Recall $\frac{d}{d\theta} \log L(\underline{x}\ \theta) = 0$

$-n + \frac{\Sigma x_i}{\lambda} = 0$

Recall $\Sigma x_i = \frac{n\bar{x}}{\lambda}$

$= -n + \frac{n\bar{x}}{\lambda} = 0$

$-n = -\frac{n\bar{x}}{\lambda}$

$\lambda 1 = \frac{\bar{x}}{\lambda} \cdot \cancel{x}$

$\lambda = \bar{x} = \hat{\lambda} = \bar{x}$ (sample mean)

Prove that $\bar{x}$ is the MLE for λ

Recall

$\frac{d^2}{d\theta^2} \log L(\underline{x}\ \theta) < 0$ ic its negative.

$\frac{d^2}{d\lambda^2} \log L(x \cdot \lambda) = \frac{d}{dx} - n + \frac{\Sigma x_i}{\lambda}$

$\frac{d}{dx} = -n + \frac{n\bar{x}}{\lambda}$

$\frac{d}{d\lambda}(-n + n\bar{x}\lambda^{-1})$

$= 0 + (-1)(1)n\bar{x}\lambda^{-2} = -n\bar{x}\lambda^{-2} = -\frac{n\bar{x}}{\lambda^2} < 0$

hence $\bar{x}$ is the MLE for λ

Obtain the MLE of parameter alpha in an exponential population given by $f(x\ \alpha) = \begin{cases} \alpha e^{-\alpha x} & \text{for } x > 0 \\ 0 \text{ elsewhere} \end{cases}$

Solution

Let $x_1\ x_2\ ..\ x_n$ be a random sample from the exponential population.

The likelihood function $L(\underline{X}, \alpha) = \prod_{i=1}^{n} \alpha e^{-\alpha x}$

$= \alpha e^{-\alpha x_1} \times \alpha e^{-\alpha x_2} \times \alpha e^{-\alpha x_n}$

$L(\underline{X} : \alpha) = \alpha^n e^{-\alpha \sum_{i=1}^{} x_i}$

Recall $\sum_{i=1}^{n} x_i = n\bar{x}$

$L(x_1\ \alpha) = \alpha^n e^{-\alpha n \bar{x}}$

Get the log likelihood function

$\text{Log } L(\underline{X}\ \alpha) = \log(\alpha^n e^{-\alpha n \bar{x}})$

$= n \log \alpha - \alpha n \bar{x}$

We get the first derivative of the log likelihood function $= 0$

$\frac{d}{d\alpha} \log L(\underline{X}\ \alpha) = \frac{d}{d\alpha}(n \log \alpha - \alpha n \bar{x})$

$\frac{d}{d} \log f(x) = \frac{f(x)'}{f(x)}$

$= \frac{n(1)}{\alpha} - n\bar{x} = 0$

$\frac{n}{\alpha} = n\bar{x}$

$1/\alpha = \bar{x} \qquad \alpha = \frac{1}{\bar{x}} \qquad \hat{a} = \frac{1}{\bar{x}} = \frac{1}{\text{sample mean}}$

Prove / verify that $\hat{\alpha} = \frac{1}{\bar{x}}$ (MLE)

$$\frac{d^2}{d\alpha^2} \log L(\underline{x}, \alpha) = \frac{d}{d\alpha} \frac{n}{\alpha} - n\bar{x}$$

$$= \frac{d}{d\alpha} n\alpha^{-1} - n\bar{x}$$

$$= (-1)(1)n\alpha^{-2} = -n\alpha^{-2} = \frac{-n}{\alpha^2} \text{ (negative)}$$

< 0 hence $\hat{\alpha} = \frac{1}{\bar{x}}$ is the MLE

Exercise,

X follows a normal distribution $N(\mu, \delta^2)$. Let X_1 X_2 upto X_n be a random sample for this population. find the MLEs of

i) μ when δ^2 is known

ii) Get the MLE of δ^2 when μ is known

iii) Get the " of μ & δ^2 when both are unknown.

Properties of Estimators

A good estimator should posses the following properties

i) unbiasedness

ii) consistency

iii) Sufficiency

iv) mean Square error

v) efficiency.

1. Unbiasedness

~~An estimator~~ Suppose $X_1, X_2 \ldots X_n$ is a random sample of size n from a distribution having a density function $f(x, \theta)$ where θ is a known parameter. Again assume that θ is an element of ~~sample~~ space parameter space Ω omega which is subset of the real life then an estimator $\hat{\theta}$ is said to be unbiased estimator of θ if the expected value of $\hat{\theta} = \theta$ ie $E(\hat{\theta}) = \theta$ for all θ members of parameter space Ω omega ~~sample~~ subset ~~of~~ real life.

The difference of the expected value of theta hat and θ is called biasedness of theta $E(\hat{\theta}) - \theta = b(\theta)$

This means that if the ~~sampling~~ mean of sampling distribution of estimator $\hat{\theta}$ is θ then $\hat{\theta}$ is an unbiased estimator of θ

Example

Suppose X_1 X_2 upto X_n is a random sample from a uniform population with density function $f(x) = \begin{cases} 1 & \lambda - \frac{1}{2} < x < \lambda + \frac{1}{2} \\ 0, & \text{elsewhere} \end{cases}$

Proove that $\bar{X}$ is an unbiased estimator of λ

Solution

$\bar{x}$ is unbiased estimator of λ

$E(\hat{\theta}) = \theta$

$E(X) = E(\bar{x}) = \lambda$

Recall $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$E(\bar{x}) = E(X) = \int_{\lambda-1/2}^{\lambda+1/2} x 1\, dx$

$= \int_{\lambda-1/2}^{\lambda+1/2} x \cdot dx = \left.\frac{x^2}{2}\right|_{\lambda-1/2}^{\lambda+1/2}$

$= \frac{(\lambda+1/2)^2}{2} - \frac{(\lambda-1/2)^2}{2}$

$(\lambda+1/2)(\lambda+1/2) = \lambda^2 + 1/2\lambda + 1/2\lambda + 1/4$

$= \lambda^2 + 1\lambda + 1/4$

$(\lambda-1/2).(\lambda-1/2) = \lambda^2 - 1/2\lambda - 1/2\lambda + 1/4$

$= \lambda^2 - \lambda + 1/4$

$\frac{(\lambda^2+\lambda+1/4)}{2} - \frac{(\lambda^2-\lambda+1/4)}{2} = \frac{\cancel{\lambda^2+\lambda}\ 2\lambda}{2} = \lambda$

$\therefore$ $\hat{\bar{X}}$ is unbiased estimator of λ

Suppose that $X_1, X_2 \ldots X_n$ is random sample population

with pdf given by $f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{for } x, \theta > 0 \\ 0 & \text{, elsewhere} \end{cases}$

show that $\bar{X}$ is unbiased estimator of θ

Solution

$E(\bar{X}) = E(X) = \theta$

$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$= \int_{0}^{\infty} x \cdot \frac{1}{\theta} e^{-x/\theta}\, dx = \theta$

Method 1

$= \frac{1}{\theta} \int_{0}^{\infty} x e^{-x/\theta} dx$

let $y = x/\theta = x = \theta y$ or $y\theta$

$\frac{dy}{dx} = \frac{1}{\theta} \Rightarrow \theta dy = dx$

$= \frac{1}{\theta} \cancel{\int_{0}^{\infty} \theta y e^{-x/\theta}}\, \theta dy \qquad \frac{1}{\theta} \int_{0}^{\infty} y\theta e^{-y} \theta dy$

$\frac{\theta^2}{\theta} \underbrace{\int_{0}^{\infty} y e^{-y} dy}_{\Gamma 2} = (n-1)! = (2-1)! = 1! = 1$

$= \frac{\theta^2}{\theta} = \theta$

Exercise.

1) Repeat above qsn using intergration by parts

2) Suppose $X_1\ X_2 \dots X_n$ is random sample of size n from distribution with mean μ and variance δ^2.
Show that $Su^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$ is unbiased estimator of δ^2

and $s^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$ is a biased estimator of δ^2

Estimable function

Estimators (properties)

i) Unbiasedness $E(\hat{\theta}) = \theta$

ii) Consistent $E(\hat{\theta}) = \theta$ or $\lim_{n \to \infty} E(\hat{\theta}) = \theta$

iii) Sufficiency $\lim var(\hat{\theta}) = 0$

iv) Efficiency - MSE

Ex 1

$X \sim N(u, \delta^2)$

$E(\bar{X}) = u$

$$(\bar{X}) = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{X_1 + X_2 + X_3 + \dots X_n}{n}$$

$$= E\left[\frac{X_1 + X_2 + X_3 + \dots + X_n}{n}\right]$$

$$= 1/n \left(E(X_1) + E(X_2) + \dots + E(X_n)\right)$$

recall $E(X) = u$

$$1/n \left(u_1 + u_2 + \dots + u_n\right) = 1/n \, (nu)$$

$$E(\bar{X}) = u$$

2. $\lim_{n \to \infty} var(\bar{X}) = 0$

$$Var(\hat{\theta}) = var(\bar{X}) = var(X_1 + X_2 + \dots + X_n)$$

$$Var(aX) = \frac{1}{a^2} var(X)$$

$var(X_1 + X_2 + \dots + X_n)$ $var(X_1) + var(X_2) + \dots + var(X_n)$

$$= \frac{1}{n^2}\left[var(X_1) + var(X_2) + \dots + var(X_n)\right]$$

$$1/n^2 \left[\delta^2 + \delta^2 + \dots + \delta_n^2\right]$$

recall

$\text{var } X = \delta^2$

$\frac{1}{n^2}(n\delta^2) = \frac{\delta^2}{n}$

$\lim_{n \to \infty} \frac{\delta^2}{n} = \frac{\delta^2}{\infty} = 0$

4) Let $X_1, X_2 \ldots X_n$ be a random sample of size n from a uniform population given by

$$f(x) = \begin{cases} 1 & , \ \lambda - \frac{1}{2} < x < \lambda + \frac{1}{2} \\ 0 & , \ \text{elsewhere} \end{cases}$$

Show that $\bar{X}$ is a consistent estimator of λ

Solution

$E(\hat{\theta}) = \theta$

This means we need to get mean of this distribution

$\text{Mean} = E(x)$

$= \int_{-\infty} x \cdot f(x)\, dx$

$E(x) = \int_{\lambda - 1/2}^{\lambda + 1/2} x \cdot dx \qquad = \frac{x^2}{2} \Big|_{\lambda - 1/2}^{\lambda + 1/2}$

$= \frac{(\lambda + 1/2)^2}{2} - \frac{(\lambda - 1/2)^2}{2}$

$\frac{(\lambda + 1/2)(\lambda + 1/2)}{2} - \frac{(\lambda - 1/2)(\lambda - 1/2)}{2}$

$\frac{2\lambda}{2} = \lambda$

$E(\bar{X}) = E(X) = \lambda$

Condition 2

$Var(\bar{X}) = 0$

$n \to \infty$

$Var(X) = E(X^2) - (E(X))^2$

$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x) dx$

$= \int_{\lambda - 1/2}^{\lambda + 1/2} x^2 dx$

$= \frac{x^3}{3} \Big|_{\lambda - 1/2}^{\lambda + 1/2}$

$\frac{(\lambda + 1/2)^3}{3} - \frac{(\lambda - 1/2)^3}{3}$

$\frac{(\lambda + 1/2)^3 (\lambda + 1/2)}{3} - \frac{(\lambda - 1/2)^2 (\lambda - 1/2)}{3}$

$\frac{(\lambda^2 + \lambda + 1/4)(\lambda + 1/2)}{3} - \frac{(\lambda^2 - \lambda + 1/4)(\lambda - 1/2)}{3}$

$\frac{\lambda^3 + 1/2\lambda^2 + \lambda^2 + 1/2\lambda + 1/4\lambda + 1/8}{3} - \frac{\lambda^3 - 1/2\lambda^2 - \lambda^2 + 1/2\lambda + 1/4\lambda - 1/8}{3}$

$$\frac{\lambda^3 + 3/2\lambda^2 + 3/4\lambda + 1/8}{3} - \frac{\lambda^3 - 3/2\lambda^2 + 3/4\lambda - 1/8}{3}$$

$$\frac{3\lambda^2 + 1/4}{3} = \lambda^2 + 1/12$$

$E(x^2) = \lambda^2 + 1/12$

$var(x) = E(x^2) - (E(x))^2$

$= \lambda^2 + 1/12 - \lambda^2$

$= 1/12$

$\lim_{n \to \infty} Var(\hat{\theta}) = 0$

$\lim_{n \to 8} 1/12 = 0.083$

$\therefore \bar{X}$ is a consistent estimator of λ

3 Sufficiency.

An estimator $T = t(X_1, X_2 \ldots X_n)$ based on a random sample X_1 X_2 and X_n of a parameter θ from a population with P.d.f $f(X, \theta)$ - is sufficient for parameter θ if the conditional distribution of X_1 X_2 upto X_n given t is independent of θ
ie $P(X_1 = x_1, X_2 = x_2 \ldots X_n = x_n | T)$ is independent of θ

An estimator $T = t(X_1 X_2 \ldots X_n)$ is a sufficient estimator of parameter θ if it contains all the information in a random sample about the parameter

Factorisation criteria / Neyman theorem
- This is a method of getting the necessary and sufficient condition for an estimator to be sufficient.
It states that $T = t(X_1 X_2 \ldots X_n)$ is a sufficient estimator for parameter θ if and only if the joint density function (likelihood function) - L of sample values X_1 X_2 upto X_n can be written in form of

$$L = L(\theta) = L(X, \theta) = L(X_1 X_2 \ldots X_n, \theta) = g(T, \theta) h(X)$$

where i) $g(T, \theta)$ depends on θ and X only through the ~~value~~ function $T = t(\underline{X})$
ii) $h(X)$ does not depend on θ
iii) $g(t, \theta)$ and $h(X)$ are positive functions ie > 0

- NB A function is ~~independent~~ independent of parameter θ if and only if (iff) it does not contain the parameter θ in its pdf as well as in its domain
Eg

$$f(a) = \begin{cases} \frac{1}{2a} & \text{for } 1-\theta < a < 1+\theta \\ 0 & \text{elsewhere} \end{cases}$$

$f(x) = \frac{1}{2a}$ for $1 < a < 2$

2^{nd} function is independent of parameter θ

Steps of finding a sufficient estimator

Step 1

Get the likelihood function ie $L = L(X\ \theta)$

$$L = \prod_{i=1}^{n} f(x_i, \theta)$$

Step 2

Check whether the likelihood function satisfies the factorisation criteria.

ie i) $L = g(T, \theta)\, h(x)$

ii) $h(x)$ is independent of parameter θ

iii) $h(x)$ & $g(T, \theta) > 0$

~~Step~~ If so $T = t(X_1, X_2 \ldots X_n)$ is a sufficient estimator of the parameter. θ

Example 1

Let $X_1, X_2 \ldots X_n$ be a random sample of size n from exponential distribution defined by $f(x) = \frac{1}{\theta} e^{-x_i/\theta}$ for $x, \theta > 0$. Find the sufficient estimator of parameter θ

Solution

Step 1

$L = L(X, \theta)$

$$L = \prod_{i=1}^{n} f(x_i, \theta)$$

$$= \frac{1}{\theta} e^{-x_1/\theta} \times \frac{1}{\theta} e^{-x_2/\theta} \times \ldots \frac{1}{\theta} e^{-x_n/\theta}$$

$$\left(1/\theta\right)^n \underbrace{e^{-\frac{\sum_{i=1} X_i}{\theta}}}_{g(T,\theta)}$$

$$L(\theta) = \left(\frac{1}{\theta}\right)^n e^{\frac{\sum_{i=1} X_i}{\theta}} \times \underbrace{1}_{hX}$$

$\sum_{i=1}^{n} X_i$ is a Sufficient estimator for θ

Note : Sufficient estimator of a parameter θ always takes the form of $\sum_{i=1}^{n} X_i$ or $\prod_{i=1}^{n} X_i$

Example 2

Let X_1 X_2 upto X_n be a random sample of size n from a population with pdf $f(x_i;\theta) = \theta x^{\theta-1}$ for $0<X<1$ $\theta>0$. Find the sufficient estimator parameter θ

Solution 1

Get the likelihood function

$$L(\theta) = \prod_{i=1}^{n} f(X_i\,\theta)$$

$$= \theta x^{\theta-1} \times \theta x_2^{\theta-1} \times \ldots \theta x_n^{\theta-1}$$

$$= \theta^n \prod_{i=1}^{n} X_i^{\theta-1}$$

$$\theta^n \prod_{i=1}^{n} X_i^{\theta-1} = \theta \left(\prod_{i=1}^{n} X_i\right)^{\theta} \times \prod_{i=1}^{n} (X_i)^{-1}$$

$$L(\theta) = \underbrace{\theta^n \left(\prod_{i=1}^{n} X_i\right)^{\theta} \times \frac{1}{\prod_{i=1}^{n} X_i}}_{g(T,\theta)\ h(x)}$$

where $h(x) = \frac{1}{\prod_{i=1}^{n} x_i}$

$$g(T, \theta) = \theta^n (\prod_{i=1}^{n} x_i$$

$h(x)$ and $g(T, \theta) > 0$

The sufficient estimator for $\theta = \prod_{i=1}^{n} x_i$

Example / exercise

Let $X_1, X_2 \ldots X_n$ be a random sample from a bernoulli distribution with parameter θ

$$f_X = \begin{cases} \theta^x (1-\theta)^{1-x} & \text{for } x = 0, 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$T = \sum_{i=1}^{n} X_i$$

2. Confidence for mean μ for t-distribution (ie when < 30

i) For one tailed test.

$$\bar{X} \pm t(\alpha, n-1) \times \frac{S}{\sqrt{n}}$$

where $\bar{X}$ = sample mean.
$t(\alpha, n-1)$ = tabulated t value at α level of significance and $(n-1)$ degree of function
S = sample standard deviation
n = sample size

$$= \left[X - t(\alpha, n-1) \times S/\sqrt{n} < \mu < \bar{X} + t(\alpha, n-1) \times \frac{S}{\sqrt{n}}\right]$$

ii) The 2 tailed test

$$\bar{X} \pm t(\alpha/2, n-1) \times S/\sqrt{n}$$

$$\left[\bar{X} - t(\alpha/2, n-1) \times S/\sqrt{n} < \mu < \bar{X} + t(\alpha/2\ n-1) \times S/\sqrt{n}\right] = (1-\alpha)$$

α = level of significance

$$S = \frac{\Sigma(X_i - \bar{X})^2}{n-1} \qquad S = \frac{\Sigma(X_i - \bar{X})^2}{n}$$

Example
The table below shows a sample of 10 patient in a heart study.

Sample size	Sample mean	Std
$n = 10$	121.2	11.1

Construct a 95% confidence interval for the true systolic

i) For one tailed test

$$\bar{x} \pm \underbrace{t(\alpha, n-1) \times s/\sqrt{n}}_{\text{margin of err}}$$

$$121.2 \pm 1.833 \times \frac{11.1}{\sqrt{10}} =$$

$$121.2 \pm 6.434064996$$

$$114.765935 < u < 127.67$$

We are 95% sure that the true systolic blood pressure lies between 114.7 and 127.6

ii) For 2 tailed test

$(t\ \alpha/2\ ; n-1)$

$(t\ 25\%, 9)$

$= 2.262$

$$121.2 \pm 2.262 \times \frac{11.1}{\sqrt{10}} =$$

$$121.2 \pm 7.939909995$$

$$129.13991 < u < 113.26009$$

We are 97.5 % sure that the true systolic blood pressure lies between 113.26 and 129.13991

iii) Test the hypothesis that
H_0 : Systolic blood pressure $= 130$
H_1 : Systolic blood pressure < 130

H_1 ✓ The systolic blood pressure is < 130

H/w Go repeat when n (Sample size) is ~~[illegible]~~ 3439 and compare the the values -
CI_s

NOTE when you increase sample size error reduces
" " decrease " " " increases

Confidence Interval for difference in means. (samples obtained from some normal population)

The $(1-\alpha)100\%$ CI for difference in means obtained from the same population is given by

$$(\bar{X} - \bar{Y}) \pm Z_{\alpha/2}\,\delta\sqrt{\frac{m+n}{mn}}$$

which is equivalent to

$$(\bar{X} - \bar{Y}) - Z_{\frac{\alpha}{2}}\,\delta\sqrt{\frac{m+n}{mn}} < u_1 - u_2 < (\bar{X} - \bar{Y}) + Z_{\frac{\alpha}{2}}\,\delta\sqrt{\frac{m+n}{mn}}$$

where $\bar{X}$ = sample mean for $x_1, x_2 \ldots x_m$

$\bar{Y}$ = sample mean for $y_1, y_2 \ldots y_n$

m = sample size for X

n = sample size for Y

δ = common standard deviation

$Z_{\alpha/2}$ = Z - score at $\alpha/2$ $\quad \alpha$ = levels of significance

Example

In 16 test runs the fuel consumption of an experimental engine had a std of 2 gallons

i) Construct a 99 % CI for

A sample of 20 light bulbs of one kind lasted an average of 530 hours of continuous use. Another sample of 30 bulbs of different kind lasted an avg 510 hours of continuous use. The 2 samples of light bulbs have a common standard deviation of 25. Construct a 99% CI for the difference of the average hours of use for the 2 kinds of light bulbs

Solution

$$(\bar{X}-\bar{Y}) - z_{\alpha/2}\,\delta\sqrt{\frac{m+n}{mn}} < u_1 - u_2 < (\bar{X}-\bar{Y}) + z_{\frac{\alpha}{2}}\,\delta\sqrt{\frac{m+n}{mn}}$$

$$(530-510) - Z_{0.5\%} \times 25\sqrt{\frac{20+20}{20 \times 30}} < (530-510) + Z_{0.5\%} \times 25\sqrt{\frac{20+30}{20 \times 30}}$$

$$20 - 2.5758 \times 25\sqrt{\frac{50}{600}} < u_1 - u_2 < 20 + 2.5758 \times 25\sqrt{\frac{50}{600}}$$

$$20 - 18.58923529 < u_1 - u_2 < 38.58923529$$

$$1.41076$$

We are 99.5% sure that the true difference between u_1 and u_2 lies between that range

4) CI for Proportions

If sampling is done from an infinite population or is done with replacement from a finite population the CI for population proportion is given by

$$P \pm Z_{\alpha/2} \sqrt{Pq/n}$$

where P is the proportion of success in the sample

$q = 1 - P$

n - sample size.

Example 1

The human resource director of oracle operation wanted to know what proportion of all persons who have ever been interviewed for a job within corporation had been hired. He settled at 95% ~~for~~ CI. Random sample of 500 interview records revealed that 76 persons in the sample had been hired. Construct the interval

Ans (13, 19) persons

(0.120, 0.180346) proportion.

Example 2.
Out of 20,000 customer ledger account a sample of 600 accounts was taken to a test
The test was for accuracy of posting and balancing where 45 mistakes were recorded. Asign limits within which the number of defective cases will lie at $\alpha = 5\%$

$P =$

5) CI for variance
The MLE for δ^2 has a chi-square (χ^2) distribution with $n-1$ degrees of freedom.
following this result, then the variance of random sample of size n from a normal distribution has a $1-\alpha$ % CI defined by

$$\left[\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}}, \frac{(n-1)S^2}{\chi^2(1-\alpha/2)}\right]$$

$$\left[\frac{(n-1)S^2}{\chi^2_{\alpha/2}} < \delta^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right]$$

Example 1.
In 16 test runs the fuel consumption of an experimental engine had std of 2 gallons
1) Construct a 99% CI for variance measuring variability of fuel consumption of the engine

Solution

$$\frac{(16-1)\,2^2}{\chi^2_{0.5\%}} < \delta^2 < \frac{(16-1)\,2^2}{\chi^2_{99.5\%}}$$

$\chi^2_{0.5\%} \to 32.8$ $\qquad$ $\chi^2_{99.5\%} \to 4.601$

$$\frac{15 \times 4}{32.8} < \delta^2 < \frac{15 \times 4}{4.601}$$

ii) Construct 99 % CI for standard deviation

Solution $\sqrt{\frac{15 \times 4}{32.8} < \delta^2 < \frac{15 \times 4}{4.601}}$

Ex
Find the 95% CI for variance and standard deviation of nicotine content in cigarettes manufactured if a sample of 20 cigarettes and a std of 1.6 milligrams