

Course project

«High-dimensional integrals for option pricing»

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Plan

- 1 Introduction to option pricing
- 2 From Black-Scholes to diffusion
- 3 Idea of the method
- 4 Experiments
- 5 Discussion

Introduction to option pricing

Options

- Contract which gives the owner of the option the right, but not the obligation, to buy or sell an underlying asset at a specified price on a specified date, depending on the form of the option
- Two basic types of options: the call and the put
- Call: gives the holder the right to buy an asset
- Put: gives the holder the right to sell an asset

Notations

- s - stock price
- $c(s, t)$ - general option value

Black-Scholes equation

Black-Scholes equation

$$\begin{aligned} \frac{\partial c(s, t)}{\partial t} + rs \frac{\partial c(s, t)}{\partial s} + \frac{1}{2} \gamma^2 s^2 \frac{\partial^2 c(s, t)}{\partial s^2} &= rc(s, t), \\ c(s, T') &= g(s), \quad s \in \mathbb{R}_+, \\ c(0, t) &= 0, \quad t \in [0; T']. \end{aligned}$$

Substitution

- New variable: $x = \ln s$
- New initial condition: $f(x) = e^{\frac{1}{\gamma^2}(r - \frac{\gamma^2}{2})x} g(e^x)$,
- New coefficients: $\sigma = \frac{1}{2}\gamma^2$, $V(x, t) = V = r + \frac{1}{2\gamma^2} \left(r - \frac{\gamma^2}{2}\right)^2$,
- New solution: $u(x, t) = e^{\frac{1}{\gamma^2}(r - \frac{\gamma^2}{2})x} c(e^x, T' - t)$

Diffusion equation

One-dimensional reaction-diffusion equation

$$\frac{\partial u(x, t)}{\partial t} = \sigma \frac{\partial^2 u(x, t)}{\partial x^2} - V(x, t)u(x, t), \quad t \in [0; T'],$$
$$u(x, 0) = f(x), \quad x \in \mathbb{R}.$$

Fast method for solving this equation was proposed in the paper:
"A low-rank approach to the computation of path integrals",
M. Litsarev, I. Oseledets, 2015.

Idea of the method

- The analytical solution is given by the Feynman-Kac formula

$$u(x, T) = \int_{C\{x, 0; T\}} f(\xi(T)) e^{-\int_0^T V(\xi(\tau), T-\tau) d\tau} \mathcal{D}_\xi$$

- Time discretization reduces this formula to n -dimensional integral ($n \gg 1$)
- But it can be computed as n one-dimensional convolutions efficiently
- Convolutions can be computed efficiently via FFT or/and low-rank approximations

Experiments: European put option

Terminal conditions:

$$g(s) = \max\{E - s, 0\}$$

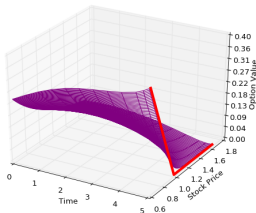


Figure: Numerical solution

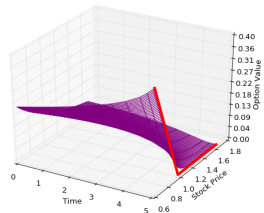


Figure: Analytical solution

Experiments: Cash-or-nothing call option

Terminal conditions:

$$g(s) = \begin{cases} B, & s \geq E \\ 0, & s < E \end{cases}$$

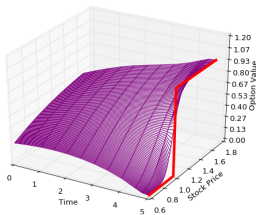


Figure: Numerical solution

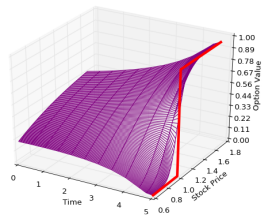


Figure: Analytical solution

Discussion

Unbounded terminal condition

- If n is big, numerical solution $\rightarrow +\infty$.
Explanation:
substitution $x = \ln s$ and one speciality of the algorithm.
- If n is small, numerical solution is not accurate and cross approximation is not useful.

Bounded terminal condition

- No such failure with big n as in the previous case.
- The bigger n , the more accurate solution.
- Cross approximation is useful: allows to reduce complexity.

More detailed analysis on that in our report.

Thanks for your attention!