Diagonalizing the correlation matrix

1) Start with an $N \times N$ correlator matrix $\mathbf{C}(t)$ (bootstrap samples)

$*1.2) \rightarrow$

2) We want to solve the following problem (generalized eigenvalue problem)

$$\mathbf{C}(t)\,\mathbf{u}^\alpha = \underbrace{\lambda^\alpha(t,t_0)}_{\substack{\text{eigenvalue} \\ \sim e^{-(t-t_0)E_\alpha}}}\,\mathbf{C}(t_0)\,\underbrace{\mathbf{u}^\alpha}_{\text{eigenvector}} \longrightarrow \text{too difficult to solve}$$

3) In order to transform it to a "normal" eigenvalue problem, we do the following transformation:

$$\mathbf{C}(t_0) = \underset{\text{Cholesky decomposition}}{\mathbf{L}\mathbf{L}^+} \longrightarrow \tilde{\mathbf{C}}(t)\,\tilde{\mathbf{u}}^\alpha = \lambda^\alpha(t,t_0)\,\tilde{\mathbf{u}}^\alpha \rightarrow \text{normal eigenvalue problem}$$

where

$$\tilde{\mathbf{C}}(t) = \mathbf{L}^{-1}\mathbf{C}(t)(\mathbf{L}^+)^{-1}, \qquad \tilde{\mathbf{u}}^\alpha = \mathbf{L}^+\mathbf{u}^\alpha$$

(the derivation is in my phd thesis)

This means that

a) Find the cholesky decomposition of $\mathbf{C}(t_0)$

b) Multiply $\mathbf{L}^{-1}\mathbf{C}(t)(\mathbf{L}^+)^{-1}$ to form $\tilde{\mathbf{C}}(t)$

c) Diagonalize $\tilde{\mathbf{C}}(t)$ to find the eigenvectors $\tilde{\mathbf{u}}^\alpha$

d) Recover the original eigenvectors $\mathbf{u}^\alpha = (\mathbf{L}^+)^{-1}\tilde{\mathbf{u}}^\alpha$

e) Reconstruct diagonalized correlation function $C_{\alpha\alpha}(t) = \mathbf{u}^{\alpha^+}(t_{ref},t_0)\,\mathbf{C}(t)\,\mathbf{u}^\alpha(t_{ref},t_0)$

f) Form the EMP for each $C_{\alpha\alpha}(t)$ $(1 \leq \alpha \leq N)$ to extract the energy

1.2) There is some pre-processing done to the correlator matrix before diagonalizing,

$$C_{\alpha\beta}(t) = \frac{1}{2} \frac{C_{\alpha\beta}(t) + C_{\beta\alpha}(t)}{\sqrt{C_{\alpha\beta}(t=0)\, C_{\beta\alpha}(t=0)}}$$

symmetrizes off-diagonal entries

normalizes correlators so $C_{\alpha\alpha}(t=0) = 1$