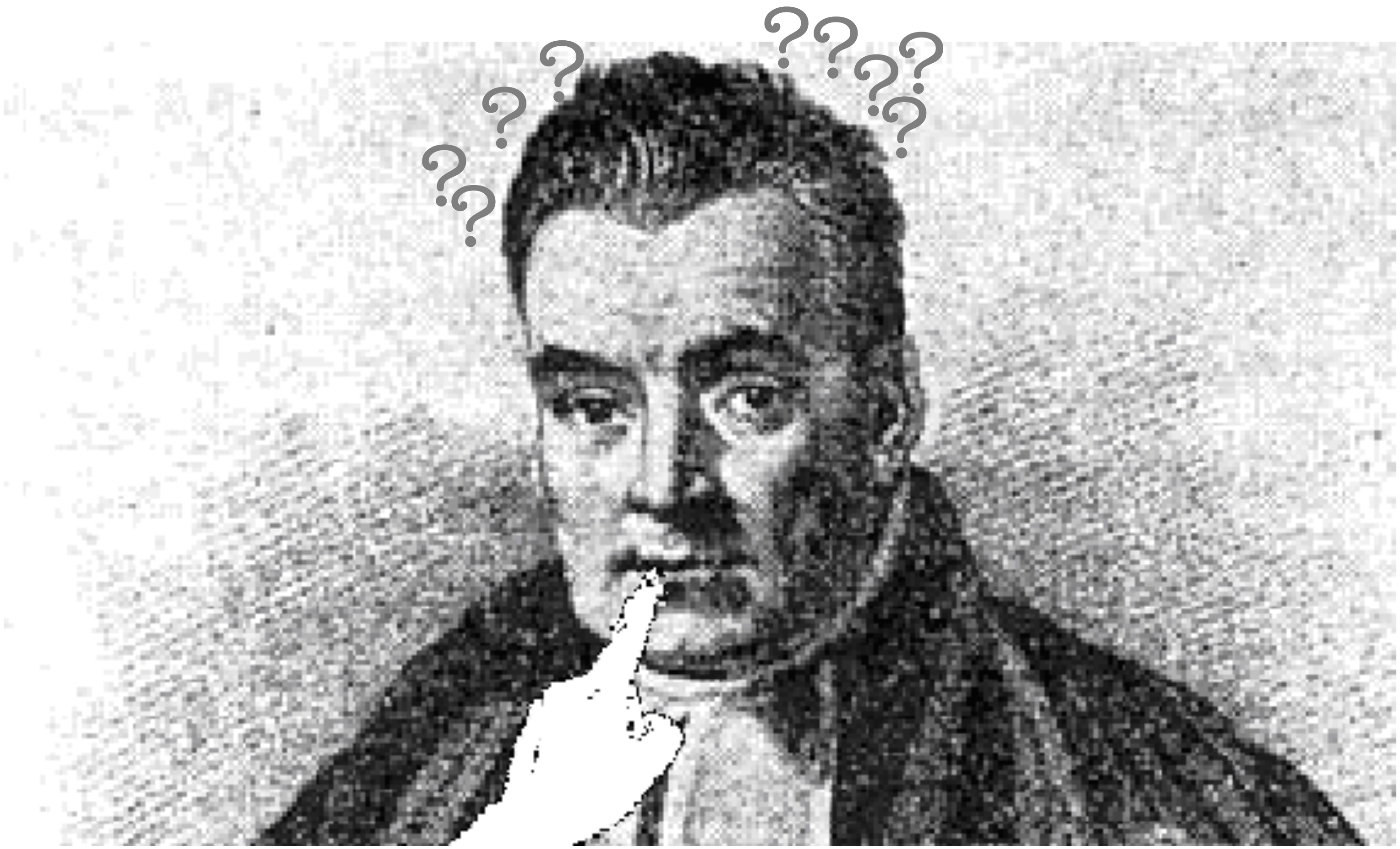
Naive Bayes

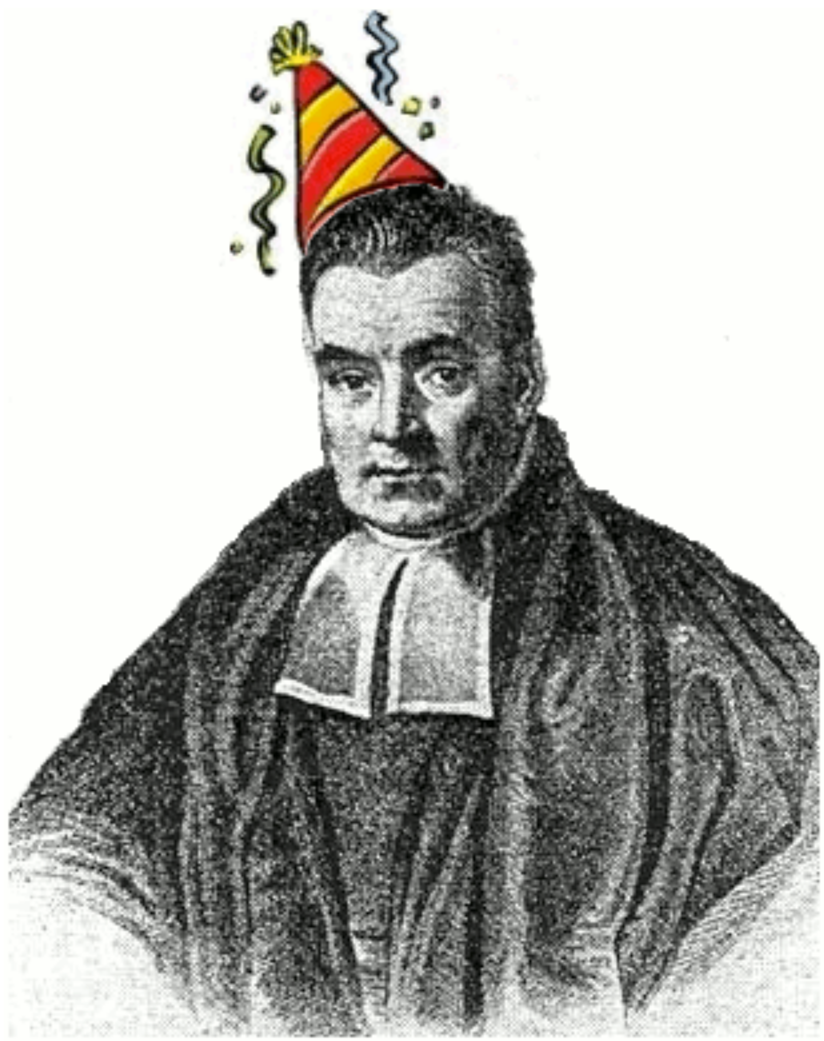
Naïve



Updating the state of knowledge

step by step

with new information



What is classification?

Deciding among hypotheses (labels),
using information we have (features)
for each example

3 Features: Votes on 3 Bills

2 Labels: Democrat / Republican

Prediction:
I know your votes, I'm trying to guess your party

3 Features: Votes on 3 Bills

2 Labels: Democrat / Republican

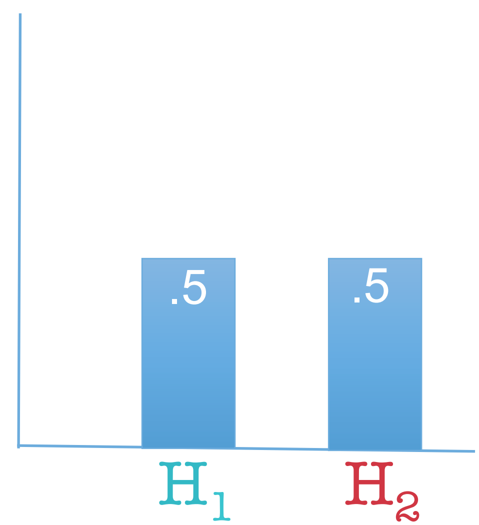
Prediction:

I know your votes, I'm trying to guess your party

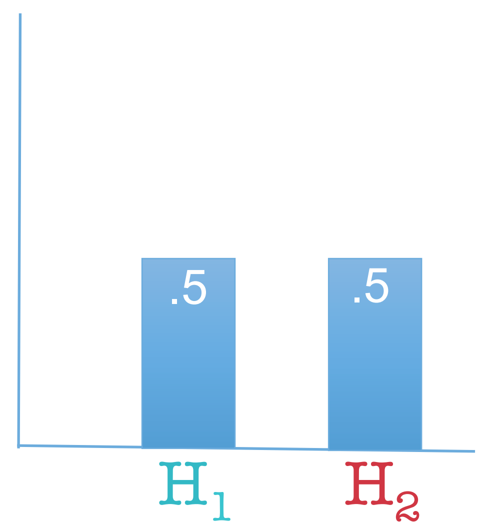
2 Labels

H_1: Democrat

H_2: Republican



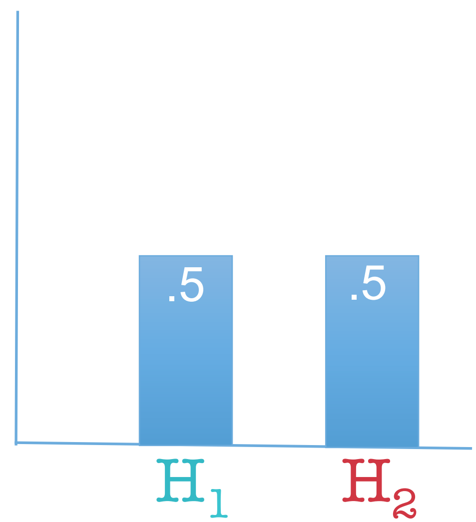
Prior: Initial belief



Prior: Initial belief

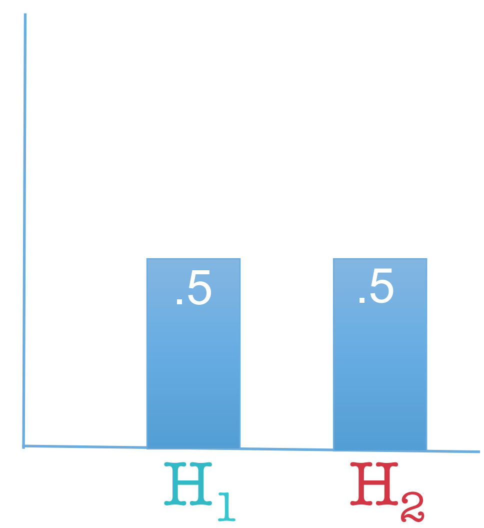
50 - 50 ?   P(Democrat) = 0.5 ?

(Uninformative prior)



Prior: Initial belief

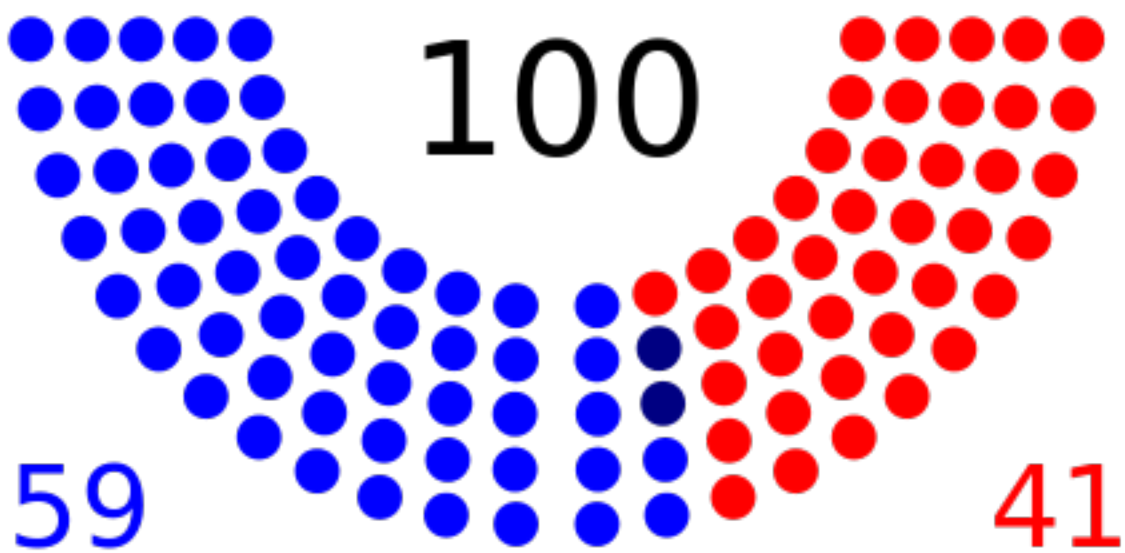
What's my best guess without any vote info?

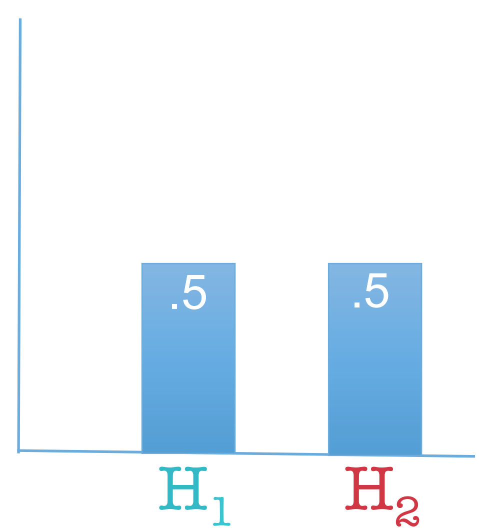


Prior: Initial belief

What's my best guess without any vote info?

I'd guess democrat since there are more of them.



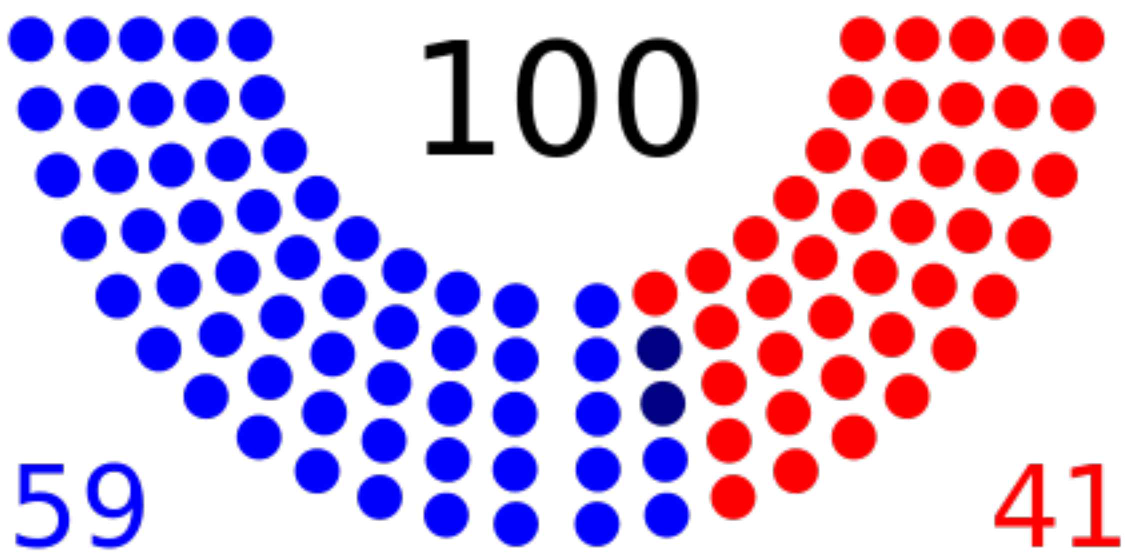


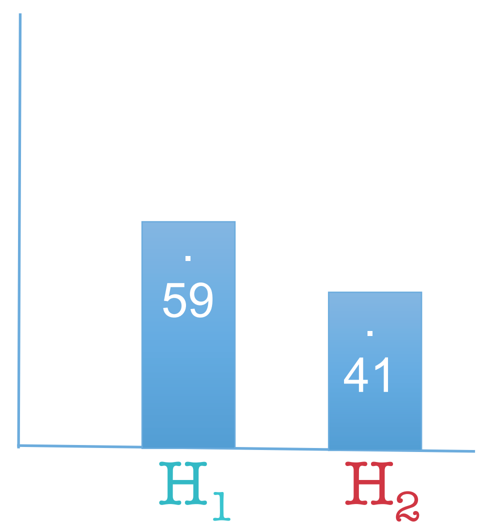
Prior: Initial belief

What's my best guess without any vote info?

I'd guess democrat since there are more of them.

P(Democrat) = 0.59





Prior: Initial belief

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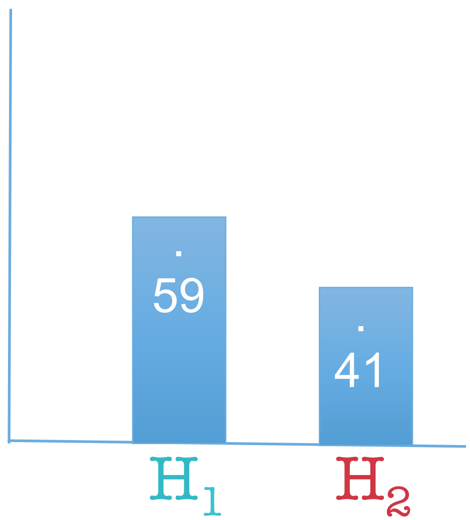
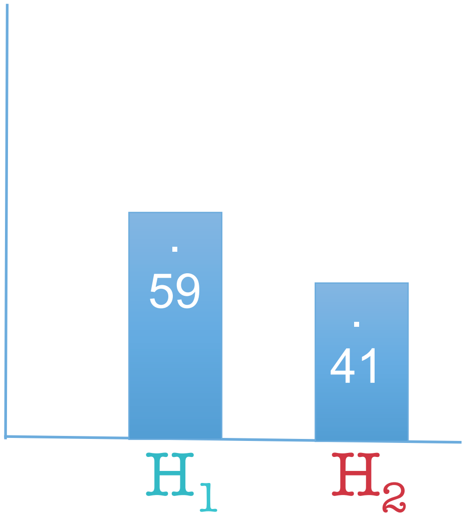


Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality



Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

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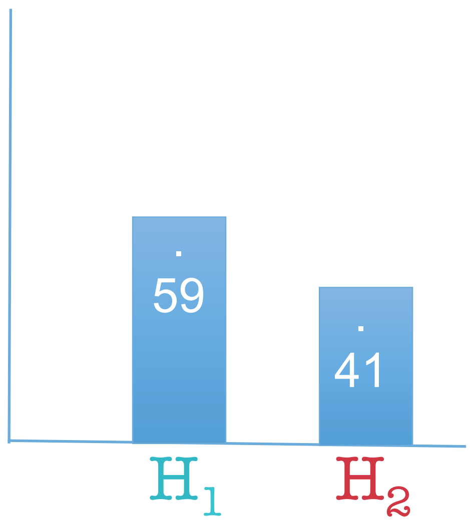
$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\,P(Dem)}{P(Y_{NN})}$$



Prior: Initial belief

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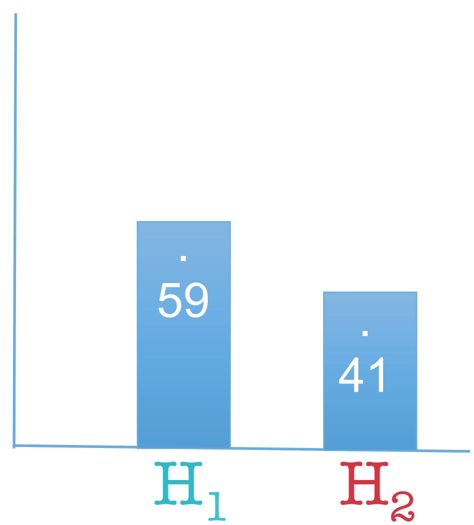
Voted YES on Net Neutrality

$$\underset{\text{posterior}}{P(Dem|Y_{NN})} = \frac{\overset{\text{likelihood}}{P(Y_{NN}|Dem)}\ \overset{\text{prior}}{P(Dem)}}{\underset{\text{evidence (normalization factor)}}{P(Y_{NN})}}$$



Prior: Initial belief

P(Democrat) = 0.59

$P(Y_{NN}|Dem)$

Prob. of voting yes on net neutrality if you're democrat

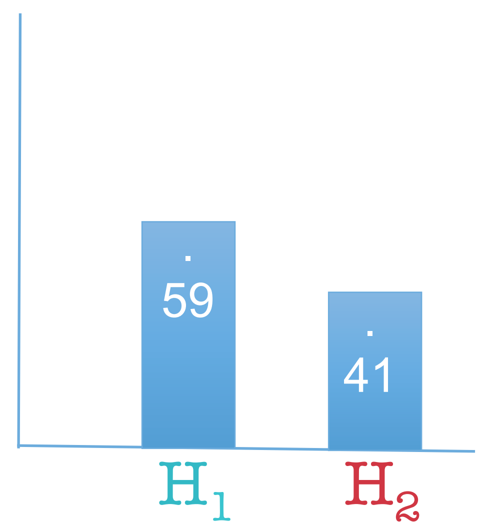
New information (feature 1):

Voted YES on Net Neutrality

posterior, likelihood, prior

$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\,P(Dem)}{P(Y_{NN})}$$

evidence (normalization factor)



Prior: Initial belief

$P(\text{Democrat}) = 0.59$

New information (feature 1):

Voted YES on Net Neutrality

$P(Y_{NN}|\text{Dem})$

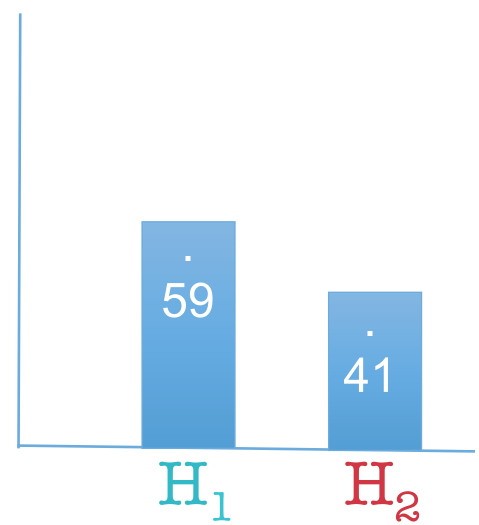
Prob. of voting yes on net neutrality if you're democrat

posterior, likelihood, prior

$$P(\text{Dem}|Y_{NN}) = \frac{P(Y_{NN}|\text{Dem})\,P(\text{Dem})}{P(Y_{NN})}$$

evidence (normalization factor)

$$P(\text{Rep}|Y_{NN}) = \frac{P(Y_{NN}|\text{Rep})\,P(\text{Rep})}{P(Y_{NN})}$$



Prior: Initial belief

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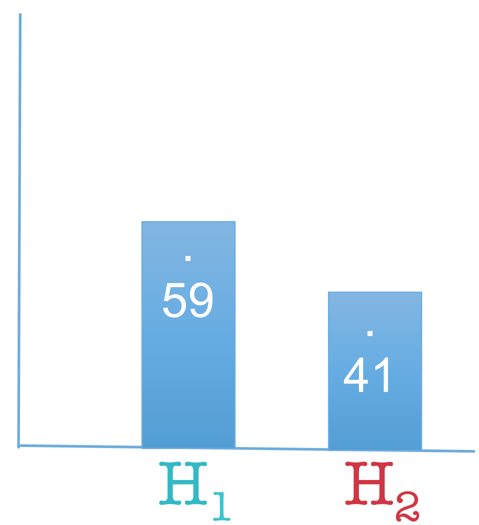
Prob. of voting yes on net neutrality if you're democrat

$$\underset{\text{posterior}}{P(Dem|Y_{NN})} = \frac{\overset{\text{likelihood}}{P(Y_{NN}|Dem)}\ \overset{\text{prior}}{P(Dem)}}{\underset{\text{evidence (normalization factor)}}{P(Y_{NN})}}$$

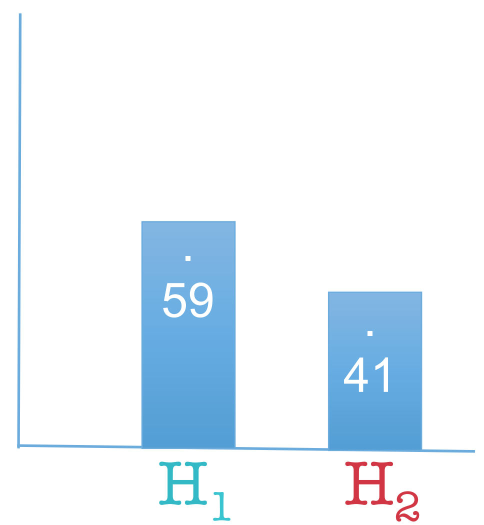
$P(Y_{NN}|Rep)$

$$P(Rep|Y_{NN}) = \frac{P(Y_{NN}|Rep)\ P(Rep)}{P(Y_{NN})}$$



$P(Y_{NN}|Dem)$
Prob. of voting yes
on net neutrality
if you're democrat

$P(Y_{NN}|Rep)$
Prob. of voting
yes
on net neutrality
if you're
republican



Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$P(Y_{NN}|\text{Dem})$
Prob. of voting yes
on net neutrality
if you're democrat

$P(Y_{NN}|\text{Rep})$
Prob. of voting
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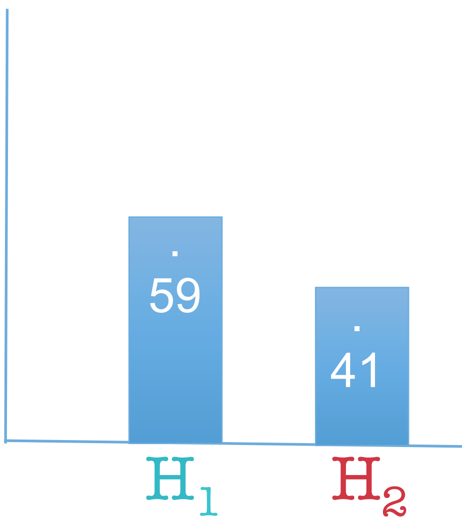


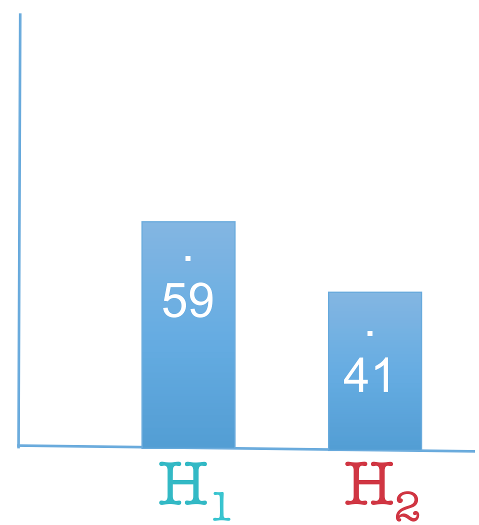
Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN}|Dem)$$

$$\approx \frac{\#\ \text{democrats that}\ Y_{NN}}{\#\ \text{all democrats}}$$

$$P(Y_{NN}|Rep)$$

$$\approx \frac{\#\ \text{republicans that}\ Y_{NN}}{\#\ \text{all republicans}}$$

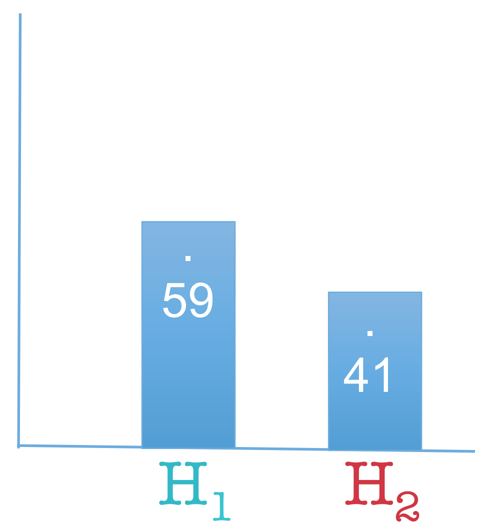


Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN}|Dem) \approx \frac{\text{\# democrats that } Y_{NN}}{\text{\# all democrats}}$$

$$P(Y_{NN}|Rep) \approx \frac{\text{\# republicans that } Y_{NN}}{\text{\# all republicans}}$$

For likelihoods of discrete data,
training/fitting means counting!
(and estimating likelihoods by dividing counts)

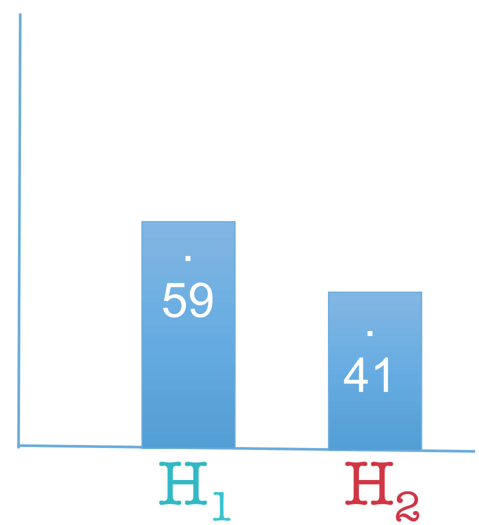


Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN}|\text{Dem}) \approx \frac{56}{59} = 0.949$$

$$P(Y_{NN}|\text{Rep}) \approx \frac{34}{41} = 0.829$$

For likelihoods of discrete data, training/fitting means counting!
(and estimating likelihoods by dividing counts)



Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality

likelihood prior

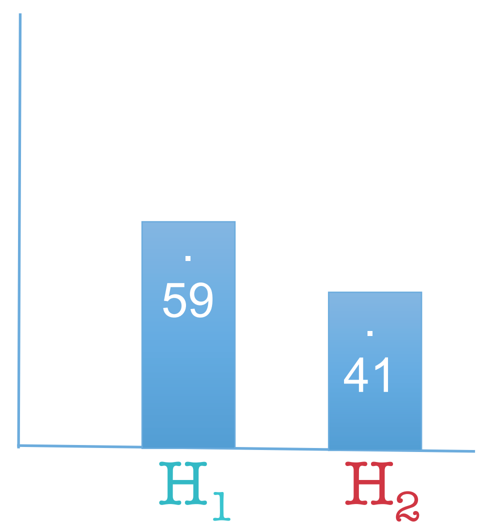
posterior

$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\,P(Dem)}{P(Y_{NN})}$$

evidence

(normalization factor)

$$P(Rep|Y_{NN}) = \frac{P(Y_{NN}|Rep)\,P(Rep)}{P(Y_{NN})}$$



Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality

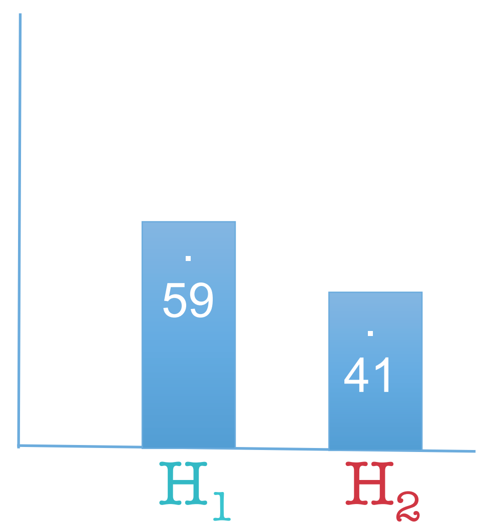
likelihood prior

posterior

$$P(Dem|Y_{NN}) = \frac{0.949 \quad * \quad 0.59}{P(Y_{NN})}$$

evidence
(normalization factor)

$$P(Rep|Y_{NN}) = \frac{0.829 \quad * \quad 0.41}{P(Y_{NN})}$$



Current belief

$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 1):

Voted YES on Net Neutrality

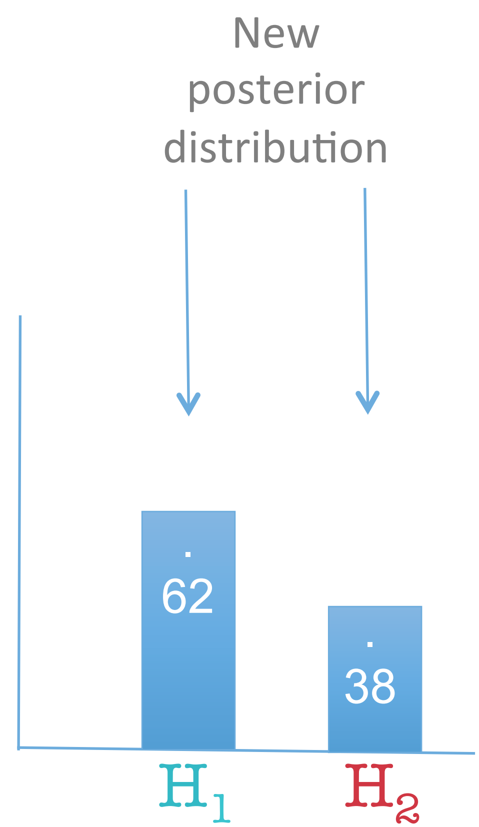
likelihood prior

posterior

$$P(\text{Dem} \mid Y_{NN}) = \frac{0.949 \;\; * \;\; 0.59}{P(Y_{NN})}$$

evidence
(normalization factor)

$$P(\text{Rep} \mid Y_{NN}) = \frac{0.829 \;\; * \;\; 0.41}{P(Y_{NN})}$$

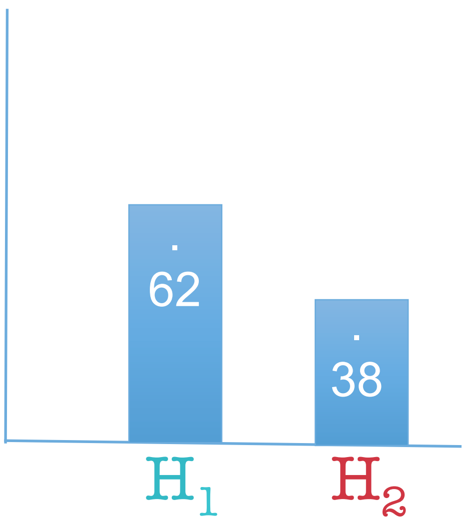


Current belief

$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts



Current belief

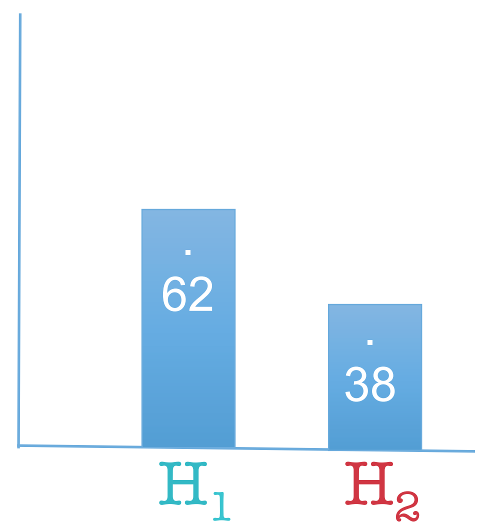
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN})}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN})}{P(Y_{TC})}$$



Current belief

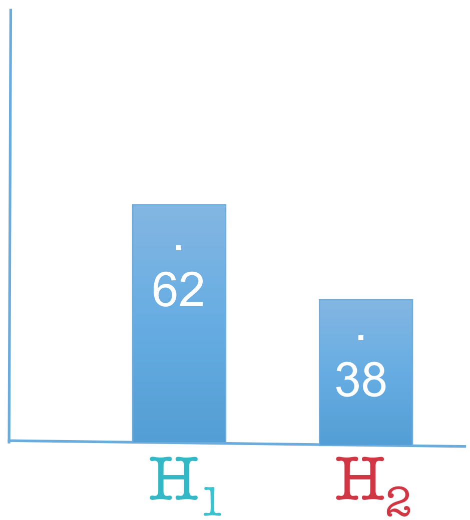
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

$P(Y_{TC} \mid \text{Dem})$

$$\approx \frac{10}{59} = 0.169$$

$P(Y_{TC} \mid \text{Rep})$

$$\approx \frac{35}{41} = 0.854$$



Current belief

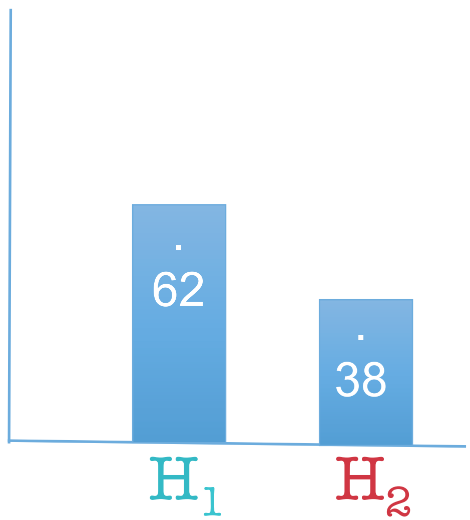
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN})}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN})}{P(Y_{TC})}$$



Current belief

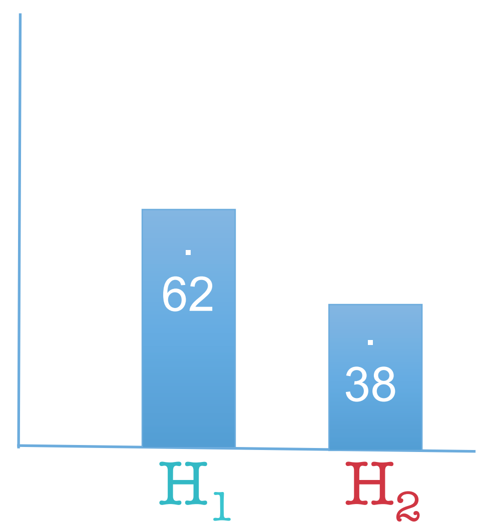
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{0.169 \quad * \quad 0.62}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{0.854 \quad * \quad 0.38}{P(Y_{TC})}$$



Current belief

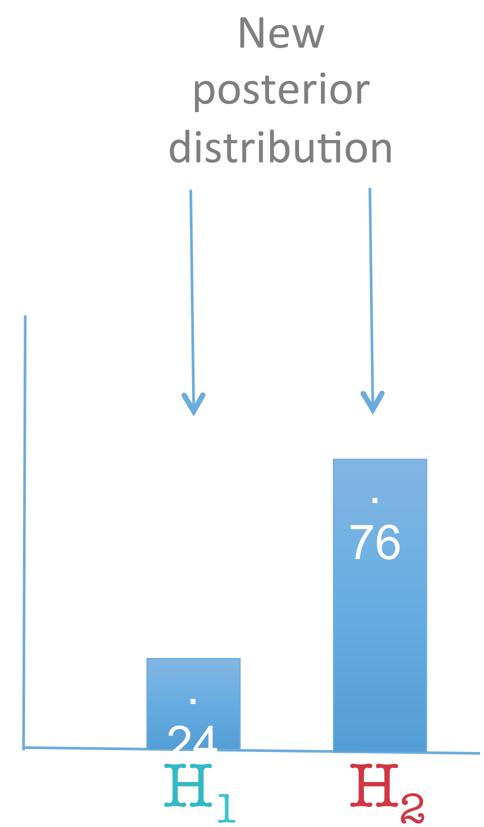
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{0.169 \quad * \quad 0.62}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{0.854 \quad * \quad 0.38}{P(Y_{TC})}$$

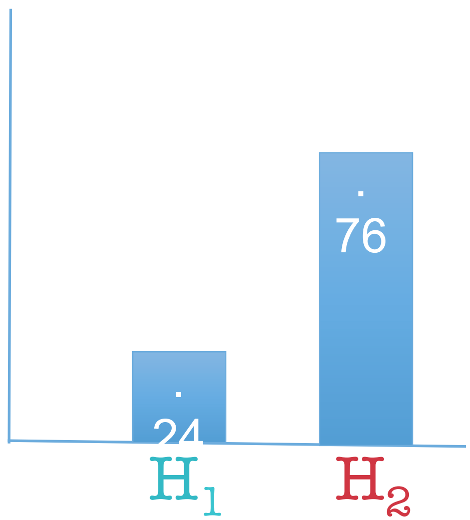


Current belief

$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns



Current belief

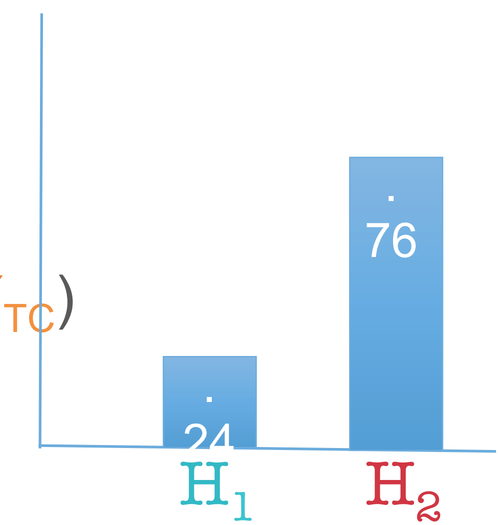
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$



Current belief

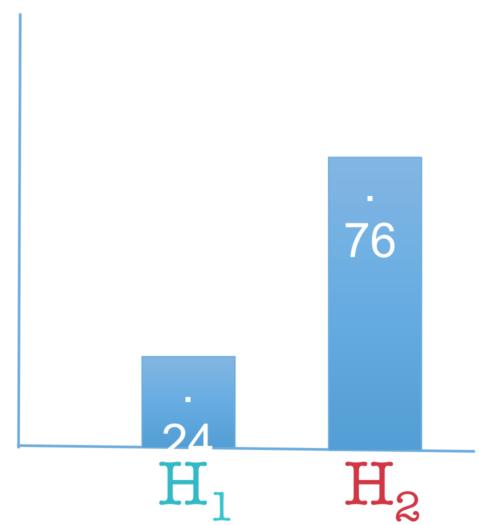
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

$P(N_{LG} \mid \text{Dem})$

$$\approx \frac{53}{59} = 0.898$$

$P(N_{LG} \mid \text{Rep})$

$$\approx \frac{23}{41} = 0.561$$



Current belief

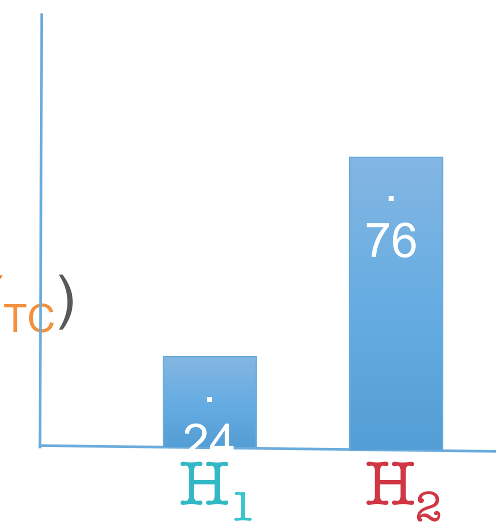
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$



Current belief

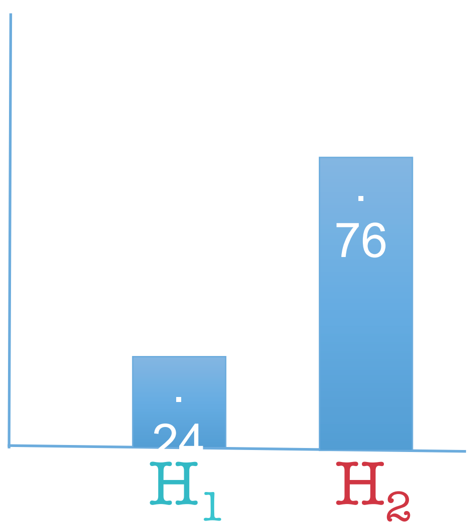
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.898 \quad * \quad 0.24}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.561 \quad * \quad 0.76}{P(N_{LG})}$$



Current belief

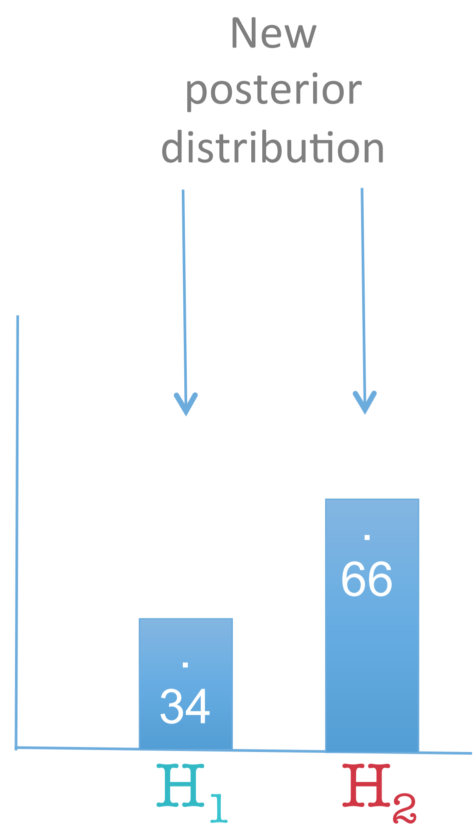
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.898 \quad * \quad 0.24}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.561 \quad * \quad 0.76}{P(N_{LG})}$$



Current belief

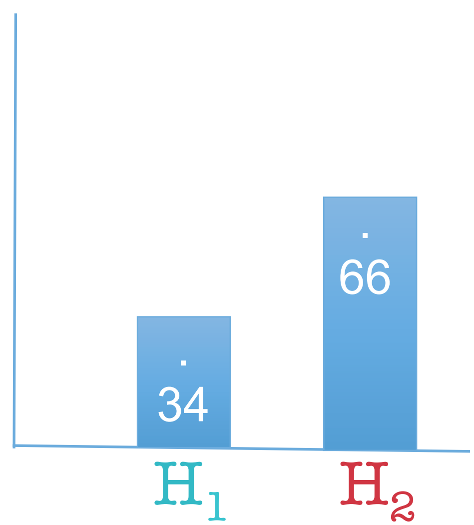
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

Classify this person that voted

Yes on Net Neutrality (Y_{NN}),

Yes on Tax Cuts (Y_{TC}),

No on License-free Guns (N_{LG})



Current belief

$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

Classify this person that voted

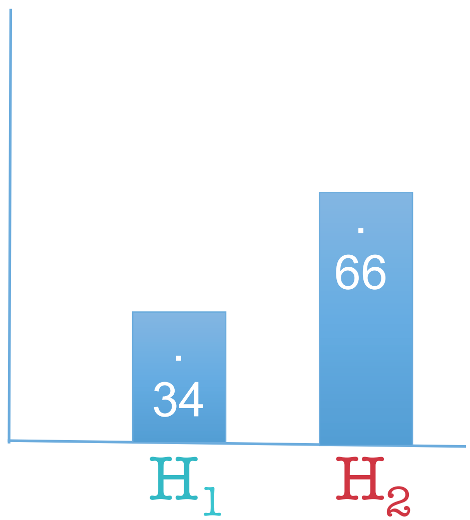
Yes on Net Neutrality (Y_{NN}),

Yes on Tax Cuts (Y_{TC}),

No on License-free Guns (N_{LG})

My strongest belief is in H_2,

I classify this person with the label Republican.



Naïve Bayes

Training:
Count and calculate the likelihood of each feature value for each class:

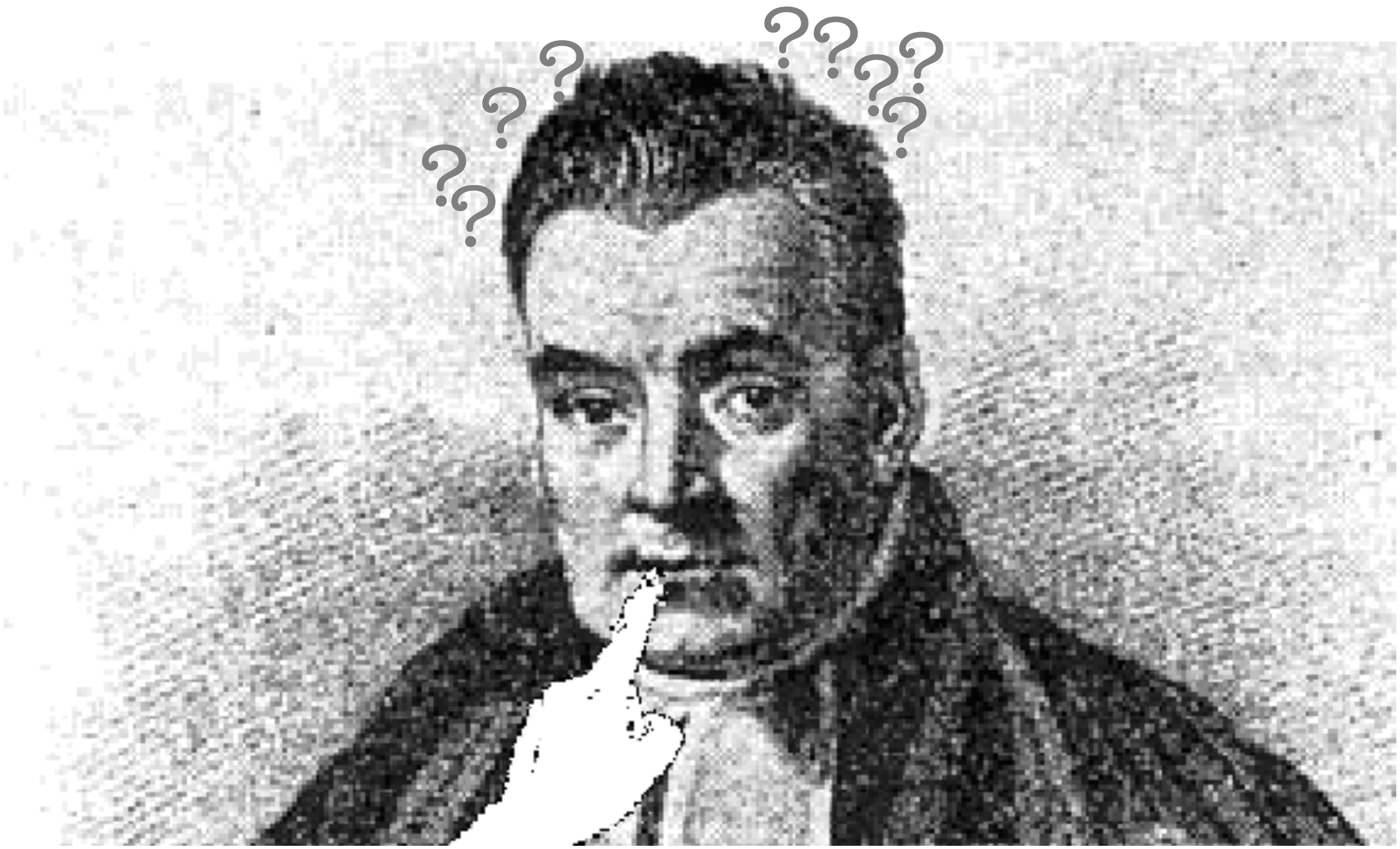
$P(Y_{NN}|Dem) \quad = 1 - P(N_{NN}|Dem)$
$P(Y_{NN}|Rep) \quad = 1 - P(N_{NN}|Rep)$

$P(Y_{TC}|Dem) \quad = 1 - P(N_{TC}|Dem)$
$P(Y_{TC}|Rep) \quad = 1 - P(N_{TC}|Rep)$

$P(Y_{LG}|Dem) \quad = 1 - P(N_{LG}|Dem)$
$P(Y_{LG}|Rep) \quad = 1 - P(N_{LG}|Rep)$

Prediction:
Use Bayes to update priors with the likelihoods,
Pick label with the highest posterior probability.

What was the naïve part?



We assumed each feature is independent

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Easier to see in a single update rather than sequential

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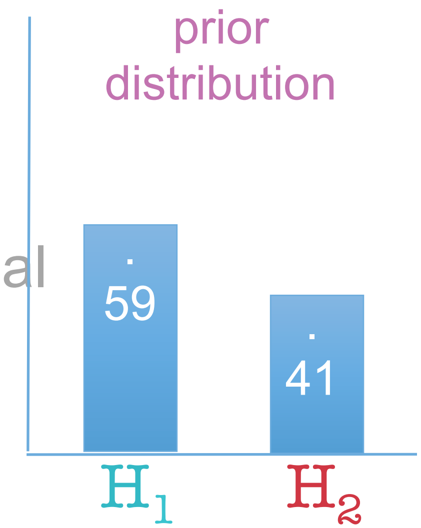
Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{P(Y_{NN},Y_{TC},N_{LG}|Dem) \quad P(Dem)}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent

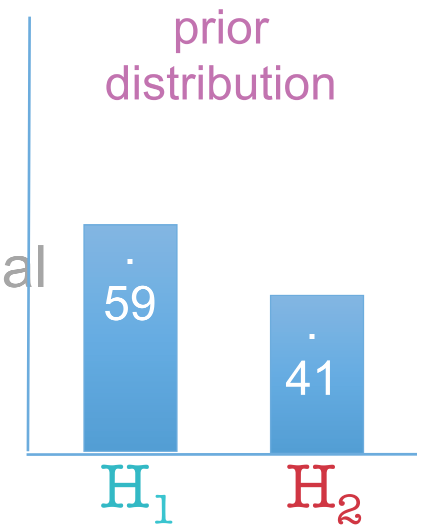


Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$\overset{\text{posterior}}{P(Dem|Y_{NN},Y_{TC},N_{LG})} = \frac{\overset{\text{likelihood}}{P(Y_{NN},Y_{TC},N_{LG}|Dem)} \; \overset{\text{prior}}{P(Dem)}}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

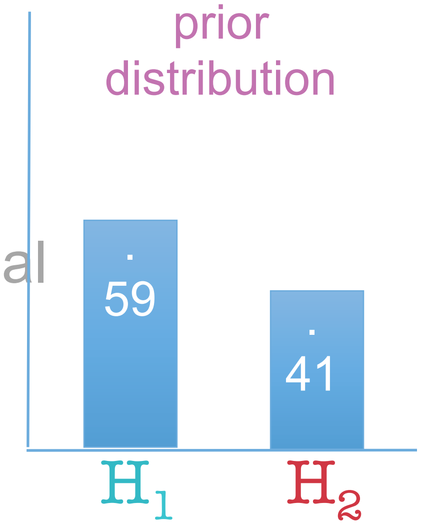
Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$\underbrace{P(Dem|Y_{NN},Y_{TC},N_{LG})}_{\text{posterior}} = \frac{\overbrace{P(Y_{NN},Y_{TC},N_{LG}|Dem)}^{\text{likelihood}} \; \overbrace{P(Dem)}^{\text{prior}}}{P(Y_{NN},Y_{TC},N_{LG})}$$

Independence Assumption:

$$P(Y_{NN},Y_{TC},N_{LG}|Dem) = P(Y_{NN}|Dem)\, P(Y_{TC}|Dem)\, P(N_{LG}|Dem)$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

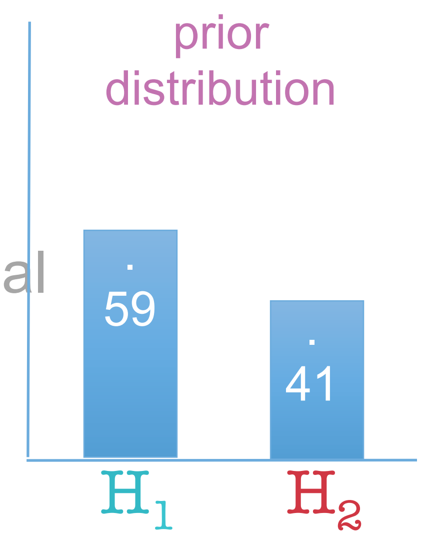
Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$\underbrace{P(Dem|Y_{NN},Y_{TC},N_{LG})}_{\text{posterior}} = \frac{\overbrace{P(Y_{NN},Y_{TC},N_{LG}|Dem)}^{\text{likelihood}}\ \overbrace{P(Dem)}^{\text{prior}}}{P(Y_{NN},Y_{TC},N_{LG})}$$

Independence Assumption:
$P(Y_{NN},Y_{TC},N_{LG}|Dem) = P(Y_{NN}|Dem)\ P(Y_{TC}|Dem)\ P(N_{LG}|Dem)$

Not even close in most cases!
Naïve Bayes still works well.

We assumed each feature is independent



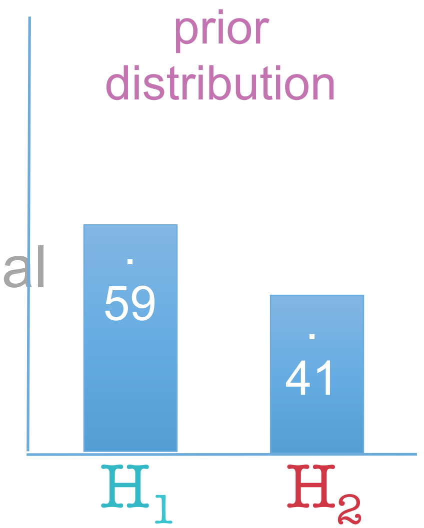
Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

likelihood — prior

posterior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{P(Y_{NN}|Dem)\,P(Y_{TC}|Dem)\,P(N_{LG}|Dem)\,P(Dem)}{P(Y_{NN},Y_{TC},N_{LG})}$$

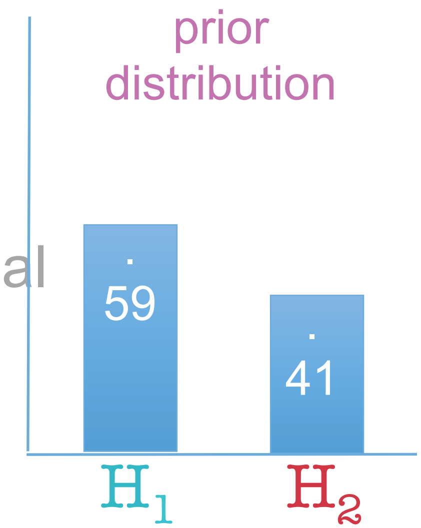


We assumed each feature is independent

Easier to see in a single update rather than sequential
Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$\underbrace{P(Dem \mid Y_{NN}, Y_{TC}, N_{LG})}_{\text{posterior}} = \frac{\overbrace{0.949 * 0.169 * 0.898}^{\text{likelihood}} \quad * \quad \overbrace{0.59}^{\text{prior}}}{P(Y_{NN}, Y_{TC}, N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

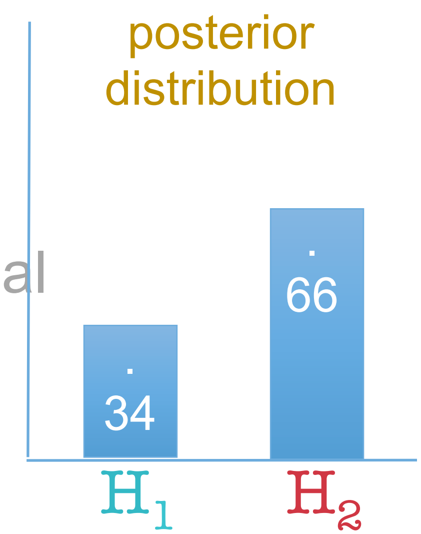
likelihood prior

posterior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{0.949 * 0.169 * 0.898 \quad * \quad 0.59}{P(Y_{NN},Y_{TC},N_{LG})}$$

$$P(Rep|Y_{NN},Y_{TC},N_{LG}) = \frac{0.829 * 0.854 * 0.561 \quad * \quad 0.41}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

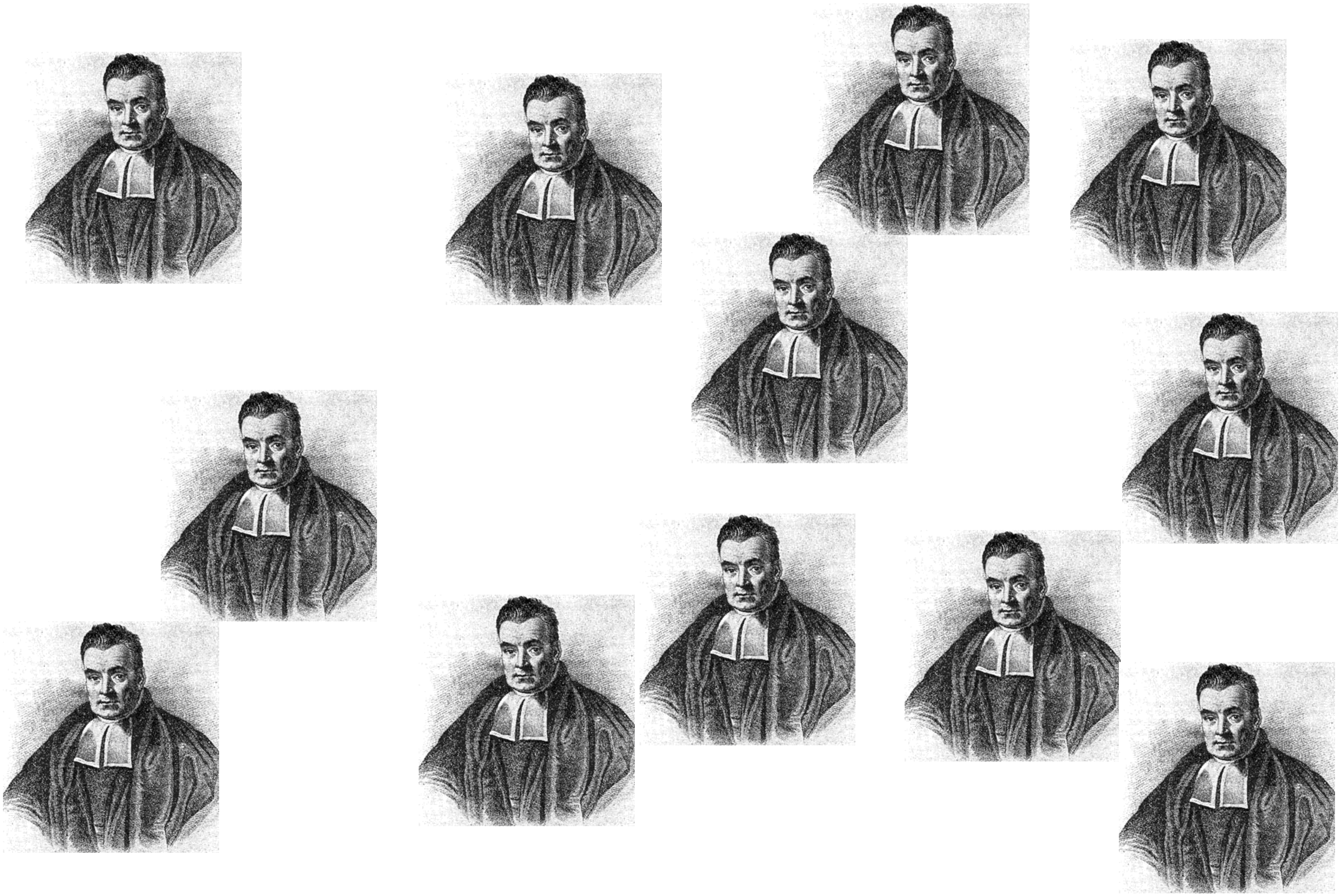
Prob. of this example having label Democrat, given the values Yes, Yes and No on the features NN, TC and LG

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = 0.34$$

$$P(Rep|Y_{NN},Y_{TC},N_{LG}) = 0.66$$

predict!

What about multiple classes?



Straightforward!

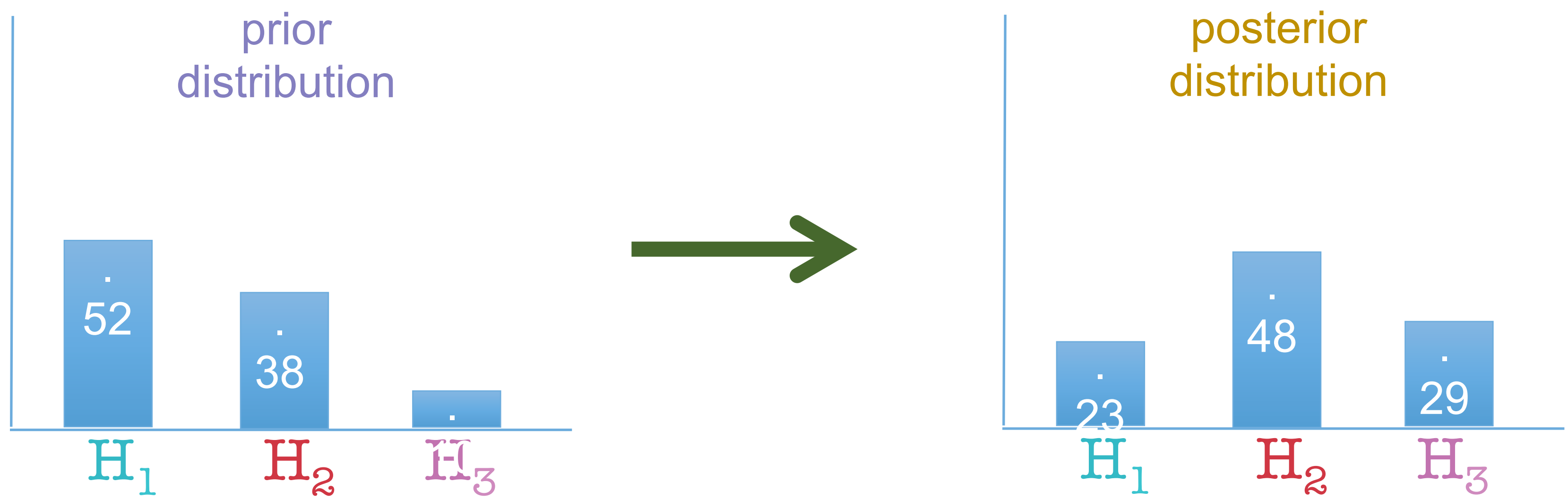
Update each hypothesis,
given the values Yes, Yes and No
on the features NN, TC and LG

$$P(Dem|Y_{NN},Y_{TC},N_{LG})$$
$$P(Rep|Y_{NN},Y_{TC},N_{LG})$$
$$P(Indep|Y_{NN},Y_{TC},N_{LG})$$

Straightforward!

Update each hypothesis,
given the values Yes, Yes and No
on the features NN, TC and LG

$$P(\text{Dem}|Y_{NN},Y_{TC},N_{LG})$$
$$P(\text{Rep}|Y_{NN},Y_{TC},N_{LG})$$
$$P(\text{Indep}|Y_{NN},Y_{TC},N_{LG})$$



Naïve Bayes: The Multinomial Approach (Text Classification example)

	Document	Class
	Hamburger NYC Hamburger	A
	Hamburger Hamburger LA	A
	Hamburger Cheeseburger	A
	Montreal Iceskate Hamburger	C
TEST	Hamburger Hamburger Hamburger Montreal Iceskate	??

1) Develop our likelihoods for ‘seeing’ each word in given the class:

$$P(\text{Hamburger} \mid A) = (5 + 1) / (8 + 6) = 3/7$$

smoothing

\# times ‘hamburger’ appeared in ‘A’ Docs

\# of unique words in all docs

\# of words in ‘A’ docs

Naïve Bayes: The Multinomial Approach

	Document	Class
	Hamburger NYC Hamburger	**A**
	Hamburger Hamburger LA	**A**
	Hamburger Cheeseburger	**A**
	Montreal Iceskate Hamburger	**C**
TEST	**Hamburger Hamburger Hamburger Montreal Iceskate**	**??**

1) Develop our likelihoods for 'seeing' each word in given the class:

P(Hamburger | A) = (5 + 1) / (8 + 6) = 3/ 7
P (Montreal| A) = (0 +1) / (8+6) = 1/14
P(Iceskate | A) = (0 +1) (8 +6) = 1/14

P(Hamburger | C)) = (1 + 1) / (3+6) = 2/9
P (Montreal| C) = (1+ 1) / (3+6) = 2 /9
P(Iceskate | C) = (1 + 1) /(3+6) = 2/ 9

P (A| test) ~ (3/7)^3*(1/14)*(1/14) * (3/4) = 0.0003
P (C | test) ~ (2/9)^3 *(2/9)*(2/9)* (1/4) = 0.0001

Naïve Bayes: The Bernouli Approach

	Document	Class
	Hamburger NYC Hamburger	**A**
	Hamburger Hamburger LA	**A**
	Hamburger Cheeseburger	**A**
	Montreal Iceskate Hamburger	**C**
TEST	**Hamburger Hamburger Hamburger Montreal Iceskate**	**??**

1) Develop our likelihoods for 'seeing' each word in given the class:

P(Hamburger | A) = (3+1) / (3+ 2) = 4/5

Constant = 2

of 'A' docs

smoothing

of 'A' docs
where 'American'
appears

Naïve Bayes: The Bernouli Approach

	Document	Class
	Hamburger NYC Hamburger	**A**
	Hamburger Hamburger LA	**A**
	Hamburger Cheeseburger	**A**
	Montreal Iceskate Hamburger	**C**
TEST	**Hamburger Hamburger Hamburger Montreal Iceskate**	**??**

1) Develop our likelihoods for 'seeing' each word in given the class:

P(Hamburger | A) = (3+1)/ (3+ 2) = 4/5
P (Montreal | A) = P(Iceskate | C) = (0+ 1) / (3+2) = 1/5
P(Cheeseburger| A) = P(NYC|A) =P(LA|A) (1+1) / (3+2) = 2/5

P(Hamburger|C) = P(Montreal)=P(Iceskate)=(1 + 1) / (1+2) = 2/3
P(NYC| C) = P(LA| C) =P(Cheeseburger| C) = (1+0)/(1+2) = 1/3

*P(A|test)=P(A)*P(Hamburger | A) *P (Montreal | A) *P(Iceskate | C) * (1-P(Cheeseburger| A))*(1-(P(NYC|A))(1-P(LA|A))*

P (A | test) ~(3/4)* (4/5)(1/5)(1/5)*(1-(2/5))*(1-(2/5))*(1-(2/5)) = .005
P (C |test) ~(1/4)* (2/3)(2/3)(2/3)*(1-(1/3))*(1-(1/3))*(1-(1/3)) = 0. 022

Naïve Bayes

Bernoulli : models the fraction of documents of class C that contain the word ‘w’ (ignores number of occurrences)

Vs.

Multinomial: models the fraction of *positions* in documents of class C that contain the word ‘w’ (keeps track of number of occurrences)

But why does Naïve Bayes work so well-
(Considering that it is Naïve) ?

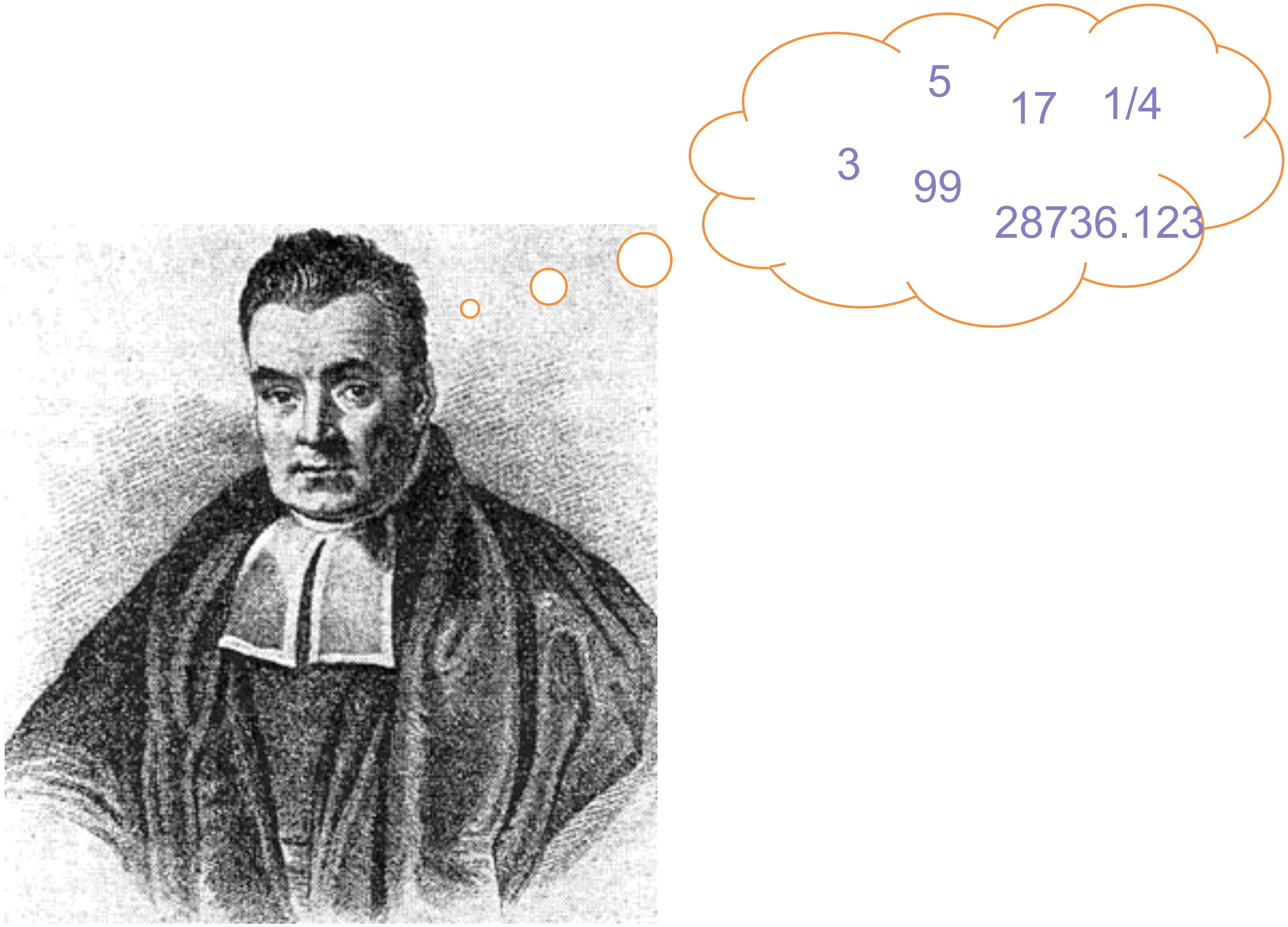
NB chooses among possible classes to find the class with the highest associated probability.

Naiveté doesn’t hurt, because correctness is based on classification, not exact predictions

Advantages of Naïve Bayes:

- Simple & Fast. Just doing a bunch of counts!
- Will converge quickly. Requires less training data
- Can handle sparse matrices
- Can handle multiple classes well

How about numeric features?



Naïve Bayes: The Gaussian Approach

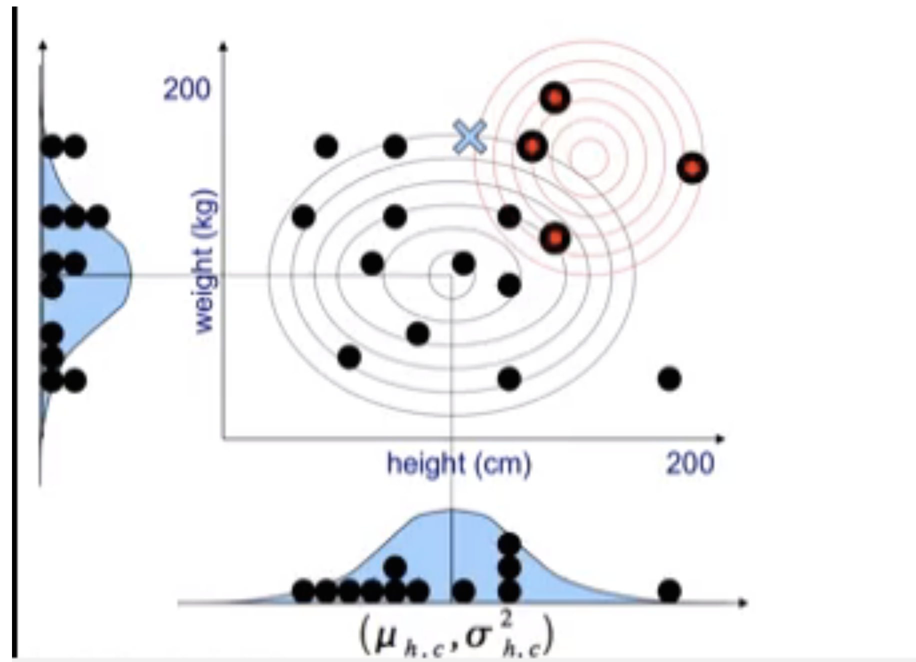
$$p(h_x|c)=\frac{1}{\sqrt{2\pi\sigma^2_{h,c}}}\exp-\frac{1}{2}\left(\frac{(h_x-\mu_{h,c})^2}{\sigma^2_{h,c}}\right)$$

$$p(w_x|c)=\frac{1}{\sqrt{2\pi\sigma^2_{w,c}}}\exp-\frac{1}{2}\left(\frac{(w_x-\mu_{w,c})^2}{\sigma^2_{w,c}}\right)$$

$$p(h_x|a)=\frac{1}{\sqrt{2\pi\sigma^2_{h,a}}}\exp-\frac{1}{2}\left(\frac{(h_x-\mu_{h,a})^2}{\sigma^2_{h,a}}\right)$$

$$p(w_x|a)=\frac{1}{\sqrt{2\pi\sigma^2_{w,a}}}\exp-\frac{1}{2}\left(\frac{(w_x-\mu_{w,a})^2}{\sigma^2_{w,a}}\right)$$

$$P(x|a)=p(h_x|a)p(w_x|a)$$



P(a|x) ~ P(x|a)*P(a)

Flavors of Bayes in `sklearn`:

Numeric Features:
Gaussian Naïve Bayes

Features that are 0 or 1 (and both matter):
Bernoulli Naïve Bayes

Features that are count-like (and only non-zero matters):
Multinomial Naïve Bayes

Which did we do?