Assignment 4

1. **Support Vector Machines with Synthetic Data**, 50 points. ¶

For this problem, we will generate synthetic data for a nonlinear binary classification problem and partition it into training, validation and test sets. Our goal is to understand the behavior of SVMs with Radial-Basis Function (RBF) kernels with different values of C and γ.

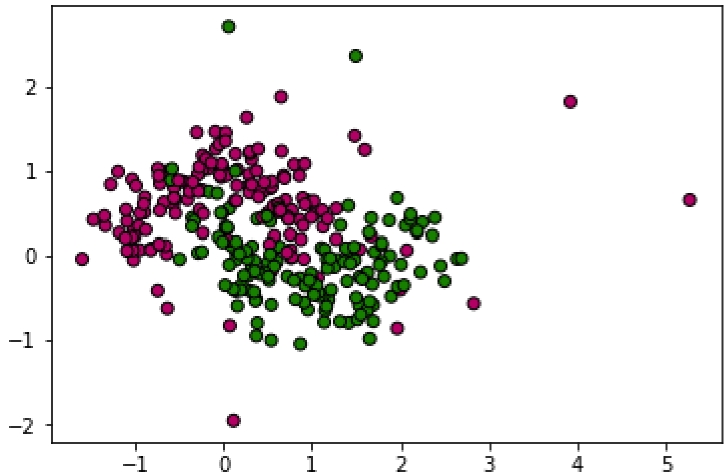
In [1]:

```
# DO NOT EDIT THIS FUNCTION; IF YOU WANT TO PLAY AROUND WITH DATA GENERATION,
# MAKE A COPY OF THIS FUNCTION AND THEN EDIT
#
import numpy as np
from sklearn.datasets import make_moons
from sklearn.model_selection import train_test_split
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap

def generate_data(n_samples, tst_frac=0.2, val_frac=0.2):
  # Generate a non-linear data set
  X, y = make_moons(n_samples=n_samples, noise=0.25, random_state=42)

  # Take a small subset of the data and make it VERY noisy; that is, generate
 outliers
  m = 30
  np.random.seed(30)  # Deliberately use a different seed
  ind = np.random.permutation(n_samples)[:m]
  X[ind, :] += np.random.multivariate_normal([0, 0], np.eye(2), (m, ))
  y[ind] = 1 - y[ind]

  # Plot this data
  cmap = ListedColormap(['#b30065', '#178000'])
  plt.scatter(X[:, 0], X[:, 1], c=y, cmap=cmap, edgecolors='k')

  # First, we use train_test_split to partition (X, y) into training and test
 sets
  X_trn, X_tst, y_trn, y_tst = train_test_split(X, y, test_size=tst_frac,
                                                random_state=42)

  # Next, we use train_test_split to further partition (X_trn, y_trn) into tra
ining and validation sets
  X_trn, X_val, y_trn, y_val = train_test_split(X_trn, y_trn, test_size=val_fr
ac,
                                                random_state=42)

  return (X_trn, y_trn), (X_val, y_val), (X_tst, y_tst)
```

In [2]:

```python
#
#  DO NOT EDIT THIS FUNCTION; IF YOU WANT TO PLAY AROUND WITH VISUALIZATION,
#  MAKE A COPY OF THIS FUNCTION AND THEN EDIT
#

def visualize(models, param, X, y):
  # Initialize plotting
  if len(models) % 3 == 0:
    nrows = len(models) // 3
  else:
    nrows = len(models) // 3 + 1

  fig, axes = plt.subplots(nrows=nrows, ncols=3, figsize=(15, 5.0 * nrows))
  cmap = ListedColormap(['#b30065', '#178000'])

  # Create a mesh
  xMin, xMax = X[:, 0].min() - 1, X[:, 0].max() + 1
  yMin, yMax = X[:, 1].min() - 1, X[:, 1].max() + 1
  xMesh, yMesh = np.meshgrid(np.arange(xMin, xMax, 0.01),
                             np.arange(yMin, yMax, 0.01))

  for i, (p, clf) in enumerate(models.items()):
    # if i > 0:
    #   break
    r, c = np.divmod(i, 3)
    ax = axes[r, c]

    # Plot contours
    zMesh = clf.decision_function(np.c_[xMesh.ravel(), yMesh.ravel()])
    zMesh = zMesh.reshape(xMesh.shape)
    ax.contourf(xMesh, yMesh, zMesh, cmap=plt.cm.PiYG, alpha=0.6)

    if (param == 'C' and p > 0.0) or (param == 'gamma'):
      ax.contour(xMesh, yMesh, zMesh, colors='k', levels=[-1, 0, 1],
                 alpha=0.5, linestyles=['--', '-', '--'])

    # Plot data
    ax.scatter(X[:, 0], X[:, 1], c=y, cmap=cmap, edgecolors='k')
    ax.set_title('{0} = {1}'.format(param, p))
```

In [3]:
```
# Generate the data
n_samples = 300    # Total size of data set
(X_trn, y_trn), (X_val, y_val), (X_tst, y_tst) = generate_data(n_samples)
```



---

a. (25 points) The effect of the regularization parameter, C

Complete the Python code snippet below that takes the generated synthetic 2-d data as input and learns non-linear SVMs. Use scikit-learn's [SVC (https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html)](https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html) function to learn SVM models with **radial-basis kernels** for fixed γ and various choices of $C \in \{10^{-3}, 10^{-2} \cdots, 1, \cdots 10^5\}$. The value of γ is fixed to $\gamma = \frac{1}{d \cdot \sigma_X}$, where d is the data dimension and σ_X is the standard deviation of the data set X. SVC can automatically use these setting for γ if you pass the argument gamma = 'scale' (see documentation for more details).

Plot: For each classifier, compute **both** the **training error** and the **validation error**. Plot them together, making sure to label the axes and each curve clearly.

Discussion: How do the training error and the validation error change with C? Based on the visualization of the models and their resulting classifiers, how does changing C change the models? Explain in terms of minimizing the SVM's objective function $\frac{1}{2}\mathbf{w}'\mathbf{w} \, + \, C \, \Sigma_{i=1}^n \, \ell(\mathbf{w} \mid \mathbf{x}_i, y_i)$, where ℓ is the hinge loss for each training example $(\mathbf{x}_i, y_i)$.

Final Model Selection: Use the validation set to select the best the classifier corresponding to the best value, C_{best}. Report the accuracy on the **test set** for this selected best SVM model. *Note: You should report a single number, your final test set accuracy on the model corresponding to $C{best}$_.*

In [4]:

```python
# Learn support vector classifiers with a radial-basis function kernel with
# fixed gamma = 1 / (n_features * X.std()) and different values of C
C_range = np.arange(-3.0, 6.0, 1.0)
C_values = np.power(10.0, C_range)

models = dict()
trnErr = dict()
valErr = dict()

for C in C_values:
  #
  #
  # Insert your code here to learn SVM models
  #
  #

visualize(models, 'C', X_trn, y_trn)

#
#
# Insert your code here to perform model selection
#
#
```

```
  File "<ipython-input-4-8875a1448a41>", line 17
    visualize(models, 'C', X_trn, y_trn)
            ^
IndentationError: expected an indented block
```

---

b. (25 points) The effect of the RBF kernel parameter, γ

Complete the Python code snippet below that takes the generated synthetic 2-d data as input and learns various non-linear SVMs. Use scikit-learn's [SVC (https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html)](https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html) function to learn SVM models with **radial-basis kernels** for fixed C and various choices of $\gamma \in \{10^{-2}, 10^{-1}\, 1, 10,\ 10^{2}\ 10^{3}\}$. The value of C is fixed to $C = 10$.

Plot: For each classifier, compute **both** the **training error** and the **validation error**. Plot them together, making sure to label the axes and each curve clearly.

Discussion: How do the training error and the validation error change with γ? Based on the visualization of the models and their resulting classifiers, how does changing γ change the models? Explain in terms of the functional form of the RBF kernel, $\kappa(\mathbf{x}, \mathbf{z}) \ = \ \exp(-\gamma \cdot \|\mathbf{x} - \mathbf{z}\|^2)$

Final Model Selection: Use the validation set to select the best the classifier corresponding to the best value, γ_{best}. Report the accuracy on the **test set** for this selected best SVM model. *Note: You should report a single number, your final test set accuracy on the model corresponding to $\gamma*{best}$_.

In [ ]:
```
# Learn support vector classifiers with a radial-basis function kernel with
# fixed C = 10.0 and different values of gamma
gamma_range = np.arange(-2.0, 4.0, 1.0)
gamma_values = np.power(10.0, gamma_range)

models = dict()
trnErr = dict()
valErr = dict()

for G in gamma_values:
  #
  #
  # Insert your code here to learn SVM models
  #
  #

visualize(models, 'gamma', X_trn, y_trn)

#
#
# Insert your code here to perform model selection
#
#
```

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2. **Breast Cancer Diagnosis with Support Vector Machines**, 25 points.

For this problem, we will use the [Wisconsin Breast Cancer (https://archive.ics.uci.edu/ml/datasets/Breast+Cancer+Wisconsin+(Diagnostic))](https://archive.ics.uci.edu/ml/datasets/Breast+Cancer+Wisconsin+(Diagnostic)) data set, which has already been pre-processed and partitioned into training, validation and test sets. Numpy's [loadtxt (https://docs.scipy.org/doc/numpy-1.13.0/reference/generated/numpy.loadtxt.html)](https://docs.scipy.org/doc/numpy-1.13.0/reference/generated/numpy.loadtxt.html) command can be used to load CSV files.

In [ ]:
```
# Load the Breast Cancer Diagnosis data set; download the files from eLearning
# CSV files can be read easily using np.loadtxt()
#
# Insert your code here.
#
```

Use scikit-learn's [SVC (https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html)](https://scikit-learn.org/stable/modules/generated/sklearn.svm.SVC.html) function to learn SVM models with **radial-basis kernels** for **each combination** of $C \in \{10^{-2}, 10^{-1}, 1, 10^1, \cdots 10^4\}$ and $\gamma \in \{10^{-3}, 10^{-2}\, 10^{-1}, 1,\ 10,\ 10^2\}$. Print the tables corresponding to the training and validation errors.

Final Model Selection: Use the validation set to select the best the classifier corresponding to the best parameter values, C_{best} and γ_{best}. Report the accuracy on the **test set** for this selected best SVM model. *Note: You should report a single number, your final test set accuracy on the model corresponding to $C*{best} and\gamma*{best}$*.

```
In [ ]: #
        #
        # Insert your code here to perform model selection
        #
        #
```

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3. **Breast Cancer Diagnosis with k-Nearest Neighbors**, 25 points.

Use scikit-learn's [k-nearest neighbor (https://scikit-learn.org/stable/modules/generated/sklearn.neighbors.KNeighborsClassifier.html)](https://scikit-learn.org/stable/modules/generated/sklearn.neighbors.KNeighborsClassifier.html) classifier to learn models for Breast Cancer Diagnosis with $k \in \{1,\ 5,\ 11,\ 15,\ 21\}$, with the kd-tree algorithm.

Plot: For each classifier, compute **both** the **training error** and the **validation error**. Plot them together, making sure to label the axes and each curve clearly.

Final Model Selection: Use the validation set to select the best the classifier corresponding to the best parameter value, k_{best}. Report the accuracy on the **test set** for this selected best kNN model. *Note: You should report a single number, your final test set accuracy on the model corresponding to $k*{best}$_.

```
In [ ]: #
        #
        # Insert your code here to perform model selection
        #
        #
```

Discussion: Which of these two approaches, SVMs or kNN, would you prefer for this classification task? Explain.