CSL 101- Discrete Mathematics
Indian Institute of Technology Bhilai
Tutorial Sheet 7

1. Give an example of a non abelian group with 6 elements.

2. Let $(G, *)$, and $(H, \#)$ be two groups. Consider the set G X H and a binary operation Δ on G X H defined as $(g_1, h_1)\Delta(g2, h2) = (g1 * g2, h1\#h2)$. Show that ($G$ X H, Δ) is a group.

3. Let p, q, and r be the propositions: p: You have the flu. q: You miss the final examination. r: You pass the course. Express each of these propositions as an English sentence. Also, find the contra-positive, converse, and inverse of each applicable proposition.
   (a) $p \to q$
   (b) $\neg q \to r$
   (c) $q \to \neg r$
   (d) $p \lor q \lor r$
   (e) $(p \to \neg r) \lor (q \to \neg r)$
   (f) $(p \land q) \lor (\neg q \land r)$

4. Explain, without using a truth table, why $(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$ is true when p, q, and r have the same truth value and it is false otherwise.

5. Explain, without using a truth table, why $(p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

6. Show that each of these conditional statements is a tautology by using truth tables and developing a series of logical equivalences.
   (a) $[\neg p \land (p \lor q)] \to q$
   (b) $[(p \to q) \land (q \to r)] \to (p \to r)$
   (c) $[p \land (p \to q)] \to q$
   (d) $[(p \lor q) \land (p \to r) \land (q \to r)] \to r$

(e) $(p \wedge q) \rightarrow p$

(f) $p \rightarrow (p \vee q)$

(g) $\neg p \rightarrow (p \rightarrow q)$

(h) $(p \wedge q) \rightarrow (p \rightarrow q)$

(i) $\neg(p \rightarrow q) \rightarrow p$

(j) $\neg(p \rightarrow q) \rightarrow \neg q$

7. Show that the following are logically equivalent:

(a) $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

(b) $\neg(p \wedge q)$ and $p \rightarrow \neg q$

(c) $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$

(d) $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$

(e) $\neg(p \oplus q)$ and $p \leftrightarrow q$

(f) $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$

(g) $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$

(h) $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$

(i) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$

(j) $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

(k) $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$

(l) $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

8. Determine whether each of these compound propositions is satisfiable.

(a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

(b) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$

(c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

(d) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$

(e) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg r)$