CSL253 - Theory of Computation

Tutorial 7

Team Members

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Question 8:

Let $A = \{\langle M \rangle \mid M$ is a DFA does not accept any string containing an odd no. of 1's$\}$.

Show that A is decidable.

Solution:

Idea

We define the regular language:

$$L_{\text{odd}} = \{w \in \{0,1\}^* \mid w \text{ contains an odd number of 1s}\}$$

Our goal is to check whether DFA M accepts any string in L_{odd}, i.e., whether:

$$L(M) \cap L_{\text{odd}} = \emptyset$$

If this intersection is empty, then M does not accept any string with an odd number of 1s. Otherwise, it does.

Construction

Since both $L(M)$ and L_{odd} are regular languages:

- Their intersection is also a regular language.
- Emptiness for DFAs is decidable.

Hence, we can construct a DFA M' such that:

$$L(M') = L(M) \cap L_{\text{odd}}$$

Then, we check if $L(M') = \emptyset$. If yes, $\langle M \rangle \in A$. Otherwise, $\langle M \rangle \notin A$.

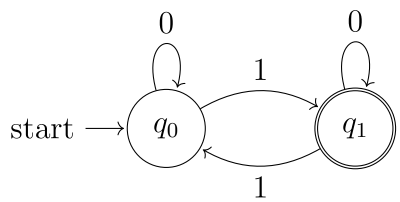
Decider for A

We describe a Turing Machine T that decides A:

1. On input $\langle M \rangle$, construct DFA D_{odd} for the language L_{odd}.
2. Construct DFA M' for the intersection $L(M) \cap L_{\text{odd}}$.
3. Test if $L(M') = \emptyset$ using the emptiness-checking algorithm for DFAs.
4. If yes, accept; otherwise, reject.

DFA for Odd Number of 1s

We now construct a DFA D_{odd} that accepts strings with an odd number of 1s.



- q_0: even number of 1s seen so far.
- q_1: odd number of 1s seen so far.

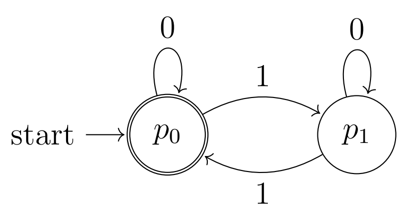
Example

Let M be a DFA that accepts strings where the number of 1s is divisible by 3. Clearly, such strings can contain odd numbers of 1s (e.g., 3).

So:

$$L(M) \cap L_{\text{odd}} \neq \emptyset \Rightarrow \langle M \rangle \notin A$$

On the other hand, suppose M accepts only strings with an even number of 1s (like the DFA below):



In this case, $L(M) \cap L_{\text{odd}} = \emptyset \Rightarrow \langle M \rangle \in A$

Conclusion

We have described a method to decide whether a DFA M accepts any string containing an odd number of 1s. Therefore, the language:

$$A = \{\langle M \rangle \mid M \text{ is a DFA which doesn't accept any string with an odd number of 1s}\}$$

is **decidable**. $\square$