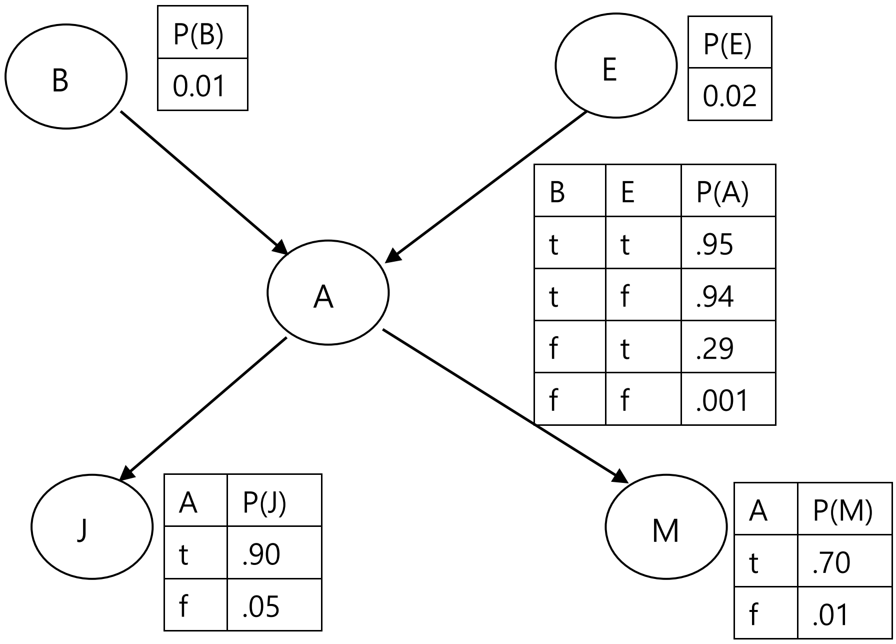
2020 Spring AI HW-4, Professor: Minkoo Kim

제출일: 6 월 4 일 11:59 p.m. AjouBb 를 통해 제출 할 것

1. In the following Bayesian Network, B, E, A, J, and M denote binary probability variables which denote ‘Burglary’, ‘Earthquake’, ‘Alarm’, ‘John's Call’, and ‘Mary's Call’, respectively, and b, e, a, j, and m represent the events which actually happen (i.e. true events), respectively. Calculate (답과 과정) the following probabilities: 1) P(a) (5 점);
2) P(m|b) (5 점).



P(B)
0.01

P(E)
0.02

B	E	P(A)
t	t	.95
t	f	.94
f	t	.29
f	f	.001

A	P(J)
t	.90
f	.05

A	P(M)
t	.70
f	.01

(Solution)

1) $\mathrm{P}(a) = P(a,b,e) + P(a,\neg b,e) + P(a,b,\neg e) + P(a,\neg b,\neg e)$
$= P(a|b,e)P(b,e) + P(a|\neg b,e)P(\neg b,e) + P(a|b,\neg e)P(b,\neg e) + P(a|\neg b,\neg e)P(\neg b,\neg e)$
= 0.95*0.01*0.02 + 0.29*0.99*0.02 + 0.94*0.01*0.98 + 0.001*0.99*0.98
= 0.0161142

2) $\mathrm{P}(m|b) = P(m,b)|P(b) = [P(m,a,b) + P(m,\neg a,b)]|P(b)$
$= [P(m|a)P(a|b)P(b)) + P(m|\neg a)P(\neg a|b)P(b]|P(b)$
$= P(m|a)P(a|b) + P(m|\neg a)P(\neg a|b)$
= 0.7* $P(a|b)$ + 0,01*(1-$P(a|b)$)

$P(a|b) = P(a|b,e)P(e) + P(a|b,\neg e)P(\neg e)$ = 0.95*0.02 + 0.94*0.98
= 0.9402

$\mathrm{P}(m|b)$= 0.7* $P(a|b)$ + 0,01*(1-$P(a|b)$) = 0.7*0.9402 + 0.01*0.0598 = 0.658738

2. 강의노트 7장 53쪽 슬라이드에 있는 질문(Question): What value does P(B|A) have if only P(A)=α and P(B)=β are given? 에 대한 답을 유도하시오. (10 점) [힌트: 강의노트를 잘 보면 교과서 어디 있는지 알 수 있음.]

(Solution) 이 답은 교과서의 연습문제 7.8 답지에 내용과 동일하며, 이 내용은 최혁중군이 제출한 내용을 인용하여 공지한 것임.

$p_1 = P(A, B)$
$p_2 = P(A, \neg B)$
$p_3 = P(\neg A, B)$
$p_4 = P(\neg A, \neg B)$

$p_1 + p_2 = \alpha$ - (1)

$p_1 + p_3 = \beta$ - (2)

$p_1 + p_2 + p_3 + p_4 = 1$ - (3)

Language function

$$L = -\sum_{1}^{4} p_i lnp_i + \lambda_1(p_1 + p_2 - \alpha) + \lambda_2(p_1 + p_3 - \beta) + \lambda_3(p_1 + p_2 + p_3 + p_4 - 1)$$

$$\frac{\partial L}{\partial p_1} = -lnp_1 - 1 + \lambda_1 + \lambda_2 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial p_2} = -lnp_2 - 1 + \lambda_1 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial p_3} = -lnp_3 - 1 + \lambda_2 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial p_4} = -lnp_4 - 1 + \lambda_3 = 0$$

$lnp_1 = \lambda_1 + \lambda_2 + \lambda_3 - 1$ - (4)

$lnp_2 = \lambda_1 + \lambda_3 - 1$ - (5)

$lnp_3 = \lambda_2 + \lambda_3 - 1$ - (6)

$lnp_4 = \lambda_3 - 1$ - (7)

(4)-(5) -> $lnp_1 - lnp_2 = \lambda_2$

(6)-(7) -> $lnp_3 - lnp_4 = \lambda_2$

$lnp_1 - lnp_2 = lnp_3 - lnp_4$ 이므로 $ln\frac{p_1}{p_2} = ln\frac{p_3}{p_4}$

$p_1 = \frac{p_2 p_3}{p_4}$ - (8)

(3)번에 (8)번을 대입하면

$p_1 + p_2 + p_3 + p_4 = \frac{p_2 p_3}{p_4} + p_2 + p_3 + p_4 = p_2\left(1 + \frac{p_3}{p_4}\right) + p_3 + p_4 = 1$ - (9)

(1)-(2) -> $p_2 = p_3 + \alpha - \beta$ - (10)

(9)번에 (10)번을 대입하면

$(p_3 + \alpha - \beta)\left(1 + \frac{p_3}{p_4}\right) + p_3 + p_4 = 1$ - (11)

(3)번에 (1)번을 대입하면

$\alpha + p_3 + p_4 = 1$ - (12)

(11)번에 (12)번을 대입하면

$(p_3 + \alpha - \beta)\left(1 + \frac{p_3}{1-\alpha-p_3}\right) = \alpha$ - (13)

$(p_3 + \alpha - \beta)(1 - \alpha - p_3 \ + p_3) = \ \alpha(1 - \alpha - p_3)$ - (14)

$(p_3 + \alpha - \beta)(1 - \alpha) = \ \alpha(1 - \alpha - p_3)$ - (15)

$p_3 + \alpha - \beta - \alpha p_3 - \alpha^2 + \alpha\beta \ = \ \alpha - \alpha^2 - \alpha p_3$ - (16)

$p_3 = \ \beta(1 - \alpha)$ – (17)

(17)을 (10)에 대입해보면

$p_2 = \ \alpha(1 - \beta)$ – (18)

(18)을 (1)에 대입해보면

$p_1 = \ \alpha\beta$ – (19)

(12)에 (17)을 대입해보면

$p_4 = (1 - \alpha)(1 - \beta)$ – (19)

(19)번 $p_1 = \ \alpha\beta$ 에서 P(A,B) = P(A)P(B)를 따른다는 것을 알 수 있다.
따라서 A,B 가 독립임을 의미한다.