

The importance of being dissimilar in Recommendation

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ABSTRACT

In recommendation scenarios, similarity measures play a fundamental role in memory-based nearest neighbors approaches. In fact, they recommend items to a user based on the similarity of either items or users in a neighborhood. In this paper, we argue that similarity between users or items, although it keeps leading importance in computing recommendations, should be paired with a value of dissimilarity (computed not just as the complement of the similarity one). We formally modeled and injected this notion in some of the most used similarity measures and evaluated our approach in a recommendation scenario showing its effectiveness with respect to accuracy and diversity results on three different datasets.

CCS CONCEPTS

• **Information systems** → **Similarity measures; Recommender systems; Top-k retrieval in databases;**

KEYWORDS

Dissimilarity, Asymmetric similarity, Collaborative Filtering

ACM Reference Format:

Vito Walter Anelli^{*}, Tommaso Di Noia^{*}, Eugenio Di Sciascio^{*}, Azzurra Ragone[•], Joseph Trotta^{*}. 2019. The importance of being dissimilar in Recommendation. In *The 34th ACM/SIGAPP Symposium on Applied Computing (SAC '19)*, April 8–12, 2019, Limassol, Cyprus. ACM, New York, NY, USA, 6 pages. <https://doi.org/10.1145/3297280.3297360>

1 INTRODUCTION

Neighborhood-based approaches have been the first family of algorithms developed for collaborative filtering recommender systems. They identify similar users or items, and they provide users with a list of items they could be interested in by exploiting the degree of similarity. Though there have been around for many years, it has been shown that neighborhood-based approaches may perform better than latent model-based methods to solve the *top-N* recommendation problem [3, 12, 16, 20]. In the *top-N* recommendation

task, the focus is on providing an accurate ranked list rather than minimizing the rating prediction error. Among neighborhood-based methods the best-known are user-kNN, item-kNN and Sparse Linear Methods (SLIM) [20]. User-based and item-based schemes have proven to be effective in different settings although they use the same logic behind the scenes. In details item-kNN and SLIM (which uses an item-based scheme) have shown to outperform user-kNN to solve the *top-N* recommendation problem [10] and several algorithms have been proposed in the literature to enhance neighbors models like GLSLIM [10] and Weighted kNN-GRU4REC [15], taking advantage of personalized models and recurrent neural networks. Moreover, we also have approaches focusing on the injection of time in neighborhood models [6] and modeling similarities by directly optimizing the pair-wise preferences error [23]. All in all, under the hood, what makes neighborhood-based methods work is a similarity measure.

Several similarity measures have been proposed and used extensively such as Jaccard [13][22] and Tanimoto [11] coefficients, Cosine Vector similarity [1, 4, 7], Pearson Correlation [14], Constrained Pearson correlation [25], Adjusted Cosine similarity [24], Mean Squared Difference similarity [25], Spearman Rank Correlation [18], Frequency-Weighted Pearson Correlation [8], Target item weighted Pearson Correlation [5]. In the vast majority of cases, all these similarity measures are based on two assumptions:

- (1) the correlation between i and j is the same correlation between j and i (symmetry of similarity);
- (2) the correlation between two entities only captures how much they are similar to each other without taking into account their degree of dissimilarity.

To the best of our knowledge, a few works have been proposed in the past years related to asymmetric similarity, and they are mainly designed for a user-based scheme. Dissimilarity was first suggested in 1999 [28] when Varian described the value of introducing diversity into search results. An Asymmetric User Similarity has been proposed in [19] where the authors underline that a similarity measure should distinguish between a user with a rich profile and a cold user. Thus, given two users u and v , they slightly modify the Jaccard index in order to consider exclusively the number of ratings of the current user (instead of the overall number of both users). In HYBR-Tyco [17] the similarity proposed by Millan [19] is combined with the Sørensen index [26]. HYBRTyco [17] is a hybrid recommender system which combines matrix factorization with an asymmetric similarity model to realize a typicality-based collaborative filtering recommender system. The same approach is exploited in another asymmetric user similarity model [21] to feed a user-user similarity

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SAC '19, April 8–12, 2019, Limassol, Cyprus

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ACM ISBN 978-1-4503-5933-7/19/04...\$15.00

<https://doi.org/10.1145/3297280.3297360>

matrix that is then completed using a matrix factorization algorithm. Additionally, both of them provide an extension for the latter similarity measure based on explicit numerical feedbacks (ratings). Despite previous works are focused on the user-based scheme, we already underlined that item-kNN shows excellent performance in *top-N* recommendation task. Moreover, when the number of users exceeds the number of items, as in most of the cases, item-based recommendation approaches require much less memory and time to compute the similarity weights than user-based ones, making them more scalable. Due to these reasons, both approaches have been considered in this work.

In this work, we investigate the effect on recommendation accuracy when we go beyond the above two assumptions and define (and include) the concepts of dissimilarity and asymmetry in similarity measures. In our proposal, we start from a probabilistic interpretation of similarity to define symmetric and asymmetric dissimilarities. The dissimilarity measures are then combined with traditional similarity values using additive and multiplicative strategies. The experimental evaluation shows that our approach outperforms the non-dissimilarity-aware counterparts improving the accuracy of results or diversity or both.

The rest of the paper is organized as follows: Section 2 presents the motivation behind our work and the proposed approach. Section 3 presents the evaluation protocol, metrics, datasets and performance of the method. Finally, in Section 4 concluding considerations are provided.

2 DISSIMILARITY IN RECOMMENDATION

2.1 Motivation

The main idea behind our proposal is that symmetric similarity may not be sufficient to capture subtle interactions between items. We assume that representing the similarity through traditional measures can lead to imperfect results as important information might not be properly considered. Let us consider some examples in an item-kNN scenario. Suppose we are dealing with a dataset containing rating data from the book domain on the following books:

| Title | Short name | Author | # Votes |
|----------------------|------------|-----------------|---------|
| A Game of Thrones | GoT | G. R. R. Martin | 100 |
| A Dance with Dragons | DwD | G. R. R. Martin | 10 |
| Shroud of Eternity | SoE | T. Goodkind | 120 |

By looking at the previous data, we see that both *A Game of Thrones* and *A Dance with Dragons* belong to the same saga *A song of ice and fire* and they are, respectively, the first and the fifth volume. *Shroud of Eternity* is the second volume of Nicci Chronicles' saga. We may assume that all the users who rated DwD also rated GoT, i.e., $U_{DwD} \subseteq U_{GoT}$. Analogously, since the topic of the book is mostly the same, we may assume that a number of readers of SoE also voted GoT, $U_{SoE} \cap U_{GoT} \neq \emptyset$. Suppose now that we have $U_{SoE} \cap U_{GoT} = 20$ and $U_{DwD} \cap U_{GoT} = 10$. If we compute the Jaccard similarity between the pairs SoE, GoT and DwD, GoT we have

$$JS(DwD, GoT) = \frac{|U_{DwD} \cap U_{GoT}|}{|U_{DwD} \cup U_{GoT}|} = 0.1$$

$$JS(SoE, GoT) = \frac{|U_{SoE} \cap U_{GoT}|}{|U_{SoE} \cup U_{GoT}|} = 0.1$$

In our opinion, much relevant information has been lost in this simple example. The scenario is shown graphically in Figure 1.

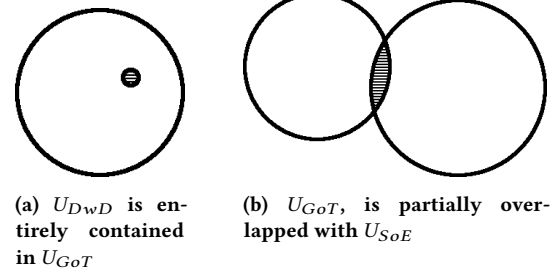


Figure 1: Representation of the motivation example

It is clear that U_{DwD} is a proper subset of U_{GoT} and, on the contrary, there are many users in U_{SoE} that have not experienced GoT. This information is mostly lost in the computation of the similarity values even though a piece of this information is retained in the denominator of the Jaccard coefficient through the overall value of $U_{GoT} \cup U_{DwD}$ and $U_{GoT} \cup U_{SoE}$.

In order to clarify the reason why we do not consider this remaining information sufficient, let us consider a more formal description of the scenario. Many similarity measures (like Jaccard in the previous example) mainly rely on a value that denotes the similarity between two items normalized by their overall weight. This can be represented as a probability. Let $\mathcal{I}(u) = \{\langle i, r_{ui} \rangle \mid u \text{ rated } i \text{ with } r_{ui}\}$ be the user profile containing the pairs item-rating and $\mathcal{U}(i) = \{\langle u, r_{ui} \rangle \mid \langle i, r_{ui} \rangle \in \mathcal{I}(u)\}$ be the set of users that experienced i . If we consider Jaccard similarity we see it represents the following probability:

$$p^{JS}(i, j) = p(\langle u, r_{ui} \rangle \in \mathcal{U}(i) \wedge \langle u, r_{uj} \rangle \in \mathcal{U}(j)) \quad (1)$$

which can be read as the probability that u experienced both i and j . Under a probabilistic lens, we may define dissimilarity measures with two different probabilities:

- the probability that a generic user u experienced the item i but never experienced the item j
- the probability that a generic user u experienced the item j but never experienced the item i

Since we are introducing an asymmetric behavior in computing the similarity between i and i as well as between j and i , we see that the two probabilities have a different role while computing a similarity measure.

In a classical memory-based Item-kNN, $sim(i, j)$ is used to compare i to different j to find out which j s are the most similar to i . In practical terms, we are interested in how much j is similar to i . If we focus on Figure 1a, we realize that DwD is more similar to GoT than the opposite situation. The reason behind this behavior is not directly related to the size of the involved sets but it depends on the probability that a user who experienced GoT did not experience DwD.

Though some interesting asymmetric similarities have been proposed in the last years [17, 21], to our knowledge, no one focused on this probability that represents a negative asymmetric dissimilarity.

2.2 Metrics

In this work, we propose a general asymmetric similarity model in which items i, j similarities are computed by taking into account the

probability that users that experienced j never experienced i . The idea behind our work is preserving the core meaning of a specific similarity, applying a corrective factor encoding the dissimilarity we mentioned before.

We introduced this correction into two binary symmetric similarities: Jaccard and Sørensen index, and in the asymmetric variant to the Jaccard coefficient proposed in [19]. We tested this correction both as an additive and as a multiplicative factor.

For the sake of completeness, we reintroduce *Jaccard coefficient similarity* (JS) that, for a memory-based item-kNN model can be expressed as:

$$JS(i, j) = \frac{|\mathcal{U}(i) \cap \mathcal{U}(j)|}{|\mathcal{U}(i) \cup \mathcal{U}(j)|} \quad (2)$$

The probability that users who experienced j never experienced i can be modeled as the complementary probability of Equation (1) w.r.t. $\mathcal{U}(i)$ over $|\mathcal{U}(i) \cup \mathcal{U}(j)|$. This probability, we name *Jaccard Asymmetric Dissimilarity* (JAD) can be formulated as follows:

$$JAD(i, j) = \frac{|\mathcal{U}(j)| - |\mathcal{U}(i) \cap \mathcal{U}(j)|}{|\mathcal{U}(i) \cup \mathcal{U}(j)|} \quad (3)$$

Again, Equation (3) can be seen in terms of probability as

$$p^{JAD}(i, j) = p(\langle u, r_{uj} \rangle \in \mathcal{U}(j)) - p^{JS}(i, j)$$

We now propose to modify the original similarity by injecting the former negative correction weighted with a parameter λ that can be easily customized. The overall similarity, *Additive Adjusted Jaccard* (AAJ), is then formulated as:

$$AAJ(i, j) = JS(i, j) - \lambda \cdot JAD(i, j) \quad (4)$$

where λ is a parameter that could depend on many factors such as the number of users and items in the dataset and the intensity of interactions between them.

Recalling that the overall formula should represent a degree of similarity between the two different items we defined the Multiplicative Adjusted Jaccard as the product of Jaccard similarity with the inverse of Jaccard Asymmetric Dissimilarity ($IJAD$). In other words, we use $\frac{1}{p^{JAD}(i, j)}$ as a corrective factor for $p^{JS}(i, j)$. We then define the *Multiplicative Adjusted Jaccard* (MAJ) as:

$$MAJ(i, j) = \frac{JS(i, j)}{JAD(i, j)} = JS(i, j) \cdot IJAD(i, j) \quad (5)$$

In the multiplicative variant, in order to avoid division by zero the minimum value for JAD is set to $\frac{1}{|\mathcal{U}(i) \cup \mathcal{U}(j)|}$. As mentioned in Section 2.1 a symmetric variant of Jaccard coefficient (named *Jaccard Symmetric Dissimilarity* (JSD)) can be used composing Equation (3) with the probability that a user u experienced the item i but never experienced the item j :

$$\begin{aligned} JSD(i, j) &= \frac{(|\mathcal{U}(j)| - |\mathcal{U}(i) \cap \mathcal{U}(j)|) + (|\mathcal{U}(i)| - |\mathcal{U}(i) \cap \mathcal{U}(j)|)}{|\mathcal{U}(i) \cup \mathcal{U}(j)|} \\ &= \frac{|\mathcal{U}(i)| + |\mathcal{U}(j)| - 2 \cdot |\mathcal{U}(i) \cap \mathcal{U}(j)|}{|\mathcal{U}(i) \cup \mathcal{U}(j)|} \end{aligned}$$

leading to the corresponding probability

$$p^{JSD}(i, j) = p^{JAD}(i, j) + p^{JAD}(j, i)$$

Thus the *Symmetric Additive Adjusted Jaccard* ($S-AAJ$) and *Symmetric Multiplicative Adjusted Jaccard* ($S-MAJ$) can be defined as follows:

$$S-AAJ(i, j) = JS(i, j) - \lambda \cdot JSD(i, j)$$

$$S-MAJ(i, j) = \frac{JS(i, j)}{JSD(i, j)}$$

In order to test our idea, we applied all the variants previously introduced for Jaccard similarity to two popular similarity measures: *Asymmetric Jaccard Similarity* (AJS) and *Sørensen coefficient* (SOR). All the derived variants are represented, respectively, in Table 1 and Table 2.

Table 1: Asymmetric Jaccard considered variants.

| Short name | Extended | Formula |
|-------------|--------------------------------------|---|
| AJS(i,j) | Asymm. Jaccard Similarity | $\frac{ \mathcal{U}(i) \cap \mathcal{U}(j) }{ \mathcal{U}(i) }$ |
| AJD(i,j) | Asymm. Jaccard Dissimilarity | $\frac{ \mathcal{U}(j) - \mathcal{U}(i) \cap \mathcal{U}(j) }{ \mathcal{U}(i) }$ |
| AAAJ(i,j) | Additive Adjusted Asymm. Jaccard | $AJS(i, j) - \lambda \cdot AJD(i, j)$ |
| MAAJ(i,j) | Multiplicative Adjus. Asymm. Jaccard | $AJS(i, j) \cdot IJAD(i, j)$ |
| S-AAAJ(i,j) | Symmetric AAAJ | $AJS(i, j) - \lambda \cdot \frac{ \mathcal{U}(i) + \mathcal{U}(j) - 2 \cdot \mathcal{U}(i) \cap \mathcal{U}(j) }{ \mathcal{U}(i) }$ |
| S-MAAJ(i,j) | Symmetric MAAJ | $AJS(i, j) \cdot IJSD(i, j)$ |

Table 2: Sørensen similarity considered variants.

| Short name | Extended | Formula |
|------------|--|--|
| SOR(i,j) | Sørensen Similarity | $\frac{ \mathcal{U}(i) \cap \mathcal{U}(j) }{ \mathcal{U}(i) + \mathcal{U}(j) }$ |
| ASD(i,j) | Asymm. Sørensen Dissimilarity | $\frac{ \mathcal{U}(j) - \mathcal{U}(i) \cap \mathcal{U}(j) }{ \mathcal{U}(i) + \mathcal{U}(j) }$ |
| AAS(i,j) | Additive Adjusted Assym Sørensen | $SOR(i, j) - \lambda ASD(i, j)$ |
| MAS(i,j) | Multiplicative Adjusted Assym Sørensen | $SOR(i, j) \cdot IJAD(i, j)$ |
| S-AAS(i,j) | Symmetric AAS | $SOR(i, j) - \lambda \frac{ \mathcal{U}(i) + \mathcal{U}(j) - 2 \cdot \mathcal{U}(i) \cap \mathcal{U}(j) }{ \mathcal{U}(i) + \mathcal{U}(j) }$ |
| S-MAS(i,j) | Symmetric MAS | $SOR(i, j) \cdot IJSD(i, j)$ |

All the above metrics have been introduced having in mind an item-kNN approach but, without loss of generality, they can be applied to user-kNN model as well.

3 EXPERIMENTAL EVALUATION

The experimental evaluation has been carried out on three publicly available datasets and with different values of k and λ .

Datasets. We evaluated the effectiveness of our approach on the three datasets shown in Table 3 belonging to different domains (Music, Books, and Movies). The Last.fm dataset [9] corresponds to transactions with Last.fm online music system released in Het-Rec 2011¹. It contains social networking, tagging, and music artist listening information from a set of 2K users. LibraryThing represents books ratings collected in the LibraryThing community website. It contains social networking, tagging, and rating information on a [1..10] scale. Yahoo!Movies (Yahoo! Webscope dataset ydata-ymovies-user-movie-ratings-content-v1_0)² contains movies

¹<http://ir.ii.uam.es/hetrec2011/>

²http://research.yahoo.com/Academic_Relations

ratings generated by Yahoo! Movies up to November 2003. It provides content, demographic, rating information, and mappings to MovieLens and EachMovie datasets.

Table 3: Datasets statistics.

| Dataset | #Users | #Items | #Transactions | Sparsity |
|---------------|--------|--------|---------------|----------|
| Yahoo! Movies | 7642 | 11,916 | 221,367 | 99.76% |
| LibraryThing | 7279 | 37,232 | 2,056,487 | 99.24% |
| Last FM | 1850 | 11,247 | 59,071 | 99.72% |

Columns corresponding to #Users, #Items and #Transactions show the number of users, number of items and number of transactions, respectively, in each dataset. The last column shows the sparsity of the dataset.

Evaluation Protocol and Experimental Setting with Parameters tuning. The evaluation protocol we adopted in our experiments is *all unrated items* [27]. With this protocol, the recommendation list is computed from a candidate list given by the cartesian product between users and items minus the items each user experimented in the training set. We performed a temporal 64-16-20 hold-out split (when temporal information is available) retaining the last 20% of ratings as test set and 16% as validation set. We measured the performance by computing Precision@N ($Prec@N$) for *top-N* recommendation lists as accuracy metric. Precision has been computed on a per-user basis, and the returned results have been averaged. As Precision needs relevant items to be computed, we set the relevance threshold to 8 over 10 for LibraryThing and Yahoo!Movies, and to 0 for Last.fm since in this latter no ratings are provided but the number of user-item transactions. We measured Diversity through *catalog coverage* (aggregate diversity in *top-N* list). The *catalog coverage*, also called diversity-in-top-N ($D@N$) [2], is measured by computing the overall number of different items recommended within the complete recommendation list. It represents the propensity of a system to recommend always the same items.

Baselines. We compared our approaches in both User-kNN and Item-kNN settings. The former finds the k-nearest user neighbors based on a similarity function and then exploits them to predict a score for each user-item pair. The latter is the item-based version of the k-nearest neighbors algorithm that uses the k-nearest items to compute the predictions. For both schemes we used the validation set to find the optimal hyper-parameters. However, we are not interested in the algorithm itself but on the similarity measures that are used to compute neighbors and predictions. As baseline to compare with, we used both symmetric and asymmetric measures, namely, *Jaccard (JS)* and *Sorensen (SOR)* (for symmetric measures) and *asymmetric Jaccard (AJS)* and *asymmetric Jaccard weighted with the Sorensen Index (ASOR)* [17] (for asymmetric measures). For all the similarities that make use of λ we evaluated them varying λ in $\{0.2, 0.4, 0.6, 0.8\}$ whereas we considered a number of *Neighbors* varying in $\{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. We ran the algorithms with all possible combinations, and we selected the best performing ones with respect to Precision@N. The best parameters for user-based and item-based schemes are represented, respectively in Table 4 and 5.

Performance of the proposed methods. Results in Table 6 show the performance of all algorithms with a user-based scheme. Concerning accuracy, it is clear that both asymmetric and symmetric multiplicative variants are the best-performing ones. MAJ and S-MAJ achieve good performance, outperforming JS. The same trend

is shown between MAAJ, S-MAAJ, and AJS. The same behavior can be observed in the Sorensen algorithms block, in which MAS outperforms SOR in LibraryThing and Yahoo!Movies datasets with the only exception of Last.fm datasets. Concerning Diversity the proposed variants constantly outperform the base variants. It is worth to note that AAJ and AAAJ algorithms that can recommend much more items with a little loss of Precision. Table 7 shows the results for the item-based scheme. Performance is much different here, and we can note that the multiplicative asymmetric variants have a worse behavior. However, the Last.fm results show that the additive variants outperform the base ones. We can observe the same behavior also in the whole asymmetric Jaccard similarity block for all the three datasets. The Jaccard similarity and the Sorensen block, for LibraryThing and Yahoo!Movies show no clear champion concerning the performance of JS/SOR and S-MAJ/S-MAS that result very similar. This behavior may be due to the tuning results that are very close to each other, and this probably prevented us from selecting the best parameters.

Experimental Setting with a fixed number of neighbors. In tables 4 and 5 we see that best values of λ and k are different depending on the adopted approach. Hence, we tested also the different algorithms with a fixed number of neighbors. In other words, we checked: How would the different algorithms perform if, i.e., we fix the number of neighbors? Once again we employed the *all unrated items* evaluation protocol to evaluate the methods. We performed a temporal 80-20 hold-out split retaining the last 20% of ratings as test set using temporal information when available.

Baselines. Also in this experiment, we compared our approaches with both User-kNN and Item-kNN settings considering, for all the algorithms, the number of neighbors fixed and set to $k = 80$. For all the similarities that make use of λ we evaluated them varying λ in $0.2, 0.4, 0.6, 0.8$. In Tables 8 and 9 we show the best results we obtained³. The best values regarding Precision and Aggregate Diversity are highlighted in bold. We computed significance tests for precision results, and we found they are statistically significant at the 0.05 level w.r.t. their respective baselines.

Performance of the proposed methods. Results in Table 8 show that our approach always outperforms baseline variants in the User-kNN scheme. In details **additive** asymmetric similarity and **multiplicative** asymmetric similarity significantly perform better than JS, SOR and ASOR for all three dataset. Among these two variants of similarity, the multiplicative variant is the best-performing one. Quite interestingly, modifying AJS, which is asymmetric in its inner nature with our asymmetric dissimilarity factor leads to an improvement irrespective of the considered dataset. It is worth noticing that, other than the accuracy improvements, aggregate diversity also increases due to the dissimilarity injection. In details, the asymmetric additive variant achieves the best results and triples catalog coverage values for LibraryThing and Yahoo!Movies.

Table 9 shows Precision and Catalog Coverage results for an item-based scheme. Obtained results are quite interesting for many reasons. First of all, it is clear that the same similarities can lead to

³The complete results are publicly available at <https://github.com/sisinflab/The-importance-of-being-dissimilar-in-Recommendation>.

Table 4: Best parameters for User-kNN scheme

| Precision - P@10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------------------|----|-----|-----|-------|-------|------|-----|-----|-----|-------|-------|-----|------|------|--------|--------|-----|-----|-----|-----|-----|----|----|-----|-----|-----|-----|-----|----|------|
| | JS | AAJ | MAJ | S-AAJ | S-MAJ | ASOR | SOR | AAS | MAS | S-AAS | S-MAS | AJS | AAAJ | MAAJ | S-AAAJ | S-MAAJ | | | | | | | | | | | | | | |
| Datasets | k | λ | k | k | λ | k | k | k | k | λ | k | k | λ | k | k | λ | k | k | λ | k | k | λ | k | k | λ | k | k | λ | k | P@10 |
| LibraryThing | 50 | 0.2 | 100 | 50 | 0.2 | 30 | 100 | 90 | 50 | 0.2 | 50 | 50 | 0.2 | 30 | 100 | 20 | 0.2 | 100 | 40 | 0.2 | 50 | 20 | 20 | 0.2 | 100 | 40 | 0.2 | 50 | 20 | 20 |
| Yahoo | 90 | 0.4 | 100 | 40 | 0.2 | 100 | 90 | 100 | 90 | 0.2 | 40 | 40 | 0.2 | 100 | 90 | 10 | 0.6 | 80 | 30 | 0.2 | 100 | 70 | 10 | 0.6 | 80 | 30 | 0.2 | 100 | 70 | 10 |
| Last FM | 90 | 0.2 | 100 | 100 | 0.2 | 50 | 90 | 90 | 100 | 0.2 | 100 | 100 | 0.2 | 50 | 90 | 30 | 0.2 | 100 | 100 | 0.2 | 50 | 20 | 30 | 0.2 | 100 | 100 | 0.2 | 50 | 20 | 20 |

Table 5: Best parameters for Item-kNN scheme

| Precision - P@10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------------------|----|-----|-----|---|-----|-----|-------|---|-------|---|------|----|-----|----|-----|----|-----|----|-------|----|-------|----|-----|----|------|----|------|----|--------|----|--------|------|
| | JS | | AAJ | | MAJ | | S-AAJ | | S-MAJ | | ASOR | | SOR | | AAS | | MAS | | S-AAS | | S-MAS | | AJS | | AAAJ | | MAAJ | | S-AAAJ | | S-MAAJ | |
| Datasets | k | λ | k | k | k | λ | k | k | k | k | k | λ | k | k | k | λ | k | k | k | k | k | k | k | λ | k | k | k | k | λ | k | k | P@10 |
| LibraryThing | 10 | 0.2 | 20 | | 20 | 0.4 | 10 | | 20 | | 70 | 10 | 0.2 | 20 | | 20 | 0.4 | 10 | | 20 | | 60 | 0.2 | 10 | | 10 | 0.2 | 10 | | 60 | | |
| Yahoo | 10 | 0.2 | 20 | | 10 | 0.2 | 10 | | 10 | | 10 | 10 | 0.2 | 10 | | 10 | 0.2 | 10 | | 10 | | 20 | 0.2 | 20 | | 10 | 0.2 | 10 | | 10 | | |
| Last FM | 10 | 0.2 | 20 | | 30 | 0.2 | 20 | | 10 | | 10 | 10 | 0.2 | 30 | | 30 | 0.2 | 20 | | 10 | | 10 | 0.2 | 40 | | 20 | 0.2 | 70 | | 10 | | |

Table 6: Comparison in terms of Precision and Aggregate Diversity for User-kNN scheme with best parameters

| Precision - P@10 | | | | | | | | | | | | | | | | | |
|----------------------------|---------|-------------|----------------|-------------|----------------|----------------|----------------|---------|----------------|-------------|---------|---------|-------------|----------------|-------------|----------------|--|
| Datasets | JS | AAJ | MAJ | S-AAJ | S-MAJ | ASOR | SOR | AAS | MAS | S-AAS | S-MAS | AJS | AAAJ | MAAJ | S-AAAJ | S-MAAJ | |
| LibraryThing | 0.03025 | 0.04005 | 0.04675 | 0.00416 | 0.03102 | 0.03565 | 0.03031 | 0.04687 | 0.04687 | 0.00416 | 0.03088 | 0.03768 | 0.03871 | 0.04670 | 0.00429 | 0.03796 | |
| Yahoo | 0.04165 | 0.05196 | 0.06284 | 0.03892 | 0.04149 | 0.04261 | 0.04151 | 0.06354 | 0.06354 | 0.03917 | 0.04170 | 0.05465 | 0.04234 | 0.06176 | 0.03705 | 0.05207 | |
| Last FM | 0.02704 | 0.02163 | 0.02532 | 0.00601 | 0.02773 | 0.02927 | 0.02747 | 0.02532 | 0.02532 | 0.00609 | 0.02695 | 0.02901 | 0.02120 | 0.03039 | 0.00592 | 0.03107 | |
| Aggregate Diversity - D@10 | | | | | | | | | | | | | | | | | |
| Datasets | JS | AAJ | MAJ | S-AAJ | S-MAJ | ASOR | SOR | AAS | MAS | S-AAS | S-MAS | AJS | AAAJ | MAAJ | S-AAAJ | S-MAAJ | |
| LibraryThing | 2735 | 7330 | 3093 | 3341 | 1953 | 2687 | 2660 | 2999 | 2999 | 3337 | 1879 | 2519 | 7443 | 3037 | 3337 | 2834 | |
| Yahoo | 698 | 2087 | 1126 | 2181 | 796 | 882 | 662 | 1074 | 1074 | 2179 | 732 | 974 | 1903 | 1147 | 2177 | 580 | |
| Last FM | 1136 | 1665 | 1485 | 1156 | 1203 | 1283 | 1097 | 1446 | 1446 | 1154 | 1145 | 764 | 1748 | 889 | 1163 | 1025 | |

Table 7: Comparison in terms of Precision and Aggregate Diversity for Item-kNN scheme with best parameters

| Precision - P@10 | | | | | | | | | | | | | | | | | | |
|----------------------------|----------------|----------------|--------------|---------|----------------|---------|----------------|---------|--------------|----------------|----------------|--------------|----------------|---------|---------|---------|--|--|
| Datasets | JS | AAJ | MAJ | S-AAJ | S-MAJ | ASOR | SOR | AAS | MAS | S-AAS | S-MAS | AJS | AAAJ | MAAJ | S-AAAJ | S-MAAJ | | |
| LibraryThing | 0.08180 | 0.07449 | 0.03189 | 0.06482 | 0.08048 | 0.07835 | 0.07949 | 0.02462 | 0.02462 | 0.06111 | 0.07894 | 0.04556 | 0.08748 | 0.02256 | 0.06293 | 0.06097 | | |
| Yahoo | 0.05105 | 0.04976 | 0.00272 | 0.04966 | 0.05187 | 0.05013 | 0.05043 | 0.00196 | 0.00196 | 0.04915 | 0.05141 | 0.01598 | 0.05031 | 0.00175 | 0.04842 | 0.02437 | | |
| Last FM | 0.02146 | 0.02549 | 0.00326 | 0.02489 | 0.02120 | 0.02052 | 0.02069 | 0.00275 | 0.00275 | 0.02403 | 0.02077 | 0.00876 | 0.02910 | 0.00240 | 0.02506 | 0.01373 | | |
| Aggregate Diversity - D@10 | | | | | | | | | | | | | | | | | | |
| Datasets | JS | AAJ | MAJ | S-AAJ | S-MAJ | ASOR | SOR | AAS | MAS | S-AAS | S-MAS | AJS | AAAJ | MAAJ | S-AAAJ | S-MAAJ | | |
| LibraryThing | 11945 | 12338 | 22097 | 11654 | 11551 | 10521 | 11774 | 21365 | 21365 | 11447 | 11450 | 18604 | 12899 | 13042 | 13604 | 16018 | | |
| Yahoo | 3262 | 5159 | 4644 | 3466 | 3091 | 3539 | 3340 | 4742 | 4742 | 4351 | 3725 | 6864 | 3946 | 3645 | 4514 | 5565 | | |
| Last FM | 2867 | 4394 | 3884 | 2920 | 2763 | 3389 | 2917 | 3648 | 3654 | 3446 | 3341 | 5150 | 3680 | 3078 | 3429 | 4447 | | |

very different results depending on the adopted scheme. In particular, asymmetric Jaccard (AJS) performs very badly for the item-kNN algorithm. Under the dissimilarities perspective, we have the same behavior, and the multiplicative approach performs badly. Quite surprisingly, the additive version can always outperform the base variants. This suggests that adopting an additive strategy for item-kNN may lead to better results. This may be due to the wide number of items pairs without any common user. Focusing on additive symmetric and asymmetric similarities we can note that aggregate diversity results reflect the same improvements observed in accuracy values. The only case that appears to behave differently is AAAJ that registered a catalog coverage lower than AJS. This happens as we considered the best performing λ for precision. In case of $\lambda \in \{0.4, 0.6\}$ we obtain aggregate diversity values of 16, 205 and 18, 071, respectively, with precision results constantly higher than AJS (0.09374 and 0.08820). We may observe another interesting pattern on the Yahoo!Movies row: the symmetric version outperforms the asymmetric one. This could be due to some datasets characteristic. By looking at the data in Table 3 we see that the ratio of the number of items to the number of users is much higher in LibraryThing and Last.fm (≈ 5 and ≈ 6) with respect to Yahoo!Movies (≈ 1.5). This suggests that the more the ratio is, the more is convenient to adopt an asymmetric scheme. However, this consideration needs to be further investigated.

4 CONCLUSION AND FUTURE WORK

In this work, we propose a method to improve the performance of neighborhood-based models, by capturing subtle interactions between users and items, which cannot be appreciated using a traditional similarity measure. We defined a dissimilarity measure, that can be used combined with traditional user-based and item-based schemes. The proposed approach takes into account the single asymmetric components, leading to an improvement in both precision and aggregate diversity results. We performed a comparative experimental evaluation using three well-known datasets, varying the tuning parameter λ and k . Experiments show that our approach outperforms competing algorithms, denoting the usefulness of incorporating symmetric and asymmetric dissimilarity in neighborhood-based models. We are currently working on an extension of our idea that takes into account also user ratings and not just set-based measures. As a further extension, we are also interested in making the approach even more personalized by weighting dissimilarity with user-centered values of λ .

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Table 8: Comparison in terms of Precision and Aggregate Diversity for User-kNN scheme

| Precision - P@10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------------------------|--------|-----------|-------------|-----------|---------------|-----------|-------------|-----------|--------|-----------|--------|-----------|-------------|-------------|---------------|---------------|--------|-----------|--------|-------------|--------|---------------|---------------|-----------|--------|-----------|------|-----------|--------|-----------|--------|-----------|
| | JS | | AAJ | | MAJ | | S-AAJ | | S-MAJ | | ASOR | | SOR | | AAS | | MAS | | S-AAS | | S-MAS | | AJS | | AAAJ | | MAAJ | | S-AAAJ | | S-MAAJ | |
| Datasets | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ |
| LibraryThing | 0.0363 | 0.2 | 0.0582 | | 0.0627 | 0.4 | 0.0364 | 0.0010 | 0.0394 | | 0.0363 | 0.2 | 0.0586 | | 0.0629 | 0.2 | 0.0405 | 0.0364 | 0.0364 | 0.2 | 0.0558 | | 0.0603 | 0.2 | 0.0057 | 0.0375 | | | | | | |
| Yahoo | 0.0437 | 0.4 | 0.0561 | | 0.0676 | 0.2 | 0.0403 | 0.0433 | 0.0442 | | 0.0438 | 0.4 | 0.0573 | | 0.0687 | 0.2 | 0.0568 | 0.0435 | 0.0607 | 0.6 | 0.0529 | | 0.0664 | 0.2 | 0.0378 | 0.0529 | | | | | | |
| Last FM | 0.0164 | 0.4 | 0.0242 | | 0.0257 | 0.2 | 0.0037 | 0.0160 | 0.0215 | | 0.0166 | 0.4 | 0.0241 | 0.0253 | 0.2 | 0.0275 | 0.0164 | 0.0248 | 0.2 | 0.0228 | | 0.0323 | 0.2 | 0.0037 | 0.0242 | | | | | | | |
| Aggregate Diversity - D@10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | JS | | AAJ | | MAJ | | S-AAJ | | S-MAJ | | ASOR | | SOR | | AAS | | MAS | | S-AAS | | S-MAS | | AJS | | AAAJ | | MAAJ | | S-AAAJ | | S-MAAJ | |
| Datasets | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ |
| LibraryThing | 2136 | 0.2 | 7367 | | 2406 | 0.4 | 2246 | 873 | 2819 | | 2083 | 0.2 | 7330 | 2292 | 0.2 | 5528 | 2171 | 1299 | 0.2 | 7501 | 2060 | 0.2 | 3331 | 1486 | | | | | | | | |
| Yahoo | 734 | 0.4 | 2070 | | 835 | 0.2 | 2187 | 825 | 936 | | 695 | 0.4 | 2048 | 786 | 0.2 | 1114 | 784 | 451 | 0.6 | 1773 | 717 | 0.2 | 2175 | 587 | | | | | | | | |
| Last FM | 1449 | 0.4 | 1345 | | 1654 | 0.2 | 1203 | 1460 | 1433 | | 1324 | 0.4 | 1324 | 1625 | 0.2 | 1410 | 1420 | 774 | 0.2 | 1665 | 1077 | 0.2 | 1207 | 941 | | | | | | | | |

Table 9: Comparison in terms of Precision and Aggregate Diversity for Item-kNN scheme

| Precision - P@10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------------------------|--------|-----------|---------------|-----------|--------------|-----------|---------------|-----------|-------------|-----------|---------------|-----------|--------|-----------|---------------|-----------|--------------|-----------|---------------|-----------|--------|-----------|--------------|-----------|---------------|-----------|-------------|-----------|---------------|-----------|--------|-----------|------|--|
| | JS | | AAJ | | MAJ | | S-AAJ | | S-MAJ | | ASOR | | SOR | | AAS | | MAS | | S-AAS | | S-MAS | | AJS | | AAAJ | | MAAJ | | S-AAAJ | | S-MAAJ | | | |
| Datasets | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | P@10 | λ | | |
| LibraryThing | 0.0869 | 0.2 | 0.0949 | | 0.0451 | 0.4 | 0.0822 | | 0.0944 | | 0.1019 | | 0.0815 | 0.2 | 0.0914 | | 0.0374 | 0.4 | 0.0826 | | 0.0901 | | 0.0598 | 0.2 | 0.1011 | | 0.0303 | 0.4 | 0.0830 | | 0.0792 | | | |
| Yahoo | 0.0331 | 0.2 | 0.0510 | | 0.0016 | 0.2 | 0.0535 | | 0.0373 | | 0.0447 | | 0.0297 | 0.2 | 0.0504 | | 0.0014 | 0.2 | 0.0531 | | 0.0352 | | 0.0046 | 0.4 | 0.0509 | | 0.0012 | 0.2 | 0.0527 | | 0.0083 | | | |
| Last FM | 0.0141 | 0.2 | 0.0248 | | 0.0036 | λ | 0.0230 | | 0.0158 | | 0.0127 | | 0.0124 | 0.4 | 0.0195 | | 0.0031 | 0.2 | 0.0230 | | 0.0155 | | 0.0036 | 0.2 | 0.0237 | | 0.0032 | 0.2 | 0.0229 | | 0.0068 | | | |
| Aggregate Diversity - D@10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | JS | | AAJ | | MAJ | | S-AAJ | | S-MAJ | | ASOR | | SOR | | AAS | | MAS | | S-AAS | | S-MAS | | AJS | | AAAJ | | MAAJ | | S-AAAJ | | S-MAAJ | | | |
| Datasets | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | D@10 | λ | | |
| LibraryThing | 9745 | 0.2 | 12004 | | 17510 | 0.4 | 12304 | | 10249 | | 10399 | | 9556 | 0.2 | 11727 | | 16557 | 0.4 | 13306 | | 9998 | | 17361 | 0.2 | 11221 | | 11132 | 0.4 | 12315 | | 14819 | | | |
| Yahoo | 3103 | 0.2 | 3718 | | 3718 | 0.2 | 3463 | | 4315 | | 2541 | | 3346 | | 3184 | 0.2 | 3935 | | 3397 | 0.2 | 3541 | | 2475 | | 2948 | 0.4 | 4092 | | 2334 | 0.2 | 3756 | | 2295 | |
| Last FM | 3508 | 0.2 | 3538 | | 3486 | 0.2 | 3462 | | 2911 | | 3779 | | 3265 | 0.4 | 305 | | 3175 | 0.2 | 3509 | | 2887 | | 2992 | 0.2 | 3935 | | 2644 | 0.2 | 3692 | | 3457 | | | |

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