# Course project «High-dimensional integrals for option pricing»

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## Plan

- Introduction to option pricing
- From Black-Scholes to diffusion
- 3 Idea of the method
- 4 Experiments
- Discussion

# Introduction to option pricing

## Black-Scholes equation

#### Black-Scholes equation

$$\frac{\partial c(s,t)}{\partial t} + rs \frac{\partial c(s,t)}{\partial s} + \frac{1}{2} \gamma^2 s^2 \frac{\partial^2 c(s,t)}{\partial s^2} = rc(s,t),$$

$$c(s,T') = g(s), \quad s \in \mathbb{R}_+,$$

$$c(0,t) = 0, \quad t \in [0; T'].$$

#### Substitution

- New variable:  $x = \ln s$
- New initial condition:  $f(x) = e^{\frac{1}{\gamma^2}(r \frac{\gamma^2}{2})x}g(e^x)$ ,
- New coefficients:  $\sigma=\frac{1}{2}\gamma^2$ ,  $V(x,t)=V=r+\frac{1}{2\gamma^2}\left(r-\frac{\gamma^2}{2}\right)^2$ ,
- New solution:  $u(x,t) = e^{\frac{1}{\gamma^2}(r-\frac{\gamma^2}{2})x}c(e^x,T'-t)$



## Diffusion equation

## One-dimensional reaction-diffusion equation

$$\frac{\partial u(x,t)}{\partial t} = \sigma \frac{\partial^2 u(x,t)}{\partial x^2} - V(x,t)u(x,t), \quad t \in [0; T'],$$
  
$$u(x,0) = f(x), \quad x \in \mathbb{R}.$$

Fast method for solving this equation was proposed in the paper: "A low-rank approach to the computation of path integrals", M. Litsarev, I. Oseledets, 2015.

## Idea of the method

## Experiments: european put option

# Experiments: something else (bounded!)

### Discussion

#### Unbounded terminal condition

- If n is big, numerical solution → +∞.
   Explanation:
   substitution x = ln s and one speciality of the algorithm.
- If n is small, numerical solution is not accurate and cross approximation is not useful.

#### Bounded terminal condition

- No such failure with big *n* as in the previous case.
- The bigger n, the more accurate solution.
- Cross approximation is useful: allows to reduce complexity.

More detailed analysis on that in our report.

Thanks for your attention!