

#### Slide 1

- Hi, my project is 'A Markovian Bus Model: How timely does the 202 need to be'

#### Slide 2

- The title in itself already has a lot of information embedded in it. So first I'm going to go through a brief introduction to this topic.

#### Slide 3

- The stress to get to school on time was one felt by thousands of students at UCSD. In March 2020, that all changed when COVID-19 hit and forced almost every in-person meeting into a virtual setting. However, once the virus is under control and UCSD opens its campus again, most students will have to return to relying on the bus for their transportation needs. Having a historically bad reputation for being late, the bus system at UCSD needs to be studied and optimized now. This way, when campus operations return to full capacity, students will be able to rely on punctual transportation and mitigate yet another source of stress in their lives.
- One of the primary bus routes that students use is the 202 route. This route is responsible for transporting passengers from surrounding neighborhoods and shopping centers to the UCSD campus.
- We will be using a Markov Chain to model the behavior of the bus traveling from one stop to the next.

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- The first question we consider is: What does the timeliness of the bus's full route and individual stops look like, when we alter the probabilities in the Markov transition matrix?
- The second question we consider is: When emphasizing the delay in the stops where most students board, how late can the bus be, so that it still arrives mostly on time at school?

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- Now that we've introduced the topic, we're going to get into some background

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- There has been abundant research on the topic of modeling and optimizing bus systems, so this list is definitely not comprehensive.
- The first 2 that I listed both proposed a model for their respective routes, in order to reduce cost and increase the quality.
- But the third paper listed is what primarily inspired my model

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- So these authors constructed a transition matrix that represents the probability of the bus arriving on time at a stop, given the timeliness of the bus at the previous stop. The timeliness of the bus was measured in minutes, such that the resulting matrix on the right has values -2, to 2, corresponding to the bus arriving at the stop two minutes early, one minute early, and so on, up to two minutes late.
- The model that I developed for the 202 bus route differs from this approach in that it only includes the states early, on time, and late in the transition matrix, as opposed to keeping track of the specific minutes.
- This is because the data for the 202 bus route is not easily accessible, and hence, it is more practical to construct a simpler version of the states of timeliness that could be extrapolated in the future.

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- Ok, so now we are going to dive into the specifics of my model.

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- First let's review what a Markov chain is.
- A Markov chain is a stochastic model describing a sequence of events in which the probability of each event is dependent only on the state attained in the previous event.
- Transition matrices are matrices containing information on the probability of transitioning from one step to the next.
- So on the bottom here, we have our transition matrix on the right, and our chain model on the left.
- So for example if it is currently raining, then the probability that it will be cloudy next is .5, the probability that it will be rainy next is .3, and sunny next is .2

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- With a transition matrix, we are able to compute the end state after  $n$  steps in the chain. All we do is multiply the transition matrix with the starting state vector.

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- Now that we've reviewed the general concept, this is the setup of my model of the 202 route.
- On the top we have the transition matrix representing the transition from one stop to the next. On the bottom we have a block matrix that combines all the transitions, representing one full route.
- And in this picture I've labelled all the stops along the route, so we start at stop 1 in UTC and go all the way to stop 11.

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- For assumptions, we have the markov property
  - The next step is only dependent on the previous step
- Time-homogeneous property

- the probability of any state transition is independent of time
- and finally, the bus is always on time at the start of the route
- and we will use this model by creating various hypothetical scenarios and applying functions to the transition matrices
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#### Slide 13

- Lastly we are going to review real quick what the stationary distribution is before I get into the model analyses.
- So the stationary distributions is a probability distribution that remains unchanged as the sequence of events progresses.
- We have a transition matrix and a row vector that must satisfy this equation, where  $\pi$  equals the stationary distribution. By transposing this equation, we just need to find the eigenvector with eigenvalue 1, in order to solve for  $\pi$  aka the stationary distribution.

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- Now we are finally going to get into the model analyses

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- Just as a refresher, I had 2 main questions in this project. Question 1 is: What does the timeliness of the bus's full route and individual stops look like, when we alter the probabilities in the Markov transition matrix?
- I created 7 hypothetical scenarios of transition matrices, which can be seen on the right here.
- And I applied 3 different functions to these scenarios.

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- For the first function, I simulated 10,000 full bus routes and tracked the end state.
- On the right you can see the count plots for each scenario.
- As you can see, the initial timeliness is not carried over to the end of the route, unless the transition matrices have extremely high probability of remaining on time as in Scenario 1 and 2.

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- For the second function, we compute the stationary distribution of  $A_{11}$  where  $A_{11} = \dots$
- And for scenario 1 it doesn't really make sense to discuss the stationary distribution because it's just the identity matrix, but the rest of the scenarios we see that they are all equal.  $\pi$  is  $[\frac{1}{3} \frac{1}{3} \frac{1}{3}]$

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- Now the last function is similar to the first function, but instead of tracking the state after the full route, we track the state after every stop. Here we get a much clearer idea of exactly how much the timeliness degrades, depending on the transition matrix used.

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- And this last function leads us into our next question.
- When emphasizing the delay in the stops where most students board, how late can the bus be, so that it still arrives mostly on time at school?
- For this question we will look at a subset of the full route: stops 2 to 8. Stop 8 is the first stop that is located at UCSD. We will have a fixed transition matrix that favors being on time for most of the stops. And for Stop 3 and 4, which are the stops most impacted by students boarding, we will vary the matrices.

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- So here we can see how the timeliness changes along the route. We are focused on if the bus has a higher chance of arriving on time rather than late. Scenario 3, 4, and 5 all satisfy this.

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- In conclusion,

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- For question 1, we saw that in the long term, the bus drifts towards having equal probability of being early, on time, or late. And since it is difficult for a Markov chain to make precise forecasts after too many steps, looking at the long term was not as informative as the next function...
- After recording the states after each stop in function 3, we can predict that .....READ

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- Finally, we also found that in question 2 when looking at the delays caused by stop 3 and 4, given our fixed transition matrices, for the bus to have a higher chance..... READ

#### Slide 24

- Now that I've completed this model, there are many possible extensions that could be made.
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- Last but not least, I just wanted to mention that although this model is much simpler than the actual structure of the bus system, it still is able to provide valuable insight into the behavior of the bus.
- Therefore, with the resources that UCSD has, it is not only possible to create a reliable bus system – it's necessary.

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- Thank you for listening, and below are my references.