Equivalently, this means that the integral operators K^Λ are self-adjoint for any compact set $\Lambda \subset E$. If K^Λ is self-adjoint, by the spectral theorem for self-adjoint and compact operators we have that $L^2(\Lambda, \lambda)$ has an orthonormal basis $(\phi_j^\Lambda)_{j\geqslant 0}$ of eigenfunctions of K^Λ. The corresponding eigenvalues $(\lambda_j^\Lambda)_{j\geqslant 0}$ have finite multiplicity (except possibly the zero eigenvalue) and the only possible accumulation point of the eigenvalues is zero. In that case, Mercer's theorem indicates that the kernel k^Λ of K^Λ can be written

$$k^\Lambda(x,y) = \sum_{n\geqslant 0} \lambda_n^\Lambda \phi_n^\Lambda(x)\overline{\phi_n^\Lambda(y)},$$

where the $(\phi_k^\Lambda)_{k\geqslant 0}$ form a Hilbert basis of $L^2(E, \lambda)$ composed of eigenfunctions of K^Λ.

Recall that K is positive if its spectrum is included in $\mathbf{R}^+$, and is of trace-class if

$$\sum_{n=1}^{\infty} |\lambda_n| < \infty,$$

its trace is then $\operatorname{Tr}(K) := \sum\limits_{n=1}^{\infty} \lambda_n$.

Definition 7. If K^Λ is of trace-class for all compacts Λ, K is said to be locally trace-class.

Hypothesis 8. Throughout this paper, our kernels will be self-adjoint, locally trace class, with spectrum contained in $[0, 1]$.

We refer to [2] and [3] for further developments on these notions.

Definition 9. A locally finite and simple point process on E is a determinantal point process if its correlation functions with respect to the reference Radon measure λ on E exist and are of the form

$$\rho_n(x_1, ..., x_n) = \det(k(x_i, x_j))_{1\leqslant i,j\leqslant n},$$

where k satisfies Hypothesis 8.

The dynamics of DPPs are described by the following fundamental theorem.

Theorem 10. Under the aforementionned assumptions, consider Mercer's decomposition of the kernel k of the determinantal point process η:

$$k(x,y) = \sum_{k=1}^{n} \lambda_k \phi_k(x)\overline{\phi_k(y)}.$$

Here, the eigenvalues are all in $[0, 1]$ and can all be chosen in $(0, 1]$; n is equal to the rank of K, which can be either finite or infinite. The (ϕ_n) form a Hilbert basis is the space $L^2(E, \lambda)$. Consider then a sequence of independent Bernoulli random variables $(B_k)_{1\leqslant k\leqslant n}$, and consider the random kernel

$$k_B(x,y) = \sum_{k=1}^{n} B_k \phi_k(x)\overline{\phi_k(y)},$$

Then the point process η_B with (random) kernel k_B has the same law as that of k :

$$k \overset{\text{Law}}{=} k_B.$$