알고리즘 Homework 2_

1. [9주차 1강 출석체크 문제] [Exercise 7.1] 3번 (10점) Sort by Distribution-counting

<sol>

	a	b	c	d
Frequencies	2	3	2	1

	a	b	c	d
Distribution values	2	5	7	8

	D[a..d]			
A[7] = b	2	**5**	7	8
A[6] = a	**2**	4	7	8
A[5] = a	**1**	4	7	8
A[4] = b	0	**4**	7	8
A[3] = c	0	3	7	8
A[2] = d	0	3	**6**	8
A[1] = c	0	3	**6**	7
A[0] = b	0	**3**	5	7

	S[0..7]							
					b			
	a							
a								
				b				
							c	
								d
						c		
			b					

2. [9주차 1강 출석체크 문제] [Exercise 7.2] 1 번 변형 (10점)

<sol>

Shift table:

c	A	B	C	D	…	O	…	Z	_
t(c)	4	2	6	6	6	1	6	6	6

or

c	A	B	O	Others
t(c)	4	2	1	6

The actual search in a particular text proceeds as follows:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]
B	E	S	S	_	K	N	E	W	_	A	B	O	U	T	_	B	A	O	B	A	B	S	
B	A	O	B	O	B																		
						B	A	O	B	O	B												
								B	A	O	B	O	B										
														B	A	O	B	O	B				
																B	A	O	B	O	B		

① i=5, t(K)=6

② i=11, t(B)=2

③ i=13, t(U)=6

④ i=19, t(B)=2

⑤ i=21, t(B)=2

⑥ i=23,, i > 22 이므로, -1.

(매치되지 않음.)

3. [10주차 2강 출석체크 문제] [Exercise 8.3] Optimal Binary Search Tree 찾기 (10점)

<sol> The table entries for the dynamic programming algorithm are computed as follows:

$$C[1,2] = \min\left\{\begin{array}{l} k=1: C[1,0]+C[2,2]+\sum_{s=1}^{2} p_s = 0+0.4+(0.2+0.4) = 1.0 \\ k=2: C[1,1]+C[3,2]+\sum_{s=1}^{2} p_s = 0.2+0+(0.2+0.4) = 0.8 \end{array}\right\} = 0.8$$

$$C[2,3] = \min\left\{\begin{array}{l} k=2: C[2,1]+C[3,3]+\sum_{s=2}^{3} p_s = 0+0.1+(0.4+0.1) = 0.6 \\ k=3: C[2,2]+C[4,3]+\sum_{s=2}^{3} p_s = 0.4+0+(0.4+0.1) = 0.9 \end{array}\right\} = 0.6$$

$$C[3,4] = \min\left\{\begin{array}{l} k=3: C[3,2]+C[4,4]+\sum_{s=3}^{4} p_s = 0+0.3+(0.1+0.3) = 0.7 \\ k=4: C[3,3]+C[5,4]+\sum_{s=3}^{4} p_s = 0.1+0+(0.1+0.3) = 0.5 \end{array}\right\} = 0.5$$

$$C[1,3] = \min\left\{\begin{array}{l} k=1: \ C[1,0]+C[2,3]+\sum_{s=1}^{3} p_s = 0+0.6+(0.2+0.4+0.1) = 1.3 \\ k=2: C[1,1]+C[3,3]+\sum_{s=1}^{3} p_s = 0.2+0.1+(0.2+0.4+0.1) = 1.0 \\ k=3: \ C[1,2]+C[4,2]+\sum_{s=1}^{3} p_s = 0.8+0+(0.2+0.4+0.1) = 1.5 \end{array}\right\} = 1.0$$

$$C[2,4] = \min\left\{\begin{array}{l} k=2: \ C[2,1]+C[3,4]+\sum_{s=2}^{4} p_s = 0+0.5+(0.4+0.1+0.3) = 1.3 \\ k=3: C[2,2]+C[4,4]+\sum_{s=2}^{4} p_s = 0.4+0.3+(0.4+0.1+0.3) = 1.5 \\ k=4: \ C[2,3]+C[5,4]+\sum_{s=2}^{4} p_s = 0.6+0+(0.4+0.1+0.3) = 1.4 \end{array}\right\} = 1.3$$

$$C[1,4] = \min\left\{\begin{array}{l} k=1: \ C[1,0]+C[2,4]+\sum_{s=1}^{4} p_s = 0+1.3+1.0 = 2.3 \\ k=2: C[1,1]+C[3,4]+\sum_{s=1}^{4} p_s = 0.2+0.5+1.0 = 1.7 \\ k=3: C[1,2]+C[4,4]+\sum_{s=1}^{4} p_s = 0.8+0.3+1.0 = 2.1 \\ k=4: \ C[1,3]+C[5,4]+\sum_{s=1}^{4} p_s = 1.0+0+1.0 = 2.0 \end{array}\right\} = 1.7$$

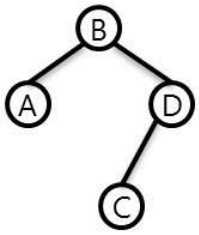
Main table

	0	1	2	3	4
1	0	0.2	0.8	1.0	1.7
2		0	0.4	0.6	1.3
3			0	0.1	0.5
4				0	0.3
5					0

Root table

	0	1	2	3	4
1	0	1	2	2	2
2		0	2	2	2
3			0	3	4
4				0	4
5					0

Optimal tree



4. [11주차 1강 출석체크 문제] [Exercise 8.4] 7번 (10점) Floyd's S.P.

<sol>

$$D^{(0)} = \begin{bmatrix} 0 & 3 & \infty & 2 & 6 \\ 5 & 0 & 4 & 2 & \infty \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & 1 & 0 & 4 \\ 5 & \infty & \infty & \infty & 0 \end{bmatrix} \qquad D^{(1)} = \begin{bmatrix} 0 & 3 & \infty & 2 & 6 \\ 5 & 0 & 4 & 2 & 11 \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & 1 & 0 & 4 \\ 5 & 8 & \infty & 7 & 0 \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & 3 & 7 & 2 & 6 \\ 5 & 0 & 4 & 2 & 11 \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & 1 & 0 & 4 \\ 5 & 8 & 12 & 7 & 0 \end{bmatrix} \qquad D^{(3)} = \begin{bmatrix} 0 & 3 & 7 & 2 & 6 \\ 5 & 0 & 4 & 2 & 11 \\ \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & 1 & 0 & 4 \\ 5 & 8 & 12 & 7 & 0 \end{bmatrix}$$

$$D^{(4)} = \begin{bmatrix} 0 & 3 & 3 & 2 & 6 \\ 5 & 0 & 3 & 2 & 6 \\ \infty & \infty & 0 & 5 & 9 \\ \infty & \infty & 1 & 0 & 4 \\ 5 & 8 & 8 & 7 & 0 \end{bmatrix} \qquad D^{(5)} = \begin{bmatrix} 0 & 3 & 3 & 2 & 6 \\ 5 & 0 & 3 & 2 & 6 \\ 14 & 17 & 0 & 5 & 9 \\ 9 & 12 & 1 & 0 & 4 \\ 5 & 8 & 8 & 7 & 0 \end{bmatrix} = D$$

5. [11주차 1강 출석체크 문제] [Exercise 9.1] 9번 (10점) Prim's MST

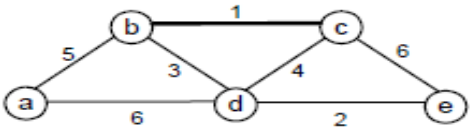
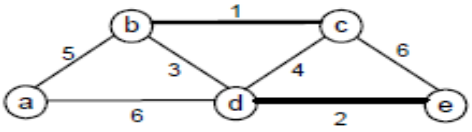
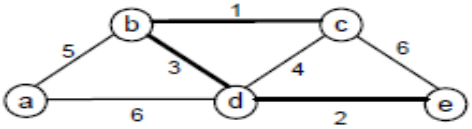
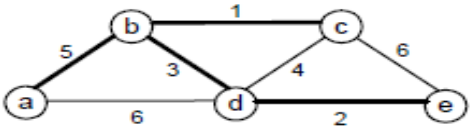
<sol>

Tree vertices	Priority queue of remaining vertices
a(-,-)	b(a,5) c(a,7) d(a,∞) **e(a,2)**
e(a,2)	**b(e,3)** c(e,4) d(e,5)
b(e,3)	**c(e,4)** d(e,5)
c(e,4)	**d(c,4)**
d(c,4)	

The minimum spanning tree found by the algorithm comprises the edges *ae*, *eb*, *ec*, and *cd*.

6. [11주차 2강 출석체크 문제] [Exercise 9.2] 1번 (10점) Kruskal's MST

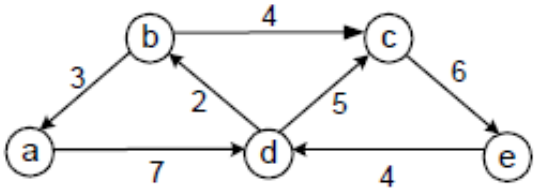
<sol>

Tree edges	Sorted list of edges (selected edges are shown in bold)	Illustration
	bc de bd cd ab ad ce 1 2 3 4 5 6 6	
bc	bc **de** bd cd ab ad ce 1 2 3 4 5 6 6	
de	bc de **bd** cd ab ad ce 1 2 3 4 5 6 6	
bd	bc de bd cd **ab** ad ce 1 2 3 4 5 6 6	
ab		

7. [11주차 2강 출석체크 문제] [Exercise 9.3] 2번 (10점) Dijkstra's S.P.

<sol>

a.



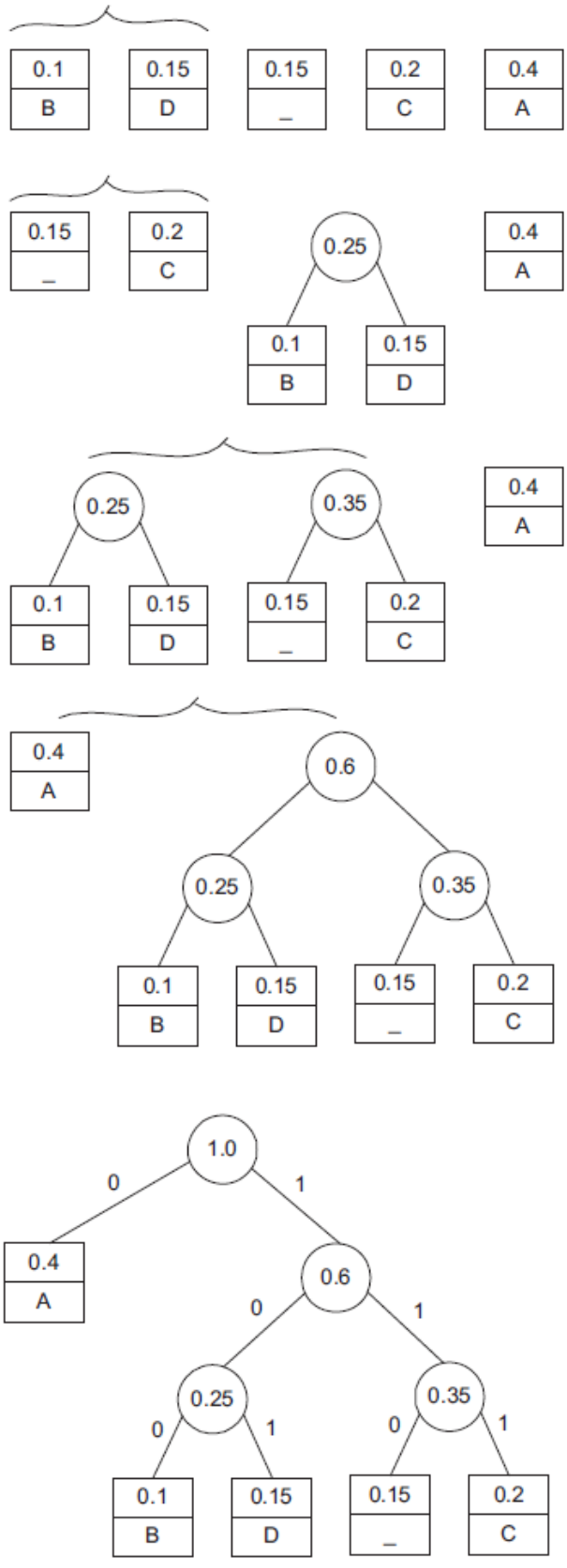
Tree vertices	Remaining vertices
a(-,0)	b(-,∞) c(−,∞) **d(a,7)** e(-,∞)
d(a,7)	**b(d,7+2)** c(d,7+5) e(-,∞)
b(d,9)	**c(d,12)** e(-,∞)
c(d,12)	**e(c,12+6)**
e(c,18)	

The shortest paths (identified by following nonnumeric labels backwards from a destination vertex to the source) and their lengths are:

from a to d: $a - d$ of length 7
from a to b: $a - d - b$ of length 9
from a to c: $a - d - c$ of length 12
from a to e: $a - d - c - e$ of length 18

8. [12주차 1강 출석체크 문제] [Exercise 9.4] 1번 (10점, 부분문제 a,b 각 5점) Huffman code

<sol> (a)



character	A	B	C	D	_
probability	0.4	0.1	0.2	0.15	0.15
codeword	0	100	111	101	110

(b) <sol> 0100011101000101

9. [12주차 2강 출석체크 제] [Exercise 10.1] 2번 변형 (10점)

<sol>

maximize $z = 2x + y + 0u + 0v$

subject to $\begin{cases} -2x + y + u = 2 \\ 3x + 2y + v = 5 \\ x >= 0, y >= 0, u >= 0, v >= 0 \end{cases}$

진입변수

↓

	x	y	u	v		θ-ratio
u	-2	1	1	0	2	2/-2=-1
퇴출변수→ v	3	2	0	1	5	5/3
	-2	-1	0	0	0	

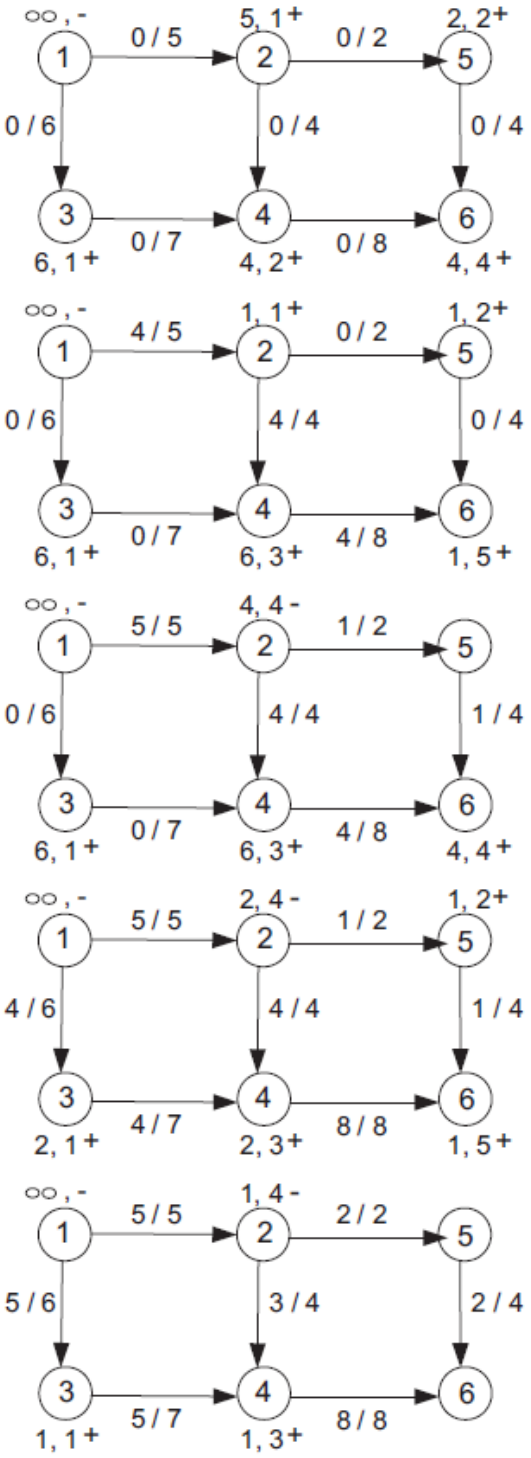
	x	y	u	v		
u	0	$\frac{7}{3}$	1	$\frac{2}{3}$	$\frac{16}{3}$	
x	1	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{5}{3}$	
	0	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{10}{3}$	->최적해

그러므로, $(x, y, u, v) = (\frac{5}{3}, 0, \frac{16}{3}, 0)$ 일 때, $z = \frac{10}{3}$ 으로 최대

→ $x = \frac{5}{3}, y = 0$일 때 $2x + y = \frac{10}{3}$ 으로 최대값이다.

10. [13주차 1강 출석체크 문제] [Exercise 10.2] 2번 (10점) Shortest-augmenting path algorithm

<sol>



Maximum flow 는 위 그림과 같다. 1에서 6으로 흐르는 최대 flow 는 10이다. Minimum cut은 {(2, 5}, (4, 6)}이다.