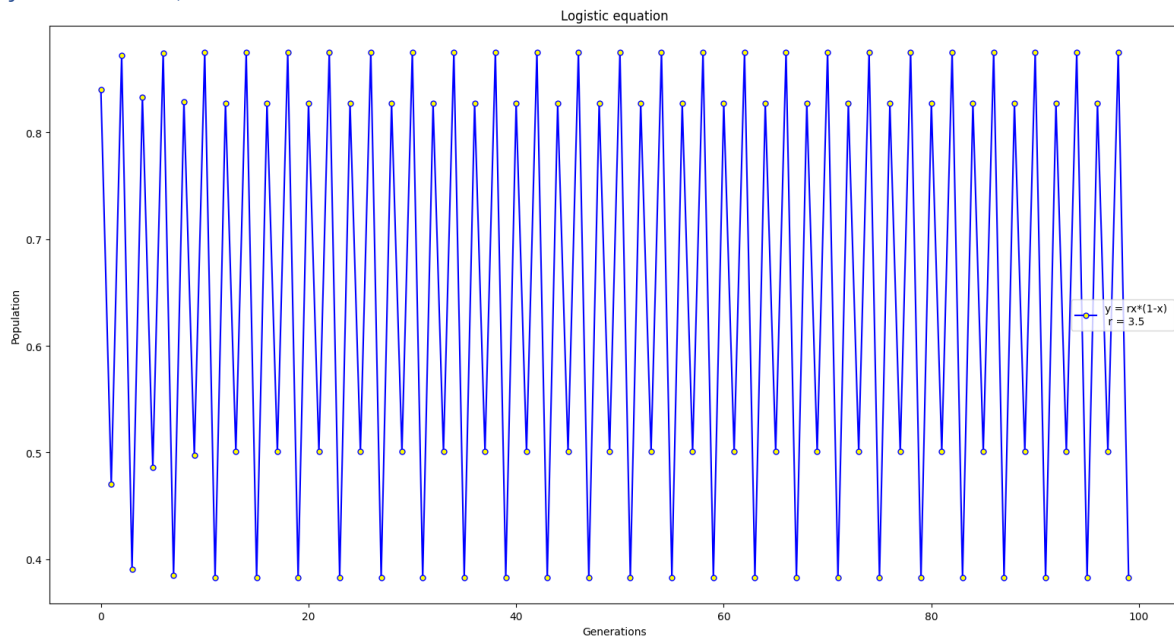
h.  $x = 0.4, r = 2.5$



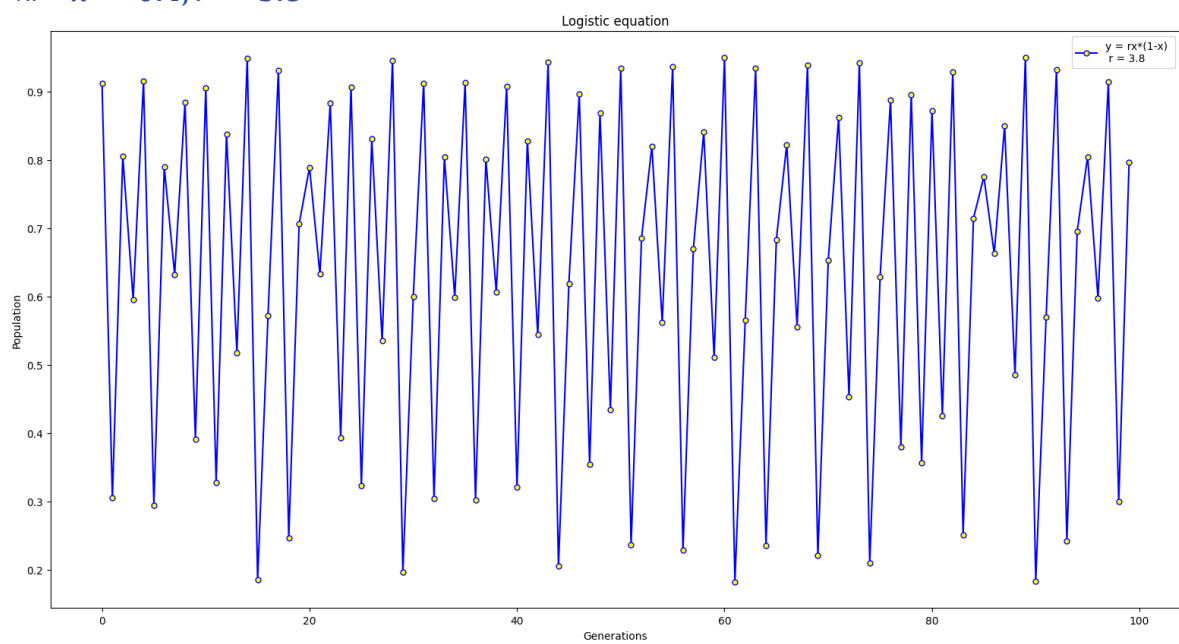
i.  $x = 0.4, r = 3.2$



j.  $x = 0.4, r = 3.5$

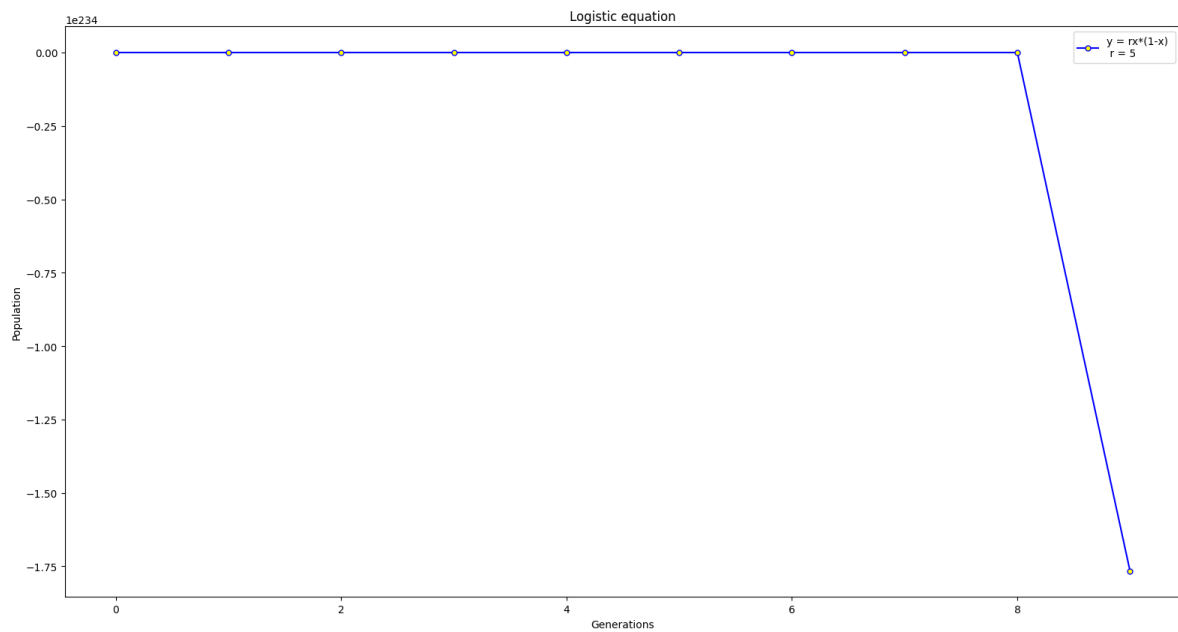


k.  $x = 0.4, r = 3.8$





1.  $x = 0.4, r = 5$

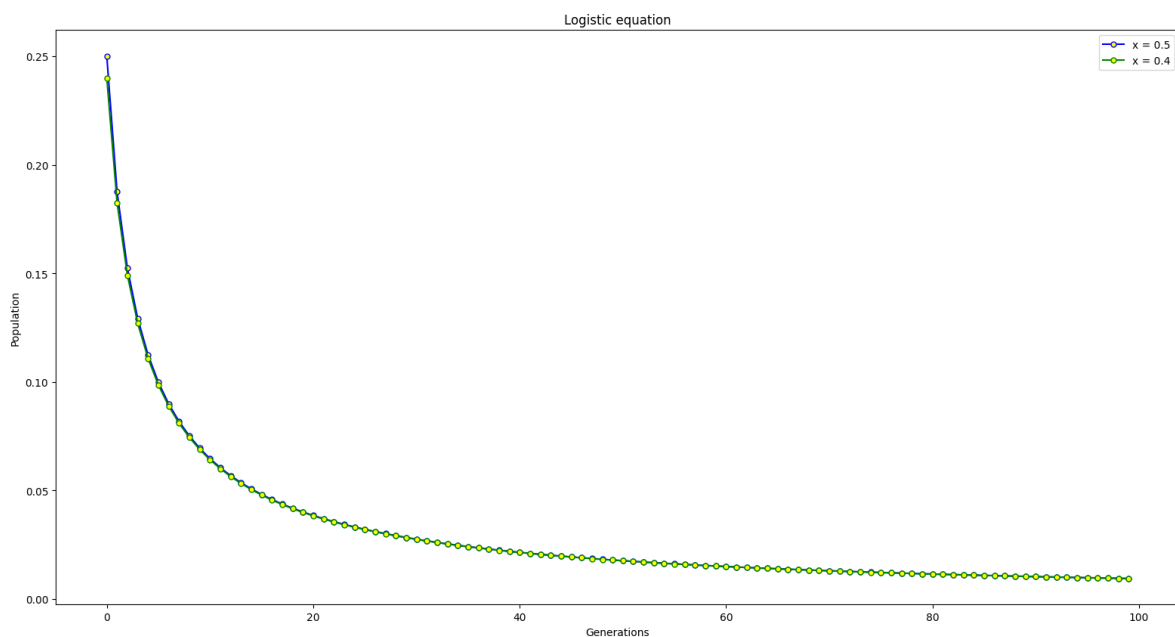


### 3. Conclusions

For answering the question: “What happens if you start from a slightly different  $x_0$  (e.g., 0.4)?” I made another plot for comparing the graphs above. I found that the principal value which we must observe was not  $x_0$ , but  $r$ . So, if we vary so slightly the value of  $x_0$  there is no a huge difference at the final result on the plots (only the time it takes to stablish the final value).

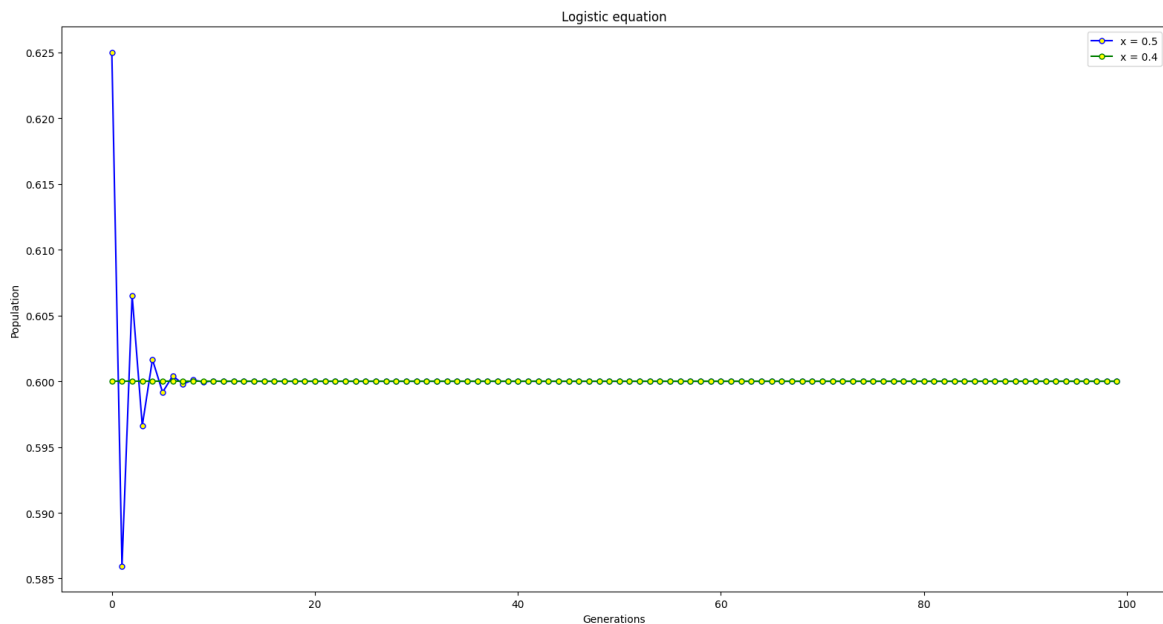
a.  $r = 1$

For  $r \leq 1$  the studied population will disappear regardless de initial number of organisms.



b.  $r = 2.5$

For  $r = 2.5$  we can observe that at some point the population will stabilize at some value. In contrast to last example ( $r = 1$ ), we can affirm that higher number of initial populations, the longer it will take for the graph to stabilize.



c.  $r = 5$

Due to the size of the equation results, it is impossible for Python to represent them. Therefore, after some investigation I found that for  $r > 4$ , values left the common interval  $[0,1]$  and the graph diverge for all values.

