lecture 3 exercises (ESSLLI)

Shravan Vasishth

August 13, 2015

Problem 5 (Conjugacy)

The Gamma distribution is defined in terms of the parameters a, b: Ga(a,b). The probability density function is:

$$Ga(a,b) = \frac{b^a \lambda^{a-1} \exp\{-b\lambda\}}{\Gamma(a)} \tag{1}$$

We have data $x_1, \dots, x_n$, with sample size n that is exponentially distributed. The exponential likelihood function is:

$$f(x_1, \dots, x_n; \lambda) = \lambda^n \exp\{-\lambda \sum_{i=1}^{n} x_i\} \tag{2}$$

It turns out that if we assume a Ga(a,b) prior distribution and the above likelihood, the posterior distribution is a Gamma distribution. Find the parameters a' and b' of the posterior distribution.

Problem 6 (GLMs)

This problem is based on the lecture 2 material. The Poisson distribution belongs to the exponential family.

$$Y \sim Po(\exp(\mu)) \quad f(y) = \frac{\mu^y \exp(-\mu)}{y!} \tag{3}$$

Write the likelihood function in the standard exponential form (as shown in the slides for the normal and binomial distributions).

Problem 7 (MCMC sampling exercise)

Suppose we have 10 successes from a sample size of 100, assuming a binomial process. Instead of a beta prior on θ, we could could use a non-conjugate prior on a transformation of θ: $logit(\theta) \sim N(\mu, \omega^2)$. Let $\omega^2 = 2.71$. Figuring out the posterior distribution of θ is not possible analytically; but MCMC methods allow us to sample from the posterior.

```
## the data:
data<-list(y=10,n=100)

cat("
model
   {
    ## likelihood
    y ~ dbin(theta,n)
    ## prior
    logit(theta) <- logit.theta
    ## precision 0.368 = 1/2.71
    logit.theta ~ dnorm(0,0.368)
   }",
     file="mcmcexample1.jag" )


track.variables<-c("theta")
library(rjags)

## Linked to JAGS 3.4.0
## Loaded modules:  basemod,bugs

mcmceg.mod <- jags.model(
  file = "mcmcexample1.jag",
                     data=data,
                     n.chains = 1,
                      n.adapt =2000 , quiet=T)

mcmceg.res <- coda.samples( mcmceg.mod,
                            var = track.variables,
                            n.iter = 10000,
                            thin = 20 )

summary(mcmceg.res)

##
## Iterations = 2020:12000
## Thinning interval = 20
## Number of chains = 1
## Sample size per chain = 500
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
```

```
##           Mean             SD       Naive SE Time-series SE
##        0.10667        0.02923        0.00131        0.00131
##
## 2. Quantiles for each variable:
##
##   2.5%    25%    50%    75%    98%
## 0.0571 0.0861 0.1045 0.1264 0.1692
```

```
densityplot(mcmceg.res)

## Error in eval(expr, envir, enclos):  could not find function "densityplot"
```

Assignment: Modify the above code so that (a) the prior for θ is $Unif(0, 1)$, and compute the posterior predictive distribution of y (call it y.pred) for 100 future observation. Does the posterior distribution of θ change? Are the predicted values reasonable?

Solution to problem 5

I'll do this later (no time!).

Solution to problem 6

We are given that

$$Y \sim Po(\exp(\mu)) \quad f(y) = \frac{\mu^y \exp(-\mu)}{y!} \tag{4}$$

$$f(y) = \exp\left[\log \frac{\mu^y \exp(-\mu)}{y!}\right] \tag{5}$$

Rewrite this as:$\exp\left[ylog\mu - \mu - \log y!\right]$. If we set $\theta = \log\mu$ then $\mu = \exp(\theta)$. Therefore,

$$\exp\left[y\log\mu - \mu - \log y!\right] = \exp\left[(y\theta) - \exp(\theta) + \log y!\right] \tag{6}$$

Let $b(\theta) = \exp(\theta)$, and $c(y, \phi) = \log y!$.

This gives us $\exp\left[\frac{y\theta - b(\theta)}{\phi/w} + c(y, \phi)\right]$.

Solution to problem 7

I'll do this later (no time!).