344.075 KV: Natural Language Processing
Sentiment Analysis with Machine Learning

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Agenda

- Principles of Machine Learning
- Sentiment Analysis
- Feature Extraction
- Dimensionality reduction with SVD

Notation

- $a \longrightarrow$ a value or a scalar

- $\boldsymbol{b} \longrightarrow$ an array or a vector
  - i^{th} element of $\boldsymbol{b}$ is the scalar b_i

- $\boldsymbol{C} \longrightarrow$ a set of arrays or a matrix
  - i^{th} vector of $\boldsymbol{C}$ is $\boldsymbol{c}_i$
  - j^{th} element of the i^{th} vector of $\boldsymbol{C}$ is the scalar $c_{i,j}$

Linear Algebra – Transpose

- $\boldsymbol{a}$ is in $1 \times d$ dimensions $\rightarrow$ $\boldsymbol{a}^{\mathrm{T}}$ is in $d \times 1$ dimensions

- $\boldsymbol{A}$ is in $e \times d$ dimensions $\rightarrow$ $\boldsymbol{A}^{\mathrm{T}}$ is in $d \times e$ dimensions

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Linear Algebra – Dot product

- $\boldsymbol{a} \cdot \boldsymbol{b}^T = c$
  - dimensions: $1 \times d \cdot d \times 1 = 1$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 5$$

- $\boldsymbol{a} \cdot \boldsymbol{B} = \boldsymbol{c}$
  - dimensions: $1 \times d \cdot d \times e = 1 \times e$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \end{bmatrix}$$

- $\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{C}$
  - dimensions: $l \times m \cdot m \times n = l \times n$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 5 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & 2 \\ 5 & -5 \\ 8 & 13 \end{bmatrix}$$

Agenda

- **Principles of Machine Learning**
- Sentiment Analysis
- Feature Extraction
- Dimensionality reduction with SVD

Statistical Learning

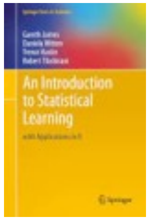
Problem definition

- N observed data points. Each data point $\boldsymbol{x}_i$ is accompanied with a label y_i

$$\mathcal{D} = \{(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \ldots, (\boldsymbol{x}_N, y_N)\}$$

- Each data point is a vector with L dimensions (features):

$$\boldsymbol{x}_i = \left\{x_{i,1}, x_{i,2}, \ldots, x_{i,L}\right\}$$



Statistical Learning

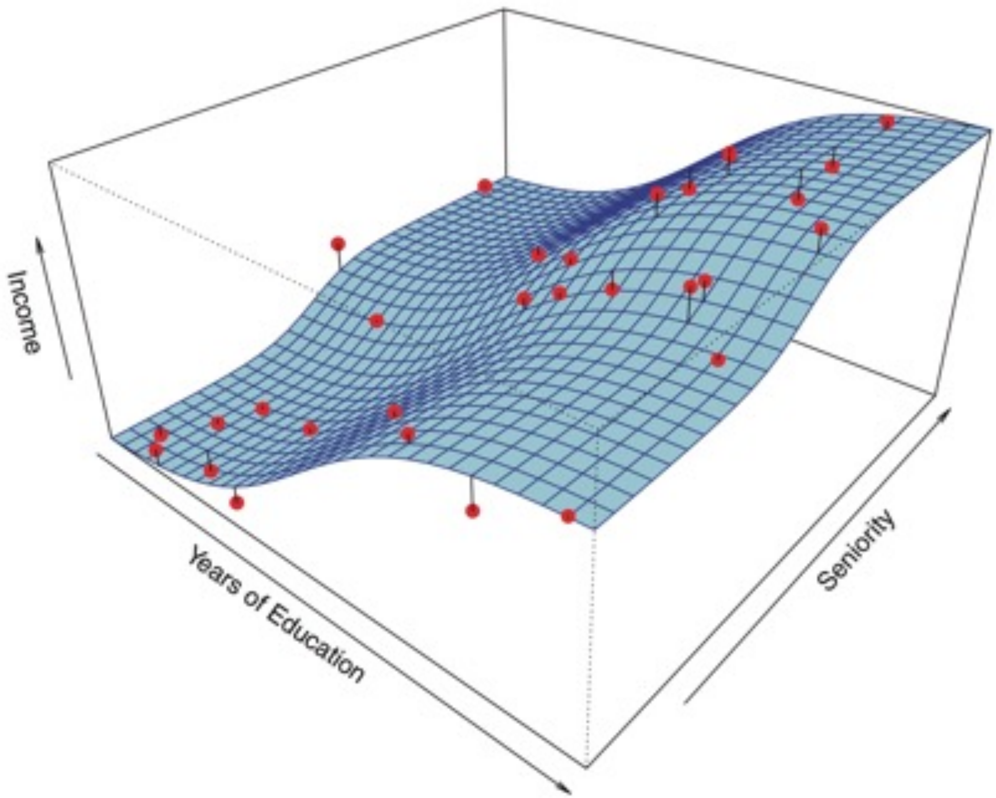
Assumption:

- The data is generated by a **TRUE but unknown function** (f^{TRUE}) such that:

$$y_i = f^{TRUE}(\boldsymbol{x}_i) + \epsilon_i$$

- $\epsilon_i > 0$
  - Called **irreducible error**
  - The error is caused by the constrains in gathering data, and measuring features
  - It means that in data-oriented approaches, there always exists some error that can't be reduced

Example f^{TRUE}



f^{TRUE} → blue surface, the true but unknown function

$\boldsymbol{x}_i$→ each red point with two features: ***Seniority*** & ***Years of Education***

y_i→ **Income** for each data point

ϵ_i→ the distance between the measured **income** of each data point and the surface

Machine Learning Model

- In machine learning, we aim to estimate f^{TRUE} by solely looking at data points
- A machine learning model defines the function f:

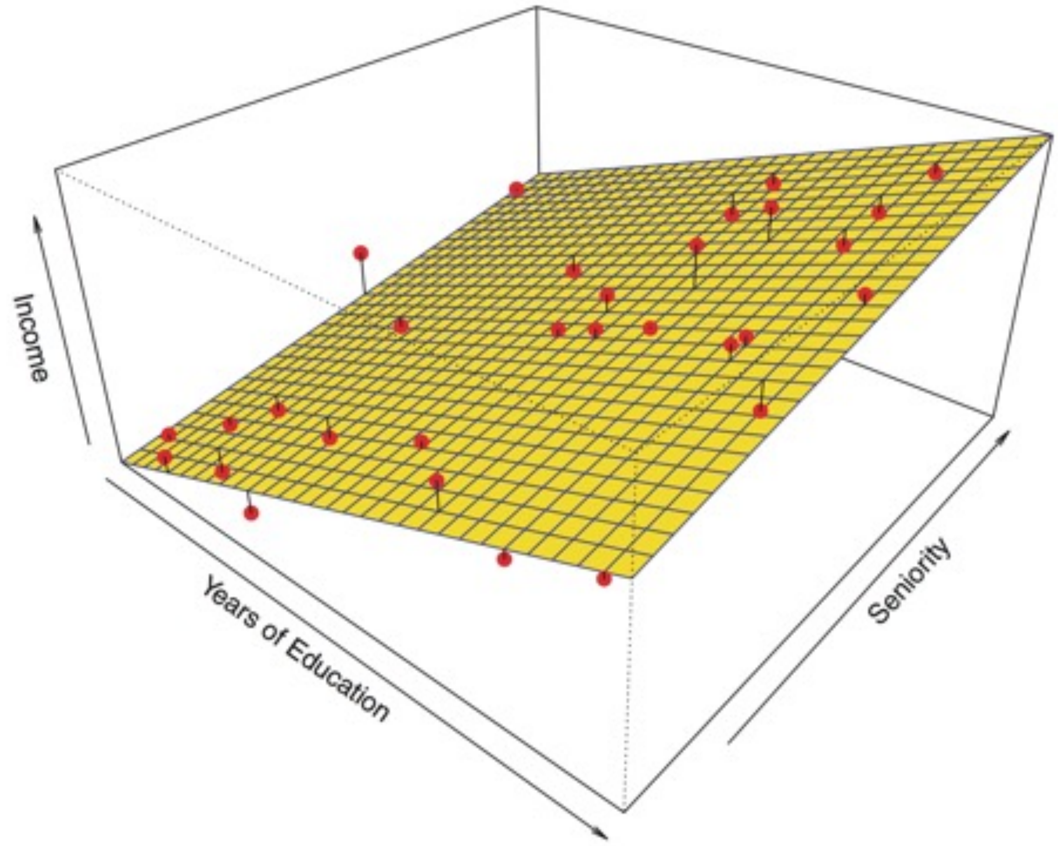
$$\hat{\boldsymbol{y}} = f(\boldsymbol{X})$$

such that $\hat{\boldsymbol{y}}$ (predicted outputs) be close to $\boldsymbol{y}$ (real outputs).

- The difference between $\hat{\boldsymbol{y}}$ and $\boldsymbol{y}$ is **reducible error**
  - Can be reduced by better models

- In statistical/machine/deep learning, we approach reducible error

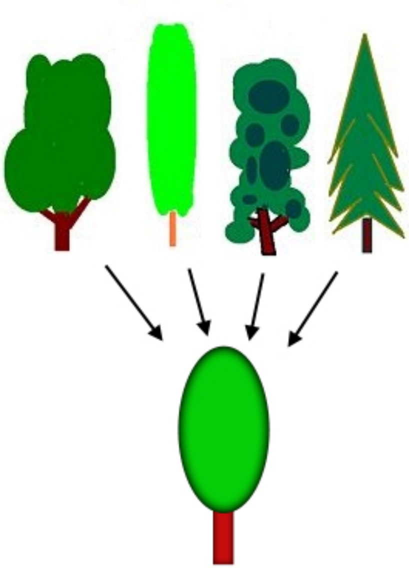
A machine learning model



f → yellow surface, machine learning model (here a linear regression)
Prediction error → the distance between the data points and the surface

Generalization

- The aim of machine learning is to create a model using **observed experiences** (training data) that generalizes to the problem domain, namely **performs** well on **unobserved instances** (test data)



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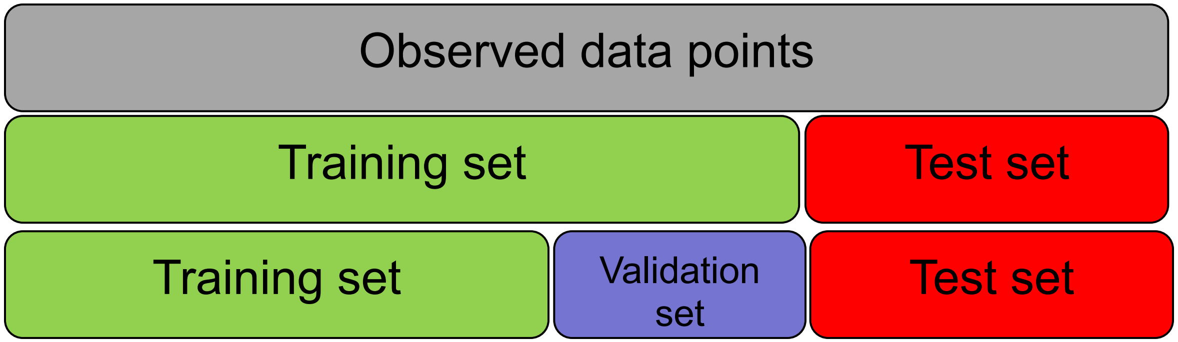
Learning the model – Splitting dataset

- To conduct machine learning experiments, we split the data into:
  - **Training set**: for training the model
  - **Validation set**: for tuning model's hyper-parameters
  - **Test set**: for evaluating model's performance

- Common train – validation – test splitting sizes
  - 60%, 20%, 20%
  - 70%, 15%, 15%
  - 80%, 10%, 10%



Learning the model

Features / Variables (X)

Labels / Output Variable (Y)

Dataset

sex	age	Pstatus	romantic	Walc
F	18	A	no	1
F	17	T	no	1
F	15	T	no	3
F	15	T	yes	1
F	16	T	no	2
M	16	T	no	2
M	16	T	no	1
F	17	A	no	1
M	15	A	no	1
M	15	T	no	1
F	15	T	no	2
F	15	T	no	1
M	15	T	no	3
M	15	T	no	2
M	15	A	yes	1
F	16	T	no	2
F	16	T	no	2
F	16	T	no	1
M	17	T	no	4

Pstatus: parent's cohabitation status ('T' - living together 'A' - apart)
Romantic: with a romantic relationship
Walc: weekend alcohol consumption (from 1 - very low to 5 - very high)

http://archive.ics.uci.edu/ml/datasets/STUDENT+ALCOHOL+CONSUMPTION#

Learning the model

Train Set

sex	age	Pstatus	romantic	Walc
F	18	A	no	1
F	17	T	no	1
F	15	T	no	3
F	15	T	yes	1
F	16	T	no	2
M	16	T	no	2
M	16	T	no	1
F	17	A	no	1
M	15	A	no	1
M	15	T	no	1
F	15	T	no	2
F	15	T	no	1
M	15	T	no	3

Test Set

M	15	T	no	2
M	15	A	yes	1
F	16	T	no	2
F	16	T	no	2
F	16	T	no	1
M	17	T	no	4

Learning the model

Train Set

sex	age	Pstatus	romantic	Walc
F	18	A	no	1
F	17	T	no	1
F	15	T	no	3
F	15	T	yes	1
F	16	T	no	2
M	16	T	no	2
M	16	T	no	1
F	17	A	no	1
M	15	A	no	1
M	15	T	no	1
F	15	T	no	2
F	15	T	no	1
M	15	T	no	3

Test Set

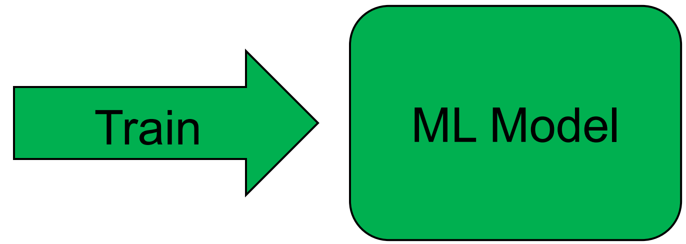
M	15	T	no	?
M	15	A	yes	?
F	16	T	no	?
F	16	T	no	?
F	16	T	no	?
M	17	T	no	?

y
2
1
2
2
1
4

Learning the model

Train Set

sex	age	Pstatus	romantic	Walc
F	18	A	no	1
F	17	T	no	1
F	15	T	no	3
F	15	T	yes	1
F	16	T	no	2
M	16	T	no	2
M	16	T	no	1
F	17	A	no	1
M	15	A	no	1
M	15	T	no	1
F	15	T	no	2
F	15	T	no	1
M	15	T	no	3



Test Set

M	15	T	no	?
M	15	A	yes	?
F	16	T	no	?
F	16	T	no	?
F	16	T	no	?
M	17	T	no	?

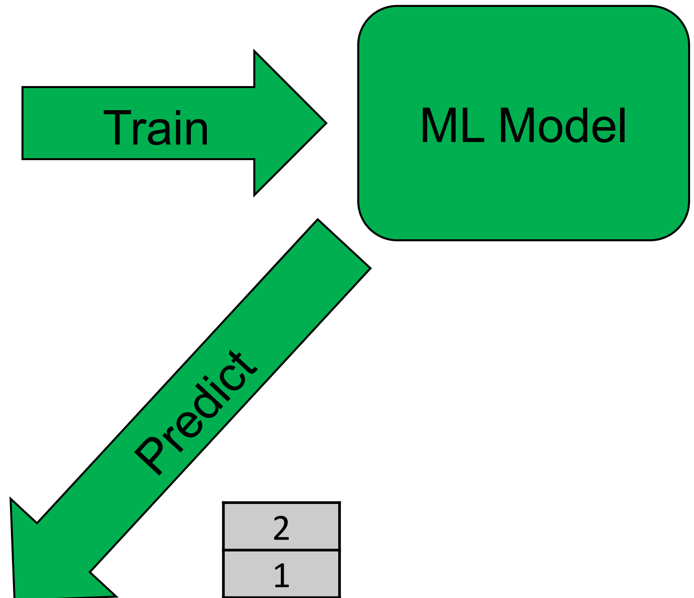
2
1
2
2
1
4

y

Learning the model

Train Set

sex	age	Pstatus	romantic	Walc
F	18	A	no	1
F	17	T	no	1
F	15	T	no	3
F	15	T	yes	1
F	16	T	no	2
M	16	T	no	2
M	16	T	no	1
F	17	A	no	1
M	15	A	no	1
M	15	T	no	1
F	15	T	no	2
F	15	T	no	1
M	15	T	no	3



Test Set

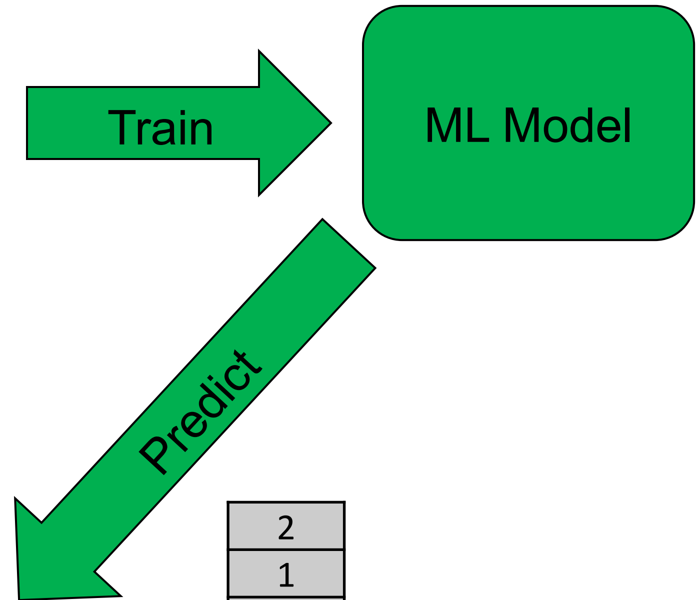
				$\hat{y}$
M	15	T	no	1
M	15	A	yes	1
F	16	T	no	2
F	16	T	no	2
F	16	T	no	3
M	17	T	no	4

y
2
1
2
2
1
4

Learning the model

Train Set

sex	age	Pstatus	romantic	Walc
F	18	A	no	1
F	17	T	no	1
F	15	T	no	3
F	15	T	yes	1
F	16	T	no	2
M	16	T	no	2
M	16	T	no	1
F	17	A	no	1
M	15	A	no	1
M	15	T	no	1
F	15	T	no	2
F	15	T	no	1
M	15	T	no	3



Test Set

M	15	T	no	1
M	15	A	yes	1
F	16	T	no	2
F	16	T	no	2
F	16	T	no	3
M	17	T	no	4

$\hat{y}$

2
1
2
2
1
4

y

Evaluation → Prediction error

Elements of machine learning

- Various ML models:
  - Linear Regression / Logistic Regression
  - Support Vector Machines (SVM)
  - Decision Tree / Random Forest
  - Neural Networks (Multi-layer Perceptron)
  - Deep Neural Networks
  - Etc.
- Model parameters
  - Each models has a set of variables, whose *optimum* values are learned from data during training
- Model hyperparameters
  - A set of specifications of each model, set before training
    - E.g. the kernel shape of SVM, or regularization weight in logistic regression

Elements of machine learning – cont.

- Loss function
  - A function that measures the discrepancies between the predicted outputs $\hat{\boldsymbol{y}}$ and real ones $\boldsymbol{y}$

- Optimization
  - The process of finding an *optimum* set of model parameters by trying to decrease the loss

- Evaluation
  - Measuring model’s prediction performance by comparing with labels

A sample ML model: Linear Regression

- In Linear Regression, f is defined as:

$$y = f(\boldsymbol{x}; \boldsymbol{w}) = w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_l x_l$$

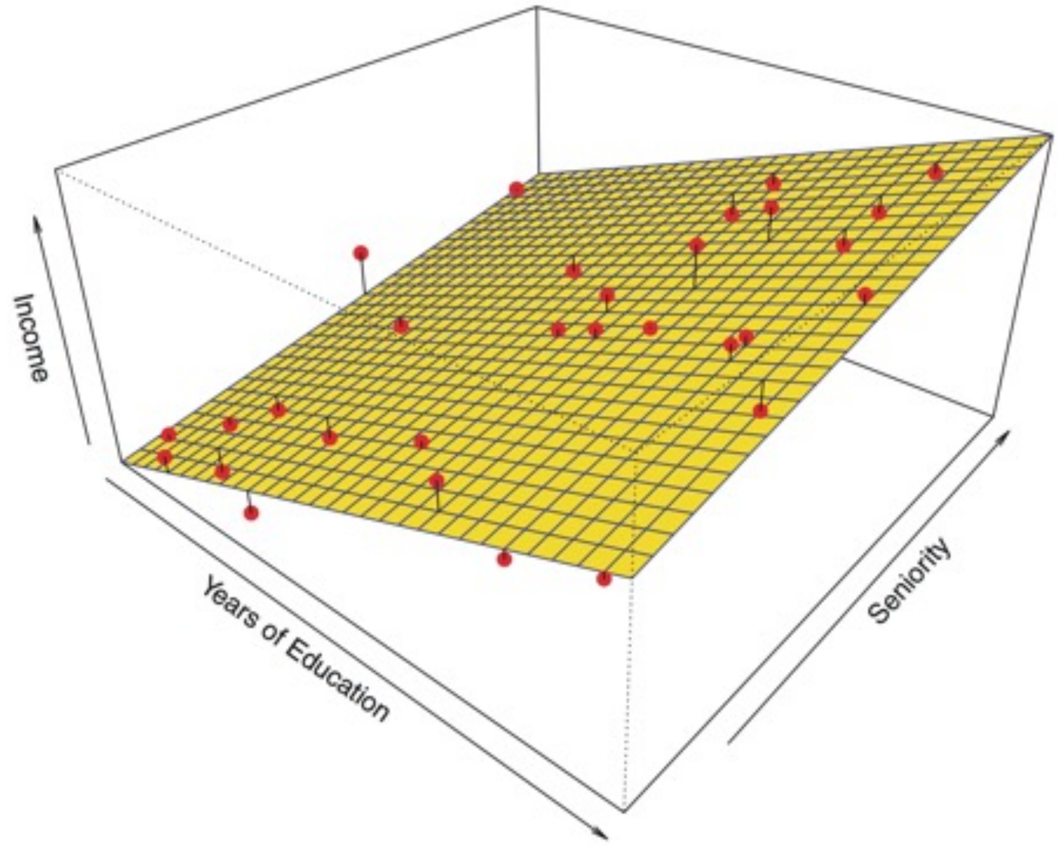
where $\boldsymbol{w} = [w_0, w_1, \ldots, w_l]$ are model parameters

- In the "income" example:

$$income = f(\boldsymbol{x}; \boldsymbol{w}) = w_0 + w_1 \times education + w_2 \times seniority$$

Model parameters are shown in red

A sample ML model: Linear Regression



$$income = f(\boldsymbol{x}; \boldsymbol{w}) = w_0 + w_1 \times education + w_2 \times seniority$$

Common Evaluation Metrics

- Classification
  - **Accuracy** $\frac{\#\ of\ correct\ predictions}{\#\ of\ samples}$
  - Precision $\frac{TP}{TP+FP}$
  - Recall $\frac{TP}{TP+FN}$
  - F-measure $\frac{2 * precision * recall}{precision + recall}$
- Regression
  - MSE
  - R-squared

Which ML model?

Model Capacity

low — high

less flexible	more flexible
less parameters	more parameters
lower variance	higher variance
higher bias	lower bias
prune to underfitting	prune to overfitting

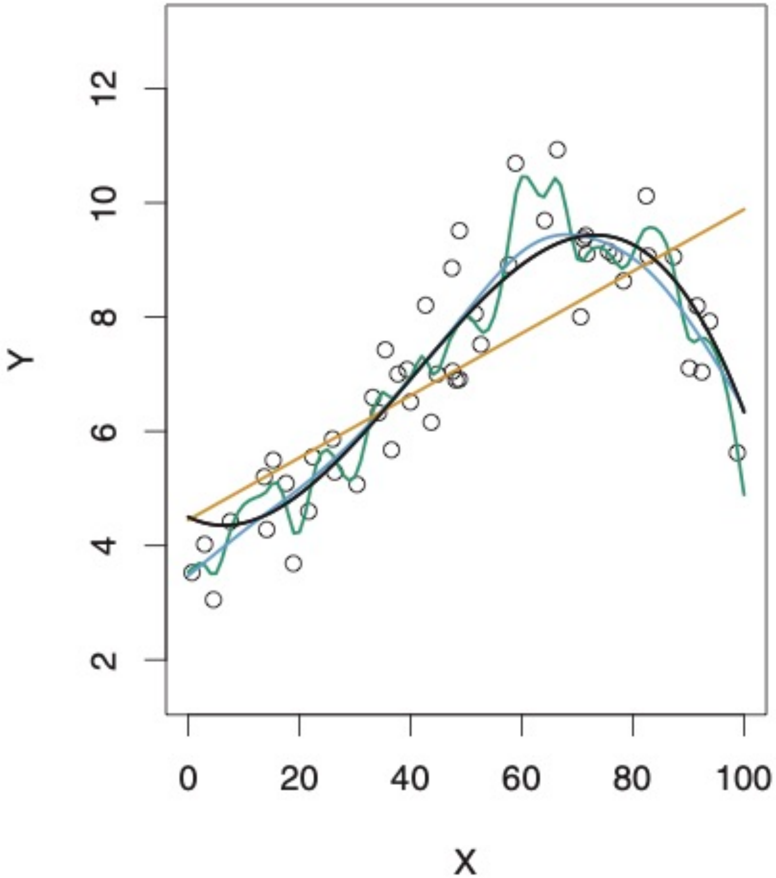
Terms of the day!

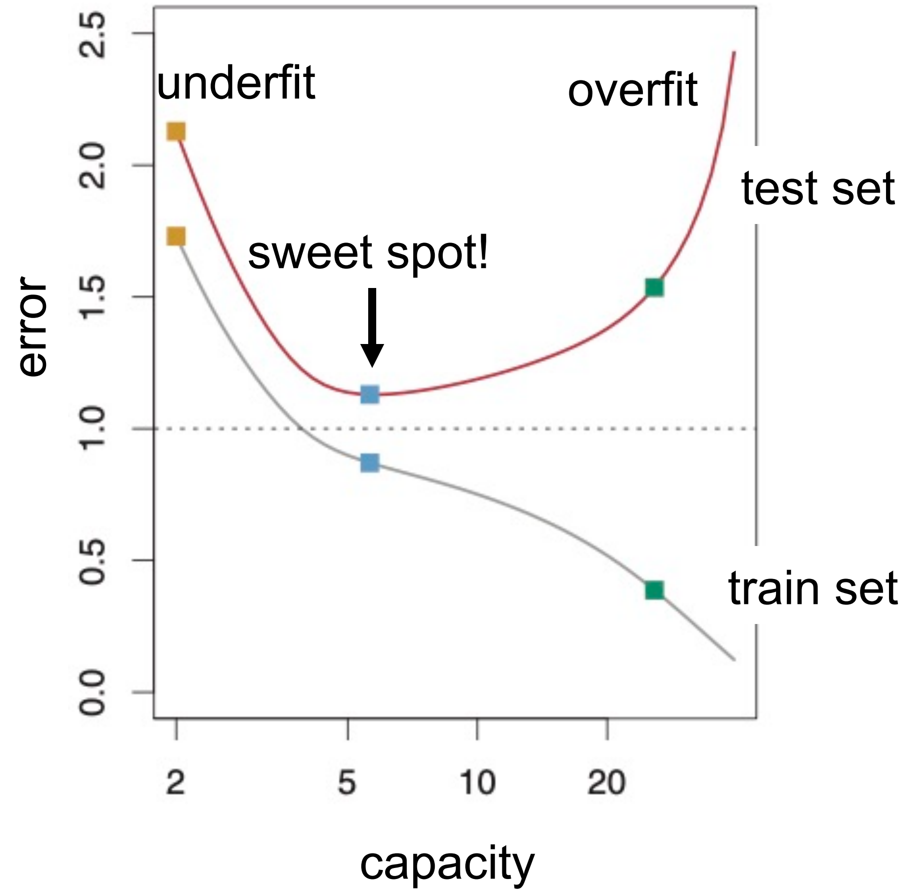
(Statistical) Bias indicates the amount of assumptions, taken to define a model. Higher bias means more assumptions and less flexibility, as in linear regression.

Variance: in what extent the estimated parameters of a model vary when the values of data points change (are resampled).

Overfitting: When the model exactly fits to training data, namely when it also captures the noise in data.

Learning Curve





<u>Models:</u>

black $\rightarrow f^{TRUE}$

orange $\rightarrow$ linear regression

blue and green $\rightarrow$ two smoothing spline models

Elements of machine learning – cont.

- Regularization
  - A regularization method introduces additional information (assumptions) to **avoid overfitting** by **decreasing variance**

Elements of machine learning – cont.

- Model selection & Hyper parameter tunning
  - Which model should we use? With what hyperparameters?

  - Select several models, and for each several sets of hyper-parameters
  - For each option, train a separate model using training set
  - Select the best performing trained models based on the evaluation result on validation set
  - Evaluate the selected model on test set → final prediction performance

Agenda

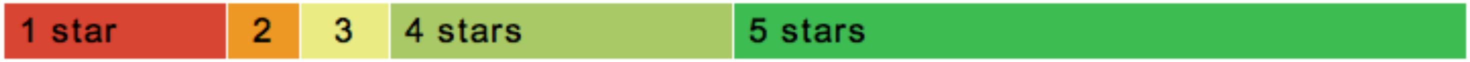
- Principles of Machine Learning
- **Sentiment Analysis**
- Feature Extraction
- Dimensionality reduction with SVD

Sentiment Analysis / Market Intelligence

HP Officejet 6500A Plus e-All-in-One Color Ink-jet - Fax / copier / printer / scanner
$89 online, **$100** nearby ★★★★☆ 377 reviews
September 2010 - Printer - HP - Inkjet - Office - Copier - Color - Scanner - Fax - 250 sh

Reviews

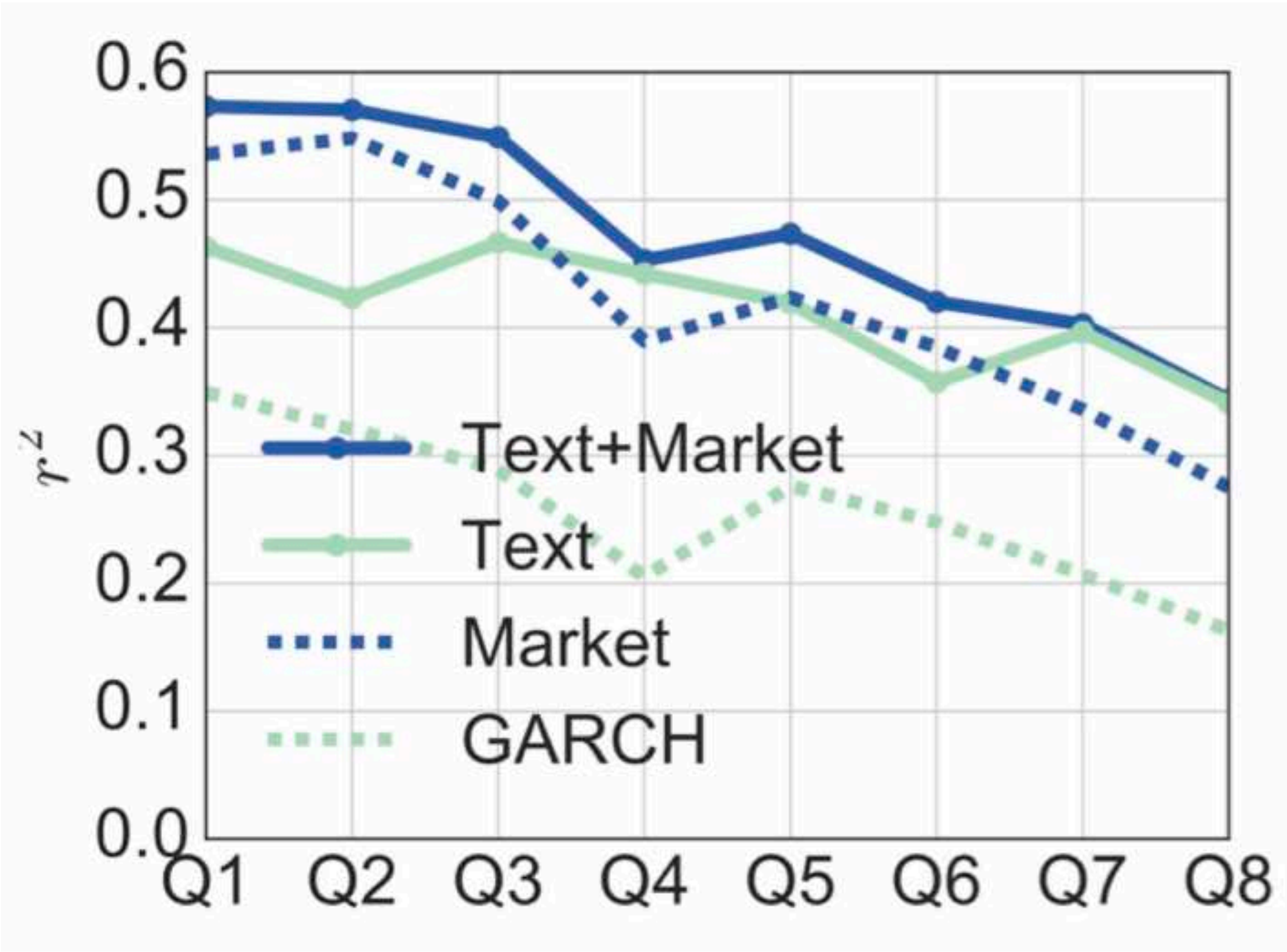
Summary - Based on 377 reviews



What people are saying

ease of use	"This was very easy to setup to four computers."
value	"Appreciate good quality at a fair price."
setup	"Overall pretty easy setup."
customer service	"I DO like honest tech support people."
size	"Pretty Paper weight."
mode	"Photos were fair on the high quality mode."
colors	"Full color prints came out with great quality."

Sentiment Analysis / Market Intelligence



Volatility Prediction using Financial Disclosures Sentiments with Word Embedding-based IR Models
Navid Rekabsaz, Mihai Lupu, Artem Baklanov, Allan Hanbury, Alexander Duer, Linda Anderson In Proc. ACL 2017

A *tough* Example!

"*This past Saturday, I bought a Nokia phone and my girlfriend bought a Motorola phone with Bluetooth. We called each other when we got home. The voice on my phone was clear, better than my previous Samsung phone. The battery life was however short. My girlfriend was quite happy with her phone. I wanted a phone with good sound quality. So my purchase was a real disappointment. I returned the phone yesterday.*"

[1]

Sentiment Analysis

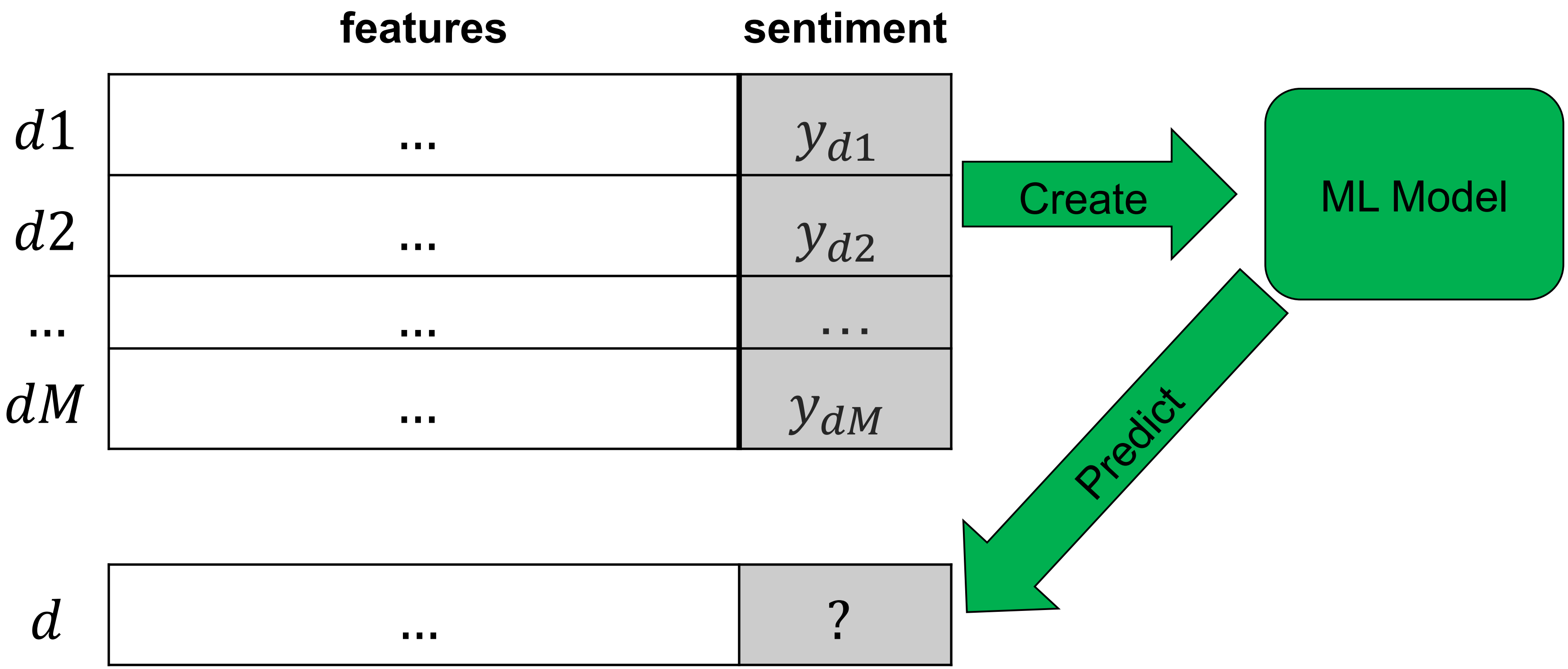
- Text- or document-level sentiment analysis assumes that whole the text expresses one sentiment about one opinion target
  - Not like the previous example!
  - In this course, we do document-level sentiment analysis using machine learning

Problem definition

- A dataset consist of M documents and their assigned sentiments (labels)

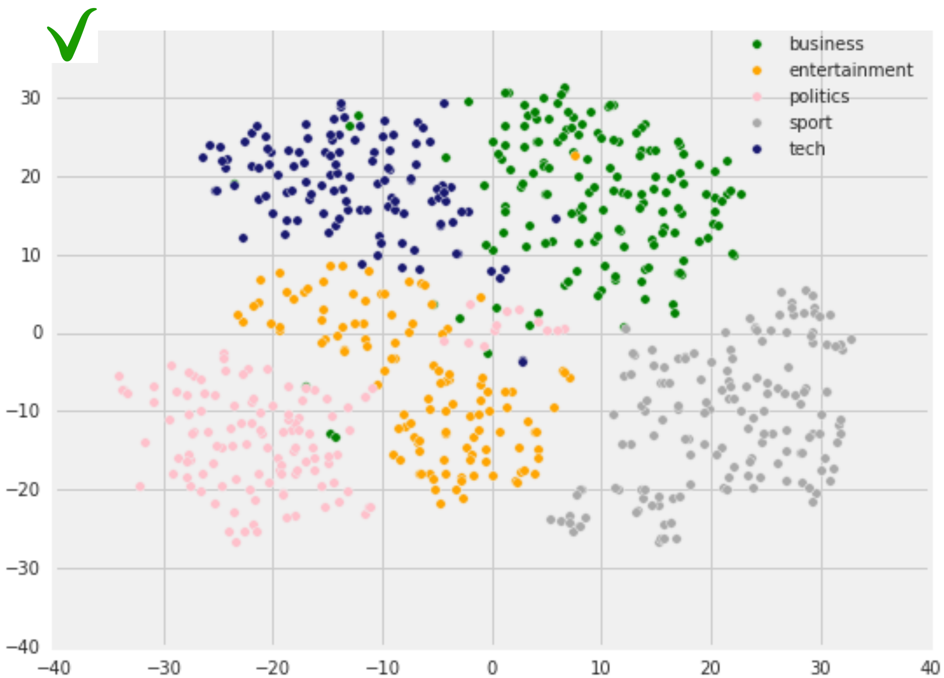
- Possible sentiment values:
  - [-1, 0, 1] → [negative, neutral, positive] (classification problem)
  - Real-valued numbers e.g. stock price (regression problem)

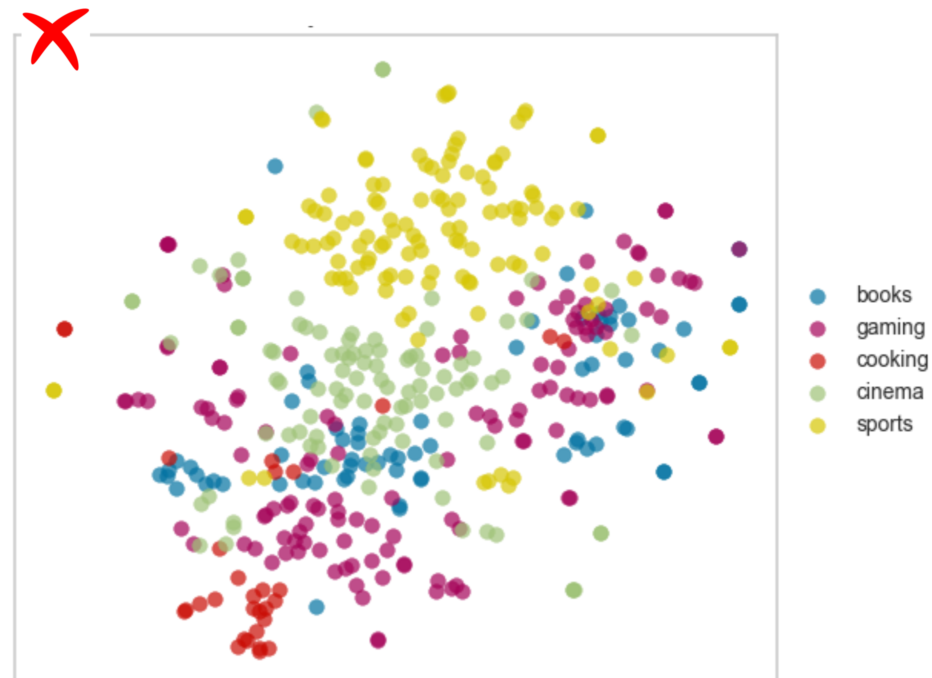
Sentiment Analysis with ML



Document Representation

- The key is in creating *good* document representations!
  - Feature extraction in classical ML
  - Representation learning in deep learning
- If representations are *well-separated* and *well-generalized*, any ML model with enough capacity can effectively classify the test data.





Two sample document representation sets projected to two-dimensional spaces

source source

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Dictionary

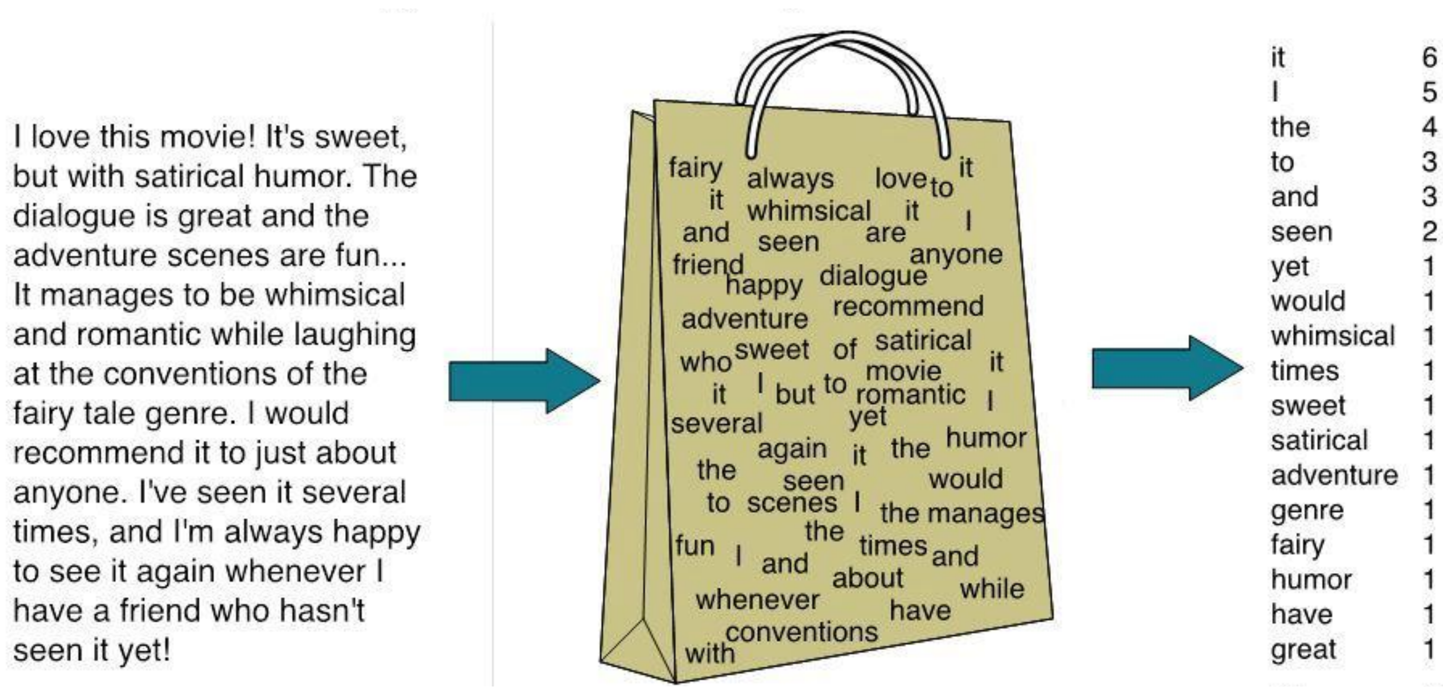
- In order to extract document features, we first create a dictionary $\mathbb{V}$ with N words:

$$\mathbb{V} = \{v1, v2, \dots, vN\}$$

- Recall the possible pre-processing steps for dictionary: removing words lower than a specific frequency, handling stop words, etc.

Bag-of-words

- We do feature extraction based on a bag-of-words (BoW) approach
- In BoW, the order and position of words is ignored and only the number occurrences of words are considered.



Document-Word Matrix

- Featurization is done in a document-word matrix
- Document are data points (rows)
- Words in the dictionary are features (dimensions, columns)
- $x_{v,d}$ is the feature value of word v in document d
- Each value $x_{v,d}$ is set using a word (term) weighting model

	$v1$	$v2$	…	vN	**sentiment (label)**
$d1$	$x_{v1,d1}$	$x_{v2,d1}$	…	$x_{vN,d1}$	y_{d1}
$d2$	$x_{v1,d2}$	$x_{v2,d2}$	…	$x_{vN,d2}$	y_{d2}
…	…	…	…	…	…
dM	$x_{v1,dM}$	$x_{v2,dM}$	…	$x_{vN,dM}$	y_{dM}

Weighting models

Term count & term frequency

(1) Term (word) count:

- One common word weighting approach is to simply count the number of occurrences of a word in a document,
  - Example: number of times ***JKU*** appears in each news document.

$$\mathrm{tc}_{v,d} = \#\ \text{of occurrences of word}\ v\ \text{in}\ d$$

(2) Term (word) frequency:

- The Importance of a word (probably) does not one-to-one increase with the number of occurrences
- Based on experimental results, logarithm is commonly used to dampen raw counts, resulting in …

$$\mathrm{tf}_{v,d} = \log(1 + \mathrm{tc}_{v,d})$$

On informativeness of less frequent words

- Words that do not appear often usually carry more information in comparison with highly frequent ones
  - For example ***JKU*** in a large news should be more informative than the word ***university*** that appears more often.

- Inverse document frequency (idf) measures the importance of words according to whole the collection of documents:

$$\mathrm{idf}_v = \log(\frac{M}{\mathrm{df}_v + 1})$$

  - M is the number of documents in the collection
  - df_v (document frequency of v) is the number of documents that contain word v

- Higher idf_v means that the word appears less often in the collection, and is therefore more informative (more important)
  - ***JKU*** has higher idf than ***university***, and ***the*** gets a very low idf.

Weighting models
Term frequency-Inverse document frequency

- **(3) $\mathbf{tf{-}idf}$** weighting model is the product of $\mathrm{tf}_{v,d}$ to idf_v

$$x_{v,d} = \mathrm{tf{-}idf}_{v,d} = \log\left(1 + \mathrm{tc}_{v,d}\right) \times \log\left(\frac{M}{\mathrm{df}_v + 1}\right)$$

tf increases with the number of occurrences <u>within the document</u>

idf increases with the rarity of the word in <u>whole the collection of documents</u>

- A well-known word weighting method!

Putting all together!

- Use a word weighting to create document-word matrix

	$v1$	$v2$	…	vN	**sentiment (label)**
$d1$	$x_{v1,d1}$	$x_{v2,d1}$	…	$x_{vN,d1}$	y_{d1}
$d2$	$x_{v1,d2}$	$x_{v2,d2}$	…	$x_{vN,d2}$	y_{d2}
…	…	…	…	…	…
dM	$x_{v1,dM}$	$x_{v2,dM}$	…	$x_{vN,dM}$	y_{dM}

- Apply a standard machine learning pipeline:
  - Separating training/validation/test sets
  - Using the training set to train models with different hyper-parameters
  - Evaluating the models on the validation set selecting the best one
  - Reporting the test set performance of the best model

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Supervised Sentiment Analysis

	$v1$	$v2$	…	vN	**sentiment (label)**
$d1$	$x_{v1,d1}$	$x_{v2,d1}$	…	$x_{vN,d1}$	y_{d1}
$d2$	$x_{v1,d2}$	$x_{v2,d2}$	…	$x_{vN,d2}$	y_{d2}
…	…	…	…	…	…
dM	$x_{v1,dM}$	$x_{v2,dM}$	…	$x_{vN,dM}$	y_{dM}

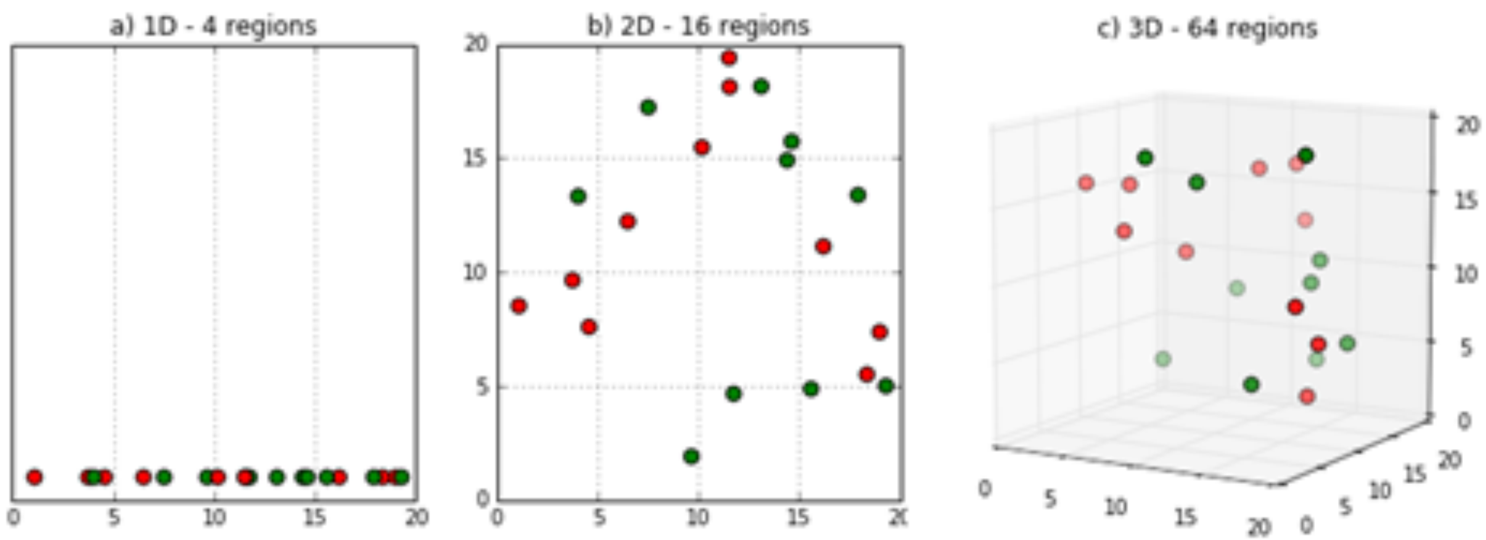
Number of rows:
$M \sim [10K - 100K]$

Number of columns:
$N \sim [20K - 500K]$

- Document vectors are
  - sparse (a lot zeros)
  - typically in very high dimensions

Curse of dimensionality

- Curse of dimensionality happens when the amount of data does not suffice to support the sparsity in dimensionality
- It causes
  - Data sparsity
  - Issues in measuring “closeness”



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Curse of dimensionality

- Why low-dimensional vectors?
  - Easier to store and load
  - More efficient when used as features in ML models
  - Usually better generalization due to the reduction of noise in data
  - Able to capture higher-order relations:
    - Synonyms like *car* and *automobile* can be merged into the same dimensions
    - Polysomy like *bank (financial institution)* and *bank (bank of river)* can be separated into different dimensions

Feature (Dimensionality) reduction

- Feature selection
  - keep some important features and get rid of the rest!

- Dimensionality reduction
  - project data from high to a low dimensional space

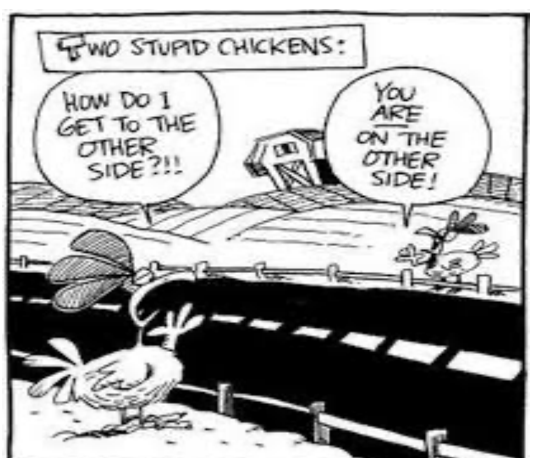
Feature selection

- During pre-processing
  - Remove stop words or very common words
    - tf–idf do it in a “soft” way *why?*
  - Remove very rare words
    - Usually done when creating dictionary
  - Stemming & lemmatization
- Features definition
  - Use only the words in a domain-specific lexicon as features
- Post-processing
  - Keep important features using some informativeness measures
  - Subset selection

Dimensionality reduction with LSA

- Latent Semantic Analysis (LSA)
  - A common method to create semantic vectors
  - Based on Singular Value Decomposition (SVD)

Semantics matters!



Singular Value Decomposition

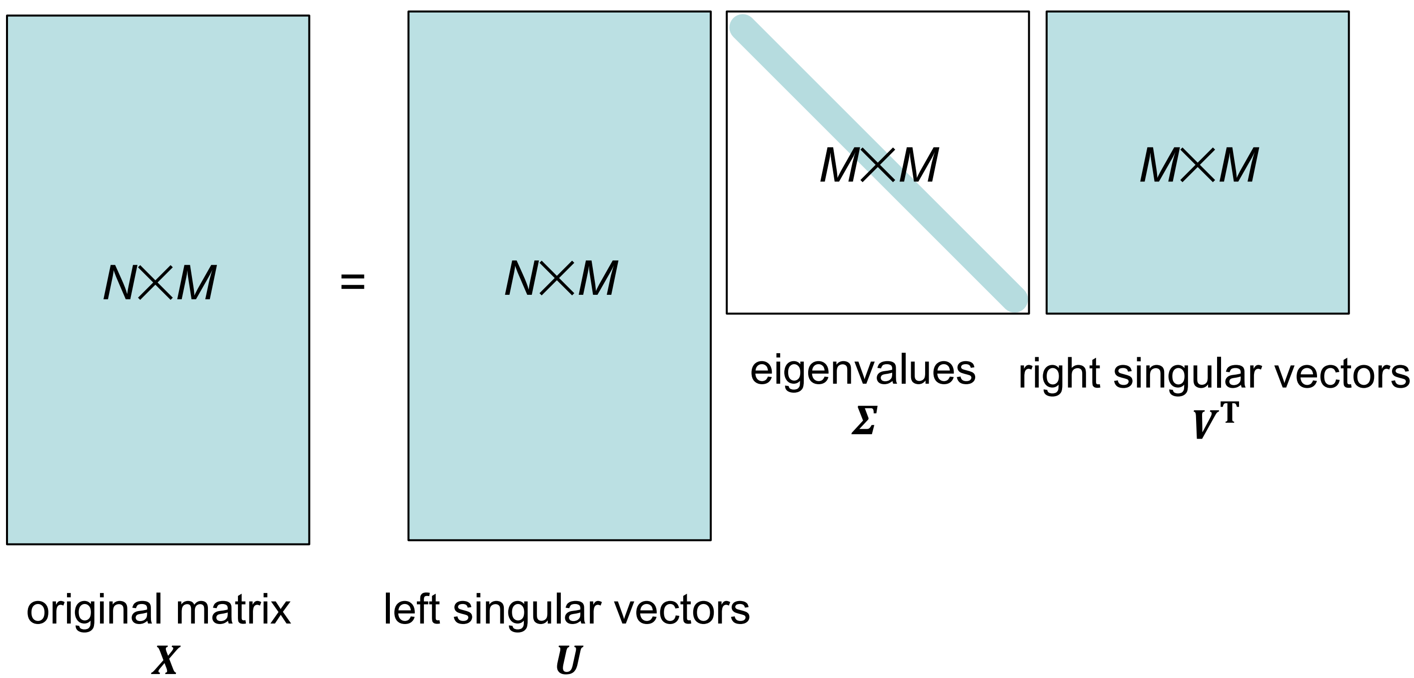
- An $N \times M$ matrix $\boldsymbol{X}$ can be factorized to three matrices:

$$\boldsymbol{X} = \boldsymbol{U\Sigma V}^{\mathrm{T}}$$

- $\boldsymbol{U}$ (left singular vectors) is an $N \times M$ unitary matrix
- $\boldsymbol{\Sigma}$ is an $M \times M$ diagonal matrix, diagonal entries
  - are eigenvalues,
  - show the importance of corresponding M dimensions in $\boldsymbol{X}$
  - are all positive and sorted from large to small values
- $\boldsymbol{V}^{\mathrm{T}}$ (right singular vectors) is an $M \times M$ unitary matrix

* The definition of SVD is simplified. Refer to https://en.wikipedia.org/wiki/Singular_value_decomposition for the exact definition

Singular Value Decomposition



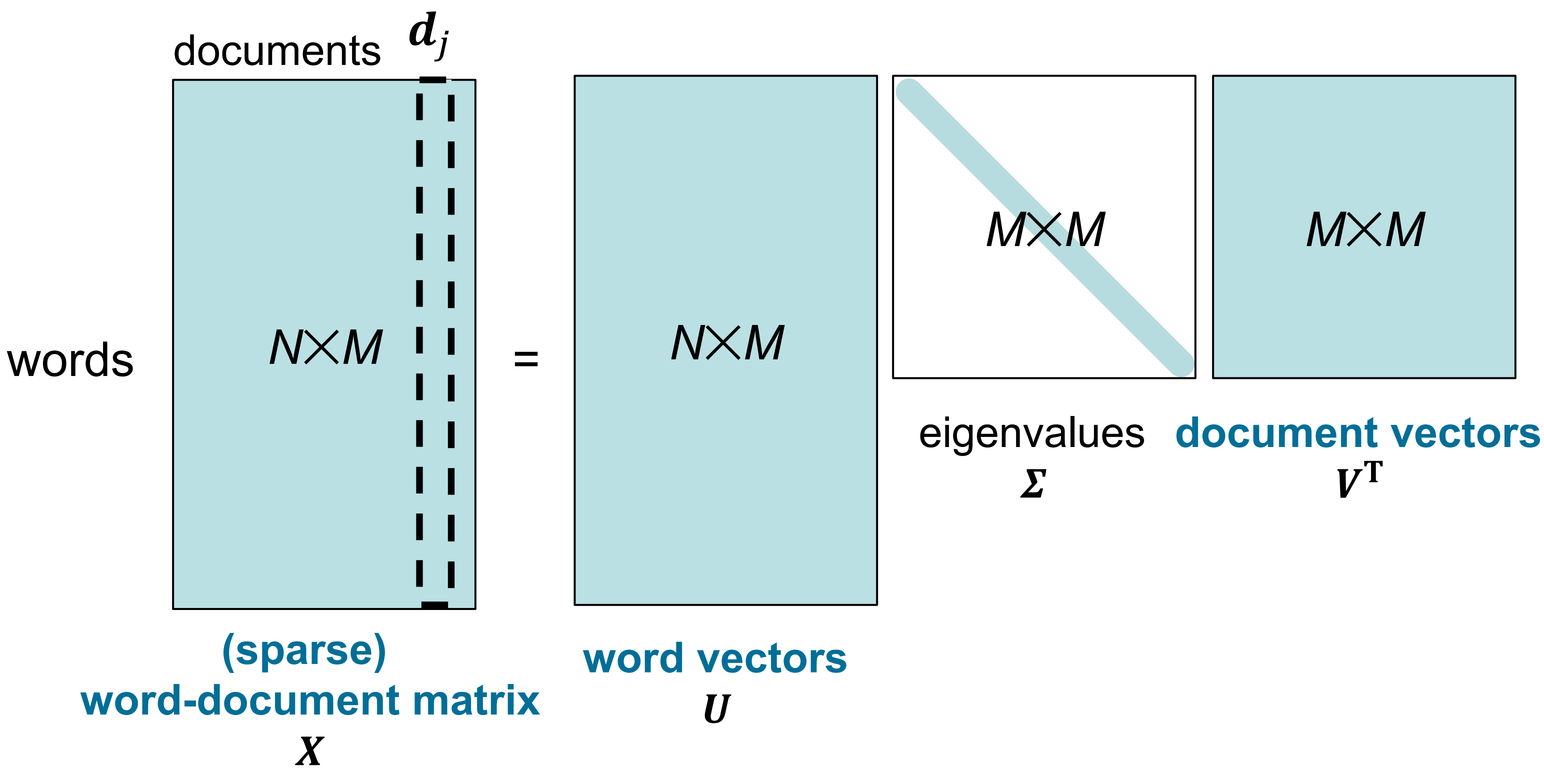
Latent Semantic Analysis

Training Time

Step 1

- apply SVD to the **word-document** matrix of training data

☞ Not the document-word matrix as we talked before! Although it is also possible to start with the document-word matrix, here we follow the common definition!



Latent Semantic Analysis

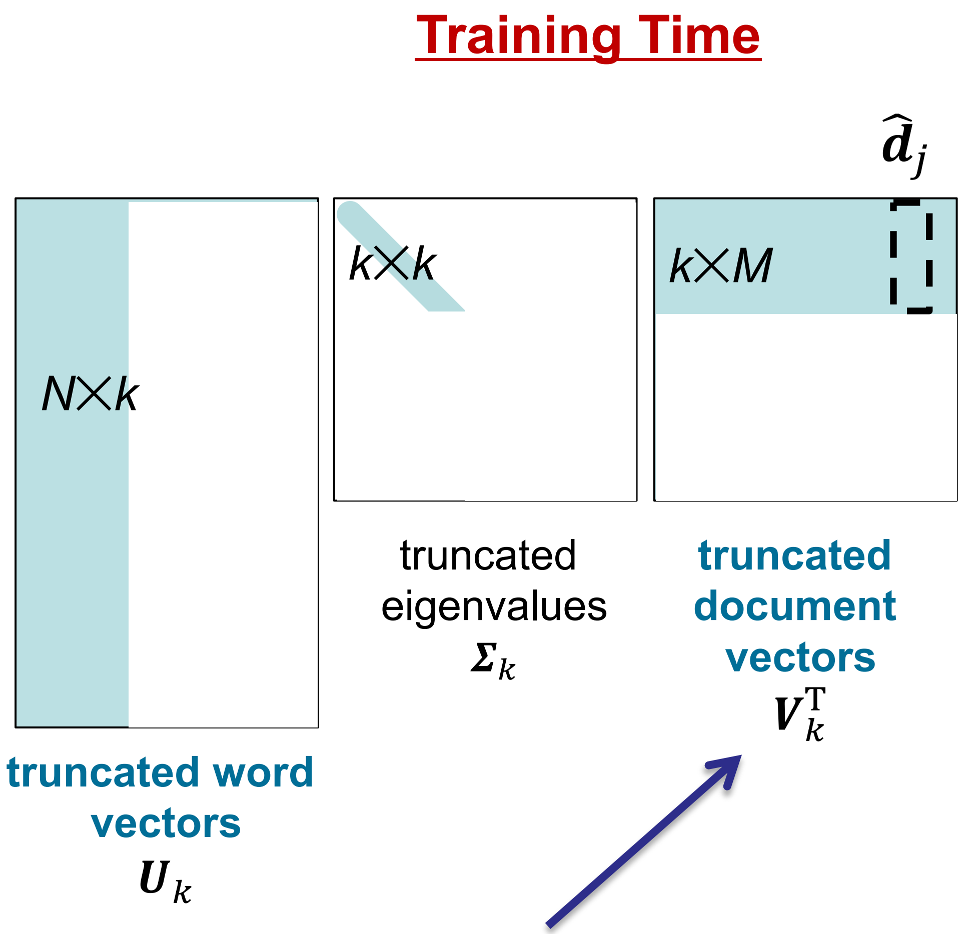
Training Time

Step 2

- keep only top k eigenvalues in $\boldsymbol{\Sigma}$ and set the rest to zero, called $\boldsymbol{\Sigma}_k$
- Truncate the $\boldsymbol{U}$ and $\boldsymbol{V}^{\mathrm{T}}$ matrices, resulting in $\boldsymbol{U}_k$ and $\boldsymbol{V}_k^{\mathrm{T}}$

- Columns in $\boldsymbol{V}_k^{\mathrm{T}}$ are the new low-dimensional document representations
  - Vectors of $\boldsymbol{V}_k^{\mathrm{T}}$ are used to train ML models

Latent Semantic Analysis



- $\boldsymbol{V}_k^{\mathrm{T}}$ is the matrix of dense low-dimensional document vectors
  - Used for training the ML models

Latent Semantic Analysis

Training Time

- Example: word-document matrix with 3 documents and 7 words
- Applying SVD to the matrix

$$\boldsymbol{X} \quad \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 5 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0.04 & 0.35 & -0.91 & 0.00 & 0.00 & 0.13 \\ 0.21 & 0.16 & 0.20 & -0.70 & -0.03 & 0.93 \\ 0.94 & -0.17 & -0.04 & 0.14 & -0.18 & -0.18 \\ 0.12 & 0.88 & 0.27 & 0.00 & 0.01 & -0.26 \\ 0.18 & -0.03 & 0.00 & 0.00 & 0.98 & 0.00 \\ 0.02 & 0.20 & 0.20 & 0.70 & 0.00 & 0.00 \end{bmatrix} \cdot \begin{bmatrix} 5.21 & 0 & 0 \\ 0 & 4.59 & 0 \\ 0 & 0 & 1.66 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.15 & 0.04 & 0.98 \\ -0.92 & -0.34 & 0.15 \\ -0.34 & 0.93 & 0.01 \end{bmatrix}$$

$$\boldsymbol{U} \qquad\qquad \boldsymbol{\Sigma} \qquad\qquad \boldsymbol{V}^{\mathrm{T}}$$

Latent Semantic Analysis

Training Time

- Keeping only the top $k = 2$ eigenvalues

New k -dimensional document representations

$$\underbrace{\begin{bmatrix} 0.04 & 0.35 \\ 0.21 & 0.16 \\ 0.94 & -0.17 \\ 0.12 & 0.88 \\ 0.18 & -0.03 \\ 0.02 & 0.20 \end{bmatrix}}_{\boldsymbol{U}_k} \cdot \underbrace{\begin{bmatrix} 5.21 & 0 \\ 0 & 4.59 \end{bmatrix}}_{\boldsymbol{\Sigma}_k} \cdot \underbrace{\begin{array}{ccc} \hat{\boldsymbol{d}}_1 & \hat{\boldsymbol{d}}_2 & \hat{\boldsymbol{d}}_3 \\ \downarrow & \downarrow & \downarrow \end{array} \begin{bmatrix} 0.15 & 0.04 & 0.98 \\ -0.92 & -0.34 & 0.15 \end{bmatrix}}_{\boldsymbol{V}_k^{\mathrm{T}}}$$

Latent Semantic Analysis

Inference Time (Validation/Test)

- Given a high-dimensional document vector $\boldsymbol{d}^*$ in $N\times1$ dimensions, we want to project it to the low-dimensional space, resulting in a new vector $\widehat{\boldsymbol{d}^*}$ with $k\times1$ dimensions

- done through this calculation:

$$\widehat{\boldsymbol{d}^*} = \boldsymbol{\Sigma}_k^{-1}\boldsymbol{U}_k^{\mathrm{T}}\boldsymbol{d}^*$$

- Exercise: check this formula by calculating if the chain of dot products from $\boldsymbol{d}^*$ to $\widehat{\boldsymbol{d}^*}$ ends up to the correct dimension

Latent Semantic Analysis

Inference Time (Validation/Test)

- Example: high-dimensional document $\boldsymbol{d}^*$

$$\boldsymbol{d}^* = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

- And the matrices calculated at train time:

$$\boldsymbol{\Sigma}_k = \begin{bmatrix} 5.21 & 0 \\ 0 & 4.59 \end{bmatrix} \quad \boldsymbol{\Sigma}_k^{-1} = \begin{bmatrix} 0.19 & 0 \\ 0 & 0.21 \end{bmatrix} \quad \boldsymbol{U}_k = \begin{bmatrix} 0.04 & 0.35 \\ 0.21 & 0.16 \\ 0.94 & -0.17 \\ 0.12 & 0.88 \\ 0.18 & -0.03 \\ 0.02 & 0.20 \end{bmatrix}$$

$$\widehat{\boldsymbol{d}^*} = \boldsymbol{\Sigma}_k^{-1} \boldsymbol{U}_k^{\mathrm{T}} \boldsymbol{d}^* = \begin{bmatrix} 0.11 \\ 0.62 \end{bmatrix}$$