# Supersymmetry In U(1)

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**Abstract**

The masses of particles found in the SM were shown to change through a supersymmetric interaction after the symmetry was broken. This was illustrated in U(1) gauge with simplified interractions, but with 2 fields;

$$\Phi_+(\phi^+, \phi^{*+}, \psi^+, \psi^{\dagger +}) \qquad \text{and} \qquad \Phi_-(\phi^-, \phi^{*-}, \psi^-, \psi^{\dagger -})$$

Masses were found to be different in all but the gaugino field, which was kept the same due to a simplification in an eigenvalue problem.

# Contents

# 1 Introduction

## 1.1 Preface

In this Dissertation I will be showing the basic principles of Supersymmetry, before extending this to a U(1) example complete with particle masses. To begin with we will see how a Lagrangian can be invariant under SUSY transformation, and what terms are needed to keep it invariant under these constraints. Then we will somewhat simplify the theory to an example in U(1), followed by the breaking of this symmetry to find the particles and their masses.
As the first 3 sections are an introduction to SUSY they can be classified as a review. The work after section 3 was done by working with existing Standard Model theories and applying them to the supersymmetric Lagrangian. It was done between my supervisor and I and not taken from any paper, although the concept has probably been explored before at some point. As such no results have been directly taken from any paper, although similarities arise to the SM due to the simplified nature of the theory.

## 1.2 Why supersymmetry

[1;2;3]

While the Standard Model excellently represents and predicts currently detectable physics, there are some problems when we look ahead at higher energy scales.
One of these is the hierarchy problem. This is the perplexingly weak strength of gravity compared to the other forces and the relatively small mass of the higgs.
Without large corrections to the higgs mass we end up with $m_H \approx m_p$ (where $m_p$ is the planck mass). For the higgs mass to be the 125Gev we have measured requires large cutoffs to the loop integrals used to calculate it. SUSY gives a natural way to fix this without using these cutoffs.
The principle of supersymmetry states that for every boson there exists a fermion that is identical to it in every way except for its spin state. A supersymmetric transformation, therefore, is one that changes a fermion into a boson and vice versa. Having these particles allows us to cancel off the mass of a scalar boson while keeping the other particles with their current cutoffs.
Ideally these particles would have the same mass as their partners, however this is not the case. If it were, we would have seen the partners of the lighter particles long ago. The solution is to break the symmetry, much like the weak force is broken to create the $W^{+/-}$. These superpartners now end up being much more massive than their counterparts, although there is enough room in the symmetry to reduce the higgs mass to its needed value even with this broken symmetry.

# 2 Supersymmetrical Invariant Lagrangian

[1;4;5;2]

## 2.1 Notation

[6;1;7]

A Lagrangian is an equation from which we can calculate a systems equations of motion. It is denoted by the symbol $L$. We can integrate a Lagrangian over a volume to obtain a Lagrangian density, denoted by $\mathcal{L}$, the relation between them being:

$$L = \int d^{n-1}\mathcal{L} \qquad \text{where n is the space-time dimension}$$

The Lagrangian is used to calculate the action. When the variation of the action is minimised we have the equations of motion for the system, i.e;

$$\mathcal{S} = \int \mathcal{L} d^4x \qquad \text{and} \qquad \delta\mathcal{S} = \int \delta\mathcal{L} d^4x = 0 \tag{2.1.1}$$

For a Lagrangian describing the Standard Model we initially start with 2 components, the scalar part and the fermionic. The scalar part refers to all the scalar particles (ie. spin 0) and comes from the massless Klein Gordon equation. The fermionic refers to fermions (spin 1/2) and comes from the massless Dirac equation, using the weyl representation. In the weyl representation the gamma zero matrix is different. This leads to the following;

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ -\mathbb{1}_2 & 0 \end{pmatrix} \qquad \text{and} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \tag{2.1.2}$$

The $\sigma^\mu$ element is a composite of all three of the standard Pauli matrices, $\sigma^i = (\sigma^x, \sigma^y, \sigma^z)$, and the 2 dimensional identity matrix, $\sigma^0$. They are related by:

$$\sigma^\mu = (\sigma^0, \sigma^i) \qquad \& \qquad \overline{\sigma}^\mu = (\sigma^0, -\sigma^i) \tag{2.1.3}$$

The significance of these is one can split the Dirac equation into a collection of either only left or right handed particles, which makes gauge interaction easier later on. The two beginning parts to our theory are therefore;

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi, \qquad \mathcal{L}_{\text{fermion}} = i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi \tag{2.1.4}$$

With $\partial_\mu$ being the 4 derivative and $\partial^\mu$ its contravariant derivative. Here we will be using the mostly minus convention, ie:

$$\partial^\nu = \partial_\mu \eta^{\mu\nu}, \qquad \text{where} \qquad \eta^{\mu\nu} = \eta_{\mu\nu} = \eta_{\nu\mu} = \text{diag}(+1, -1, -1, -1)$$

Not included in eq.(2.1.4) are the spinor indices. Every non scalar field has one of these indices and the Pauli matrices have 2. With spinor indices eq.(2.1.4) looks as follows:

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi, \qquad \mathcal{L}_{\text{fermion}} = i\psi^\dagger_{\dot{\alpha}} (\overline{\sigma}^\mu)^{\dot{\alpha}\alpha} \partial_\mu \psi_\alpha \tag{2.1.5}$$

There is a lot to mention with this notation. Firstly, spinors are usually denoted by the first Greek letters $(\alpha, \beta, \gamma)$ while 4 dimensional tensors are denoted by the later symbols $(\mu, \nu)$. The dot above a spinor is used in a fields conjugate and denotes that it is right handed. A lower index can be raised or lowered by the following operation:

$$\xi_\alpha = \epsilon_{\alpha\beta} \xi^\beta, \qquad \xi^{\dagger\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} \xi^\dagger_{\dot{\beta}}, \tag{2.1.6}$$

and

$$\epsilon_{\alpha\beta} \epsilon^{\beta\gamma} = \epsilon^{\gamma\beta} \epsilon_{\beta\alpha} = \delta^\gamma_\beta, \qquad \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\beta}\dot{\gamma}} = \epsilon^{\dot{\gamma}\dot{\beta}} \epsilon_{\dot{\beta}\dot{\alpha}} = \delta^{\dot{\gamma}}_{\dot{\beta}} \tag{2.1.7}$$

By convention, repeated spinor indices can be contracted in the following order:

$$^\alpha{}_\alpha \qquad \text{and} \qquad {}_{\dot{\alpha}}{}^{\dot{\alpha}} \tag{2.1.8}$$

Lastly the Pauli matrices have a set index configuration, they are:

$$(\sigma^\mu)_{\alpha\dot{\alpha}} \qquad \text{and} \qquad (\overline{\sigma}^\mu)^{\dot{\alpha}\alpha} \tag{2.1.9}$$

## 2.2 Supersymmetric Invariance

[1;4;2;5]

We want to show that a massless scalar field and weyl fermion field are invariant under supersymmetric transformations. To do this we start with the scalar field and its SUSY transformation. Under the principle of supersymmetry a bosonic field should be turned into a fermionic one and vice versa, the simplest way to do this is by:

$$\delta\phi = \epsilon^\alpha \psi_\alpha, \qquad \delta\phi^* = \epsilon^{\dagger\dot{\alpha}} \psi^\dagger_{\dot{\alpha}} \tag{2.2.1}$$

Note that $\psi$ is anticommuting along with $\epsilon$, which is an infinitesimal weyl fermion object parametrizing supersymmetry transformations. It will be treated as a constant for now, ie, $\partial_\mu \epsilon^\alpha = 0$. If we apply the above equations to eq.(2.1.5) first, we obtain:

$$\begin{aligned} \delta\mathcal{L}_{\text{Scalar}} &= -\partial^\mu \delta\phi^* \partial_\mu \phi - \partial^\mu \phi^* \partial_\mu \delta\phi \\ &= -\partial^\mu \epsilon^{\dagger\dot{\alpha}} \psi^\dagger_{\dot{\alpha}} \partial_\mu \phi - \partial^\mu \phi^* \partial_\mu \epsilon^\alpha \psi_\alpha \\ &= -\epsilon^\dagger_{\dot{\alpha}} \partial^\mu \psi^{\dagger\dot{\alpha}} \partial_\mu \phi - \epsilon^\alpha \partial^\mu \psi_\alpha \partial_\mu \phi^* \end{aligned} \tag{2.2.2}$$

The change for a fermionic field is more complicated than the scalar one. Our motivation here is to show that the change in the action disappears. The simplest way to do this is this is to get $\delta\mathcal{L}$ into a total derivative. As the scalar part naturally has 2 derivatives the fermionic variation must have a second derivative in it. In order to remove the Pauli matrix in the Dirac equation the variation must also have a second Pauli matrix, thus the simplest way we can transform it and still cancel looks as follows;

$$\delta\psi_\alpha = -i(\sigma^\nu \epsilon^\dagger)_\alpha \partial_\nu \phi, \qquad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\nu)_{\dot\alpha}\partial_\nu\phi^* \tag{2.2.3}$$

There is also an $i$ infront to cancel with the $i$ in the dirac equation. Applying these to the fermionic Lagrangian we obtain:

$$\begin{aligned}
\delta\mathcal{L}_{\text{Fermion}} &= i\delta\psi^\dagger_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\partial_\mu\psi_\alpha + i\psi^\dagger_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\partial_\mu\delta\psi_\alpha \\
&= ii(\epsilon\sigma^\nu)_{\dot\alpha}\partial_\nu\phi^*(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\partial_\mu\psi_\alpha - i\psi^\dagger_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\partial_\mu i(\sigma^\nu\epsilon^\dagger)_\alpha\partial_\nu\phi \\
&= -\epsilon^\gamma(\sigma^\nu)_{\gamma\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\beta}\partial_\mu\psi_\beta\partial_\nu\phi^* + \psi^\dagger_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}(\sigma^\nu)_{\alpha\dot\beta}\epsilon^{\dagger\dot\beta}\partial_\nu\partial_\mu\phi \\
&= -\epsilon\sigma^\nu\overline{\sigma}^\mu\partial_\mu\psi\partial_\nu\phi^* + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\nu\partial_\mu\phi
\end{aligned} \tag{2.2.4}$$

The indices above are summed in such a way that we can remove them. Ideally we would want the fermionic and scalar variations to cancel up to the total derivative mentioned before. This can be done with 2 partial derivatives. However to remove the Pauli matrices we must use the Fierz identities, which are:

$$[\sigma^\nu\overline{\sigma}^\mu + \sigma^\mu\overline{\sigma}^\nu]_\alpha{}^\beta = -2\eta^{\mu\nu}\delta^\beta_\alpha \qquad \text{and} \qquad [\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu]^{\dot\alpha}{}_{\dot\beta} = -2\eta^{\nu\mu}\delta^{\dot\beta}_{\dot\alpha} \tag{2.2.5}$$

One can split $\sigma^\nu\overline{\sigma}^\mu$ (or its opposite with the bar on the other matrix) into its symmetric and antisymmetric parts and use it with the above identities to remove some of the matrices, ie;

$$\sigma^\nu\overline{\sigma}^\mu = \frac{1}{2}\left(\sigma^\nu\overline{\sigma}^\mu + \sigma^\mu\overline{\sigma}^\nu\right) = \frac{1}{2}(-2\eta^{\mu\nu}\delta^\beta_\alpha) = -\eta^{\mu\nu}\delta^\beta_\alpha \tag{2.2.6}$$

The product rule can be used to move some terms into a total derivative, ie;

$$\begin{aligned}
(f(x)g(y))' &= f(x)'g(y) + f(x)g(y)' \\
f(x)'g(y) &= (f(x)g(y))' - f(x)g(y)' \\
f(x)g(y)' &= (f(x)g(y))' - f(x)'g(y)
\end{aligned} \tag{2.2.7}$$

Eq.(2.2.5) is used from the first to the second line. Then from the second to first line eq.(2.2.6) is used on the 2 last terms to remove the Pauli matrices. Lastly from the third to last line the product rule identities in eq.(2.2.7) are used to get everything that isn't cancelled into a total derivative;

$$\begin{aligned}
\delta\mathcal{L}_{\text{Fermion}} &= -\epsilon\sigma^\nu\overline{\sigma}^\mu\partial_\mu\psi\partial_\nu\phi^* + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\nu\partial_\mu\phi \\
&= (-\partial_\mu(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\nu\phi^*) + \epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\mu\partial_\nu\phi^*) + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\nu\partial_\mu\phi \\
&= -\partial_\mu(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\nu\phi^*) - \epsilon\eta^{\mu\nu}\psi\partial_\nu\partial_\mu\phi^* - \psi^\dagger\eta^{\nu\mu}\epsilon^\dagger\partial_\mu\partial_\nu\phi \\
&= \epsilon\partial^\mu\psi\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi - \partial_\mu\left(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\nu\phi^* + \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi\right)
\end{aligned} \tag{2.2.8}$$

Now we have an equation that cancels the scalar Lagrangian, leaving only a total derivative. When applied to the variation of our action, eq.(2.1.1), we know that the volume integral of a total derivative must be 0, thus showing that the SUSY transformations leave our Lagrangian invariant. For our algebra to close the commutator of two transformations must be another symmetry of the system. From eq.(2.2.1) and eq.(2.2.3), we can see:

$$\begin{aligned}
(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\phi &= \delta_{\epsilon_2}(\epsilon_1^\alpha\psi_\alpha) - \delta_{\epsilon_1}(\epsilon_2^\alpha\psi_\alpha) \\
&= -\epsilon_1^\alpha(i(\sigma^\mu\epsilon_1^\dagger)_\alpha\partial_\mu\phi) + i\epsilon_1^\alpha(i(\sigma^\mu\epsilon_2^\dagger)_\alpha\partial_\mu\phi) \\
&= i(-\epsilon_1^\alpha(\sigma^\mu\epsilon_2^\dagger)_\alpha)\partial_\mu\phi + i(\epsilon_2^\alpha(\sigma^\mu\epsilon_1^\dagger)_\alpha)\partial_\mu\phi \\
&= i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\phi
\end{aligned} \tag{2.2.9}$$

Here we see that the commutator of 2 supersymmetric transformations is a total derivative with respect to the original scalar field. To show this is indeed a symmetry and not a fluke we must show this holds for the fermionic fields also;

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha = -i(\sigma^\mu\epsilon_1^\dagger)_\alpha\epsilon_2^\beta\partial_\mu\psi_\beta + i(\sigma^\mu\epsilon_2^\dagger)_\alpha\epsilon_1^\beta\partial_\mu\psi_\beta \tag{2.2.10}$$

For the next step we will need an identity known as the Fierz rearrangement identity, which states;

$$\chi_\alpha(\xi\eta) = -\xi_\alpha(\eta\chi) - \eta_\alpha(\chi\xi) \tag{2.2.11}$$

Where $\chi, \xi, \eta$ represent spin functions. To get eq.(2.2.10) to look like eq.(2.2.9) we first set $\chi = \sigma^\mu \epsilon^\dagger_{1/2}$ , $\xi = \epsilon_{2/1}$ and $\eta = \partial_\mu \psi$. Then we obtain;

$$\begin{aligned} -i(\sigma^\mu \epsilon_1^\dagger)_\alpha \epsilon_2^\beta \partial_\mu \psi_\beta &= -i(-\epsilon_{2\alpha}(\partial_\mu \psi^\gamma (\sigma^\mu \epsilon_1^\dagger)_\gamma - \partial_\mu \psi_\alpha (\sigma^\mu \epsilon_1^{\dagger\gamma})\epsilon_{2\gamma}) \\ &= -i(-\epsilon_{2\alpha}(\partial \psi^\gamma (\sigma^\mu)_{\gamma\dot\beta} \epsilon_1^{\dagger\dot\beta}) - \partial_\mu \psi_\alpha \epsilon^{\gamma\delta} (\sigma^\mu \epsilon_1^\dagger)_\delta \epsilon_{2\gamma}) \\ &= -i(\epsilon_{2\alpha} \epsilon^\dagger_{1\dot\beta} (\overline{\sigma}^\mu)^{\dot\beta\gamma} \partial_\mu \psi_\gamma + \partial_\mu \psi_\alpha \epsilon^{\delta\gamma} (\sigma^\mu \epsilon_1^\dagger)_\delta \epsilon_{2\gamma}) \\ &= -i(\epsilon_{2\alpha}(\epsilon_1^\dagger \overline{\sigma}^\mu \partial_\mu \psi) + \partial_\mu \psi_\alpha (\sigma^\mu \epsilon_1^\dagger)_\delta \epsilon_2^\delta) \\ &= -i(\epsilon_{2\alpha}(\epsilon_1^\dagger \overline{\sigma}^\mu \partial_\mu \psi) + (\epsilon_2 \sigma^\mu \epsilon_1^\dagger)\partial_\mu \psi_\alpha) \end{aligned} \tag{2.2.12}$$

For the first term the identity $\chi^\gamma(\sigma^\mu)_{\gamma\dot\gamma}\epsilon^{\dagger\dot\gamma} = -\epsilon^\dagger_{\dot\gamma}(\overline{\sigma}^\mu)^{\dot\gamma\gamma}\chi_\gamma$ [7] was used. The process can be repeated for the second term of eq.(2.2.10) and when added together one obtains;

$$[\delta_{\epsilon_2}, \delta_{\epsilon_1}]\psi_\alpha = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\psi_\alpha + i(\epsilon_{1\alpha}(\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) - i(\epsilon_{2\alpha}(\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi) \tag{2.2.13}$$

The first term is the variation we want, and the second and third terms vanish on shell through applying the equations of motion for our conjugate fermionic field, ie:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^\dagger)} - \frac{\partial \mathcal{L}}{\partial \psi^\dagger} = i\overline{\sigma}^\mu \partial_\mu \psi = 0 \tag{2.2.14}$$

This works on shell, i.e for non virtual particles. But for virtual particles (like propagators) we would need it to also close off shell. The simplest way to do this is to add a new field. A complex scalar field added in this way without any kinetic terms would disappear under the equations of motion, while still being able to close the algebra we need. This kind of field is often referred to as an auxiliary field, and looks as follows;

$$\mathcal{L}_{\text{auxiliary}} = F^* F \tag{2.2.15}$$

Note that from dimensional analysis $F$ must have units of $[\text{mass}]^2$ and that its equations of motion are $F = F^* = 0$, suggested. In order to close eq.(2.2.13) $F$ must transform linearly in $\psi$. The other values are derived much like the fermionic variation, it needs to have a derivative and a Pauli matrix;

$$\delta F = -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi \qquad \text{and} \qquad \delta F^* = i\partial_\mu \psi^\dagger \overline{\sigma}^\mu \epsilon \tag{2.2.16}$$

Which gives:

$$\delta\mathcal{L}_{\text{auxiliary}} = -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi F^* + i\partial_\mu \psi^\dagger \overline{\sigma}^\mu \epsilon F \tag{2.2.17}$$

These can be shown to vanish on shell through eq.(2.2.14). By adding terms to how our fermionic field transformations linear in $F$ we can get the terms we need to cancel the terms off shell up to a total derivative;

$$\delta\psi_\alpha = -i(\sigma^\nu\epsilon^\dagger)_\alpha \partial_\nu \phi + \epsilon_\alpha F \qquad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\nu)_{\dot\alpha}\partial_\nu\phi^* + \epsilon_{\dot\alpha}F^* \tag{2.2.18}$$

Which now transforms as follows along the lines of eq.(2.2.4);

$$\delta\mathcal{L}_{\text{Fermion}} = -\epsilon\sigma^\nu\overline{\sigma}^\mu\partial_\mu\psi\partial_\nu\phi^* + \psi^\dagger\overline{\sigma}^\mu\sigma^\nu\epsilon^\dagger\partial_\nu\partial_\mu\phi + i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi F^* - i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\epsilon F + \partial_\mu(i\psi^\dagger\overline{\sigma}^\mu\epsilon F) \tag{2.2.19}$$

Returning to finish eq.(2.2.13) off shell we start from the initial commutator;

$$\begin{aligned} [\delta_{\epsilon_2}, \delta_{\epsilon_1}]\psi_\alpha &= \delta_{\epsilon_2}(-i(\sigma^\mu\epsilon_1^\dagger)_\alpha\partial_\mu\phi + \epsilon_{1\alpha}F) - \delta_{\epsilon_1}(-i(\sigma^\mu\epsilon_2^\dagger)_\alpha\partial_\mu\phi + \epsilon_{2\alpha}F) \\ &= -i(\sigma^\mu\epsilon_1^\dagger)_\alpha\partial_\mu\epsilon_2\psi_\alpha + \epsilon_{1\alpha}(-i\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) + i(\sigma^\mu\epsilon_2^\dagger)_\alpha\partial_\mu\epsilon_1\psi_\alpha - \epsilon_{2\alpha}(-i\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi) \end{aligned} \tag{2.2.20}$$

Finally we see that the first and third terms are our usual transformations, eq.(2.2.12) and that the second and final term are exactly what we need to cancel the other terms off shell to a total derivative. We can thus conclude that;

$$[\delta_{\epsilon_2}, \delta_{\epsilon_1}]X = i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu X \tag{2.2.21}$$

Where $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$.

## 2.3 Supercurrents

[6;8;3;9;10]

Invariance under a symmetry implies a conserved Noether current and a conserved charge. The current in this case is called the supercurrent and is denoted by $J^\mu_\alpha$. It is an anticommuting four vector that carriers a spinor index. Using the Noether procedure;

$$^{[1;6]}\sum_X \delta X \frac{\partial \mathcal{L}}{\partial(\partial_\mu X)} - K^\mu = \epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} \tag{2.3.1}$$

With $\delta\mathcal{L} = \partial_\mu K^\mu$, therefore, $-K^\mu = \epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\nu\phi^* + \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi$. We have both $J^\mu$ and $J^{\mu\dagger}$ instead of just the one current, this is due to the current and its hermitian conjugate being conserved separately, due to having completely left or right handed fields. Where one would normally just use one current we instead have to use the superposition of both of them. This procedure is now applied to our fields $X$;

$$\begin{aligned}
\epsilon J^\mu + \epsilon^\dagger J^{\dagger\mu} &= \delta\phi\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} + \delta\phi^*\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^*)} + \delta\psi\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} - K^\mu \\
&= \epsilon\psi(-\partial^\mu\phi^*) + \epsilon^\dagger\psi^\dagger(-\partial^\mu\phi) + (-i(\sigma^\nu\epsilon^\dagger)_\alpha\partial_\nu\phi(i\psi^\dagger\overline{\sigma}^\mu)^\alpha) - K^\mu \\
&= \epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\nu\phi^* + (\sigma^\nu)_{\alpha\dot{\alpha}}\epsilon^{\dagger\dot{\alpha}}\psi^\dagger_{\dot{\beta}}(\overline{\sigma}^\mu)^{\dot{\beta}\alpha}\partial_\nu\phi \\
&= \epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\nu\phi^* + \epsilon^{\dagger\dot{\alpha}}\psi^\dagger_{\dot{\beta}}(\overline{\sigma}^\mu)^{\dot{\beta}\alpha}(\sigma^\nu)_{\alpha\dot{\alpha}}\partial_\nu\phi \\
&= \epsilon^\alpha(\sigma^\nu\overline{\sigma}^\mu\psi)_\alpha\partial_\nu\phi^* + \epsilon^{\dagger\dot{\alpha}}(\psi^\dagger\overline{\sigma}^\mu\sigma^\nu)_{\dot{\alpha}}\partial_\nu\phi
\end{aligned} \tag{2.3.2}$$

We would hope that if we added in the auxiliary fields the current would remain conserved, and this will now be shown to be true. The only contributions to the Noether current would be from the variation in our fermionic field and from the auxiliary part of the total derivative, $K^\mu$;

$$\delta\psi\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} - K^\mu = \epsilon_\alpha F(i\psi^\dagger\overline{\sigma}^\mu)^\alpha - i(\psi^\dagger\overline{\sigma}^\mu)^\alpha\epsilon_\alpha F = 0 \tag{2.3.3}$$

Now we can read off the supercurrents. Note that they are conserved separately for the current and its hermitian conjugate. The currents with their conserved charges are as follows:

$$\begin{aligned}
J^\mu_\alpha &= (\sigma^\nu\overline{\sigma}^\mu\psi)_\alpha\partial_\nu\phi^* & J^{\dagger\mu}_{\dot{\alpha}} &= (\psi^\dagger\overline{\sigma}^\mu\sigma^\nu)_{\dot{\alpha}}\partial_\nu\phi \\
\partial_\mu J^\mu_\alpha &= 0 & \partial_\mu J^{\dagger\mu}_{\dot{\alpha}} &= 0 \\
Q_\alpha &= \sqrt{2}\int d^3\vec{x}J^0_\alpha & Q^\dagger_{\dot{\alpha}} &= \sqrt{2}\int d^3\vec{x}J^{\dagger 0}_{\dot{\alpha}}
\end{aligned} \tag{2.3.4}$$

$Q_\alpha$ and $Q^\dagger_{\dot{\alpha}}$ are generators of supersymmetric algebra, with $\sqrt{2}$ a normalization factor. A Noether current acting on a quantised quantum mechanical operator generates a translation, ie;

$$[\epsilon Q + \epsilon^\dagger Q^\dagger, X] = -i\sqrt{2}\delta X \tag{2.3.5}$$

For our system to be a truly quantised system this needs to remain true. In order to show this we need the following canonical equal time relations;

$$\begin{gathered}
[\phi(\vec{x}),\phi(\vec{y})] = [\phi^*(\vec{x}),\phi^*(\vec{y})] = 0 \\
[\pi(\vec{x}),\phi^*(\vec{y})] = [\phi(\vec{x}),\pi^(\vec{y})*] = 0 \\
^{[10]}[\pi(\vec{x}),\phi(\vec{y})] = [\phi^*(\vec{x}),\pi^(\vec{y})*] = i\delta^{(3)}(\vec{x}-\vec{y}) \\
\{\psi_\alpha(\vec{x}),\psi^\dagger_{\dot{\beta}}(\vec{y})\} = (\sigma^0)_{\alpha\dot{\alpha}}\delta^{(3)}(\vec{x}-\vec{y}) \\
\{\psi_\alpha(\vec{x}),\psi_\beta(\vec{y})\} = \{\psi^\dagger_{\dot{\alpha}}(\vec{x}),\psi^\dagger_{\dot{\beta}}(\vec{y})\} = 0
\end{gathered} \tag{2.3.6}$$

Where $\pi$ is $\partial\mathcal{L}/\partial_0\phi = -\partial_0\phi^*$ and $\pi^*$ is its conjugate. We begin with the field $\phi$;

$$\begin{aligned}
[\epsilon Q + \epsilon^\dagger Q^\dagger, \phi] &= \epsilon Q\phi + \epsilon^\dagger Q^\dagger\phi - \phi\epsilon Q - \phi\epsilon^\dagger Q^\dagger \\
&= [\epsilon Q,\phi] + [\epsilon^\dagger Q^\dagger,\phi]
\end{aligned} \tag{2.3.7}$$

As $\epsilon$ and its conjugate commute with $\phi$. Note that for the fermionic field we would instead obtain anticommutation relations due to $\psi$ and $\epsilon$ anticommuting. Continuing;

$$
\begin{aligned}
[\epsilon Q,\phi]+[\epsilon^\dagger Q^\dagger,\phi] &= \sqrt{2}\int d^3\vec{x}[\epsilon(\sigma^\nu\overline{\sigma}^0\psi)\partial_\nu\phi^*,\phi]+\sqrt{2}\int d^3\vec{x}[\epsilon^\dagger(\psi^\dagger\overline{\sigma}^0\sigma^\nu)\partial_\nu\phi,\phi] \\
&= \sqrt{2}\int d^3\vec{x}\epsilon(\sigma^0\overline{\sigma}^0\psi)[\partial_0\phi^*,\phi]-\sqrt{2}\int d^3\vec{x}\epsilon(\sigma^i\overline{\sigma}^0\psi)[\nabla\phi^*,\phi] \\
&= -\sqrt{2}\int d^3\vec{x}\epsilon(\sigma^0\overline{\sigma}^0\psi)i\delta^{(3)}(\vec{x}-\vec{y}) \\
&= -i\sqrt{2}\epsilon\psi \\
&= -i\sqrt{2}\delta\phi
\end{aligned}
\tag{2.3.8}
$$

In the third line the minus comes from $\partial_0\phi^*$ being negative $\pi$. The second term has a commutator of $\nabla\phi^*$ and $\phi$ which is zero. For $\phi^*$ the second term from the first line is used instead, however one arrives at the same result. If we do the same thing for $\psi$ keeping in mind we are now dealing with anticommutators;

$$
\begin{aligned}
\{\epsilon Q,\psi\}+\{\epsilon^\dagger Q^\dagger,\psi\} &= \sqrt{2}\int d^3\vec{x}\{\epsilon(\sigma^\nu\overline{\sigma}^0\psi)\partial_\nu\phi^*,\psi\}+\sqrt{2}\int d^3\vec{x}\{\epsilon^\dagger(\psi^\dagger\overline{\sigma}^0\sigma^\nu)\partial_\nu\phi,\psi\} \\
&= -\sqrt{2}\int d^3\vec{x}\epsilon^\dagger(\overline{\sigma}^0\sigma^\nu)\partial_\nu\phi\{\psi^\dagger,\psi\} \\
&= -\sqrt{2}\int d^3\vec{x}\epsilon^\dagger(\overline{\sigma}^0\sigma^\nu)\partial_\nu\phi(\sigma^0)\delta^{(3)}(\vec{x}-\vec{y}) \\
&= -\sqrt{2}\epsilon^\dagger\sigma^\nu\partial_\nu\phi \\
&= -i\sqrt{2}\delta\psi
\end{aligned}
\tag{2.3.9}
$$

Here once everything has been moved past in the first term we find the anticommutators of $\psi^\dagger$ and $\psi$ which are zero. For the second term we get a negative from moving the $\psi^\dagger$ into the anticommutator. One can actually confirm eq.(2.2.21) in a similar manner. Below it is show just for the scalar field $\phi$;

$$
\begin{aligned}
&\left[\epsilon_2 Q+\epsilon_2^\dagger Q^\dagger,[\epsilon_1 Q+\epsilon_1^\dagger Q^\dagger,\phi]\right]-\left[\epsilon_1 Q+\epsilon_1^\dagger Q^\dagger,[\epsilon_2 Q+\epsilon_2^\dagger Q^\dagger,\phi]\right] \\
=&-i\sqrt{2}\left([\epsilon_2 Q+\epsilon_2^\dagger Q^\dagger,\delta_{\epsilon 1}\phi]-[\epsilon_1 Q+\epsilon_1^\dagger Q^\dagger,\delta_{\epsilon 2}\phi]\right) \\
=&-i\sqrt{2}\left([\epsilon_2 Q+\epsilon_2^\dagger Q^\dagger,\epsilon_1\psi]-[\epsilon_1 Q+\epsilon_1^\dagger Q^\dagger,\epsilon_2\psi]\right)
\end{aligned}
\tag{2.3.10}
$$

From eq.(2.3.7) we can extrapolate that we will be rearranging these terms into anticommutators by swapping around the anticommuting terms. Therefore if we assume that any terms not dependant on a $Q^\dagger$ disappear via eq.(2.3.6) we can shorten the calculation, this is done in the third line below;

$$
\begin{aligned}
&-i\sqrt{2}\left([\epsilon_2 Q+\epsilon_2^\dagger Q^\dagger,\epsilon_1\psi]-[\epsilon_1 Q+\epsilon_1^\dagger Q^\dagger,\epsilon_2\psi]\right) \\
=&-i\sqrt{2}\left(\epsilon_2 Q\epsilon_1\psi-\epsilon_1\psi\epsilon_2 Q-\epsilon_1 Q\epsilon_2\psi+\epsilon_2\psi\epsilon_1 Q+\epsilon_2^\dagger Q^\dagger\epsilon_1\psi-\epsilon_1\psi\epsilon_2^\dagger Q^\dagger-\epsilon_1^\dagger Q^\dagger\epsilon_2\psi+\epsilon_2\psi\epsilon_1^\dagger Q^\dagger\right) \\
=&-i\sqrt{2}\left(\epsilon_1\epsilon_2^\dagger Q^\dagger\psi+\epsilon_1\epsilon_2^\dagger\psi Q^\dagger+\epsilon_1^\dagger\epsilon_2 Q^\dagger\psi+\epsilon_1^\dagger\epsilon_2\psi Q^\dagger\right) \\
=&i\sqrt{2}\left(\epsilon_1\epsilon_2^\dagger\{Q^\dagger,\psi\}+\epsilon_1^\dagger\epsilon_2\{Q^\dagger,\psi\}\right) \\
=&2i\left(\epsilon_1\epsilon_2^\dagger\sigma^\nu\partial_\nu\phi+\epsilon_1^\dagger\epsilon_2\sigma^\nu\partial_\nu\phi\right) \\
=&2\left(\epsilon_1\sigma^\nu\epsilon_2^\dagger-\epsilon_2\sigma^\nu\epsilon_1^\dagger\right)i\partial_\nu\phi
\end{aligned}
\tag{2.3.11}
$$

One can now use the stress energy tensor to calculate the spacetime momentum operator, which will be used to show the exact supersymmetric relations. To begin with we define the stress energy tensor as:

$$
{}^{[6]}\Theta^\rho{}_\sigma=\sum_X\frac{\partial\mathcal{L}}{\partial(\partial_\rho X)}\partial_\sigma X-\mathcal{L}\delta^\rho_\sigma
\tag{2.3.12}
$$

The Hamiltonian density and the three momentum operator can be easily defined from the spacetime momentum operator, which in turn is obtained from the stress energy tensor:

$$
\Theta^0{}_\sigma=P^\sigma=(\mathcal{H},\vec{P})
\tag{2.3.13}
$$

The Hamiltonian for our system is therefore;

$$
\begin{aligned}
H = \int \mathcal{H} d^3\vec{x} &= \int \Theta^0{}_0 d^3\vec{x} \\
&= \int d^3\vec{x} \left( \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)}\partial_0\phi + \frac{\partial \mathcal{L}}{\partial(\partial_0\phi^*)}\partial_0\phi^* + \frac{\partial \mathcal{L}}{\partial(\partial_0\psi)}\partial_0\psi - \mathcal{L}\delta^0_0 \right) \\
&= \int d^3\vec{x} \left( -\dot{\phi}^*\dot{\phi} - \dot{\phi}\dot{\phi}^* + i\psi\sigma^0\dot{\psi} + \dot{\phi}^*\dot{\phi} - \nabla\phi\nabla\phi^* - i\psi^\dagger\sigma^0\dot{\psi} + i\psi^\dagger\overline{\sigma}^i\partial_i\psi \right) \\
&= \int d^3\vec{x} \left( -\pi\pi^* - \nabla\phi\nabla\phi^* - i\psi^\dagger\sigma^i\nabla\psi \right)
\end{aligned}
\tag{2.3.14}
$$

As we have a Lagrangian density the three momentum must also be integrated over the volume, although the operation is very similar;

$$
\begin{aligned}
P = \int \Theta^0{}_i d^3\vec{x} &= \int d^3\vec{x} \left( \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)}\nabla\phi + \frac{\partial \mathcal{L}}{\partial(\partial_0\phi^*)}\nabla\phi^* + \frac{\partial \mathcal{L}}{\partial(\partial_0\psi)}\nabla\psi - \mathcal{L}\delta^0_i \right) \\
&= \int d^3\vec{x} \left( \dot{\phi}^*\nabla\phi + \dot{\phi}\nabla\phi^* + i\psi^\dagger\sigma^0\nabla\psi - 0 \right) \\
&= \int d^3\vec{x} \left( \pi\nabla\phi + \pi^*\nabla\phi^* + i\psi^\dagger\sigma^0\nabla\psi \right)
\end{aligned}
\tag{2.3.15}
$$

The momentum operator generates spacetime translations according to:

$$[P^\mu, X] = i\partial^\mu X \tag{2.3.16}$$

The Jacobi identities are;

$$[[A,B],C] = [A,[B,C]] - [B,[A,C]] \tag{2.3.17}$$

We can use the above 2 identities to rearrange eq.(2.3.10) into;

$$\left[ [\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger], X] \right] = 2\left( \epsilon_1\sigma_\nu\epsilon_2^\dagger - \epsilon_2\sigma_\nu\epsilon_1^\dagger \right)[P^\nu, X] \tag{2.3.18}$$

Assuming that all terms vanish on shell from their EOM, we can remove the $X$ dependence;

$$[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger] = 2(\epsilon_1\sigma_\nu\epsilon_2^\dagger - \epsilon_2\sigma_\nu\epsilon_1^\dagger)P^\nu \tag{2.3.19}$$

To get the supersymmetric algebra relations the above equation must be expanded, first the left hand side;

$$
\begin{aligned}
[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger] &= \epsilon_2 Q_2 \epsilon_1 Q_1 + \epsilon_2 Q_2 \epsilon_1^\dagger Q_1^\dagger + \epsilon_2^\dagger Q_2^\dagger \epsilon_1 Q_1 + \epsilon_2^\dagger Q_2^\dagger \epsilon_1^\dagger Q_1^\dagger \\
&- \epsilon_1 Q_1 \epsilon_2 Q_2 - \epsilon_1 Q_1 \epsilon_2^\dagger Q_2^\dagger - \epsilon_1^\dagger Q_1^\dagger \epsilon_2 Q_2 - \epsilon_1^\dagger Q_1^\dagger \epsilon_2^\dagger Q_2^\dagger \\
&= -\epsilon_2\epsilon_1 Q_2 Q_1 + \epsilon_1\epsilon_2 Q_1 Q_2 - \epsilon_2\epsilon_1^\dagger Q_2 Q_1^\dagger + \epsilon_1\epsilon_2^\dagger Q_1 Q_2^\dagger \\
&- \epsilon_2^\dagger\epsilon_1 Q_2 \dagger Q_1 + \epsilon_1^\dagger\epsilon_2 Q_1^\dagger Q_2 - \epsilon_2^\dagger\epsilon_1^\dagger Q_2^\dagger Q_1^\dagger + \epsilon_1^\dagger\epsilon_2^\dagger Q_1^\dagger Q_2^\dagger \\
&= \epsilon_1\epsilon_2\{Q_1, Q_2\} + \epsilon_1\epsilon_2^\dagger\{Q_1, Q_2^\dagger\} + \epsilon_1^\dagger\epsilon_2\{Q_1^\dagger, Q_2\} + \epsilon_1^\dagger\epsilon_2^\dagger\{Q_1^\dagger, Q_2^\dagger\}
\end{aligned}
\tag{2.3.20}
$$

If we compare this to the right hand side of eq.(2.3.19) we can see that only way for each side to be the same is for the following anticommutation relations to exist;

$$
\begin{gathered}
\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot{\alpha}}, Q^\dagger_{\dot{\beta}}\} = 0 \\
\text{and} \\
\{Q_\alpha, Q^\dagger_{\dot{\alpha}}\} = \{Q^\dagger_{\dot{\alpha}}, Q_\alpha\} = -2\sigma^\nu_{\alpha\dot{\alpha}} P_\nu
\end{gathered}
\tag{2.3.21}
$$

## 2.4 Chiral Multiplets

[1;4;2;3]

So far we have considered a single chiral multiplet. This multiplet is a collection of a complex scalar and a chiral fermion, ie $\Phi(\phi, \psi)$ and its conjugate. Now we must extend this to multiple chiral multiples. We do this to see how they interact together, but without gauge interactions. We will see that these are restricted by keeping our Lagrangian invariant under a supersymmetric transformation. To start we simply give each component of a field an index $i$, i.e;

$$\mathcal{L}_{\text{free}} = -\partial^\mu \phi^{*i}\partial_\mu \phi_i + i\psi^{\dagger i}\overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i}F_i \tag{2.4.1}$$

We sum over repeated indices with $\phi_i$ and $\psi_i$ carrying lower indices by convention. Their complex conjugates carry the raised index. We can now redefine our supersymmetric transformations;

$$\begin{aligned} \delta\phi_i &= \epsilon\psi_i & \delta\phi^{*i} &= \epsilon^\dagger\psi^{\dagger i} \\ \delta(\psi_i)_\alpha &= -i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi_i + \epsilon_\alpha F_i & \delta(\psi^{\dagger i})_{\dot\alpha} &= i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^{*i} + \epsilon_{\dot\alpha}F^{*i} \\ \delta F_i &= -i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi_i & \delta F^{*i} &= i\partial_\mu\psi^{\dagger i}\overline{\sigma}^\mu\epsilon \end{aligned} \tag{2.4.2}$$

If we assume that the total $\mathcal{L}$ will be $\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$, where $\mathcal{L}_{\text{int}}$ is a new interactions part that we will derive below. To begin, each term must have mass dimensions $\leq 4$, this leaves the following terms;

$$\mathcal{L}_{int} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF^i + x^{ij}F_iF_j\right) - U + \ldots \tag{2.4.3}$$

Here the omitted terms are the complex conjugates. $W^{ij}$,$W^i$, $x^{ij}$ and $U$ are polynomials in $\phi_i, \phi^{*i}$.
In order for this interacting Lagrangian to be invariant $U$ must disappear, this is due to there not being any terms able to cancel it into a total derivative. I.e. U can only depend on $\phi$ and thus cannot contain a derivative term. The same argument can be made for $x^{ij}$ as the auxiliary fields it is coupled to share the same problem. Thus they must be set to 0. We are now left with;

$$\mathcal{L}_{int} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i\right) + \ldots \tag{2.4.4}$$

Note that $W^{ij}$ is symmetric under $i \leftrightarrow j$ transformations. If we now take the variation of our Lagrangian we may be able to deduce $W$. In order for everything to remain invariant all our terms must vanish on shell. First we split them up into three sections, parts that depend on 4 spinors, parts that depend on derivatives of fields and lastly parts linear in $F^i, F^{*i}$. The latter part can be ignored as the terms cancel each other out.

$$\delta\mathcal{L}_{\text{int}} = -\frac{1}{2}\delta W^{ij}\psi_i\psi_j - \frac{1}{2}W^{ij}\delta\psi_i\psi_j - \frac{1}{2}W^{ij}\psi_i\delta\psi_j + W^i\delta F_i \tag{2.4.5}$$

As stated earlier, $W$ is polynomial in $\phi_i$ and its conjugate. If we use an ansats to guess its initial value:

$$W^{ij} = A^{ijk}\phi_k + B^{ij}{}_k\phi^{*k} + M^{ij} + \ldots \tag{2.4.6}$$

Here the two generic coupling constants $A$ and $B$ are just that, but the third has been called $M$ in hindsight to its actual significance. Next we try to discern these constants' values, and insert them into the variation of $W$;

$$\frac{\partial W^{ij}}{\partial\phi_k} = A^{ijk} \quad \text{and} \quad \frac{\partial W^{ij}}{\partial\phi^{*k}} = B^{ij}{}_k \tag{2.4.7}$$

And;

$$\delta W^{ij} = A^{ijk}\delta\phi_k + B^{ij}{}_k\delta\phi^{*k} \tag{2.4.8}$$

Putting this all together we get:

$$\begin{aligned} \delta\mathcal{L}_{\text{free}}|_{\text{spinor}} = -\frac{1}{2}\delta Wij\psi_i\psi_j &= \left(-\frac{1}{2}\frac{\partial W^{ij}}{\partial\phi_k}\delta\phi_k\psi_i\psi_j - \frac{1}{2}\frac{\partial W^{ij}}{\partial\phi^{*k}}\delta\phi^{*k}\psi_i\psi_j\right) \\ &= \left(-\frac{1}{2}\frac{\partial W^{ij}}{\partial\phi_k}(\epsilon\psi_k)\psi_i\psi_j - \frac{1}{2}\frac{\partial W^{ij}}{\partial\phi^{*k}}(\epsilon^\dagger\psi^{\dagger *k})\psi_i\psi_j\right) \end{aligned} \tag{2.4.9}$$

The first term of this equation can vanish due to the symmetric nature of $ijk$. When applied with eq.(2.2.11) we see;

$$\begin{aligned} \chi_\alpha(\xi\eta) &= -\xi_\alpha(\eta\chi) - \eta_\alpha(\chi\xi) \\ (\psi_i)(\psi_j\psi_k) &= -\psi_j(\psi_k\psi_i) - \psi_k(\psi_i\psi_j) \end{aligned} \tag{2.4.10}$$

These only vanish if our Lagrangian is totally invariant. As this can only happen with $\delta\phi_i$ and not its conjugate, we must therefore discard the conjugate term. What we are left with is:

$$W^{ij} = y^{ijk}\phi_k + M^{ij} \tag{2.4.11}$$

Another substitution has been made here, letting $A = y$, as it will be referring to the Yukawa coupling. One more thing to note is that we can represent $W^{ij}$ by:

$$W^{ij} = \frac{\partial^2}{\partial\phi_i\partial\phi_j}W \tag{2.4.12}$$

Working backwards from this while accounting for permutation we can obtain an expression for $W$,

$$\begin{aligned} W = \int d\phi_i \int d\phi_j W^{ij} &= C_1 y^{ijk}\phi_i\phi_j\phi_k + C_2 M^{ij}\phi_i\phi_j \\ &= \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}M^{ij}\phi_i\phi_j \end{aligned} \tag{2.4.13}$$

The next thing to do is deal with the sections containing total derivatives, they are:

$$\begin{aligned} \delta\mathcal{L}_{\text{free}}|_{\partial_\mu} &= -\frac{1}{2}W^{ij}\delta\psi_i\psi_j - \frac{1}{2}W^{ij}\psi_i\delta\psi_j + W^i\delta F_i \\ &= -\frac{W^{ij}}{2}\left((-i\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi_i\psi_j + \psi_i(-i\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi_j + \epsilon_\alpha F_i\psi_j + \psi_i\epsilon_\alpha F_j\right) - iW^i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi_i \\ &= \frac{W^{ij}}{2}\left(\partial_\mu\phi_i\psi_j i\sigma^\mu\epsilon^\dagger + \partial_\mu\phi_j\psi_i i\sigma^\mu\epsilon^\dagger\right) + iW^i\partial_\mu\psi_i\sigma^\mu\epsilon^\dagger \\ &= iW^{ij}\partial_\mu\phi_i\psi_j i\sigma^\mu\epsilon^\dagger + iW^i\partial_\mu\psi_i\sigma^\mu\epsilon^\dagger \end{aligned} \tag{2.4.14}$$

Applying the equations of motion to the above equation we arrive at:

$$\begin{aligned} iW^{ij}\partial_\mu\phi_j\sigma^\mu\epsilon^\dagger - \partial_\mu(iW^i\sigma^\mu\epsilon^\dagger) &= 0 \\ W^{ij}\partial_\mu\phi_j &= \partial_\mu(W^i) \end{aligned} \tag{2.4.15}$$

and as;

$$W^{ij} = \frac{\partial^2}{\partial\phi_i\partial\phi_j}W \qquad \text{then maybe} \qquad W^i = \frac{\partial}{\partial\phi_i}W \tag{2.4.16}$$

If we want eq.(2.4.15) to be a total derivative then what we speculated before must be true, thus we get;

$$W^i = \frac{\partial W}{\partial\phi_i} = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k \tag{2.4.17}$$

And so we have derived what will be referred to as the superpotential, for reasons that are about to become evident. At the beginning of this section it was mentioned that the terms linear in $F$ can be omitted, and a few have been omitted.
One can now include a linear term into the Lagrangian of the form $Li\phi_i$ as it will remain invariant under any transformation. However this is not helpful unless we are trying to show spontaneous symmetry breaking, and as such will be omitted for the current discussion.
Using the equations of motion for our auxiliary fields one can replace them by the following substitutions;

$$F_i = -W_i^* \qquad \text{and} \qquad F^{*i} = -W^i \tag{2.4.18}$$

Our $\mathcal{L}_{\text{int}}$ is therefore;

$$\begin{aligned} \mathcal{L}_{\text{int}} &= -\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i - \frac{1}{2}W_{ij}^*\psi^{\dagger i}\psi^{\dagger j} \\ &= -\frac{1}{2}W^{ij}\psi_i\psi_j - \frac{1}{2}W_{ij}^*\psi^{\dagger i}\psi^{\dagger j} - W^iW_i^* \end{aligned} \tag{2.4.19}$$

We call the last term our scalar superpotential $V(\phi, \phi^*)$. Adding everything together we now have a full Lagrangian for an interacting chiral theory, and it looks as follows;

$$\begin{aligned} \mathcal{L} = &-\partial^\mu\phi^{*i}\partial_\mu\phi_i + i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i - \frac{1}{2}M^{ij}\psi_i\psi_j - \frac{1}{2}M_{ij}^*\psi^{\dagger i}\psi^{\dagger j} - \frac{1}{2}y^{ijk}\phi_i\psi_j\psi_k - \frac{1}{2}y_{ijk}^*\phi^{*i}\psi^{\dagger j}\psi^{\dagger k} \\ &- M_{ik}^*M^{kj}\phi^{*i}\phi_j - \frac{1}{2}M^{in}y_{jkn}^*\phi_i\phi^{*j}\phi^{*k} - \frac{1}{2}M_{in}^*y^{jkn}\phi^*i\phi_j\phi_k - \frac{1}{4}y^{ijn}y_{kln}^*\phi_i\phi_j\phi^{*k}\phi^{*l} \end{aligned} \tag{2.4.20}$$

Now that there is a full Lagrangian with a mass matrix, $M$, we can use the equations of motion for our numerous fields. We are only interested with terms linear in the field being varied;

$$\begin{aligned}
&\text{For } \phi : \\
&\qquad M^*_{ik}\phi^{*i} - \partial_\mu\partial^\mu\phi^{*i} + \ldots = 0 \\
&\text{For } \phi^* : \\
&\qquad M^*_{ik}\phi_j - \partial_\mu\partial^\mu\phi_i + \ldots = 0 \\
&\text{For } \psi : \\
&\qquad M^{ij}\psi_j - \partial_\mu\psi^\dagger i\overline{\sigma}^\mu + \ldots = 0 \\
&\text{For } \phi^* : \\
&\qquad M^*_{ij}\psi^{\dagger j} - i\overline{\sigma}^\mu\partial_\mu\psi_i + \ldots = 0
\end{aligned} \tag{2.4.21}$$

The first 2 equations are already linearised, however for the last two we need to decouple them from their conjugates. To begin with we take the last equation and multiply from the left by $\sigma^\nu\partial_\nu$. We can also use eq.(2.2.3) afterwards;

$$\begin{aligned}
M^*_{ij}\psi^{\dagger j} &= i\overline{\sigma}^\mu\partial_\mu\psi_i \\
\sigma^\nu\partial_\nu M^*_{ij}\psi^{\dagger j} &= \sigma^\nu\partial_\nu i\overline{\sigma}^\mu\partial_\mu\psi_i \\
M^*_{ij}\partial_\nu\psi^{\dagger j}\sigma^\nu &= i\sigma^\nu\overline{\sigma}^\mu\partial_\nu\partial_\mu\psi_i \\
M^*_{ij}(-iM^{ij}\psi_k) &= i\eta^{\mu\nu}\partial_\nu\partial_\mu\psi_i \\
-M^*_{ik}M^{kj}\psi_j &= \partial^\mu\partial_\mu\psi_j + \ldots
\end{aligned} \tag{2.4.22}$$

If this is repeated for the other term one finds;

$$-\psi^{\dagger i}M^*_{ik}M^{kj} = \partial^\mu\partial_\mu\psi^{\dagger j} + \ldots \tag{2.4.23}$$

We will show the significance of these equations and mass matrices later, when we are finding all the particle masses. For now we can see rearranging this into the Klein Gordon equation;

$$(\Box + M^*_{ik}M^{kj})\psi_j = 0 \tag{2.4.24}$$

Here we can see that the mass term is given by $M^*_{ik}M^{kj} = |M|^{2j}_i$.

## 2.5 Gauge Interactions

[1;4;5;2;9]

Next the discussion will move onto the gauge interactions, the derivations of which will be left much more brief. Most of the interactions can be deduced or shown to be true via the steps outlined before. To begin with we add a normal gauge iteration in the form of a yang mills field strength tensor and a kinetic term with weyl fermion gauginos, $\lambda^a$. The $a$ here runs over the adjoint representation of the group, ie. 1 for U(1) and 2 for SU(2). Lastly due to off shell degrees of freedom we need to add a bosonic auxiliary field, denoted by $D^a$ which has a mass dimension of 2, like our other auxiliary field. Therefore our gauge Lagrangian density should look as follows;

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\overline{\sigma}^\mu\mathcal{D}_\mu + \frac{1}{2}D^aD^a \tag{2.5.1}$$

Where the yang mills field strength tensor is given by:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \tag{2.5.2}$$

And the covariant derivative is represented by:

$$\mathcal{D}_\mu X = \partial_\mu X + gf^{abc}A^a_\mu X \tag{2.5.3}$$

There are a number of new terms introduced here, firstly note that $D^a$ transforms as an adjoint of the gauge group and that $(D^a)^* = D^a$, hence the final term. Here $f^{abc}$ is a totally antisymmetric structure constant of the gauge group and $g$ is the gauge coupling constant. The massless gauge boson field is represented by $A^a_\mu$. In the case of an abelian group we set $f^{abc}$ to 0.

Each of our fields must transform supersymmetrically just like our fermionic and scalar fields. Using the assumptions that each field should be linear in $\epsilon, \epsilon^\dagger$ and have mass dimensions of $-1/2$ we can make the following guesses for an abelian group;

$$\begin{aligned}
\delta A^a_\mu &= -\frac{1}{\sqrt{2}}\left(\epsilon^\dagger\overline{\sigma}_\mu + \lambda^\dagger\overline{\sigma}_\mu\epsilon\right) \\
\delta\lambda^a_\alpha &= \frac{i}{2\sqrt{2}}(\sigma^\mu\overline{\sigma}^\nu\epsilon)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha D^a \\
\delta D^a &= \frac{i}{\sqrt{2}}\left(-\epsilon^\dagger\overline{\sigma}^\mu\mathcal{D}_\mu\lambda^a + \mathcal{D}_\mu\lambda^{\dagger a}\overline{\sigma}^\mu\epsilon\right)
\end{aligned} \tag{2.5.4}$$

The factors of $\sqrt{2}$ are here to keep the action invariant over integration. The gauge fields follow the fermionic and scalar and obey eq.(2.2.21), although with a covariant derivative instead of a normal partial one.

## 2.6 Full gauge theory

[1;4;5;2;9]

Now that we know the gauge interactions we can promote all the partial derivatives to covariant ones and see what interactions come out. We now must introduce $T$ and can use the identity $[T^a, T^b] = if^{abc}T^c$ to show that $T$ is the generator of our symmetry. In a U(1) gauge $T = 1$ and for SU(2) $T$ is equal to half the Pauli matrices. We can deduce that the covariant derivatives of our remain fields are:

$$\begin{aligned}
\mathcal{D}_\mu\phi_i &= \partial\phi_i - igA^a_\mu(T^a\phi)_i \\
\mathcal{D}_\mu\phi^{*i} &= \partial\phi^{*i} + igA^a_\mu(\phi^*T^a)^i \\
\mathcal{D}_\mu\psi_i &= \partial\psi_i - igA^a_\mu(T^a\psi)_i
\end{aligned} \tag{2.6.1}$$

Applying these derivatives gives us almost all the necessary interactions between the gauge field and our bosonic and fermionic fields we would want. The only missing ones are between $\phi_i$ and $\psi_i$ and the gaugino and bosonic auxiliary field. These will have to be added manually and are, with normalization factors already added;

$$-\sqrt{2}g(\phi^*T^a\psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a\phi) + g(\phi^*T^a\phi)D^a \tag{2.6.2}$$

It is also necessary at this point to add another term to the variation of the auxiliary field;

$$\delta F_i = -i\epsilon^\dagger\overline{\sigma}^\mu\mathcal{D}_\mu\psi_i + \sqrt{2}g(T^a\phi)_i\epsilon^\dagger\lambda^{\dagger a} \tag{2.6.3}$$

We have now all the elements to write down the full interacting Lagrangian, but first we can remove the auxiliary bosonic field with its equations of motion, with $D^a = -g(\phi^*T^a\phi)$ our Lagrangian is:

$$\begin{aligned}
\mathcal{L} = &-\mathcal{D}^\mu\phi^{*i}\mathcal{D}_\mu\phi_i + i\psi^{\dagger i}\overline{\sigma}^\mu\mathcal{D}_\mu\psi_i - \frac{1}{2}M^{ij}\psi_i\psi_j - \frac{1}{2}M^*_{ij}\psi^{\dagger i}\psi^{\dagger j} - \frac{1}{2}y^{ijk}\phi_i\psi_j\psi_k - \frac{1}{2}y^*_{ijk}\phi^{*i}\psi^{\dagger j}\psi^{\dagger k} \\
&- M^*_{ik}M^{kj}\phi^{*i}\phi_j - \frac{1}{2}M^{in}y^*_{jkn}\phi_i\phi^{*j}\phi^{*k} - \frac{1}{2}M^*_{in}y^{jkn}\phi^*i\phi_j\phi_k - \frac{1}{4}y^{ijn}y^*_{kln}\phi_i\phi_j\phi^{*k}\phi^{*l} - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} \\
&+ i\lambda^{\dagger a}\overline{\sigma}^\mu\mathcal{D}_\mu\lambda^a + \frac{1}{2}g^2(\phi^*T^a\phi)(\phi^*T^a\phi) - \sqrt{2}g(\phi^*T^a\psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a\phi) - g^2(\phi^*T^a\phi)(\phi^*T^a\phi)
\end{aligned} \tag{2.6.4}$$

# 3 Applying to a U(1) gauge theory

[11;12;13;14]

## 3.1 Creating the Lagrangian

[11;12] Now that we have our unbroken Lagrangian we can apply it to a simple example. Imagine 2 interacting chiral fields, with positive and negative charge, ie:

$$\Phi_+(\phi_+, \psi_+) \qquad \text{and} \qquad \Phi_-(\phi_-, \psi_-) \tag{3.1.1}$$

For a U(1) theory we can set the structure constant, $f^{abc}$, to 0. It is useful now to define a superpotential with gauge terms, i.e;

$$V(\phi, \phi^*) = W^*_iW^i + \frac{1}{2}g^2(\phi^*T^a\phi)^2 \tag{3.1.2}$$

By using the simplest gauge transformation we can show that the charges of our U(1) are ± 1;

$$\begin{aligned} \phi &\to \phi' = U(\lambda)\phi \\ \psi &\to \psi' = U(\lambda)\psi \\ \phi^* &\to \phi^{*\prime} = \phi^* U^\dagger(\lambda) \end{aligned} \tag{3.1.3}$$

Where;

$$U = e^{-i\lambda Q} \tag{3.1.4}$$

Here Q is the charge and $\lambda$ is a gauge parameter. When applied to our superpotential we find these terms;

$$\begin{aligned} M^*_{ik}M^{kj}\phi^{*i}U^\dagger(\lambda)U(\lambda)\phi_k =& M^*_{+k}M^{k+}\phi^{*+}e^{i\lambda Q}e^{-i\lambda Q}\phi_+ + M^*_{+k}M^{k-}\phi^{*+}e^{i\lambda Q}e^{-i\lambda Q}\phi_- \\ &+ M^*_{-k}M^{k+}\phi^{*-}e^{i\lambda Q}e^{-i\lambda Q}\phi_+ + M^*_{-k}M^{k-}\phi^{*-}e^{i\lambda Q}e^{-i\lambda Q}\phi_- \end{aligned} \tag{3.1.5}$$

The simplest way to get these terms to be invariant is to make the charges $Q = \pm q$. Note this is unlike a U(2) theory like the weak force, that can have charges of order 1/3. This means that there can be no Yukawa coupling term due to each term having an innate positive or negative charge. This also removes any mass term from the superpotential that has 2 of the same charge. ie;

$$\frac{1}{2}M^{++}\psi_+\phi_+ \qquad M^{++}\text{must be 0 to account for the invariance} \tag{3.1.6}$$

Through the same mechanism we set all the Yukawa couplings to 0. Lastly we can also let $a = 1$ and $T = 1$ for the gauge counting and group generator and are left with the following Lagrangian;

$$\begin{aligned} \mathcal{L} =& -\partial^\mu\phi^{*+}\partial_\mu\phi_+ - \partial^\mu\phi^{*-}\partial_\mu\phi_- + ig\partial^\mu\phi^{*+}A_\mu\phi_+ + ig\partial^\mu\phi^{*-}A_\mu\phi_- \\ &- ig\partial_\mu\phi_+A_\mu\phi^{*+} - ig\partial_\mu\phi_-A_\mu\phi^{*2} - g^2A_\mu\phi_+A_\mu\phi^{*+} - g^2A_\mu\phi_-A_\mu\phi^{*-} \\ &+ i\psi^{\dagger 1}\overline{\sigma}^\mu\partial_\mu\psi_+ + i\psi^{\dagger 2}\overline{\sigma}^\mu\partial_\mu\psi_2 + g\psi^{\dagger +}\overline{\sigma}^\mu A_\mu\psi_+ + g\psi^{\dagger 2}\overline{\sigma}^\mu A_\mu\psi_2 \\ &- \frac{1}{2}M^{+-}\psi_+\psi_- - \frac{1}{2}M^*_{12}\psi^{\dagger +}\psi^{\dagger -} - \frac{1}{2}M^{21}\psi_-\psi_+ - \frac{1}{2}M^*_{21}\psi^{\dagger -}\psi^{\dagger +} - M^*_{1k}M^{k1}\phi^{*+}\phi_+ - M^*_{2k}M^{k2}\phi^{*-}\phi_- \\ &- \frac{1}{2}g^2\phi^{*+}\phi_+\phi^{*+}\phi_+ - \sqrt{2}g\phi^{*1}T\psi_1\lambda - \sqrt{2}g\lambda^\dagger\psi^{\dagger +}\phi_+ \\ &- \frac{1}{2}g^2\phi^{*-}\phi_-\phi^{*-}\phi_- - \sqrt{2}g\phi^{*-}\psi_-\lambda - \sqrt{2}g\lambda^\dagger\psi^{\dagger -}\phi_- \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\lambda^\dagger\overline{\sigma}^\mu\partial_\mu\lambda \end{aligned} \tag{3.1.7}$$

So we have the full Lagrangian with the assumption that the a fields conjugate is its antiparticles, as its charge is effectively reversed. Next we can move on to identifying the particles and interactions. Note that any indices left (like $k$ in line 4) are also summed over 1 and 2. As $M_{++} = 0$ this means that $M^*_{+-}M_{-+} = |M|^2_+$.

## 3.2 Interactions in Diagrammatic Form

[6;1;4]

In the order they appear in the Lagrangian we have the scalar terms, the fermionic terms, the mass coupling terms and the gauge interactions. From this we can draw a number of Feynman diagrams for the interactions. This can also highlight the types of particles we can see in such a theory. Firstly we will be looking at the mixing of the gauge boson and our scalar particles (fig 1). The first two terms of the Lagrangian are omitted as there are no vertices.

The next two terms deal with quartic interactions, with the first 2 terms of lines 5 and 6 of the Lagrangian and the last two terms of line 2. The field strength tensors in the last line would have been included with these diagrams however they have no vertices and so they give rise to no diagrams. The quartic interactions are important in the superpotential and will be integral when breaking the symmetry. The first 2 terms (fig 2(a,b)) are used directly in the superpotential and give us part of the mass of the scalar boson. It is from the mixing of the gauge and scalar boson in the next 2 terms (fig 2(c,d)) that we obtain the gauge mass after symmetry breaking has occurred.

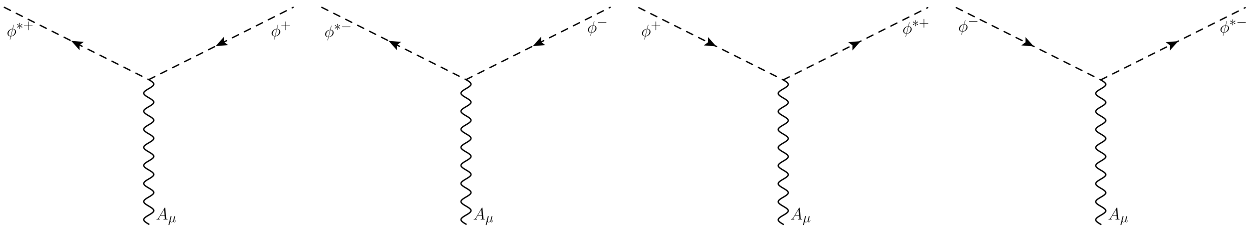


Figure 1: The terms here are in order as they appear in the Lagrangian. Each of the terms has a gauge field coupling to a scalar and its conjugate, although the positions of the bosons are reversed for the latter two.

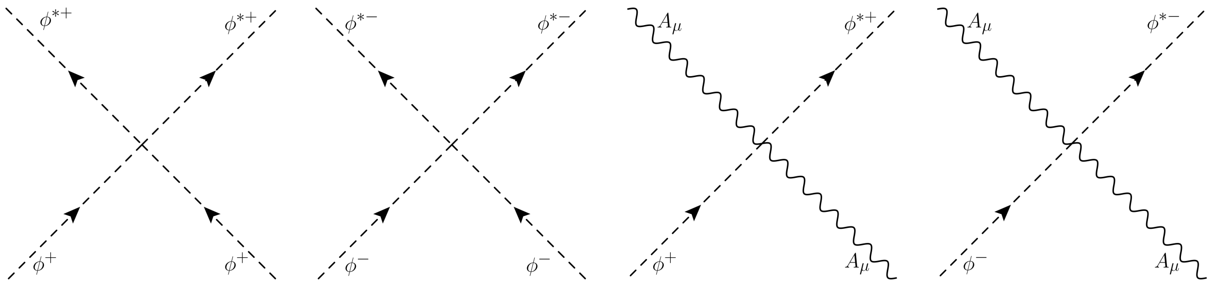


Figure 2: The first 2 terms couple together 4 scalar fields and the latter two couple 2 gauge fields with a scalar field and its conjugate.

Theses next feynman diagrams deal with the remaining cubic interactions in our Lagrangian. The first 2 terms come from the last 2 terms in line 3 while the remaining terms come from the last 2 terms of lines 5 and 6. The terms with the gaugino interactions (fig 3(c-f)) become important in the last section in finding the new broken masses for all the fermionic fields.

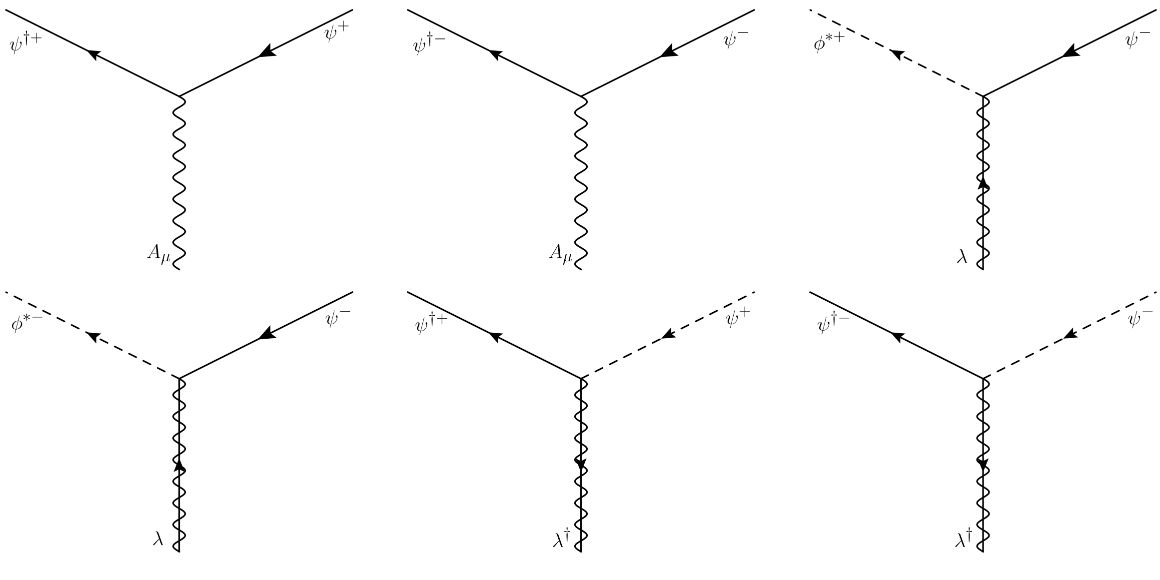


Figure 3: The first 2 terms show the interaction of a fermionic field and its conjugate with the bosonic gauge field. The next 2 terms show the interactions of the positive and negative fermionic and conjugate scalar fields with each other and the gaugino. The last 2 terms show the same interaction but conjugated.

These last 4 terms come from the non-gauge superpotential, represented in line 4. The third and fourth terms are mirror images of the first and second terms of this line. These terms are our traditional mass terms and give the scalar boson and weyl fermions mass before breaking the symmetry. Note that the fermionic mass terms are coupled together and must be decoupled before a mass can be found, see eq.(2.4.22).

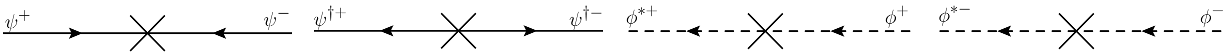


Figure 4: Here we see the fermion fermion interaction of our 2 fields and the interaction of the same scalar field with its conjugate.

## 3.3 The Equations of Motion and Particle Masses

[14;6]

After reviewing the particle content of our U(1) theory we can use the equations of motion to find each of the particles masses, as seen through eq.(2.4.21). To begin with each of the equations of motion is as follows:

For $\phi_\pm$;

$$-ig\partial^\mu\phi_i^* A_\mu - \sqrt{2}g\lambda^\dagger\psi_i^\dagger - g^2\phi_i^{2*}\phi_i - M^*_{ik}M^{ki}\phi_i^* - \Box\phi_i^* + \partial_\mu(igA_\mu\phi_i^*) = 0 \tag{3.3.1}$$

For $\phi^*_\pm$;

$$-ig\partial_\mu\phi_i A_\mu - \sqrt{2}g\psi_i\lambda - g^2\phi_i^*\phi_i^2 - M^*_{ik}M^{ki}\phi_i - \Box\phi_i + \partial_\mu(igA_\mu\phi_i) = 0 \tag{3.3.2}$$

For $\psi_\pm$;

$$-\sqrt{2}g\phi_i^*\lambda - M^{ij}\psi_j - i\partial_\mu\psi_i^\dagger\overline{\sigma}^\mu + g\psi_i^\dagger\overline{\sigma}^\mu A_\mu = 0 \tag{3.3.3}$$

For $\psi^\dagger_\pm$;

$$-\sqrt{2}g\lambda^\dagger\phi_i - M^*_{ij}\psi^{\dagger j} + i\overline{\sigma}^\mu\partial_\mu\psi_i + g\psi^{\dagger i}\overline{\sigma}^\mu A_\mu = 0 \tag{3.3.4}$$

For $\lambda$;

$$-\sqrt{2}g(\phi^{*1}\psi_+) - i\partial_\mu\lambda^\dagger\sigma^\mu + \sqrt{2}g(\phi^{*2}\psi_=) = 0 \tag{3.3.5}$$

For $\lambda^\dagger$;

$$-\sqrt{2}g(\psi_+^\dagger\phi_+) + i\sigma^\mu\partial_\mu\lambda - \sqrt{2}g(\psi^{\dagger-}\phi_-) = 0 \tag{3.3.6}$$

For $A_\mu$;

$$\begin{aligned} &ig\partial^\mu\phi^{*+}\phi_+ + ig\partial^\mu\phi^{*-}\phi_- - ig\partial_\mu\phi_+\phi^{*+} - ig\partial_\mu\phi_-\phi^{*-} - g^2\phi_+A_\mu\phi^{*+} - g^2\phi_-A_\mu\phi^{*-} \\ &- g^2A_\mu\phi_+\phi^{*+} - g^2A_\mu\phi_-\phi^{*-} + g\psi^{\dagger+}\overline{\sigma}^\mu\psi_+ + g\psi^{\dagger-}\overline{\sigma}^\mu\psi_- + \partial_\mu F^{\mu\nu} = 0 \end{aligned} \tag{3.3.7}$$

In order to calculate the particle masses, we must try to get the equation to obey the Klein Gordon equation as in eq.(2.4.23). We can ignore any gauge couplings to do this, and any non linear terms. The scalar fields and their conjugates are already in KG form, and so can be read off;

$$\begin{aligned} (\Box + |M|^2_+)\phi^*_+ = 0 \qquad &\text{and} \qquad (\Box + |M|^2_+)\phi_+ = 0 \\ (\Box + |M|^2_-)\phi^*_- = 0 \qquad &\text{and} \qquad (\Box + |M|^2_-)\phi_- = 0 \end{aligned} \tag{3.3.8}$$

For the fermionic fields we will have to multiply both sides $i\sigma^\nu\partial_\nu$ and use eq.(2.2.3) but we obtain similar equations, the process is the same as in eq.(2.4.21);

$$\begin{aligned} (\Box + M^{+-}M^*_{-+})\psi_+ = 0 \qquad &\text{and} \qquad (\Box + M^{-+}M^*_{+-})\psi_+^\dagger = 0 \\ (\Box + M^{-+}M^*_{+-})\psi_- = 0 \qquad &\text{and} \qquad (\Box + M^{+-}M^*_{-+})\psi_-^\dagger = 0 \end{aligned} \tag{3.3.9}$$

For both the gauge fields we find the particles mass parameter to be zero. This is what one would expect of the bosonic field, although it is not necessarily true for the gaugino. For the gaugino we use the same first step of multiplying by $i\sigma^\nu\partial_\nu$ followed by applying the Fierz identity. However we find its mass parameter is zero so we obtain:

$$\Box\lambda = 0 \qquad \text{and} \qquad \Box\lambda^\dagger = 0 \tag{3.3.10}$$

Lastly after removing gauge couplings we are left with only $\partial_\mu F^{\mu\nu} = 0$ for the gauge boson field. This is simple enough on its own, however applying the Lorentz gauge we can make it look like the gaugino terms and obtain the classical wave equation:

$$\Box A_\mu = 0 \tag{3.3.11}$$

# 4 The Vacuum Expectation Value (VEV)

[11;12;15;16;17;18]

In a previous section we introduced the scalar superpotential as the sum of all the independent scalar field terms, ie.;

$$V(\phi, \phi^*) = W_i^* W^i + \frac{1}{2} g^2 (\phi^* T^a \phi)^2 \tag{4.0.1}$$

As mentioned before the fact that we are working in U(1) removes the generators of the group, $T$. However it also allows us to effectively split this system into 2 potentials, $V_+$ and $V_-$. They are;

$$\begin{aligned} V_+ &= |M|^2_+ |\phi|^2_+ + \frac{1}{2} g^2 |\phi|^4_+ \\ V_- &= |M|^2_- |\phi|^2_- + \frac{1}{2} g^2 |\phi|^4_- \end{aligned} \tag{4.0.2}$$

Where $|M|^2_+ = M^*_{+k} M^{k+}$. As was discussed in the introduction, supersymmetry must be a spontaneously broken symmetry. In order for this to occur our Lagrangian must be invariant under transformation, which it is, and have a non-zero VEV. In order to find the VEV we must first find the minima of our system, they are given by;

$$\begin{aligned} \frac{\partial V_+}{\partial |\phi|_+} &= |M|^2_+ |\phi|_+ + 2g^2 |\phi|^3_+ = 0 \\ \frac{\partial V_+}{\partial |\phi|_-} &= |M|^2_- |\phi|_+ + 2g^2 |\phi|^3_- = 0 \end{aligned} \tag{4.0.3}$$

In a Lagrangian with a mass that could be positive or negative there would be 2 results, firstly, for $|M|^2 > 0$, $|\phi|^2_+ = |\phi|^2_- = 0$. However for $M^2 < 0$ we would get;

$$|\phi|^2_+ = \frac{-|M|^2_+}{g^2} \qquad \text{and} \qquad |\phi|^2_- = \frac{-|M|^2_-}{g^2} \tag{4.0.4}$$

This is not a valid solution due to the mass not being able to be a negative number. Furthermore if we were to take the second derivative and compute the mass matrix, as we will later, we would find that the mass would be imaginary, which is also something that $|M|^2$ is not able to be. Thus our Lagrangian has no VEV, and so remains an unbroken symmetry. To remedy this we must add soft breaking terms to our Lagrangian. These terms are also invariant under transformation but will give use a non-zero VEV.

## 4.1 Spontaneous Symmetry Breaking

[11;12;15;16;17;18;1]

The terms we will be adding to our Lagrangian;

$$V_{\text{soft}} = M^2_+ |\phi|^2_+ + M^2_- |\phi|^2_- + M^2_{+-}(\phi_+ \phi_- + \phi^{*+} \phi^{*-}) \tag{4.1.1}$$

The last term in this removes our ability to split up the potential into small parts. However we will still have 2 VEV's of $v_+$ and $v_-$. Thus the new potential is;

$$V = (|M|^2_+ + M^2_+)|\phi|^2_+ + (|M|^2_+ + M^2_-)|\phi|^2_+ + \frac{1}{2} g^2 (|\phi|^4_+ + |\phi|^4_+) + M^2_{+-}(\phi_+ \phi_- + \phi^{*+} \phi^{*-}) \tag{4.1.2}$$

We now find the minima again through the same operation as before;

$$\begin{aligned} \frac{\partial V_+}{\partial |\phi|_+} &= (|M|^2_+ + M^2_+)|\phi|_+ + 2g^2 |\phi|^3_+ = 0 \\ \frac{\partial V_-}{\partial |\phi|_-} &= (|M|^2_+ + M^2_+)|\phi|_+ + 2g^2 |\phi|^3_+ = 0 \end{aligned} \tag{4.1.3}$$

and so the VEV's are;

$$v_+^2 = |\phi|_+^2 = \frac{-(|M|_+^2 + M_+^2)}{g^2}$$
$$v_-^2 = |\phi|_-^2 = \frac{-(|M|_-^2 + M_-^2)}{g^2} \tag{4.1.4}$$

We now have a spontaneously broken symmetry with a valid VEV. What follows next will require some diagrams;

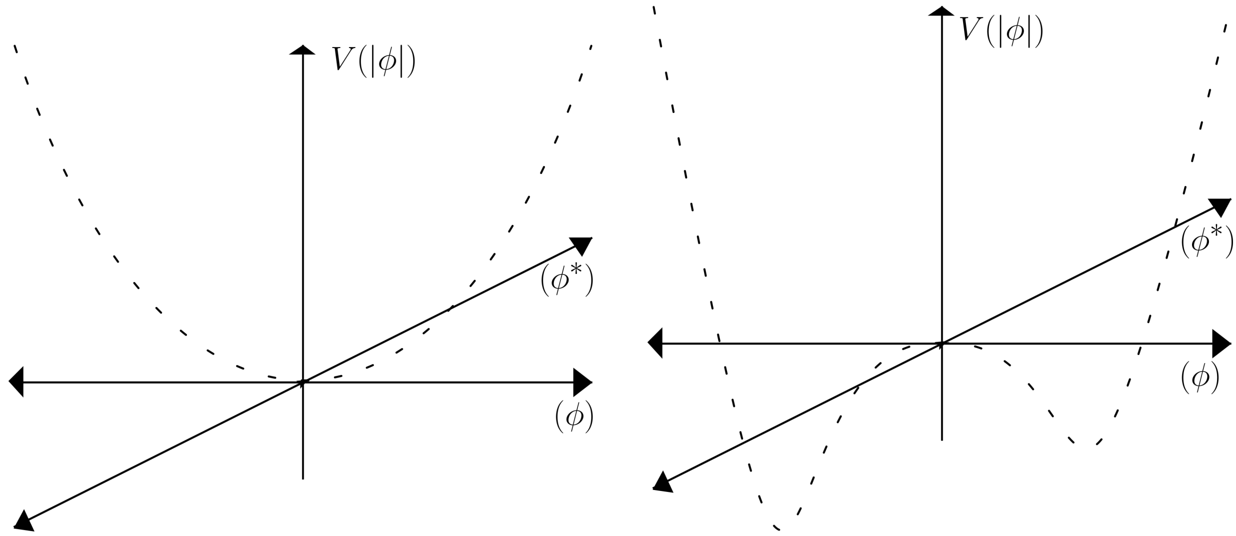


Figure 5: On the left is the unbroken case seen before adding a soft term, and the second is the softly broken symmetry.

Here we can see that the broken VEV forms a circle over the real and imaginary planes, such that $|\phi|^2 = v^2$. And if;

$$\phi = (\phi_1 + i\phi_2) \qquad \text{then} \qquad \phi^* = (\phi_1 - i\phi_2) \tag{4.1.5}$$

It follows that;

$$|\phi|^2 = \phi\phi^* = \phi_1^2 + \phi_2^2 \tag{4.1.6}$$

It is helpful to represent this on a circle (fig6).
Next we will redefine our field so that it starts at the VEV instead of 0. So our new shifted coordinates are;

$$\begin{aligned} \phi_1(x) &= v_+ + \chi(x) \\ \phi_2(x) &= \zeta(x) \end{aligned} \tag{4.1.7}$$

Here we have only shifted the VEV in the real $\phi$ direction. With these coordinates we can apply these to eq.(4.1.6);

$$\begin{array}{llll} & \phi_+ = v_+ + \chi_+ + i\zeta_+ & \text{and} & \phi_- = v_- + \chi_- + i\zeta_- \\ \therefore & |\phi|_+^2 = (v_+ + \chi_+)^2 + \zeta_+^2 & \text{and} & |\phi|_-^2 = (v_- + \chi_-)^2 + \zeta_-^2 \end{array} \tag{4.1.8}$$

We can now reintroduce these into our Lagrangian and re-express $\phi$ as 2 real fields, $\chi$ and $\zeta$.

# 5 Particle Masses

[11;12;15;16;17;18;1]

With our redefined fields we now have an intrinsic potential added, $v_+$. What we will see is that this can be converted into a mass term, and will add the extra masses we need to our fields. We begin, however, with the scalar field;

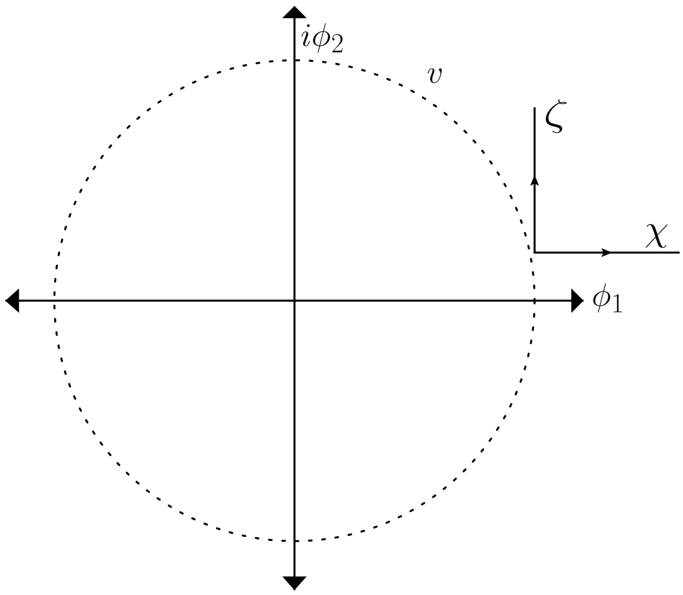


Figure 6: A representation of $|\phi|^2$ on a circle with axis of real and imaginary $\phi$, and the distance shifted along the real axis.

## 5.1 Scalar Masses

[11;12;1]

There a number of ways to find the masses in our fields. In the previous sections we used the EOM and rearranged to find the Klein Gordon equation, however we can also obtain the mass of a particle from its potential. If we define the minimum of the potential we have found as;

$$\left.\frac{\partial V}{\partial \phi^i}\right|_{\phi^i(x)=\phi_0^i(x)} = 0 \tag{5.1.1}$$

Where $\phi^a$ is a constant field that minimises $V$. Now we can expand around $V$ at its minimum;

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi-\phi_0)^i(\phi-\phi_0)^j \left.\left(\frac{\partial^2 V}{\partial\phi^i\partial\phi^j}\right)\right|_{\phi_0} + \quad \dots \tag{5.1.2}$$

The term infront of the quadratic term is the mass of our system, i.e;

$$m_{ij}^2 = \left.\frac{\partial^2 V}{\partial\phi^i\partial\phi^j}\right|_{\phi_0} \tag{5.1.3}$$

We can now apply this method to our shifted potential, with hindsight we know that only the quadratics terms are of interest. We begin by subbing eq.(4.1.8) into eq.(4.1.2);

$$\begin{aligned} V_{tot} =&(M|_+^2 + M_+^2)((v_+ + \chi_+)^2 + \zeta_+^2) + (|M|_-^2 + M_-^2)((v_- + \chi_-)^2 + \zeta_-^2) \\ &+ \frac{1}{2}g^2[((v_+ + \chi_+)^2 + \zeta_+^2)^2 + ((v_- + \chi_-)^2 + \zeta_-^2)^2] \\ &+ M_{+-}^2[(v_+ + \chi_+ + i\zeta_+)(v_- + \chi_- + i\zeta_-) - +(v_+ + \chi_+ - i\zeta_+)(v_- + \chi_- - i\zeta_-)] \end{aligned} \tag{5.1.4}$$

Expanding the brackets we obtain;

$$\begin{aligned} V_{tot} =&(|M|_+^2 + M_+^2)\chi_+^2 + (|M|_+^2 + M_+^2)\zeta_+^2 + (|M|_-^2 + M_-^2)\chi_-^2 + (|M|_-^2 + M_-^2)\zeta_-^2 \\ &+ \frac{1}{2}g^2[2v_+^2\zeta_+^2 + 2v_+^2\chi_+^2 + 4v_+^2\chi_+^2] + \frac{1}{2}g^2[2v_-^2\zeta_-^2 + 2v_-^2\chi_-^2 + 4v_-^2\chi_-^2] + \mathcal{O}(\chi,\zeta) \end{aligned} \tag{5.1.5}$$

Here there are only quadratic terms included, which are the ones relevant to us. Using eq.(5.1.3) first for $\chi$;

$$\begin{aligned} \frac{\partial^2 V}{\partial\chi_+^2} &= 2(|M|_+^2 + M_+^2) + 3g^2v_+^2 = m_{\chi+}^2 \\ \frac{\partial^2 V}{\partial\chi_-^2} &= 2(|M|_-^2 + M_-^2) + 3g^2v_-^2 = m_{\chi-}^2 \end{aligned} \tag{5.1.6}$$

However we know from eq.(4.1.4) that $g^2v^2 = -(|M|^2 + M^2)$, therefore;

$$\begin{aligned} m^2_{\chi_+} &= g^2 v_+^2 \\ m^2_{\chi_-} &= g^2 v_-^2 \end{aligned} \tag{5.1.7}$$

Thus we have shown what was mentioned earlier, that the VEV can form a new mass term. Next we repeat the operation for $\zeta$;

$$\begin{aligned} \frac{\partial^2 V}{\partial \zeta_+^2} &= 2(|M|_+^2 + M_+^2) + 2g^2v_+^2 = m^2_{\zeta_+} \\ \frac{\partial^2 V}{\partial \zeta_-^2} &= 2(|M|_-^2 + M_-^2) + 2g^2v_-^2 = m^2_{\zeta_-} \end{aligned} \tag{5.1.8}$$

If we use the same process as before we find that these terms cancel out and that $\zeta$ is massless.
We have found that one set of fields, $\chi_\pm$, are our higgs bosons and $\zeta_\pm$ are two goldstone bosons.

## 5.2 Gauge Boson Mass

[11;12;1]

Due to us operating in U(1) we have only one gauge boson, $A_\mu$, which makes this process quite short.
The shift of our scalar fields has affected the scalar-gauge interactions. As mentioned in section 3.2 we have in our Lagrangian the terms;

$$g^2 A_\mu A^\mu |\phi|_+^2 + g^2 A_\mu A^\mu |\phi|_-^2 \tag{5.2.1}$$

The new fields will give a mass term from these in the form of;

$$g^2 v_+^2 A_\mu A^\mu + g^2 v_-^2 A_\mu A^\mu + \dots \tag{5.2.2}$$

The omitted terms here are ones with interactions between our new fields and the gauge boson. One can now redo the EOM for $A_\mu$ like it was done in section(3.3). The terms in the Lagrangian relevant to this are;

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (g^2v_+^2 + g^2v_-^2) A^\mu A_\mu + \dots \tag{5.2.3}$$

Then the EOM are;

$$\begin{aligned} \partial_\mu F^{\mu\nu} + (g^2v_+^2 + g^2v_-^2)A_\mu &= 0 \\ \partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu + (g^2v_+^2 + g^2v_-^2)A_\mu &= 0 \\ (\Box + (g^2v_+^2 + g^2v_-^2))A_\mu &= 0 \end{aligned} \tag{5.2.4}$$

And so our gauge field has now been given mass by the VEV of our scalar field just like the $Z^\pm$ & $Z^0$ bosons in the SM, i.e;

$$m_\gamma^2 = g^2v_+^2 + g^2v_-^2 \tag{5.2.5}$$

## 5.3 The Gaugino and Fermionic Masses

[1;19]

Just like the terms of our gauge interaction, the coupling to the scalar and gaugino/fermionic terms gives rise to a new mass matrix, however there is no quadratic coupling here. Instead the gaugino and fermionic fields form a new basis, the terms in question are;

$$\begin{aligned} & -\sqrt{2}g\phi_+^*\psi_+\lambda - \sqrt{2}g\phi_-^*\psi_-\lambda - \sqrt{2}g\lambda^\dagger\psi_+^\dagger\phi_+ - \sqrt{2}g\lambda^\dagger\psi_-^\dagger\phi_- \\ \rightarrow \quad & -\sqrt{2}gv_+\psi_+\lambda - \sqrt{2}gv_-\psi_-\lambda - \sqrt{2}g\lambda^\dagger\psi_+^\dagger v_+ - \sqrt{2}g\lambda^\dagger\psi_-^\dagger v_- + \quad \dots \end{aligned} \tag{5.3.1}$$

We can perform this change of basis and mix these terms together with the fermionic mass parameters;

$$M_{+-}\psi_-\psi_+ + M_{+-}^*\psi_-^\dagger\psi_+^\dagger \tag{5.3.2}$$

First we take the equations of motion for the non conjugate fields;

$$
\begin{aligned}
\text{For } \lambda; \qquad & \sqrt{2}gv_+\psi_+ + \sqrt{2}gv_-\psi_- + \ldots = 0 \\
\text{For } \psi_+; \qquad & M_{+-}\psi_- + \sqrt{2}gv_+\lambda + \ldots = 0 \\
\text{For } \psi_-; \qquad & M_{+-}\psi_+ + \sqrt{2}gv_-\lambda + \ldots = 0
\end{aligned}
\tag{5.3.3}
$$

The dots here represent terms not in our new basis $(\lambda, \psi_+, \psi_-)$ and its conjugate. If we now express these equations as an eigenvector problem we get;

$$
\begin{pmatrix} 0 & \sqrt{2}gv_+ & \sqrt{2}gv_- \\ \sqrt{2}gv_+ & 0 & M_{+-} \\ \sqrt{2}gv_- & M_{+-} & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ \psi_+ \\ \psi_- \end{pmatrix} = 0
\tag{5.3.4}
$$

However this matrix has no neat solution. In order to get eigenvalues out we have to make one simplification of making $M_{+-}$ and its conjugate zero. With this simplification it is straightforward enough to find the eigenvalues, and the eigenvectors in order to diagonalise our matrix;

$$
\begin{aligned}
\text{Det}(M - e\mathbb{1}) &= \begin{pmatrix} -e & \sqrt{2}gv_+ & \sqrt{2}gv_- \\ \sqrt{2}gv_+ & -e & 0 \\ \sqrt{2}gv_- & 0 & -e \end{pmatrix} \\
&= -e^3 + 2e(g^2v_+^2 + g^2v_-^2) \\
&= 0
\end{aligned}
\tag{5.3.5}
$$

Therefore we have eigenvalues of $e = 0$ and $e_\pm = \pm\sqrt{2g^2v_+^2 + 2g^2v_-^2}$ and eigenvectors of;

$$
\begin{aligned}
v_1 &= \left(0, -\frac{v_-}{v_+}, 1\right) \\
v_2 &= \left(-\frac{\sqrt{g^2v_+^2 + g^2v_-^2}}{g^2v_-^2}, \frac{v_-}{v_+}, 1\right) \\
v_3 &= \left(\frac{\sqrt{g^2v_+^2 + g^2v_-^2}}{g^2v_-^2}, \frac{v_-}{v_+}, 1\right)
\end{aligned}
\tag{5.3.6}
$$

We can now diagonalise our initial matrix by multiplying on the left by an eigenvector matrix with the eigenvectors in the columns and its inverse on the right. After doing this operation we are left with;

$$
\begin{pmatrix} 0 & 0 & 0 \\ 0 & e_- & 0 \\ 0 & 0 & e_+ \end{pmatrix} \begin{pmatrix} \lambda \\ \psi_+ \\ \psi_- \end{pmatrix}
\tag{5.3.7}
$$

Multiplying out and adding the terms in the dots before we obtain the following EOM;

$$
\begin{aligned}
\text{For } \lambda; \qquad & i\partial_\mu\lambda^\dagger\overline{\sigma}^\mu = 0 \\
\text{For } \psi_+; \qquad & i\partial_\mu\psi_+^\dagger\overline{\sigma}^\mu + \psi_+e_- = 0 \\
\text{For } \psi_-; \qquad & i\partial_\mu\psi_-^\dagger\overline{\sigma}^\mu + \psi_-e_+ = 0
\end{aligned}
\tag{5.3.8}
$$

We still have our conjugate fermionic fields, and in order to solve the EOM into a KG equation we need to solve these simultaneous equations;

$$
\begin{aligned}
\text{For } \lambda^\dagger; \qquad & \sqrt{2}gv_+\psi_+^\dagger + \sqrt{2}gv_-\psi_-^\dagger + \ldots = 0 \\
\text{For } \psi_+^\dagger; \qquad & M_{+-}^*\psi_-^\dagger + \sqrt{2}gv_+\lambda^\dagger + \ldots = 0 \\
\text{For } \psi_-^\dagger; \qquad & M_{+-}^*\psi_+^\dagger + \sqrt{2}gv_-\lambda^\dagger + \ldots = 0
\end{aligned}
\tag{5.3.9}
$$

In order to solve the next eigenvalue problem we will again have to set $M_{+-}^*$ to zero. When we do this the matrix is identical to the one before, and so it has the same eigenvalues and eigenvectors. The equations of motion therefore end up being;

$$
\begin{aligned}
\text{For } \lambda^\dagger; \qquad & -i\overline{\sigma}^\mu \partial_\mu \lambda = 0 \\
\text{For } \psi_+^\dagger; \qquad & -i\overline{\sigma}^\mu \partial_\mu \psi_+ + \psi_+^\dagger e_- = 0 \\
\text{For } \psi_-^\dagger; \qquad & -i\overline{\sigma}^\mu \partial_\mu \psi_- + \psi_+^\dagger e_+ = 0
\end{aligned}
\tag{5.3.10}
$$

Initially we find that the mass for the gaugino and its conjugate are both zero and so remain unchanged. This is a direct consequence of removing $M_{+-}$, if it had remain in the equation the gaugino would have a mass. In order to solve the remaining 4 fields we must multiply each equation by $\sigma^\nu \partial_\nu$ and use eq.(2.2.6) to remove the $\sigma^\mu$ matrices and add a contracted 4-vector derivative. Firstly this will be illustrated with the gaugino fields;

$$
\begin{aligned}
\sigma^\nu \partial_\nu i \partial_\mu \lambda^\dagger \overline{\sigma}^\mu \qquad & \rightarrow \qquad i\eta^{\mu\nu} \partial_\nu \partial_\mu \lambda^\dagger = \Box \lambda^\dagger = 0 \\
\sigma^\nu \partial_\nu i \partial_\mu \lambda \overline{\sigma}^\mu \qquad & \rightarrow \qquad i\eta^{\mu\nu} \partial_\nu \partial_\mu \lambda = \Box \lambda = 0
\end{aligned}
\tag{5.3.11}
$$

And now with the remaining fermionic fields;

$$
\begin{aligned}
\psi_+ : \qquad & i\eta^{\mu\nu} \partial_\nu \partial_\mu \psi_+^\dagger + \sigma^\nu \partial_\nu \psi_+ e_- = 0 \\
\psi_- : \qquad & i\eta^{\mu\nu} \partial_\nu \partial_\mu \psi_-^\dagger + \sigma^\nu \partial_\nu \psi_+ e_+ = 0 \\
\psi_+^\dagger : \qquad & -i\eta^{\mu\nu} \partial_\nu \partial_\mu \psi_+ + \sigma^\nu \partial_\nu \psi_+^\dagger e_- = 0 \\
\psi_-^\dagger : \qquad & -i\eta^{\mu\nu} \partial_\nu \partial_\mu \psi_- + \sigma^\nu \partial_\nu \psi_-^\dagger e_- = 0
\end{aligned}
\tag{5.3.12}
$$

From here all we need to do is rearrange eq.(5.3.10) and eq.(5.3.8) so that we can sub in for the second term in each of the above lines, and we will obtain a KG equation like in section 3.3;

$$
\begin{aligned}
(\Box + (e_- e_-))\psi_+^\dagger &= 0 \\
(\Box + (e_+ e_+))\psi_-^\dagger &= 0 \\
(\Box + (e_- e_-))\psi_+ &= 0 \\
(\Box + (e_+ e_+))\psi_- &= 0
\end{aligned}
\tag{5.3.13}
$$

Where $e_+ e_+ = e_- e_- = 2g^2(v_+^2 + v_-^2)$. Thus all the remaining fermionic particles have the same mass. This would again not happen if we had not set $M_{+-}$ to zero, as up untill the last expression we still had separate masses for left and right handed particles, like in section 3.3. However each particle and its antiparticle would have the same mass even if the eigenvalues were different. This is to be expected.

# 6 Conclusion

In essence the goals set out in the preface were to;

**a.** Show invariance under supersymmetric transformations.

**b.** Extend the theory constructed in **a.** to U(1)

**c.** Break the symmetry and find the particle masses.

For **a.** we started with a simple Lagrangian of a scalar field and a weyl fermion field. Next SUSY transformations were derived and applied to the fields and they were shown to be invariant under the variation of the action. This was show to be true in eq.(2.2.8).
After this we wanted to show that the commutator of two SUSY transformations was infact another symmetry of the theory. This is often known as algebraic closure. In order to show that this closed off-shell (eq(2.2.13)) an auxiliary field needed to be added. This auxiliary field ended up being important in forming the superpotential and from it the masses of the particles in the system.
A symmetry implies a conserved current, and the supercurrent was found using Noethers theorem. The rest of section 2.3 was spent finding the commutation relations of the supercharges. These were important as they are the generators of susy transformations.
The next step was to extend the theory to multiple interacting fields. Many interactions were discarded including a standard potential, thus the axillary field was needed in order to give us a scalar potential later on. Adding interactions was concluded with eq.(2.4.20). The last part of a complete theory for the SM is

the gauge interaction, and this was covered much more briefly. First the various interaction were added, then the derivatives were replaced with covariant derivatives and lastly 3 terms were added that could not be derived another way (eq.(2.6.2)).The final Lagrangian is represented in eq.(2.6.4).

With the review section completed we moved on to a simplified version of the theory. We limited everything to a U(1) gauge, with the generators $T^a$ set to 1 and with only one gauge parameter, $a = 1$ aswell. Furthermore we added a second chiral field, so that we have a $+$ and a $-$ for the interacting indices. This allows us to set a charge $Q$ for global symmetry transformation, and as a result, many terms disappear from the Lagrangian. These disappearances are necessitated by gauge invariance, as in eq.(3.1.6). The full Lagrangian is then shown in eq.(3.1.7).
After these simplifications the interactions are illustrated in feynman diagrams and some comments were made as to the nature of the interactions. Then the masses of the unbroken symmetry were found. The gaugino and the gauge boson were found to be massless. The masses of $\phi_+$ and its conjugate were found to be the same and the mass of $\phi_-$ and its conjugate were also found to be the same. The masses of the fermions were found to carry similarities aswell with there only being 2 separate masses, consistent with these being antiparticles. This concluded looking at an unbroken symmetry.

The final 2 sections looked at the VEV and its implication in SUSY breaking, along with the masses of the newly broken particles. We saw that in an unbroken SUSY the minima of the superpotential was zero due to the mass being necessarily real and greater than zero. Terms were then added in the form of $V_{soft}$, allowing the minima of the new potential to have non zero value, and thus a VEV (eq.(4.1.4)). With the newly broken minima a circle on the real-imaginary plane, $v_\pm$ was added to our field $\phi$ and so a shifted field was created. These shifted fields were called $\chi$ and $\zeta$ and were both real.
Next the new particles masses were found, this was done by finding the second derivative of the potential, as shown in eq.(5.1.3). What was found was that the newly created VEV was a component of a new mass, and through interactions with the original $\phi$ field we were then able to find new masses for all our particles. Furthermore the 2 new shifted fields were found to have different masses. The field $\chi$ was found to be our classic higgs mass, like before, and the $\zeta$ field was found to be massless, and corresponded to the goldstone bosons.
First the gauge bosons were found to have mass through 2 terms, as seen in section 5.2. Just like the gauge bosons in the SM they only had mass after breaking the symmetry, although unlike the SM there is only one gauge boson in a U(1) theory.
The last thing that was done was finding the fermionic masses. This included the gaugino mass as it had now been mixed through the shifted fields. The EOM were taken and an eigenvalue problem was created. The mass $M_{+-}$ and its conjugate had to be set to 0 in order to solve this problem. This resulted in a massless gaugino, and the same mass for all the remaining fermions of $2g^2(v_+^2 + v_-^2)$.

Had the eigenvalue problem been solvable the gaugino and its conjugate would have been given a new mass, and so would have proved the statement set out in section 1. Namely that in order for SUSY to exist the symmetry must be broken in such a way that all SM particles be given a new, greater, mass. This was found to be true in almost all cases, but is best shown in the gauge example, where the mass is greater than just normal symmetry breaking.

Given more time there is another area of research with this dissertation. It is possible to use this U(1) theory as an extension to the SM. The boson created in this theory would be a spin-1 particle, and one could calculate the interactions between this theory and the SM particles. It may even be possible to then link these extended approaches to string-motivated examples. [15]

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