CSL253 - Theory of Computation

Tutorial 7

Team Members

1. S.S.V.Rahul - 12341980
2. Shanmukha siva venkata sai - 12340700
3. Katari Venu - 12341110

Question 6

Let INFINITEDFA $= \{\langle A \rangle \mid A$ is a DFA and $L(A)$ is an infinite language$\}$. Show that INFINITEDFA is decidable.

Problem 6: Decidability of $\texttt{INFINITE}_{\texttt{DFA}}$

Show that $\texttt{INFINITE}_{\texttt{DFA}}$ is decidable.

On input $\langle A \rangle$, where A is a DFA:

1. Let k be the number of states of A.
2. Construct a DFA D that accepts all strings of length $\geq k$.
3. Construct a DFA M such that

$$L(M) = L(A) \cap L(D)$$

where:

- $L(A)$: the original language accepted by DFA A,
- $L(D)$: the set of all strings of length $\geq k$, where k is the number of states in A.

So, $L(M)$ means the set of all strings that:

- Are accepted by A, and
- Have length $\geq k$.

- This implies: if the intersection is not $\emptyset$, then there exists a string of length $\geq k$. But since A has k states, according to the Pumping Lemma, there must be a loop in the DFA.
- Therefore, we can pump (i.e., repeat) that loop any number of times to generate strings of arbitrary length, meaning A accepts an infinite number of strings.

4. Test whether $L(M) = \emptyset$ using the `EDFA` decider T:

   - If T says $L(M) = \emptyset$ (i.e., no string of length $\geq k$ is accepted), then A is finite, so **reject**.
   - Otherwise, A accepts infinitely many strings (since some strings of length $\geq k$ are accepted), so **accept**.

$$\Rightarrow \text{If } T \text{ accepts, reject; if } T \text{ rejects, accept.}$$

`EDFA` is a decidable language

Decider T:

On input $\langle A \rangle$, where A is a DFA:

1. Mark the start state q_0.
2. Repeat until no new states get marked:
   - Explore all states that can be reached from the start state.
3. Mark any state that has an incoming transition from an already marked state:
   - If a state is reachable from a marked state, we also mark it.
4. Final check:
   - If no accept state is marked, it means no accepting state is reachable. So $L(A) = \emptyset$ $\Rightarrow$ **accept**.
   - Otherwise, if at least one accept state is marked, then A accepts some strings. So $L(A) \neq \emptyset \Rightarrow$ **reject**.