$P_0 = \frac{D_N}{r_E - g} = \frac{4.5}{(10-3)\%} = \frac{4.5 \times 100}{7} = 64.28$

Q: Titan industries has 217 million shares outstanding & expects earning at the end of this year of \$860 million. Titan plans to pay out 50% of its shares. If Titan's earnings are expected to grow by 7.5% per year and payout rate remain constant, determine Titan's share price for DDM, P_0... DDM... assuming an equity cost of capital of 10%

$P_0 = \frac{D}{r-g}$

$= \frac{\frac{50}{100} \times 860}{(10-7.5)\%} = \frac{430}{2.5} \times 100 = 17200$

$P_0 = \frac{17200}{217} = \underline{79.26}$ per share

Constant Dividend Growth Model:

$P_0 = \frac{Div_1}{r_E - g}$ To increase P_0: $Div\uparrow, r_E\downarrow, g\uparrow$

$\Rightarrow r_E = \underbrace{\frac{Div_1}{P_0}}_{\text{divident yield}} + g \rightarrow$ capital gain rate

Q: Consolidated Edison, Inc. is a regulated utility company that services the New York City area. Suppose they plans to pay \$ 2.60 per share in dividends in the coming year. If its equity cost of capital is 6% and dividends are expected to grow by 2% per year in the future, estimate the value of the company's stock.

$Div = 2.6$, $r_E = 6\%$, $g = 2\%$

$P_0 = \frac{Div_1}{r_E - g} = \frac{2.6}{0.06-0.02} = \frac{2.6 \times 100}{4} = \underline{\underline{65}}$

Dividend versus investment:

$Div_1 = \underbrace{\frac{\text{Earnings}}{\text{Share Outstanding}}}_{EPS} \times$ Divident Payout Rate

Change in earning = New invest * Return on NI

New invest = Earning × RR (Retuntion rate)

$\frac{\text{Change in earning}}{\text{earning}} = \cancel{\text{earning}} \times \text{Ret R} \times \text{Retn NI} / \cancel{\text{earning}}$

$g = RR \times$ Retuntion on NI

Q: Suppose crane could cut its dividend payout rate of 75% for the foreseeable future and use the retained earnings to open a n... divident P/0 = 75%

RR = 25%

Divident = EPS × PPG rate = \$6

r_E = Div yield + g = 6/60 + 0 = 0.1 = 10%

New divident = 6 × D P/0 = 6 × 3/4 = 4.5

g = Ret Rate × Return on NI = 25% × 12% = $\underline{\underline{3\%}}$

Dividend Discount Model

$$P_0 = \frac{Div_1}{1+r_E} + \frac{Div_2}{(1+r_E)^2} + \dots + \frac{Div_N}{(1+r_E)^N} + \frac{P_N}{(1+r_E)^N}$$

The price of the stock is equal to the present value of the expected future dividends it will pay

Constant dividend growth

dividend grow at constant rate g.

$$P_0 = \frac{Div_1}{r_E - g}$$

$$r_E = \underbrace{\frac{D_1}{P_0}}_{\text{Dividend yield}} + \underbrace{g}_{\text{Capital gain rate}}$$

Q Consolidated Edison, Inc is a regulated utility company that services the New York City Area. Suppose Con Edison plans to pay \$2.60 per share in dividends in the coming year. If its equity cost of capital is 6% and dividends are expected to grow by 2% per year in the future, estimate the value of Con Edison's stock

$$P_0 = \frac{2.60}{0.06 - 0.02} = \frac{2.6}{0.04} = 65$$

Constant Dividend Growth Model:

$P_0 = \frac{Div_1}{r_E - g}$ To increase P_0: Div ↑, r_E ↓, g ↑

$\Rightarrow r_E = \underbrace{\frac{Div_1}{P_0}}_{\text{divident yield}} + g \rightarrow$ capital gain rate

Q: Consolidated Edison, Inc. is a regulated utility company that services the New York City area. Suppose they plans to pay $ 2.60 per share in dividends in the coming year. If its equity cost of capital is 6% and dividends are expected to grow by 2% per year in the future, estimate the value of the company's stock.

Div = 2.6, r_E = 6%, g = 2%

$P_0 = \frac{Div_1}{r_E - g} = \frac{2.6}{0.06-0.02} = \frac{2.6 \times 100}{4} = \underline{\underline{65}}$

Dividend versus investment:

$Div_1 = \underbrace{\frac{\text{Earnings}}{\text{Share Outstanding}}}_{EPS} \times$ Divident Payout Rate

Change in earning = New invest * Return on NI

New invest = ~~Eo~~ Earning × RR (Retention rate)

$\frac{\text{Change in earning}}{\text{earning}}$ = earning × Ret R × Retn NI / earning

g = RR × Retention on NI

Q: Suppose crane could cut its dividend payout rate of 75% for the foreseeable future and use the retained earnings to open a n . . .

Dividend = EPS × PPG rate = $6 divident P/O = 75%

RR = 25%

r_E = Div yield + g = 6/60 + 0 = 0.1 = 10%

New divident = 6 × D P/O = 6 × 3/4 = 4.5

g = Ret Rate × Return on NI = 25% × 12% = $\underline{\underline{3\%}}$

$N = 9\text{yrs}$, $YTM = 10\%$

$B_0 = 80\left[\frac{1}{0.1} - \frac{1}{0.1(1.1)^9}\right] + 1000\left(\frac{1}{(1.1)^9}\right) = \underline{\underline{884.819}}$

$YTM = 8\%$, $N = 9\text{yrs}$

$C\% < YTM = Pr\downarrow$ Discount

$C\% > YTM = Pr\uparrow \rightarrow$ Premium

Q: $YTM = 16\%$, $C = 14\%$, $FV = 1000$, $N = 7\text{yrs}$, semi annually
$\Downarrow 8\%$ $\Downarrow 7\%$ $\Downarrow 14$

$B_0 = 70\left[\frac{1}{0.08} - \frac{1}{0.08(1.08)^{14}}\right] + 1000\left[\frac{1}{1.08^{14}}\right] = 917.55$

Perpetuity: $PV = \frac{C}{r}$

Q: You want to endow an annual MBA graduation party at your alma mater. You budget $30,000 per year forever for the party. If the university earns 8% per year on its investments, and if the first party is in one year's time, how much will you need to endow the party?

30k 30k

$P = \frac{C}{r} = \frac{30000}{8} \times 100 = 375000$

after 1 year:

$\frac{375000}{1.08}$

375k 30k 30k

if it starts after n years: $P = \frac{x}{(1+r)^n}$

Annuities

Q: You are the winner of the $30 state lottery. You can take your prize money either as:

a) 30 payments of $1 million per year (starting today) or

b) $15 million paid today

if interest rate is 8%. What to select.

eg: $FV = 100$, coupon $= 13\%$, $N = 1$

MP = a) 90 b) 100 c) 110

yield $= \frac{CF}{\text{Bond MP}} \times 100 = \frac{13}{90} \times 100 = 14.44\%$

$\frac{13}{100} \times 100 = 13\%$

$\underbrace{\text{yield \& market price}}_{\text{inverse relation}}$

$\frac{13}{110} \times 100 = 11.81\%$

Approx

$YTM = \frac{CF + [(FV - MP)/N]}{\left(\frac{FV+MP}{2}\right)} \times 100$

eg: $FV = 1000$ coupon rate $= 9\%$ market price $= 900$ $N = 6$ yrs

$YTM = \frac{90 + (1000-900)/6}{\frac{1900}{2}} \times 100 = \frac{(90 + 16.66) \times 2}{19} = \frac{213.33}{19} = \underline{\underline{11.228\%}}$

When req. rate of return < yield to mature } Buy

$\Rightarrow B_0 > MP$

Q: $C\% = 8\%$, ~~N = 8%~~, $N = 10$ yrs, Principal par $= 1000$, $YTM = 8\%$

$B_0 = ?$

$B_0 = CF\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right] + FV\left[\frac{1}{(1+r)^n}\right]$

$= 80\left[\frac{1}{0.08} - \frac{1}{1.08(1.08)^{10}}\right] + 1000\left[\frac{1}{1.08^{10}}\right] = \underline{\underline{1000}}$

Bond discount

$C\% < YTM = P \downarrow$ Discount

$C\% > YTM \rightarrow BP \uparrow$ Premium

$YTM = 16\%$.
$C\% = 14\%$
$N = 7$ yrs   Semi annually
$\hookrightarrow$ So $YTM = 8\%$. $C\% = 7\%$. $N = 14$ yrs

$B_0 = 917.5$

Perpetuity

$$PV = \frac{c}{r} \quad \Big| \quad PV = \frac{c}{r+g}$$

Q. $C = 30000$
$r = 8\%$

$$P = C/r = \frac{30000}{8} \times 100 = 3,75,000$$

Now, the party is end of 2yrs

0	1	2	3	4	…
		30K	30K	30K	

3,75,000 (using previous)

Convert to present value by dividing

$$\frac{3,75,000}{(1+r)^n} = \frac{3,75,000}{(1.08)^1}$$

$$B_0 = \underbrace{\left[\frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_n}{(1+r)^n}\right]}_{\text{present value annuality factor}} + \underbrace{\left[\frac{FV}{(1+r)^n}\right]}_{\text{face value cash flow}}$$

coupon rate ~~r%~~ × Face value = ~~Interest~~ cash flow value

~~r% × FV = CF~~

eg: Face value = 1000 Required rate of return $(r) \to 12\%$

Coupon rate = 6% Time period maturity (N) = 3

Bond price = ?

$CV = \frac{6}{100} \times 1000 = 60$

$$B_0 = \underbrace{\frac{60}{(1+0.12)} + \frac{60}{(1+0.12)^2} + \frac{60}{(1.12)^3}}_{144} + \underbrace{\frac{1000}{(1.12)^3}}_{712}$$

$B_0 = \underline{\underline{855}}$

current price of the bond / Intrinsic value

IV / B_0 > MP → Buy

IV / B_0 < MP → sell

$$B_0 = CF\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right] + FV\left(\frac{1}{(1+r)^n}\right)$$

$$B_0 = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{FV}{(1+r)^n}$$

eg. par value = 1000 coupon = 15% Bond Market price = 900

N = 10 yrs req. rate of return = 16%.

CF = 150

$$B_0 = 150\left[\frac{1}{0.16} - \frac{1}{0.16(1.16)^{10}}\right] + \frac{1000}{(1.16)^{10}}$$

$= 150 \times 4.83 + 226.68 = 951.18$

900 < 951

MP < B_0 Buy

<u>Yield to maturity</u> : Required rate of return that makes current price of bond to market price is called yield to maturity.

$P = PV(\text{future cash flow})$ ↳ divident

$-P_0$ ——— $Div_1 + P_1$

$P_0 = \frac{Div + P_1}{1 + r_E}$ r_E → equity cost of capital

$\underbrace{r_E}_{\text{equity cost of capital}} = \underbrace{\frac{Div_1}{P_0}}_{\text{yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{capital gain rate}}$ ← capital gain

Q: Suppose you expect Walgreen company to pay dividends of \$1.40 per share and trade for \$80 per share at the end of the year. If investments with equivalent risk to Walgreen's stock have an expected return of 8.5%, what is the most you would pay today for Walgreen's stock? What dividend yield & capital gain rate would you expect at this price?

$Div = 1.4$, $P_1 = 80$, $r_E = 8.5\%$

$P_0 = \frac{1.4 + 80}{1 + 0.085} = \$75.02$

Divident yield $= \frac{1.4}{P_0} = \frac{1.4}{75.02} = 0.0186$

Capital gain rate $= \frac{80 - 75.02}{75.02} = 0.0663$

0 ——— 1 ——— 2

$-P_0$ ——— Div_1 ——— $Div_2 + P_2$

$P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2 + P_2}{(1 + r_E)^2}$

Derivation: $P_1 = \frac{Div_2 + P_2}{1 + r_E} \Rightarrow P_0 = \frac{Div_1}{1 + r_E} + \frac{P_1}{1 + r_E} = \frac{Div_1}{1 + r_E} + \frac{1}{1 + r_E}\left[\frac{Div_2 + P_2}{1 + r_E}\right]$

Dividend-Discount Model:

$P_0 = \frac{Div_1}{1 + r_E} + \frac{Div_2}{(1 + r_E)^2} + \cdots + \frac{Div_N}{(1 + r_E)^N} + \frac{P_N}{(1 + r_E)^N}$

The price of the stock is equal to present value of the expected future dividends it will pay.

Annuities:

$$\boxed{PV = C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)}$$

0 ——— 1 ——— 2 30

1 million, $N = 29$

$PV = 1 \times \frac{1}{0.08}\left(1 - \frac{1}{(1.08)^{29}}\right) = 11.15$ mil

total $= 1 + 11.15 = 12.15$ million

Future value of annuity

$FV = PV \times (1+r)^N = \frac{C}{r}\left(1 - \frac{1}{(1+r)^N}\right) \times (1+r)^N = C \times \frac{1}{r}\left((1+r)^N - 1\right)$

Q: Paul is 35 years old and he has decided to invest in retirement plan. At the end of each year until he is 65, he will save \$10,000 in a retirement account. If the account earns 10% per year, how much will Ellen have saved at age 65?

$FV = \frac{10000}{0.1} \times \left((1.1)^{30} - 1\right)$

Growing perpetuity: $PV = C / r - g$

Q: In the previous MBA question, asked to pay more, cost rise by 4%.

$PV = \frac{30000}{8-4} \times 100 = 7,500,000$

Growing annuity

$PV = C \times \frac{1}{r - g}\left(1 - \left(\frac{1+g}{1+r}\right)^N\right)$

Loan or Annuity payment

$C = \frac{P \rightarrow \text{principal amount (PV)}}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)}$

Q: A biotech firm plans to buy a new machinery for \$500,000. The firm pay 20% of the purchase as a down payment and finance the remainder by taking 48 months, interest. Find C; interest 0.5% per month

$P = 500000\left(1 - \frac{20}{100}\right) = 400000$ $C = \frac{4,00,000}{\frac{100}{0.5}\left(1 - \frac{1}{(1.005)^{48}}\right)}$

Q. Suppose you expect Walgreen Company to pay dividends of \$1.40 per share and trade for \$80 per share at the end of the year. If investments with equivalent risk to Walgreen stock have an expected return of 8.5%. What is the most you would pay today for Walgreen st. What dividend yield & capital gain rate would you expect

$$P_0 = \frac{Div_1 + P_1}{(1+r_E)}$$

$$= \frac{1.40 + 80}{1.085} = 75.02$$

$$\text{Yield} = \frac{Div_1}{P_0} = \frac{1.40}{75.02} = 0.0187$$

$$\text{Capital gain rate} = \frac{P_1 - P_0}{P_0} = \frac{4.98}{75.02} = 0.0663$$

$\boxed{\text{Multi-Year investor}}$

0	1	2
$-P_0$	$Div_1 + P_1$	$Div_2 + P_2$

$$P_0 = \frac{(Div_1)}{1+r_E} + \frac{(Div_2 + P_2)}{(1+r_E)^2}$$

$$P_1 = \frac{Div_2 + P_2}{1+r_E}$$

Q Titan Industries has 217 million shares outstanding and expects earnings at the end of this year of \$ 860 million. Titan plans to pay out 50% of its earnings in total, paying 30% as a dividend and using 20% to repurchase shares. If Titan's earnings are expected to grow by 7.5% per year and these payout rates remain constant, determine Titan's share price assuming an equity cost of capital of 10%

$$P_0 = \frac{D}{r-g}$$

$$= \frac{\frac{50}{100} \times 860}{10 - 7.5} = \frac{430 \times 100}{2.5}$$ 17200 million

$C\% = 8\%$
$N = 10$ yrs
Principal $= 1000$
$YTM = 8\%$

$$= \left[\cancel{\frac{80}{1.8} + \frac{80}{(1.8)^2} + \frac{80}{(1.8)^3}} \right.$$

$$B_0 = 80 \left[\frac{1}{0.08} - \frac{1}{0.08\,(1.08)^{10}} \right] + 1000 \left(\frac{1}{(1.08)^{10}} \right)$$

$$= 1000$$

$N = 9$ yrs
$YTM = 10\%$

$$B_0 = 80 \left[\frac{1}{0.1} - \frac{1}{0.1\,(1.1)^9} \right] + 1000 \left(\frac{1}{(1.1)^9} \right)$$

$$= 884.81$$

$C\% < YTM \longrightarrow$ Bond price $\downarrow$

$N = 10$ yrs Par Value $= 1000$

Coupon $= 15\%$

req. rate of r $= 16\%$

Bond market Price $=$ ₹900

$$= \frac{150}{\cancel{1}}\left[\frac{1}{0.16} - \frac{1}{0.16(1.16)^{10}}\right] + 1000\left(\frac{1}{(1.16)^{10}}\right)$$

$725 + 227$

952.

coupon $= 12\%$ $r = 13\%$

Bond Market P $= 960$

$$120\left[\frac{1}{0.13} - \frac{1}{0.13(1.13)^{10}}\right] + 1000\left[\frac{1}{(1.13)^{10}}\right]$$

$651 + 295$

$= 946$

Present Value of Annuity factor . PVAF

$$B_0 = \underbrace{\left[\frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_n}{(1+r)^n}\right]}_{PVAF} + \left[\frac{FV}{(1+r)^n}\right]$$

Example

FV = 1000

Coupon rate = 6%

required rate of return = 12%

Time period = 3

CF = 6% of 1000 = 60 ₹

$$B_0 = \left[\frac{60}{1.12} + \frac{60}{(1.12)^2} + \frac{60}{(1.12)^3}\right] + \left[\frac{1000}{(1.12)^3}\right]$$

$$= 855.89$$

$\frac{B_0}{IV} > MP \Rightarrow$ Buy

$\frac{B_0}{IV} < MP \Rightarrow$ Sell

Q. interest rate = 10%

100 Rs in one year — 90.9 today

PV = 100 + (100/1.1) = 100 + 90.91 = 190.91

NAP if interest rate is a) 5% b) 12%

a) 195.23

b) 189.28

FV → maturity

Time → 2

Coupon rate → r

CF - cash flow

$$PV = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{FV}{(1+r)}$$

→ Price of Bond.

Annuities.

$$C \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)$$

Q. You are the winner of 30$ million state lottery. You can take the prize money as

- (a) 30 payments of $1 million per year (starting today)
- (b) $15 mil. paid today

- If the interest rate is 8%, which option should you take?

$$PV = 1 \times \frac{1}{0.08}\left(1 - \frac{1}{(1.08)^{29}}\right)$$

$$= 11.15 \text{ mil}$$

$$11.15 \text{ mil} + 1 \text{ mil} = 12.15 \text{ mil}$$

Future value of an annuity

$$FV = C \times \frac{1}{r}\left((1+r)^N - 1\right)$$

Q. Paul is 35 yr old. He decided to invest in retirement plan, At end of each year until he is 65, he will save 10,000$ in a retirement account. If the account earns 10% per year, how much will ~~[illegible]~~ he have saved at age 65?

Dividend versus investment

$$DIV_t = \frac{Earnings_t}{Shares\ Outstanding_t} \times Dividend\ Payout\ Rate_t$$

Δ in earning = New invst. * Return on NI

New Invst = Earning × RR (Retention rate)

$$g\ \boxed{= \frac{\Delta \text{ in earning}}{\text{earning}}} = \frac{Earning \times Ret.R \times Return\ on\ NI}{Earning}$$

g = RR × Return on NI

Example Crane goods expects to have eps 6 \$ in coming year. Rather investing these, firm plans to pay out.

Suppose Crane could cut its dividend payout rate to 75%. The ret on invst is expected 12%. Assuming its equity capital is unchanged what effect would this new policy have on stock price

Dividend = EPS × Div P/O

= 6 \$

$$r_E = \text{Div yield} + g = \frac{6}{60} + 0 = 10\%$$

$$10000 \times \frac{1}{0.10}\left((1.10)^{30} - 1\right)$$

Growing perpetuity

$$PV = C/r-g$$

Growing annuity

$$PV = C \times \frac{1}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^N\right)$$

Annuity Payment : Loan Payment

$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)}$$

Internal Rate of Return

Interest rate thats sets NPV of cash flow = 0
for bond, IRR = yield

$$P_0 = \frac{Div_1 + P_1}{(1+r_E)}$$

r_E - equity cost of capital

$$r_E = \underbrace{\frac{Div_1}{P_0}}_{\text{Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gain Rate}}$$

Capital Gain

New Dividend $= 6 \times \frac{3}{4} = 4.5$

$RR = 1 - \text{Div P/o rate} = 1 - 0.75 = 0.25$

$NI = 12\%$

$g = 0.25 \times 12 = 3\%$

$P_0 = \frac{D_N}{10\% - 3\%} = \frac{4.5}{7\%} = \frac{4.5}{7} \times 100 = \frac{450}{7} = 64$

Case II

ret on NI = 8%

$g = 0.25 \times 8 = 2\%$

$P_0 = \frac{4.5}{8\%} = \frac{450}{8} = 56$

coupon Rate
(10%)

Face value ← Bondholder
→ investor

F →

100 → N = 3
maturity

$$\underbrace{\frac{1}{10} \quad \frac{2}{10} \quad \frac{3}{10}}_{\text{interest income}} + \underbrace{\frac{3^{rd}}{100}}_{\text{maturity value}}$$

Bond Prices
↓
$B_0 = PV(\;)$ → true value → Intrinsic value (IV)

$IV \gtreqless B_{\text{Market price (MP)}} = B_{MP}$

Face Value (FV)
coupon rate
Required rate of return (r)
MP
Time period → maturity (N)

Annuity

$$B_0 = PV\left(\frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3}\right) + \left(\frac{100}{(1+r)^3}\right)$$

$$+ \cancel{B_0}\left(\cancel{\frac{100}{(1+r)}}\right)$$

$$\boxed{B_0 = CF\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right] + FV\left(\frac{1}{(1+r)^n}\right)}$$

Yield to maturity.
rate of interest which will make your return equal to market price.

FV = 100 Coupon = 13% N = 1
(a) MP = 90 (b) MP = 100 (c) 110

$$\text{Yield} = \frac{CF}{\text{Bond } MP} \times 100$$

(a) $Y = \frac{13}{90} \times 100$ (b) $\frac{13}{100} \times 100$ (c) $\frac{13}{110} \times 100$

14.4% 13% 11.8%

Approx

$$\boxed{YTM = \frac{CF + \left[(FV - MP)/N\right]}{\left[\frac{FV + MP}{2}\right]} \times 100}$$

FV = 1000 Coupon rate = 9%
MP = 900 N = 6 yr.

$$\frac{90 + [100/6]}{950} \times 100$$

= 11.27%

Buy → Req Return less than YTM