# ISE 5113 Advanced Analytics and Metaheuristics
# Homework #2

Instructor: Charles Nicholson

Assigned: February 11, 2016
Due: February 22, 2016

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**Requirement details**

**Due:** February 22, 2016 at 11:59 PM

1. Homeworks should be submitted in a clean, clear, concise electronic format. You must show your logic, work, and/or code where appropriate.
2. Homeworks are to be completed in teams of two. If team members disagree on an answer, you can record solutions corresponding to each member (please clearly mark which solution belongs to which team member)
3. Any code (e.g. AMPL) is part of your solution – make sure to provide comments on what your code is doing. Keep it clean and clear!
4. For any mathematical programming problem, in addition to solving the problem and responding to the questions, please ensure you clearly define the following elements in the your answer: (i) **any necessary assumptions**, (ii) **decision variables**, (iii) **objective and objective function**, and (iv) **constraints**.
5. When formulating a network flow problem you **must provide a graphical depiction** in which the appropriate information is clearly noted (e.g. arc capacities, costs, node requirements, etc.)
6. Required software: AMPL

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**Question 1**: Totally Unimodular Matrices (4 points)

Are the following matrices totally unimodular or not?

$$A_1 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} -1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}$$

**Question 2**: CHRISTMAS PARTY AND SUMMER CAMP (12 points)

The intention of this question is that you formulate parts (a) and (b) as integer programs and in part (c) decide whether or not the constraint matrices of your formulations are TU. That is, will they be TU for *every* instance of these problems.

(a) For their annual Christmas party the thirty members of staff of the thriving company Ipopt were invited/obliged to dine together and then spend the night in a fancy hotel. The administrative assistant had the unenviable task of allocating the staff two to a room. Knowing the likes and dislikes of everyone, she drew up a list of all the compatible pairs. How could you help her to fill all fifteen rooms?

(b) Recently a summer camp was organized for an equal number of English and French children. After a few days, the children had to participate in an orienteering competition in pairs, each pair made up of one French and one English child. To allocate pairs, each potential pair was asked to give a weight from 1 to 10 representing their willingness to form a pair. Formulate the problem of choosing pairs so as to maximize the sum of the weights.

(c) If you have a LP solver available, can you help either the administrative assistant or the camp organizer or both?

**Question 3**: INFORMS ANNUAL MEETING AND THE DEMAND FOR CLEAN TABLE CLOTHS (24 points)

*Lenny's Lace and Linens* must provide tablecloths for the nightly dinner reception at the INFORMS Annual Meeting this year. The Annual Meeting will last 5 days, and each day they require a different number of clean tablecloths for the dinner guests. The catering company can buy new tablecloths at a price of \$8 dollars each, or launder the used ones.

Laundering can be done at *Willie's Washateria* which cleans and returns the tablecloths the next morning. They charge \$3.75 for each tablecloth.

Another laundromat, *Carl's Casino and QuikClean*, is much cheaper but slower. At a price of \$1.25 each, the tablecloths will be cleaned and ready in two days.

Finally, *Jimmy's Janitors R Us* is in charge of clean-up each day. For a cost of only \$0.75 each, they will "clean" the tablecloths. JJrUs's can only handle 20 tablecloths a day. They take 3 days to return the tablecloths.

*Lenny's Lace and Linens* has no tablecloths in inventory and any leftover tablecloths have no value. The daily clean table cloth requirements are in listed in Table 1.

Table 1: INFORMS Daily Table Cloth Needs

| Day | Requirements |
|---|---|
| 1 | 400 |
| 2 | 375 |
| 3 | 515 |
| 4 | 395 |
| 5 | 620 |

The problem is to decide how to meet INFORMS's table cloth requirement at a minimum cost. Model and solve this as a network flow problem.

**Question 4**: Silos (15 points)

A farmer owns 4 commodities which he must store in 7 different silos. Each silo can contain at most one commodity. Associated with each commodity and silo combination is a loading cost. Each silo has a finite capacity so some commodities may have to be split over several silos. Formulate and solve this problem to find a minimum cost storage plan.

| Commodity | Silo-Loading Cost per Ton | | | | | | | Amount of Commodity (tons) |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| A | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 75 |
| B | 2 | 3 | 3 | 3 | 1 | 5 | 5 | 50 |
| C | 4 | 4 | 3 | 2 | 1 | 5 | 5 | 25 |
| D | 1 | 1 | 2 | 2 | 3 | 5 | 5 | 80 |
| Silo Capacity (tons) | 25 | 25 | 40 | 60 | 80 | 100 | 100 | |

**Question 5**: Mix Tape (15 points)

Back in the olden days, people recorded music on a piece of technology known as a cassette tape (see image below). The device could store music on both "sides" of the device; generally referred to as Side A and Side B. The music storage amount was quite small with respect to current standards and is related to the length of the tape held in the cassette device.


Figure 1: Ancient music storage device

The "Oldie-but-Goodie" company is in the process of releasing a cassette tape with 10 songs on it. The length of tape required for each song is given in the table below. The tape will have 2 sides, A and B. Formulate and solve an integer program to assign songs to each side of the tape so that the total length of the tape is minimized.

| Song | Length (cm.) |
|---|---|
| 1 | 54 |
| 2 | 68 |
| 3 | 73 |
| 4 | 45 |
| 5 | 29 |
| 6 | 56 |
| 7 | 35 |
| 8 | 37 |
| 9 | 55 |
| 10 | 71 |

**Question 6**: Boomer Global Air Services (30 points)

Li Chase set about the task of preparing a fuel plan for her upcoming four-leg flight to Boston, MA; New York City, NY; Dallas, TX; and back. Like the other 13 corporate pilots she worked with, Chase enjoyed flying a lot more than doing paperwork. But unlike some of her colleagues Chase rather enjoyed the challenge of constructing a fuel plan.

Boomer Global Air Services (BGAS) operated four aircraft to serve the transportation needs of the corporate headquarters of the Boomer, Inc. Located on a 1,415-acre campus in Moline, IL the headquarters housed the executive and administrative staff of Boomer's divisions along with a wide array of company-wide functions. Close to 2,400 Boomer employees worked at headquarters. Company executives routinely visited BGAS to fly to company factories, marketing facilities, and customer locations throughout the world.

The company's largest and most expensive aircraft, the Gulfstream GV, had a range of 6,000 nautical miles. Purchased in 2007, it was flown throughout the world, including the growth areas of India and China. It could carry up to 13 passengers, a flight attendant, and two or three pilots. It burned fuel at a rate of approximately 450 gallons per hours. The firm owned and operated two Cessna Citation X aircraft (CE750), which it had purchased in 2008 and 2010. The CE750 (Figure 2) was the fastest nonmilitary plane in the world and often went from Moline to as far as South America, Europe, and western Russia – a larger plane than most small jets. Its fuel burn rate of 310 gallons per hour coupled with its 13,000 pound capacity tank meant that it required a fuel stop to reach these more distant destinations. It carried up to eight passengers and two pilots.

Figure 2: Cessna Citation X aircraft

The company's newest aircraft was a 2013 Cessna Citation Sovereign (CE680). Used only within North America, this craft carried up to eight passengers and burned fuel at approximately 270 gallons per hour.

Each of the four aircraft was budgeted for 650 flight hours per year, and BGAS had an annual budget of $22 million– less than 0.1% of company sales. The department consisted of 14 pilots (including the department manager and two pilot managers), six maintenance technicians, and four support staff members who were responsible for scheduling and office support.

## The Upcoming Flight

In two days, the CEO and CFO of Boomer, Inc. had a trip scheduled for Moline to Boston, the New York City areas, Dallas, and then back to Moline. The purpose of the trip was to pick up some key analysts and mutual fund managers in Boston and New York and show then the new Boomer factory in Dallas and the new Boomer distribution center in Moline. They would be picking up two passengers in Boston and four in New York.

As usual, BGAS would use the airport in Teterboro, New Jersey, as their destination in the New York

City area; it was the closest airport to Wall Street, Manhattan, and the Lincoln Tunnel. Each U.S. airport carried a four-letter identifier beginning with the letter K. The upcoming four-leg flight would go from KMLI to KBOS to KTEB to KDAL and back to KMLI. Pilots at BGAS were responsible for creating and filing their own flight plans with the U.S. Federal Aviation Administration (FAA).

One element of the flight plan was the takeoff and landing weight of the aircraft. To calculate these, one started with the basic operating weight (BOW) of the aircraft and added the weight of the passenger and fuel. The BOW included the structure of the aircraft, a stocked galley, emergency equipment, and the crew. The only weight components that varied from flight to flight were passengers and fuel. The only component the varied from takeoff to landing on a given flight was fuel. (For the purposes of this case study, we ignore the possibility of executive skydiving.)

### Tinkering with Tankering

This meant that one of Chase's first tasks was to determine a fueling plan for the upcoming flights. Coming up with a fuel plan was not a joyful task for pilots because there was no straightforward way to calculate how much fuel to take on or "upload" at the beginning of each leg. One question was whether or not to "tanker."

Tankering referred to a practice in which extra fuel was uploaded initially to avoid having to purchase higher-priced fuel at destination airports. BGAS operated its own fuel farm at Moline's KMLI, which kept its fuel costs low. Fuel at KMLI at the time cost $3.97 a gallon. In contrast, fuel purchased at KBOS cost $8.35 a gallon. As a simple example of tankering, Chase could decide to upload enough fuel at KMLI to carry him through both of the first two legs, thereby avoiding buying fuel at KBOS. In essence, BGAS would carry or tanker from KMLI the fuel needed to fly from KBOS to KTEB.

One factor that worked against the tankering was ramp fees. Ramp fees were fixed fees charged to each landing jet by the destination airport's general-aviation terminal; the fees covered the costs of operating the terminal. The ramp fee at KBOS was $800. The fee was waived with the purchase of 500 or more gallons of fuel.

To begin the process of constructing a fuel plan, Chase assembled the information in **Table 2**. The fuel burn numbers were fairly easy to calculate based on flight miles and aircraft. (The burn numbers included the fuel used during taxiing at the departing airport.) Although the calculation was more complicated than just multiplying miles by average gallons per mile (because extra fuel was used at takeoff), most pilots could do the calculation in their heads. Fuel prices, ramp fees, and minimum gallons needed to waive the ramp fees could all be found on the Internet.

Table 2: Flight Details

| Leg | Depart | Arrive | Miles | Duration (hh:mm) | Fuel burn incl. taxi (pounds) | Fuel price ($/gallon) | Ramp fee | Minimum gallons to waive fee |
|---|---|---|---|---|---|---|---|---|
| 1 | KMLI | KBOS | 890 | 2:00 | 4,800 | $3.97 | | |
| 2 | KBOS | KTEB | 176 | 0:40 | 2,000 | $8.35 | $800 | 500 |
| 3 | KTEB | KDAL | 1,202 | 2:55 | 5,300 | $7.47 | $450 | 300 |
| 4 | KDAL | KMLI | 628 | 1:35 | 3,100 | $6.01 | $400 | 350 |

In addition to the cost of fuel and ramp fees, Chase needed to consider the limitation of the CE750 **Table 3**). The fuel tank capacity was a firm physical limit, and the departure ramp and lading weight limit were structural limits developed by the manufacturer and approved by the FAA during aircraft certification. To calculate departing ramp or arrival weight, Chase added BOW to the weight of the fuel and the weight of the passengers (passenger weight calculations were based on a company-mandated figure of 200 pounds per person, including luggage).

Table 3: Aircraft Limitations (in pounds)

| Aircraft | CE750 | GV |
|---|---|---|
| Maximum Ramp Weight | 36,400 | 90,900 |
| Maximum Landing Weight | 31,800 | 75,300 |
| BOW* | 22,200 | 48,800 |
| Fuel Tank Capacity | 13,000 | 41,300 |

*BOW = basic operating rate of the aircraft, including crew and excluding the weight of fuel and passengers.

There were two final considerations. The company specified that aircraft always land with at least 2,400 pounds of fuel. Any fuel plan Chase would have to be one in which the weight of fuel at arrival met or exceeded 2,400 pounds. This "safety stock" was there to ensure jets had enough fuel to make it to an alternate airport should there be bad weather at the destination airport. The second consideration was that the company dictated immediately bringing the fuel level up to 7,000 pounds upon arrival back at KMLI. The rationale for this was that the aircraft would always be ready to go at a moment's notice. This meant the Chase's fuel plan should begin with the CE750 containing 7,000 pounds of fuel. (For flights using the larger Gulfstream GV aircraft, the policy was always to land with at least 4,500 pounds of fuel and bring its fuel level up to 8,700 pounds upon arrival at KMLI.)

As Chase prepared to put pencil to paper to create a fuel plan for the upcoming KMLI to KBOS to STEB to KDAL to KMLI trip, he paused to ponder why aircraft gauges measured fuel in pounds and yet fuel was sold in gallons. Like every other pilot at BGAS, he knew the importance of the number 6.7 – the weight in pounds of a gallon of jet fuel.

(a) Formulate the problem and solve for an optimum fuel plan for Chase for the upcoming trip.
   i. Compare your results with a no "tankering" solution.
   ii. What are the most important limitations of the model? How might these be addressed?

(b) Suppose the BGAS department manager wished to modify the model to require that "if you buy any gas, you must buy at least 100 gallons".
   i. Formulate the problem with this modification.
   ii. How does this change the solution and cost?