Tarea 2

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Given the following definition,

$x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$, then

$$\frac{\partial y}{\partial x} = \begin{pmatrix} \partial y_1/\partial x_1 & \partial y_1/\partial x_2 & \cdots & \partial y_1/\partial x_n \\ & \ddots & & \\ & & & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

Prove that given $y = Ax$, we have $\frac{\partial y}{\partial x} = A$

Desarrollamos:

$$\frac{dA}{dx} = \frac{d\begin{pmatrix} a_{11} & \cdots & a_{13} \\ & \ddots & \\ & & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}{d\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}$$

$$= \frac{\begin{pmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{pmatrix}}{d\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}$$

$$\begin{pmatrix} \dfrac{d(a_{11}x_1 + a_{12}x_2 + a_{13}x_3)}{dx_1} & \cdots & \dfrac{d(a_{11}x_1 + a_{12}x_2 + a_{13}x_3)}{dx_3} \\ \vdots & \ddots & \\ \dfrac{d(a_{31}x_1 + a_{32}x_2 + a_{33}x_3)}{dx_1} & & \dfrac{d(a_{31}x_1 + a_{32}x_2 + a_{33}x_3)}{dx_3} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = A$$

2. Given $d = y^T A x$, prove $\frac{\partial d}{\partial x} = y^T A$

$$\underbrace{\underbrace{(1 \times N)(N \times m)}_{1 \times m}\underbrace{(m \times 1)}_{m \times j}}_{1 \times 1}$$

$1 \times 1 \rightarrow$ d es un número

$$d = (y_1 \ y_2 \cdots y_N)\begin{pmatrix} a_{11} & a_{12} \cdots & a_{1m} \\ \vdots & \ddots & \\ a_{n1} & & a_{nM} \end{pmatrix}\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$$

$$\rightarrow \frac{\partial d}{\partial x} = \frac{\partial \left[(y_1 \ y_2 \cdots y_N)\begin{pmatrix} a_{11} & a_{12} \cdots & a_{1m} \\ \vdots & \ddots & \\ a_{n1} & & a_{nM} \end{pmatrix}\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} \right]}{\partial \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}}$$

$$= \frac{\partial \left(y_1 a_{11} + \cdots y_N a_{n1}, \ldots, y_1 a_{1m} + \cdots + y_N a_{nm} \right)\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}}{\partial \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}}$$

$$= \frac{\partial \left((y_1 a_{11} + \cdots y_N a_{N1}) x_1 + \cdots + (y_1 a_{1m} + \cdots + y_N a_{Nm}) x_m \right)}{\partial \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}}$$

$$\left(\frac{\partial \left((y_1 a_{11} + \cdots y_N a_{N1}) x_1 + \cdots + (y_1 a_{1m} + \cdots + y_N a_{Nm}) x_m \right)}{\partial x_1}, \ldots, \frac{\partial \left((y_1 a_{11} + \cdots y_N a_{N1}) x_1 + \cdots + (y_1 a_{1m} + \cdots + y_N a_{Nm}) x_m \right)}{\partial x_m} \right)$$

$$= \left(y_1 a_{11} + \cdots y_N a_{n1}, \ldots, y_1 a_{1m} + \cdots + y_N a_{Nm} \right)$$

$$= y^T A \Big|_x \quad \blacksquare$$