Proposition 24. Denoting $\mathfrak{S}$ the original Bergman DPP, we have

$$\mathfrak{S}^R_{N_R} \xrightarrow[R\to 1^-]{} \mathfrak{S}$$

in distribution.

Proof.

Denote k^R_N the restricted-truncated (to N points) kernel and k the asymptotic one. Consider a compact $\Lambda \subset D(0,1)$. Recall that the Bergman kernel $k(x,y) = \frac{1}{\pi}\frac{1}{(1-x\overline{y})^2}$ is defined on the *open* disc $D(0,1)$ (this relation would not make sense on some points of the boundary).

Since $D(0,1)$ is open, denoting $m := \max\limits_{z\in\Lambda} |z|$, we have $m < 1$. So, we have

$$\begin{aligned}
\left\| k^R_{N_R} - k \right\|_{\infty,\Lambda} &= \left\| \frac{1}{\pi} \sum_{k=N_R}^{\infty} (k+1) x^k \overline{y}^k \right\|_{\infty,\Lambda} \\
&\leqslant \frac{1}{\pi} \sum_{k=N}^{\infty} (k+1) \left\| x^k \overline{y}^k \right\|_{\infty,\Lambda} \\
&\leqslant \frac{1}{\pi} \sum_{k=N}^{\infty} (k+1) m^{2k} \xrightarrow[N\to\infty]{} 0
\end{aligned}$$

We conclude using Theorem 25. $\square$

Theorem 25. (Proposition 3.10, [18])

Let $(K_n)_{n\geqslant 0}$ be integral operators with nonnegative continuous kernels $k_n(x,y)$. Assume that K_n are bounded hermitian locally trace class integral operators. Assume that $(k_n)_{n\geqslant 0}$ converges to a kernel k uniformly on each compact as $n \to \infty$. Then, the kernel k defines an integral operator K that is also a bounded hermitian locally trace class.

Then the determinantal measures, i.e. the DPP probability laws μ_n associated to the DPP induced by the integral operators K_n weakly converge to the determinantal probabilty measure μ induced by K.

IV The Bergman DPP on an annulus

We now consider a variant of the previous restriction. A sight at Figure 1 in the introduction shows that the points in the Bergman DPP are very highly repelled from the center, and very intensely concentrated at the border of the disc they're living in. The following was shown in [17] :

Theorem 26. The law of the set of the moduli $\{|X_k|, k \geqslant 1\}$ of the points that come out from the Bergman determinantal point process is exactly the law of the set

$$\{U_k^{1/(2k)}, k \geqslant 1\}$$

where $(U_k)_{k\geqslant 1}$ is a sequence of independent, uniform in $[0,1]$ random variables.

Remark 27. (Conjecture)