Consequently,

$$\langle \phi_n^R, \phi_m^R \rangle_{L^2(\mathcal{B}(0,R))} = \delta_{n,m} \frac{\sqrt{(n+1)(m+1)}}{\pi} \frac{1}{R^{n+m+2}} 2\pi \frac{R^{2n+2}}{2n+2} = \delta_{n,m}.$$

This prooves that this family is an orthonormal family of eigenfunctions of the integral operator K^R associated to k^R, and we have hence found its Mercer decomposition. □

Remark 14. The fact that the eigenvalues and eigenfunctions of the Bergman DPP are explicitely computable makes it very manipulatable. Let us mention that restricting any DPP on any compact rarely ever gives birth to such well-behaving results. The only DPP that was ever discovered to enjoy such properties if the Ginibre DPP as shown in [8].

We will now make use of Optimal transport tools and show that, when choosing the right number of points (which we will exhibit), the law induced by the truncated version of this kernel is close (in the Wassertein sense) to the non-truncated restriction.

Let X and Y be two Polish spaces. Let μ and ν be probability measures on X and Y respectively. Denote $\Pi(\mu, \nu)$ the set of probability measures on $X \times Y$, the first marginal of which is μ and the second ν. If c is a lower semi-continuous function from $X \times Y$ to $\mathbf{R}^+$, the Monge-Kantorovitch problem asks to find

$$\inf_{\gamma \in \Pi(\mu,\nu)} \int_{X \times Y} c(x, y) \mathrm{d}\gamma(x, y).$$

We refer to [25] and [26] for proper introductions to the topic of Optimal transport.

Definition 15. Observing that the cardinality of the symmetric difference $\mathrm{d}(\xi, \zeta) = |\xi \Delta \zeta|$ induces a distance on the set of configurations on a subset of $\mathbf{C}$, the Kantorovitch-Rubinstein distance is the Wasserstein-1 distance induced by d, that is

$$\mathcal{W}_{KR}(\mu, \nu) = \inf_{\substack{\mathrm{law}(\xi)=\mu \\ \mathrm{law}(\zeta)=\nu}} \mathbf{E}(|\xi \Delta \zeta|) = \inf_{\substack{\mathrm{law}(\xi)=\mu \\ \mathrm{law}(\zeta)=\nu}} \mathbf{E}(\mathrm{d}(\xi, \zeta)).$$

It is a distance on the set of point process laws. See [5] for a thorough study of Wasserstein distances on configuration spaces. The following result exhibits a good compromise as to the number of points the Bergman ought to be truncated to.

Theorem 16. Let

$$N_R := \sum_{n=0}^{\infty} R^{2n+2} = \frac{R^2}{(1-R)(1+R)}$$

Denote $\mathfrak{S}^R$ the law of the restricted Bergman to a compact ball of radius R centered at 0 and $\mathfrak{S}_\alpha^R$ the law of its truncation to α points.

If we truncate it to βN_R points, we have

$$\mathcal{W}_{KR}(\mathfrak{S}^R, \mathfrak{S}_{\beta N_R}^R) \leqslant N_R e^{-2\beta g(R)} \tag{2}$$

where