Course project
«Optimization approaches to community detection»

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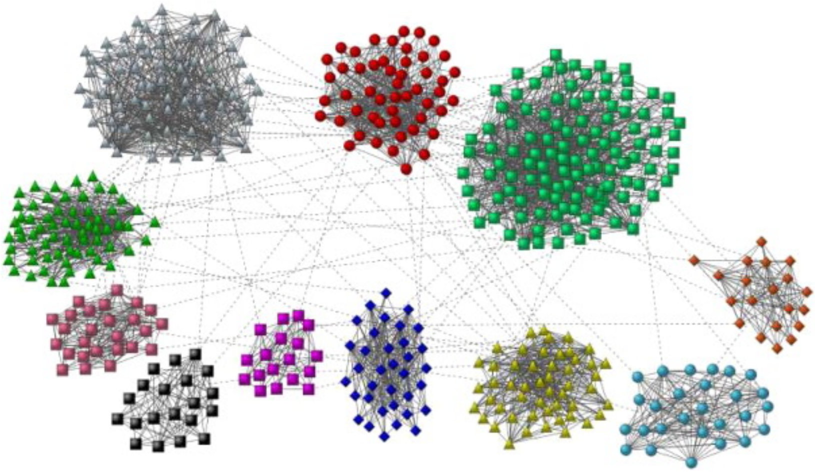
Plan

1. Introduction to community detection
2. Algorithms
   - Spectral method
   - Modularity-based method
   - Conjugate gradients method
   - Semidefinite relaxations
3. Experimental results

Example



Notations

Assumption

We consider **undirected unweighted** graphs **without loops** with n nodes. The nodes are enumerated as $\{1, ..., n\}$.
Graph is given by its $n \times n$ adjacency matrix A.

Goal of community detection

Find **partition** of nodes into **non-overlapping** clusters.
The number of clusters is k.
The clusters are denoted as $\{\mathcal{C}_1, ..., \mathcal{C}_k\}$.

Subsets and Cuts

Measuring sizes of subsets

Let $V_1, \dots, V_k$ be subsets of vertices. Then:

- $|V_i| = \{\text{number of vertices in } V_i\}$
- $vol(V_i) = \sum\limits_{i \in V_i} d_i$

MinCut

Define $W(S_1, S_2) := \sum\limits_{i \in S_1, j \in S_2} a_{ij}$. Then the MinCut problem is:

$$cut(V_1, \dots, V_k) = \frac{1}{2} \sum_{i=1}^{k} W(V_i, \overline{V_i}) \to \min_{V_1, \dots, V_k}$$

Balancing Cuts

RatioCut and Normalized Cut

The MinCut solution separates one individual vertex from the rest.
The following objectives are considered:

$$RatioCut(V_1, \ldots, V_k) = \sum_{i=1}^{k} \frac{cut(V_i, \overline{V_i})}{|V_i|} \to \min_{V_1, \ldots, V_k}$$

$$Ncut(V_1, \ldots, V_k) = \sum_{i=1}^{k} \frac{cut(V_i, \overline{V_i})}{vol(V_i)} \to \min_{V_1, \ldots, V_k}$$

Balancing conditions lead to NP-hard problem. Spectral clustering is a way to solve relaxed versions of those problems

Relaxed problem

Types of Laplacians

- Unnormilized Laplacian: $L = D - A$
- Symmetric Laplacian: $L_{sym} = D^{-\frac{1}{2}} L D^{-\frac{1}{2}} = I - D^{-\frac{1}{2}} A D^{-\frac{1}{2}}$
- Random walk Laplacian: $L_{rw} = D^{-1} L = I - D^{-1} A$

Idea

- Solving relaxed problem is equivalent to considering eigenvectors corresponding to k smallest eigenvalues of Laplacian that describe cluster properties of given graph

- Ulrike von Luxburg «A Tutorial on Spectral Clustering», 2007

Modularity-based method

Formulating an optimization problem

Modularity-based method

Conjugate gradients method

Formulating an optimization problem

Conjugate gradients method

Semidefinite relaxations

Formulating an optimization problem

Semidefinite relaxations