

Hamiltonian Cycle Problem

Overview

A Hamiltonian cycle in a graph is a cycle that visits every vertex exactly once and returns to the starting vertex. Unlike the Eulerian cycle, which covers every edge once, the Hamiltonian cycle focuses on covering all vertices exactly once. This problem is fundamental in graph theory.

Problem Statement

Given a graph $G = (V, E)$, determine whether there exists a cycle that visits every vertex exactly once and returns to the starting point. Such a cycle is called a Hamiltonian cycle.

Example

Consider a square graph with 4 vertices $\{1, 2, 3, 4\}$ connected in a cycle: Edges = $\{(1,2), (2,3), (3,4), (4,1)\}$. In this case, the Hamiltonian cycle is: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$. If we take a star graph with one center vertex and 4 leaves, there is no Hamiltonian cycle because the leaves cannot be visited exactly once and return back without repetition.