Corollary 11. If the n above is finite, then the point process has a finite number of points almost surely. By independence, the Borel-Cantelli lemmas imply that the number of points is almost surely finite if and only if K is of trace class. Since the integral operators that we will consider will all be locally of trace-class, the restrictions of our DPPs to compact subsets will all have a finite number of points almost surely.

For more details, we refer to [10] and [18].

Definition 12. The Bergman determinantal point process is the determinantal point process on the open unit disc centered at the origin of the complex plane with kernel

$$k(x, y) = \frac{1}{\pi} \frac{1}{(1 - x\overline{y})^2}.$$

See [17] and [11] for a thorough study.

The key motivation of this article is the algorithm that was introduced and studied in [7], [8]. It is recalled down below. Its goal is to simulate a DPP restricted to a ball centered at the origin and of radius R. It assumes that the set

$$I = \{n \geqslant 0, B_n = 1\},$$

of "active" Bernoulli random variables has been computed, and computes the positions of the points $(X_k)_{k \in I}$. Recall that this makes sense because restricting a DPP to a compact reduces the number of points to almost surely finite.

Since simulating a countable infinity of Bernoulli random variables is unfeasible, we introduce the truncation to N points of the restricted DPP, which is defined by its kernel

$$k_N^\Lambda = \sum_{k=0}^{N-1} \lambda_n^\Lambda \phi_n^\Lambda(x) \overline{\phi_n^\Lambda(y)},$$

which is the only one that's numerically feasible.

III The Bergman DPP on a disc

The big question is now to know to what extent this truncation greatly affects the simulation of the DPP law (or if the error is only a minor issue). We provide the theoretical results that answer the last question in the negative for the Bergman DPP (and partially for general DPPs). This is the question that was asked in [6].

Notation. Throughout this paper and unless expressly stated otherwise, R and r denote real numbers in the interval $(0, 1)$ such that $r < R$.

Let us first contruct the restriction of the Bergman DPP to a compact ball centered at the origin with radius R.

Proposition 13. Denote $\mathcal{B}(0, R)$ the compact ball centered at 0 with radius R. Mercer's decomposition of the kernel $k^R(x, y)$ of the Bergman determinantal point process restricted to $\mathcal{B}(0, R)$ is

$$k^R(x, y) = \sum_{n \geqslant 0} \lambda_n^R \phi_n^R(x) \overline{\phi_n^R(y)},$$