CSL 101 DISCRRETE MATHEMATICS

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Graph Isomorphism

All that matters is the *connections*.

Graphs with the same connections are *isomorphic*.

Informally, two graphs are isomorphic if they are the same after *renaming*.

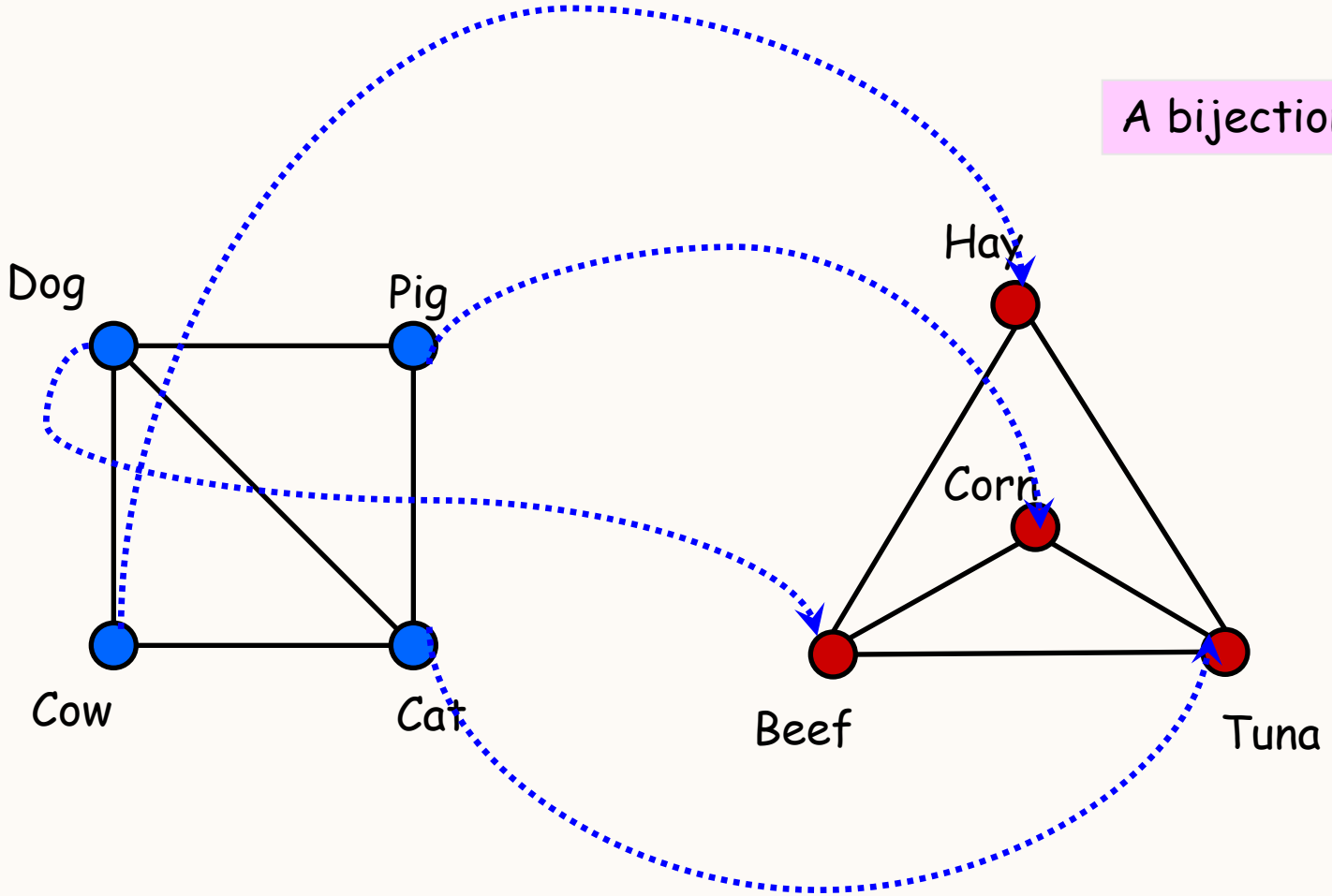
G_1 *isomorphic* to G_2 means there is an *edge-preserving vertex matching*.

relation preserving

renaming function

Graph isomorphism has applications like checking fingerprint, testing molecules…

Are These Isomorphic?



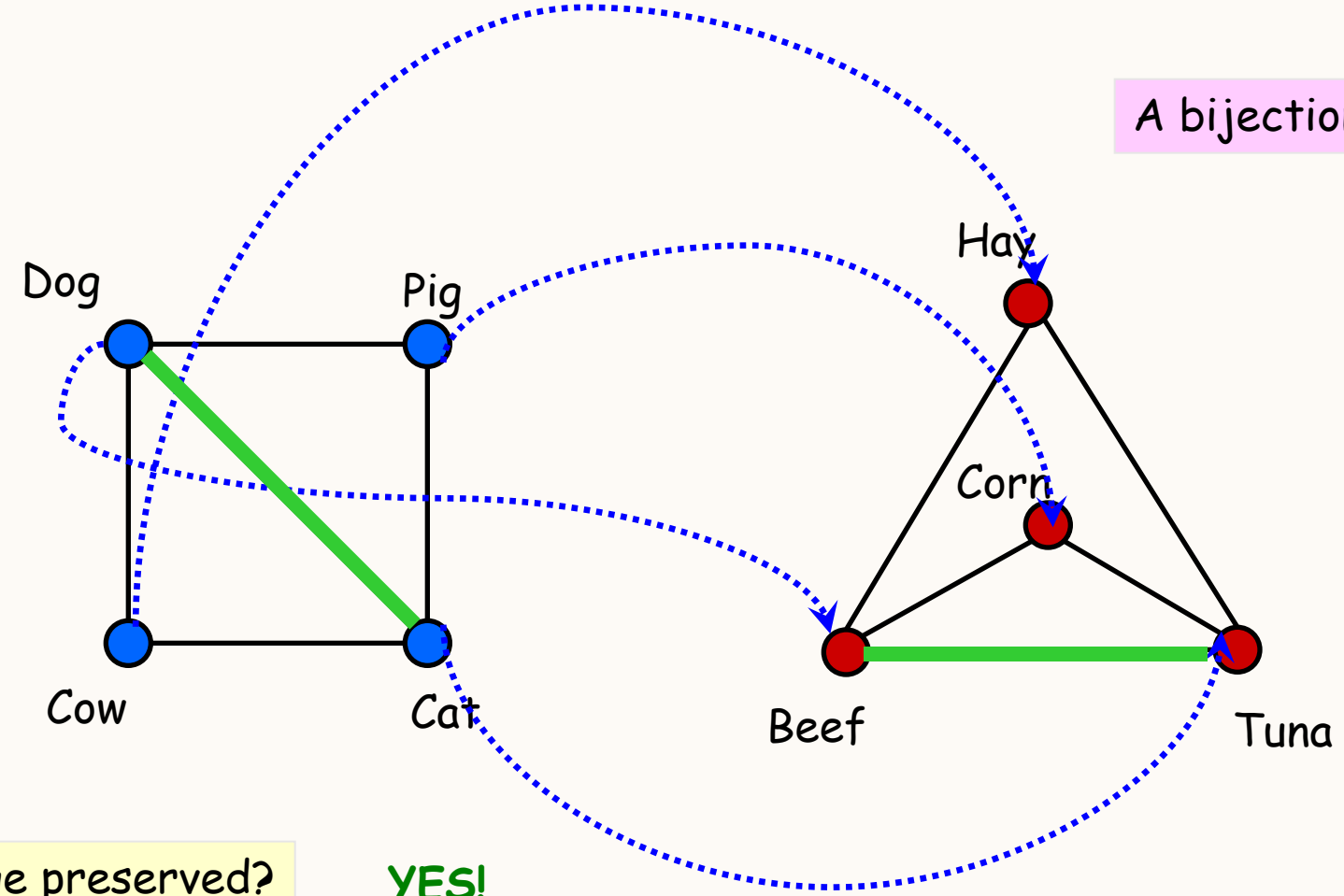
f (Dog) = Beef

f (Cat) = Tuna

f (Cow) = Hay

f (Pig) = Corn

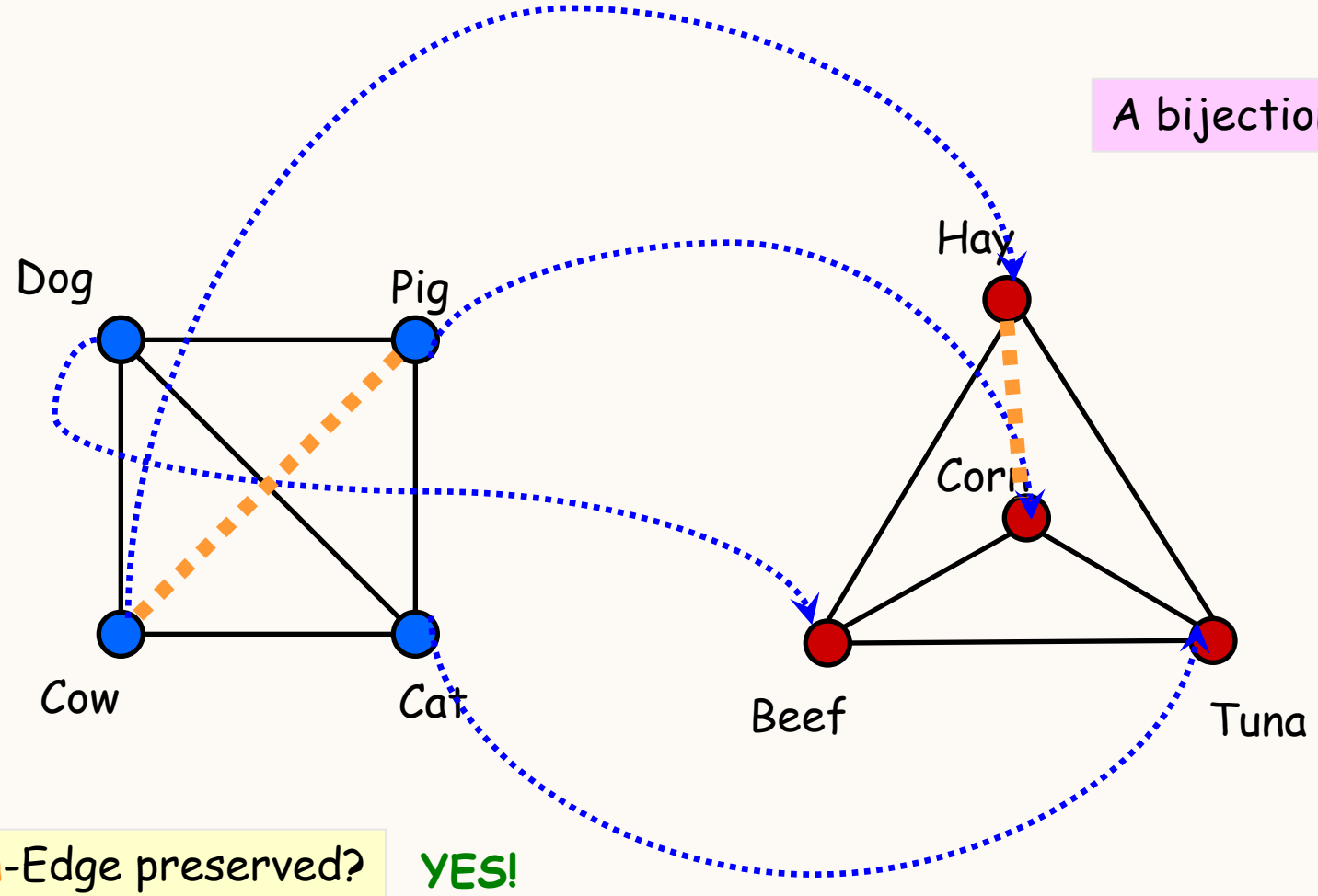
Are These Isomorphic?



Edge preserved? YES!

If there is an edge in the original graph, there is an edge after the mapping.

Are These Isomorphic?



Non-Edge preserved? YES!

If there is no edge in the original graph, there is no edge after the mapping.

Graph Isomorphism

G_1 *isomorphic* to G_2 means there is an *edge-preserving vertex matching.*

∃ *bijection* $f: V_1 \rightarrow V_2$

u — v in E_1 iff $f(u)$ — $f(v)$ in E_2

uv is an edge in G1

f(u)f(v) is an edge in G2

- If G1 and G2 are isomorphic, do they have the same number of vertices? YES
- If G1 and G2 are isomorphic, do they have the same number of edges? YES
- If G1 and G2 are isomorphic, do they have the same degree sequence? YES
- If G1 and G2 have the same degree sequence, are they isomorphic? NO

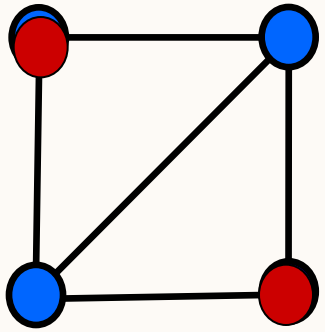
Exercise

How to show two graphs are isomorphic?

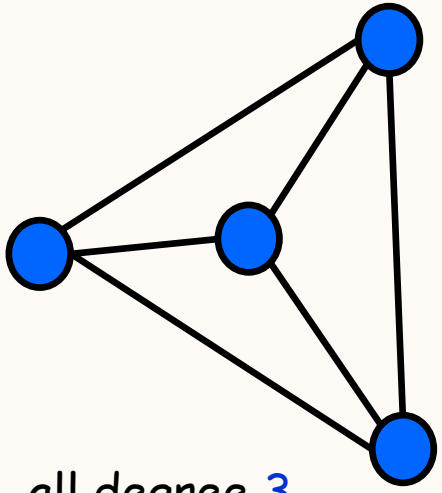
Find a mapping and show that it is edge-preserving.

How to show two graphs are non-isomorphic?

Find some isomorphic-preserving properties
which is satisfied in one graph but not the other.



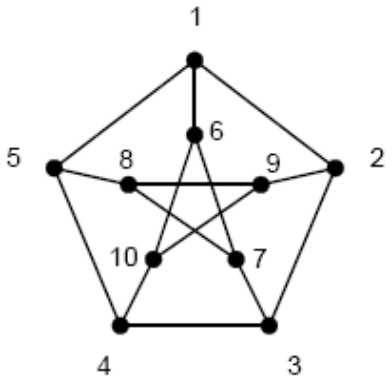
degree 2



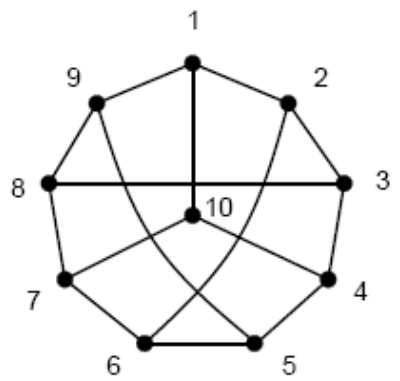
all degree 3

Non-isomorphic

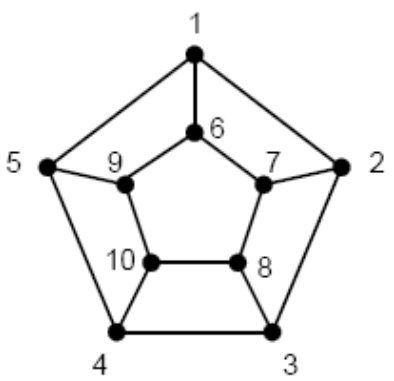
Exercise



G_1

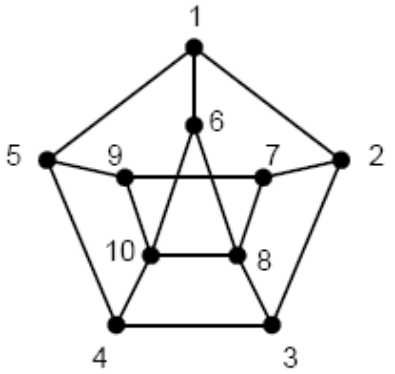


(a) G_2

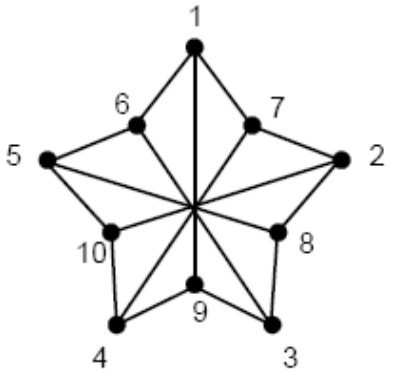


(b) G_3

Which is isomorphic to G1?



(c) G_4



(d) G_5

Testing graph isomorphism is not easy –

No known general method to test graph-ismorphism which is much more efficient than checking all possibilities.