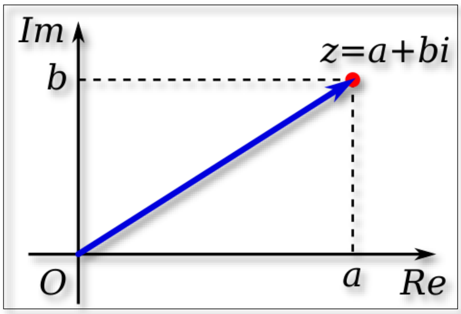
THE STORY OF COMPLEX NUMBERS

1. Definition of complex numbers :

In mathematics, a complex number is an element of a number system that extends the real numbers with a specific element denoted “i” called the imaginary unit and satisfying the equation (i^2=-1) every complex number can be expressed in the form (a+ ib) where a and b are real numbers. Because no real number satisfies the above equation.



A complex number can be visually represented as a pair of numbers (a, b) forming a vector on a diagram called an Argand diagram, representing the complex plane. Re is the real axis, Im is the imaginary axis, and i is the "imaginary unit"

2. The need for a Complex Number

In the fifteenth century, Europe experienced intellectual excitement with Western civilization awakening from the Middle Ages. Johannes Gutenberg invented the printing press in 1450, and universities at Bologna, Paris, Oxford, and elsewhere became centers of higher education and scholarship. Italian artists like Raphael and Michaelangelo began extraordinary artistic careers, while Leonardo da Vinci gave meaning to the term Renaissance man. Christopher Columbus' discovery of the Americas in 1492 demonstrated the frontiers of knowledge beyond the classical world.

And so it was in mathematics. Italian Luca Pacioli (ca. 1445-1509) presented a cubic equation ($ax^3+bx^2+cx+d=0$) that he believed was unsolvable.But SCipione del Ferro (1465-1526) discovered a formula that solved the "depressed cubic." After his death his student

Antonio Fior (ca. 1506-?) challenged Niccolo Fontana (1499-1557), nicknamed “Tartaglia”, through these solutions.Tartaglia won.

Gerolamo Cardano (1501-1576) shared Tartaglia's secret given by promise of confidentiality, with his brilliant young protégé Ludovico Ferrari (1522-1565), and together the two of them made astounding progress. For instance, Cardano discovered how to solve the general cubic equation. And so, in the year 1545, there appeared Cardano's mathematical masterpiece “Ars Magna”.Accordingly the concave cube ($x^3 + mx = n$) as solutions,

$$x = \sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} - \sqrt[3]{-\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$

When solving equations some times getting square roots of negative numbers was problematic. At that time Rafael Bombelli (ca. 1526 1573), in his 1572 treatise Algebra brought some idea about complex numbers.

3. Birth

Bombelli so established the groundwork for the theory of complex numbers. Numerous textbooks, even for college students, imply that the solution of quadratic equations, particularly the equation $x^2 + 1 = 0$, gave rise to complex numbers.complex numbers were introduced because of the cubic equation rather than the quadratic equation.

4. Growth

Despite doubts about the meaning and legitimacy of complex numbers for two and a half centuries, they were extensively used and theoretical work was conducted. Albert Girard(1620) suggested that an equation of degree n may have n roots in 1620, but these statements were vague and unclear. Rene Descartes coined the term "imaginary" for the new numbers, stating that although every equation has as many roots as indicated by its degree, no real numbers correspond to some of these imagined roots. Gottfried Wilhelm von Leibniz

(1646-1716) focused on the meaning and application of complex numbers, viewing them as a divine sanctuary and often used alongside imaginary numbers in the process of integration. The controversy surrounding complex numbers was resolved by Leonhard Euler(1707-1783) in the late 18th century. Complex numbers were used by Johann Lambert for map projection, Jean D'Alembert in hydrodynamics, and Euler, D'Alembert, and Joseph-Lonis Lagrange in incorrect proofs of the fundamental theorem of algebra. Karl Friedrich Gauss(1777-1855) gave the first essentially correct proof of the fundamental theorem of algebra in 1797, but claimed that "the true metaphysics of square root of (-1) is elusive."

The logical justification of the laws of operation with negative and complex numbers became a pressing pedagogical issue at Cambridge University during the nineteenth century. By 1831, Gauss published his results on the geometric representation of complex numbers as points in the plane, dispelling much of the mystery surrounding complex numbers. William Rowan Hamilton(1805-1865) gave an essentially rigorous algebraic definition of complex numbers as pairs of real numbers in 1833, and Augustin-Louis Cauchy provided a rigorous and abstract definition of complex numbers in terms of congruence classes of real polynomials modulo $x^2+ 1$.

5. Maturity

During the late 19th century, the mystery and distrust of complex numbers disappeared, but some textbook writers still held a lack of confidence in them into the 20th century. They often supplemented imaginary number proofs with non-involved ones, allowing complex numbers to be viewed differently.

Complex numbers could now be viewed in the following ways:

1. Points or vectors in the plane

2. Ordered pairs of real numbers

3. Operators (i.e., rotations of vectors in the plane)

4. Numbers of the form a + bi, with a and b real numbers

5. Polynomials with real coefficients modulo $x^2 + 1$

6. Matrices of the form

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix},$$

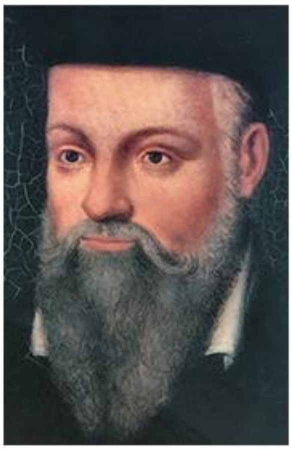
with a and b real numbers

7. An algebraically closed, complete field

Complex numbers are crucial in mathematics for understanding concepts, results, and theories. 19th-century developments allowed for deeper understanding of their role in algebra, analysis, geometry, and number theory. Masters like Gauss, Riemann, and Hadamard have made significant contributions, highlighting the importance of complex numbers in various contexts and perspectives.

- **Pioneers in constructing complex numbers**

1. Jerome Cardan (1501-1576)



2. Gottfried Wilhelm von Leibniz (1646-1716)

3. Leonhard Euler (1707-1783)

4. Karl Friedrich Gauss (1777-1855)

5. William Rowan Hamilton (1805-1865)

6. George P61ya (1887-1985)