Morphological Image Processing

Digital Image Processing

Mathematic Morphology

- Used to extract image components that are useful in the representation and description of region shape, such as
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

Mathematic Morphology

- Mathematical framework used for:
  - pre-processing:
    - noise filtering, shape simplification, ...
  - enhancing object structure:
    - skeletonization, convex hull...
  - Segmentation
    - watershed,…
  - quantitative description
    - area, perimeter, ...

Basic Set Theory

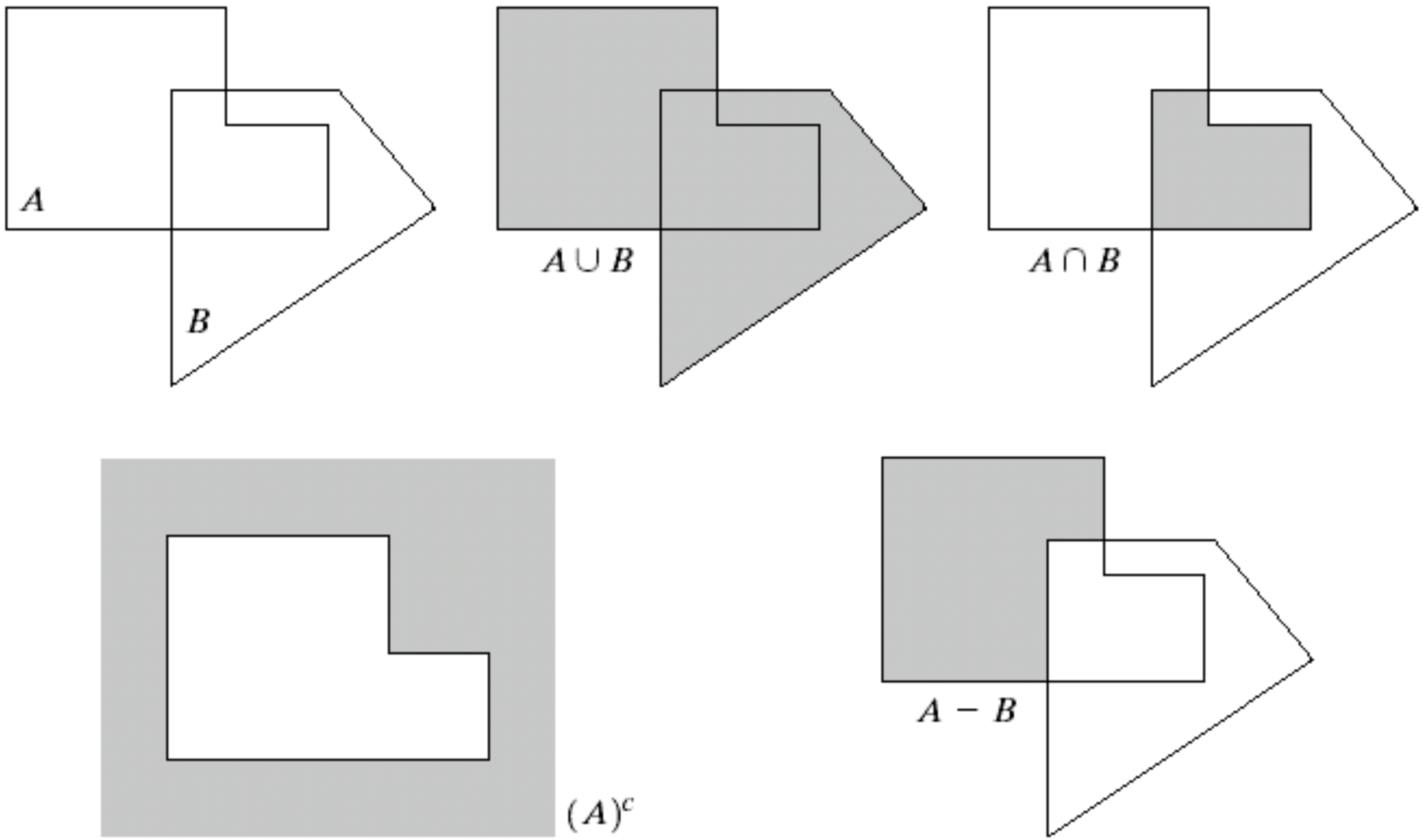
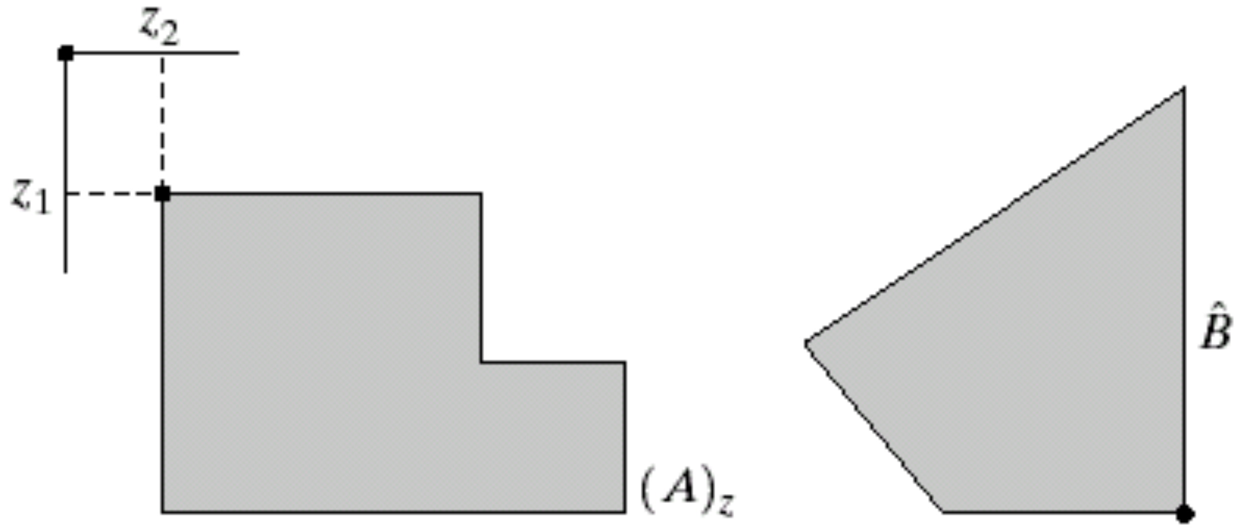


FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Reflection and Translation

$$\hat{B} = \{w | w - b, for\ b \in B\}$$
$$(A)_z = \{c | c \in a + z, for\ a \in A\}$$



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Logic Operations

p	q	p **AND** q **(also** $p \cdot q$**)**	p **OR** q **(also** $p + q$**)**	**NOT** (p) **(also** $\bar{p}$**)**
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Example

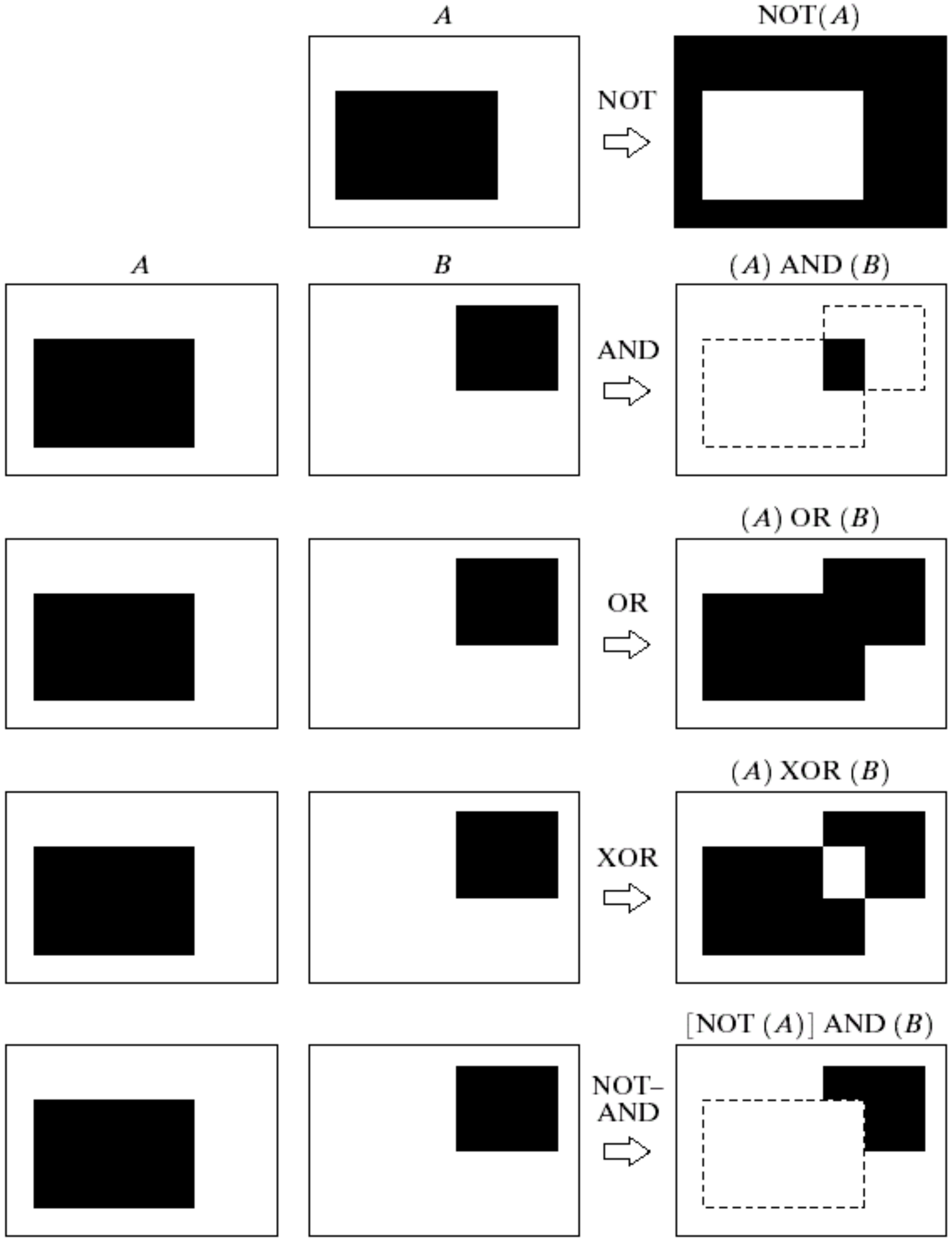
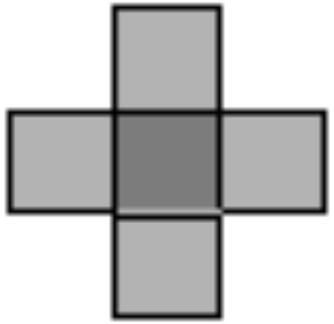
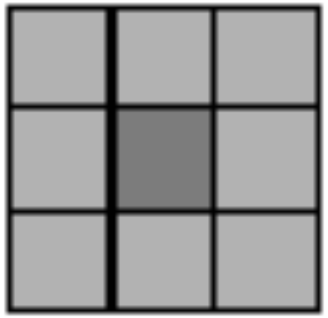


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Structuring Element (SE)

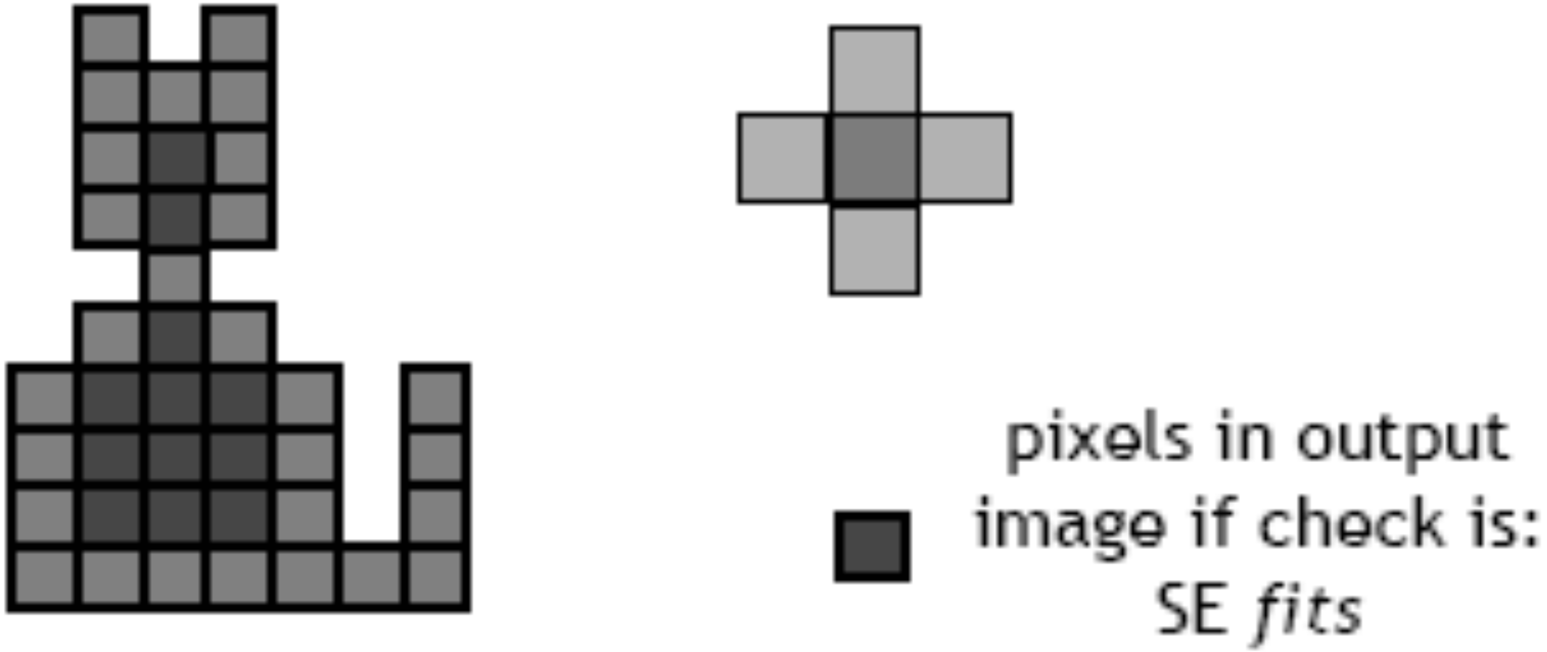
- Small set to probe the image under study
  for each SE, define origin, shape and size must be adapted to geometric properties for the objects





Basic Idea

- In parallel for each pixel in binary image:
  - check if SE is ”satisfied”
  - output pixel is set to 0 or 1 depending on used operation



Basic morphological operations

- Erosion 

- Dilation 

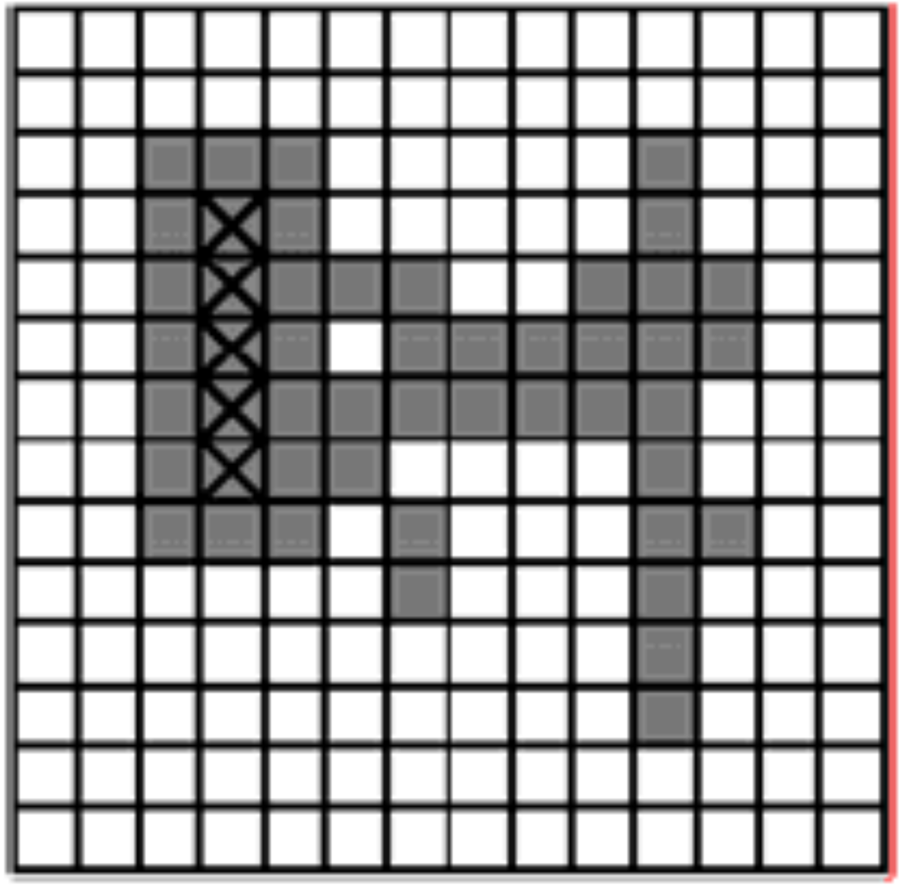
Erosion

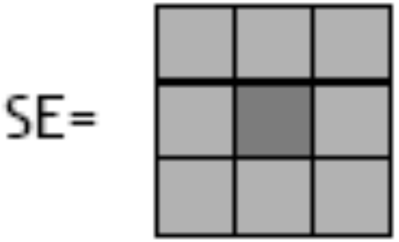
- Does the structuring element fit the set?
- Erosion of a set A by structuring element B: all z in A such that B is in A when origin of $B = z$

$$A \ominus B = \{z | (B)z \subseteq A\}$$

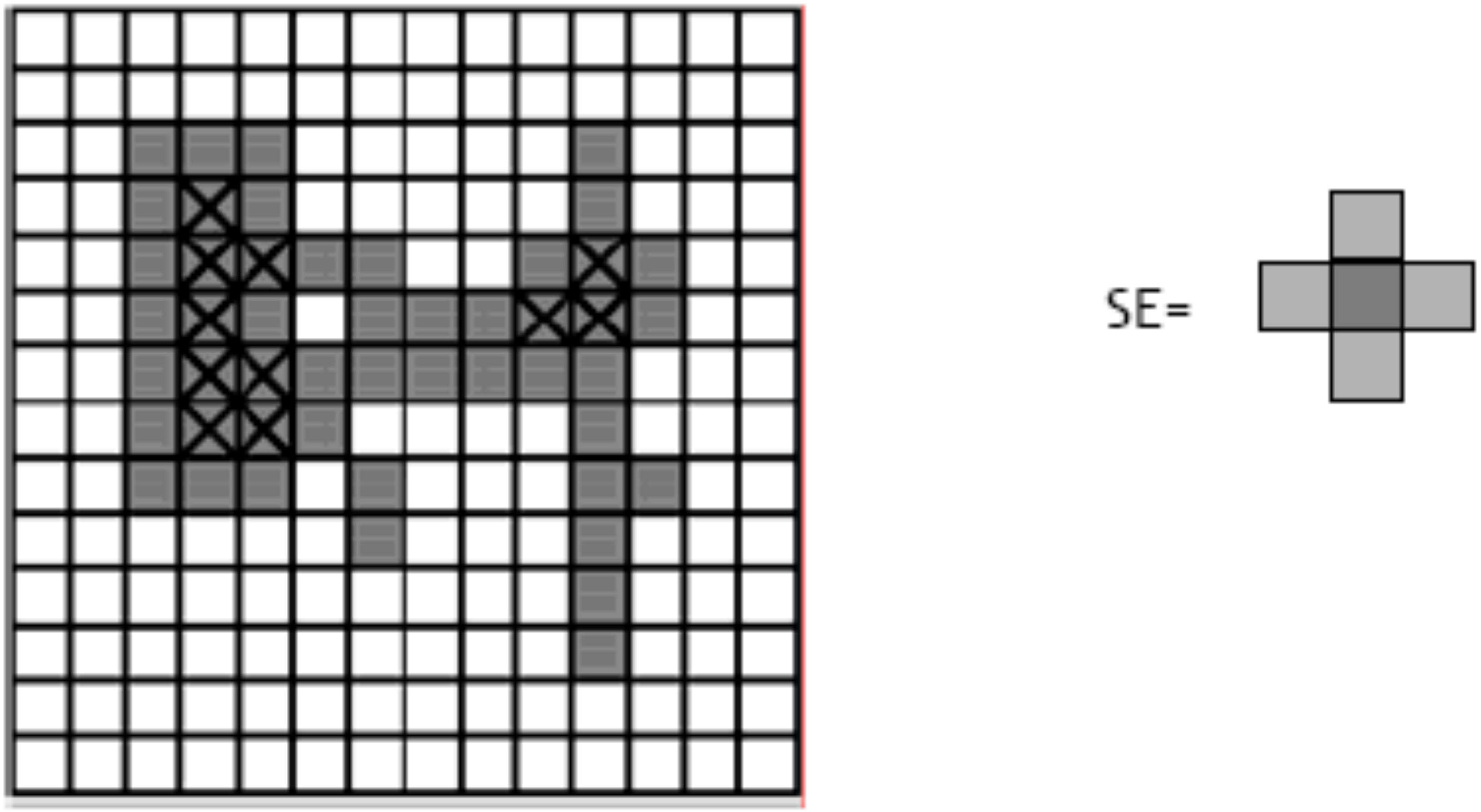
- Shrink the object

Erosion



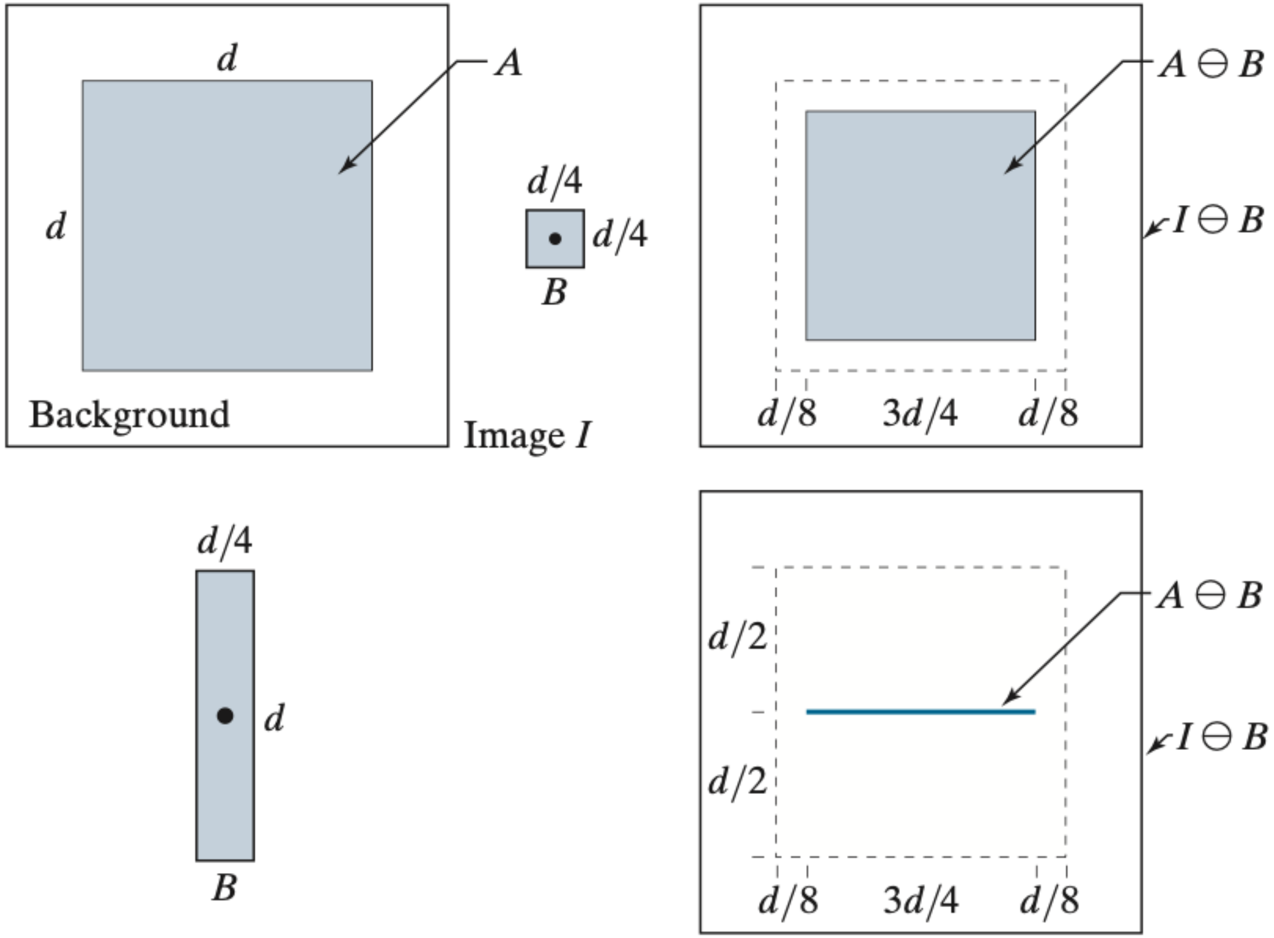


Erosion



Erosion

$$A \ominus B = \{z | (B)z \subseteq A\}$$



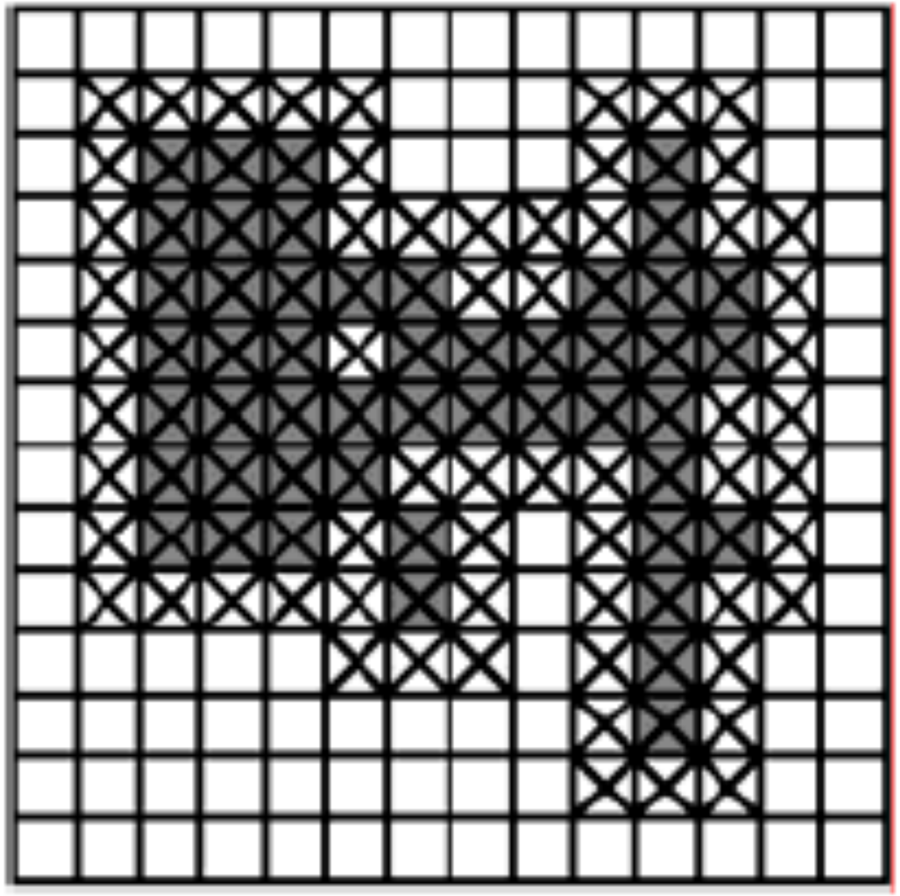
Dilation

- Does the structuring element hit the set?
- Dilation of a set A by structuring element B: all z in A such that B hits A when origin of $B = z$

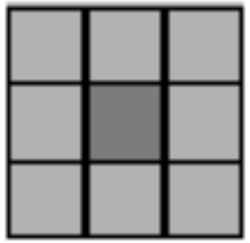
$$A \oplus B = \{z | (B)z \cap A \neq \emptyset\}$$

- Growing the object

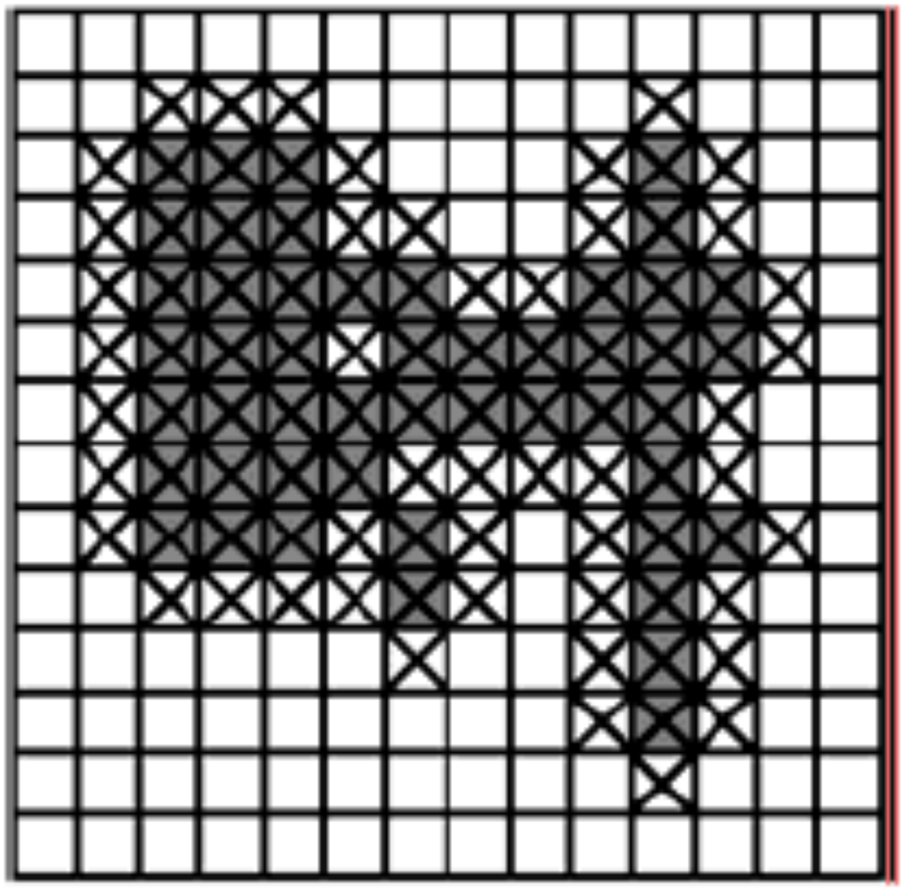
Dilation

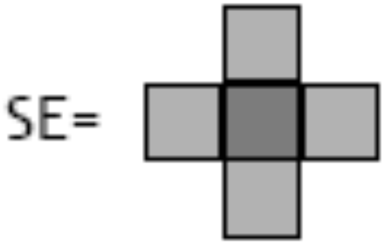


SE=



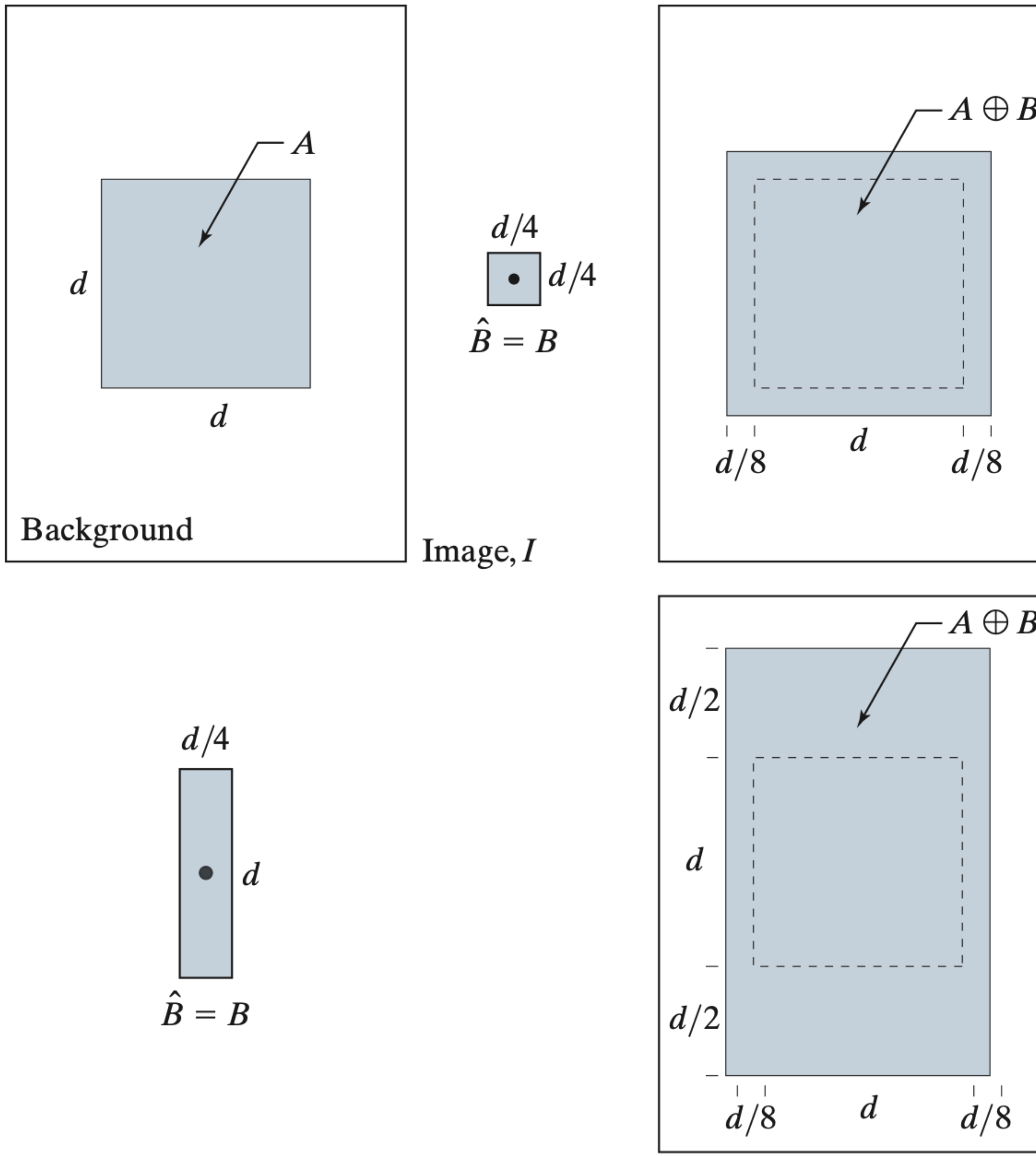
Dilation



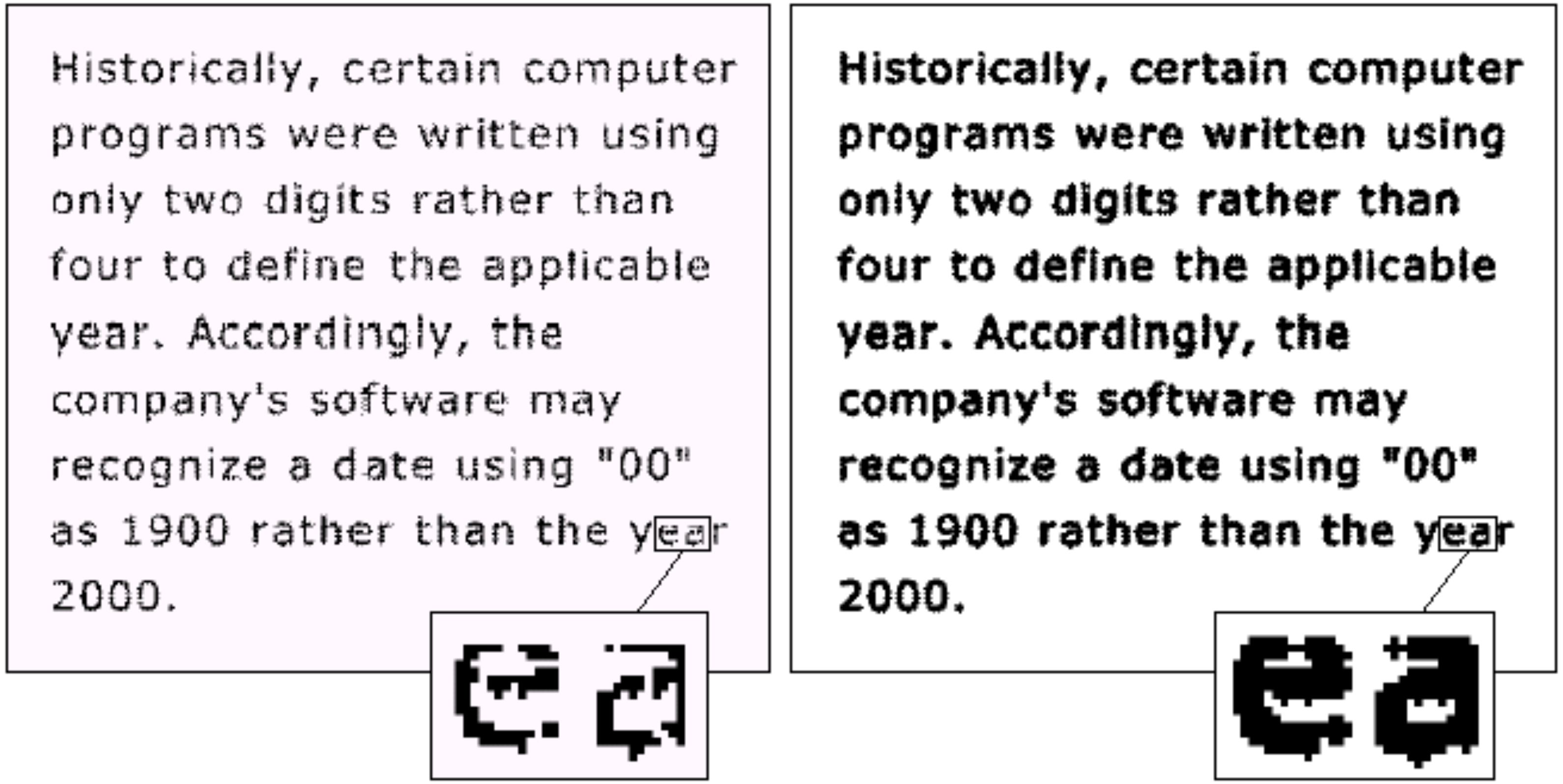


Dilation

$$A \oplus B = \{z \mid (B)z \cap A \neq \emptyset\}$$



Dilation : Bridging gaps



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Useful

- Erosion
  - removal of structures of certain shape and size, given by SE
- Dilation
  - filling of holes of certain shape and size, given by SE

Duality

- Erosion and dilation are duals of each other

  - $(A \ominus B)^c = A^c \oplus B^c$

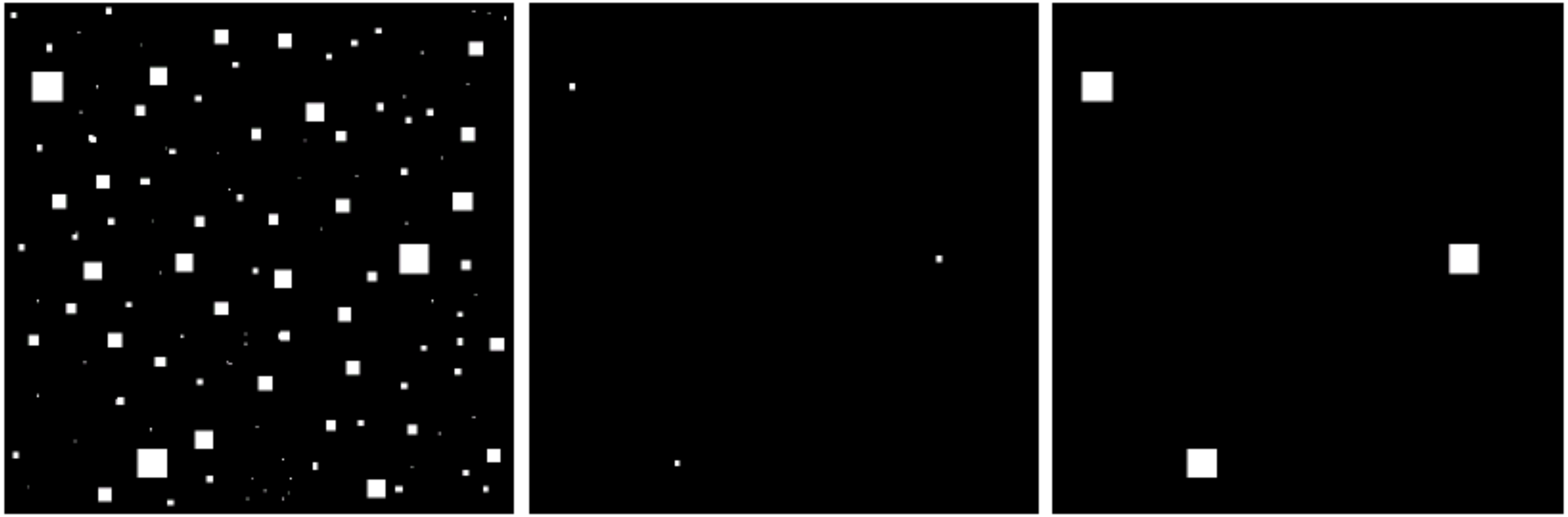
  - $(A \oplus B)^c = A^c \ominus B^c$

Combining Erosion and Dilation

- WANTED:
  - remove structures / fill holes
  - without affecting remaining parts

- SOLUTION:
  - combine erosion and dilation
  - (using same SE)

Erosion : Eliminating Irrelevant Detail



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element B = 13x13 pixels of gray level 1

Opening

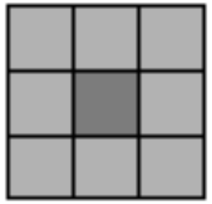
- Erosion followed by dilation denoted: $\circ$

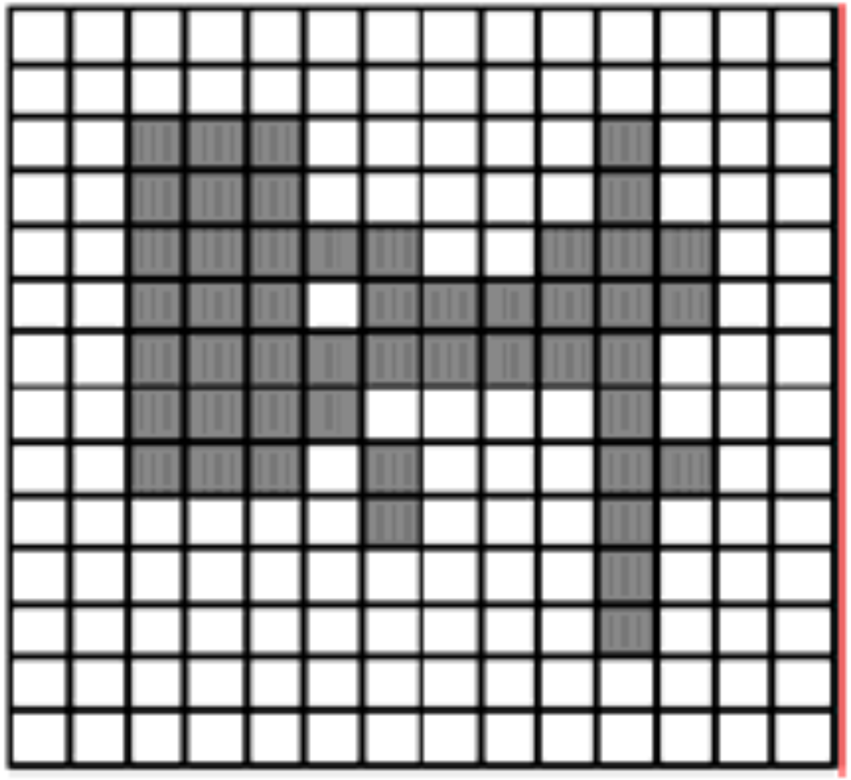
$$A \circ B = (A \ominus B) \oplus B$$

- Eleminates protrusions
- Breaks necks
- Smooth contour

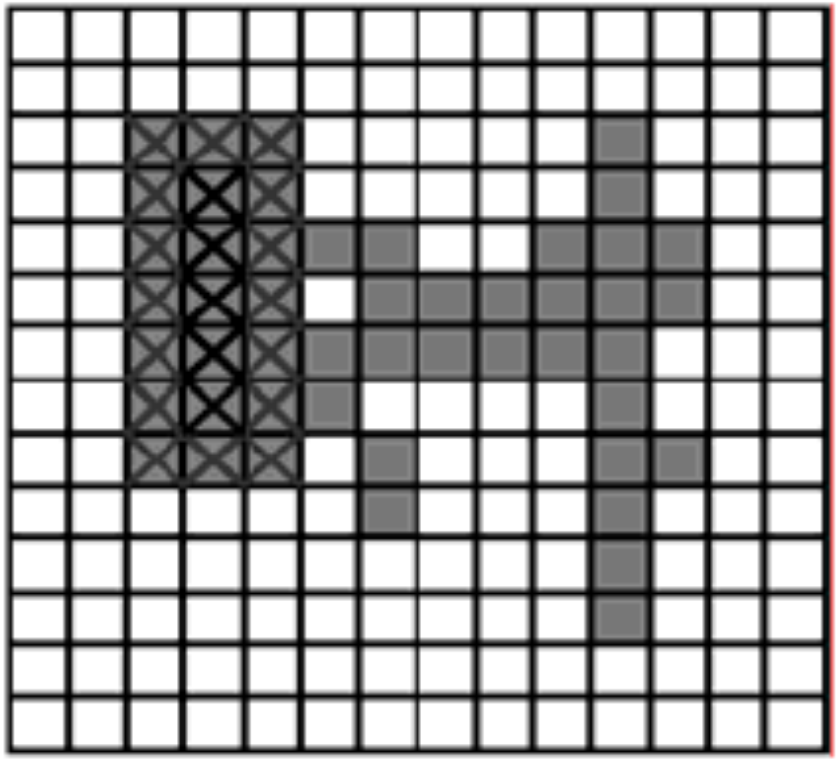
Opening

B=



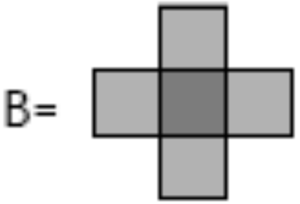


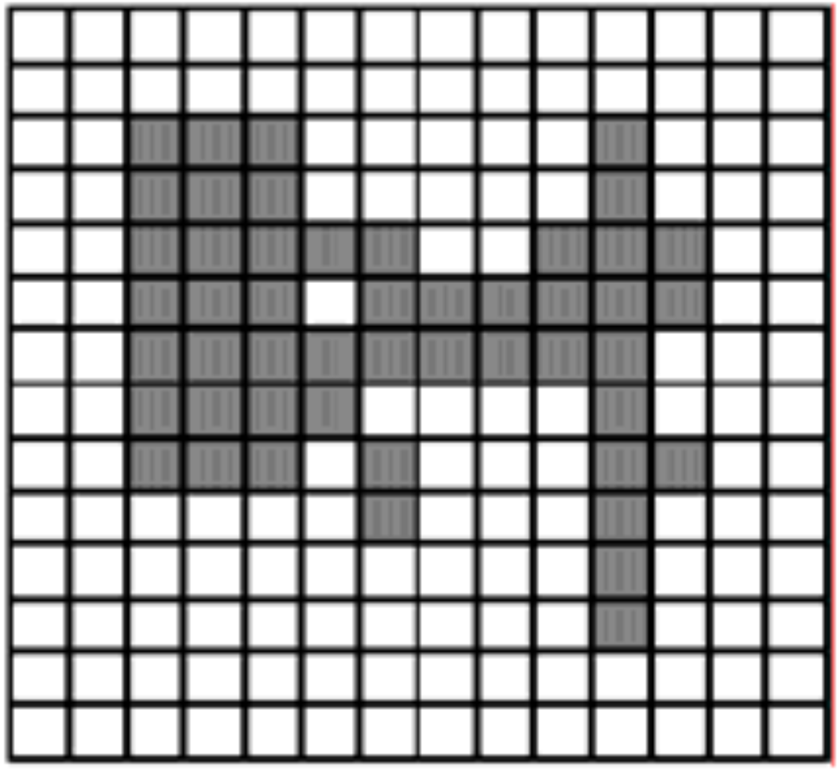
A



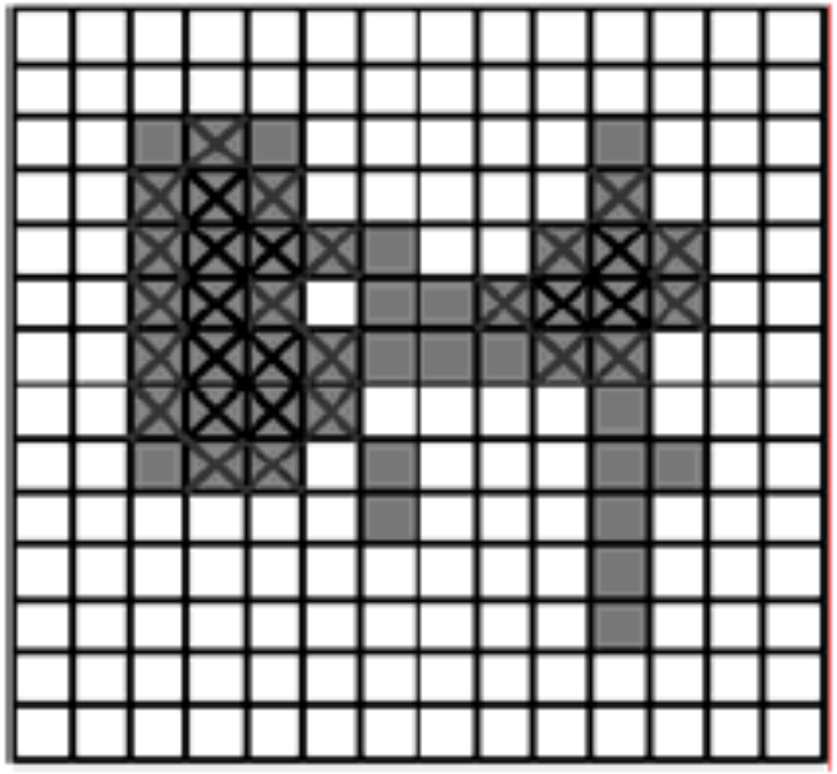
$A \ominus B$ $A \circ B$

Opening



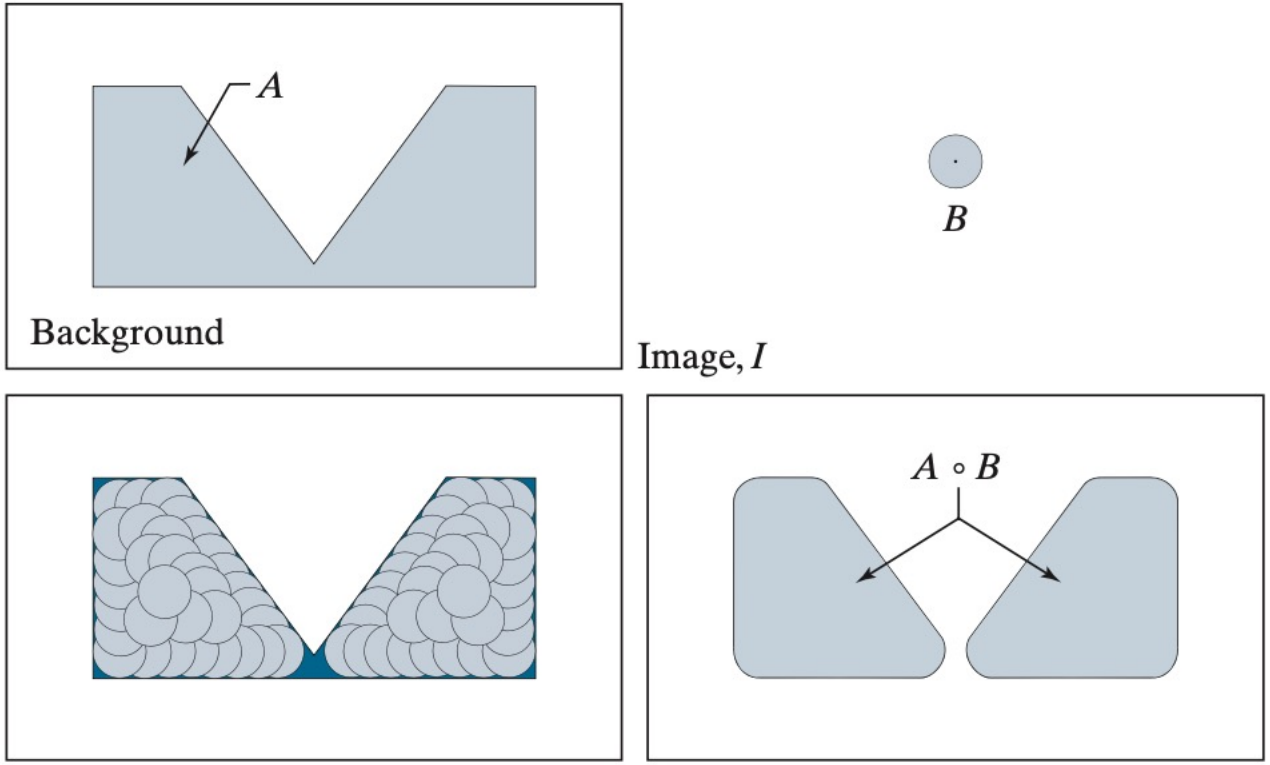


A



$A \ominus B$ $A \circ B$

Opening



$$A \circ B = (A \ominus B) \oplus B$$
$$A \circ B = \cup \{(B)_z | (B)_z \subseteq A\}$$

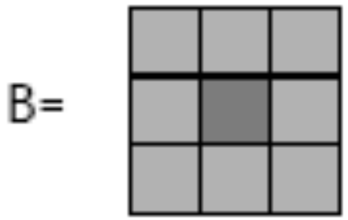
Closing

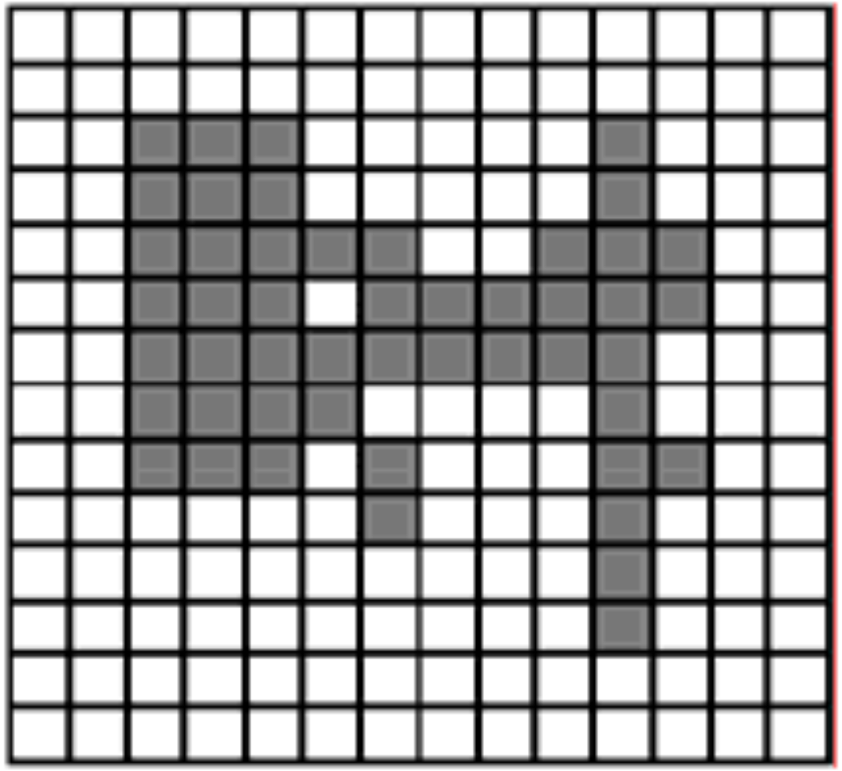
- dilation followed by erosion, denoted •

$$A \cdot B = (A \oplus B) \ominus B$$

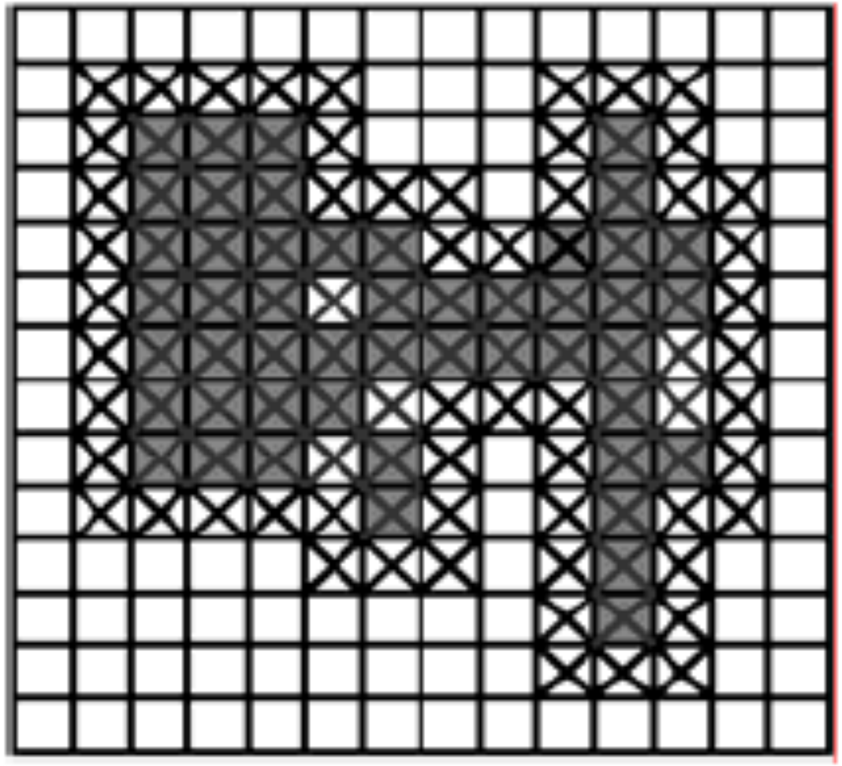
- Smooths contour
- Fuses narrow breaks and long thin gulfs
- Eliminates small holes
- Fills gaps in the contour

Closing





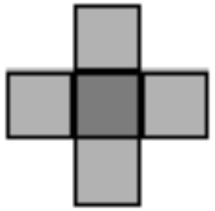
A

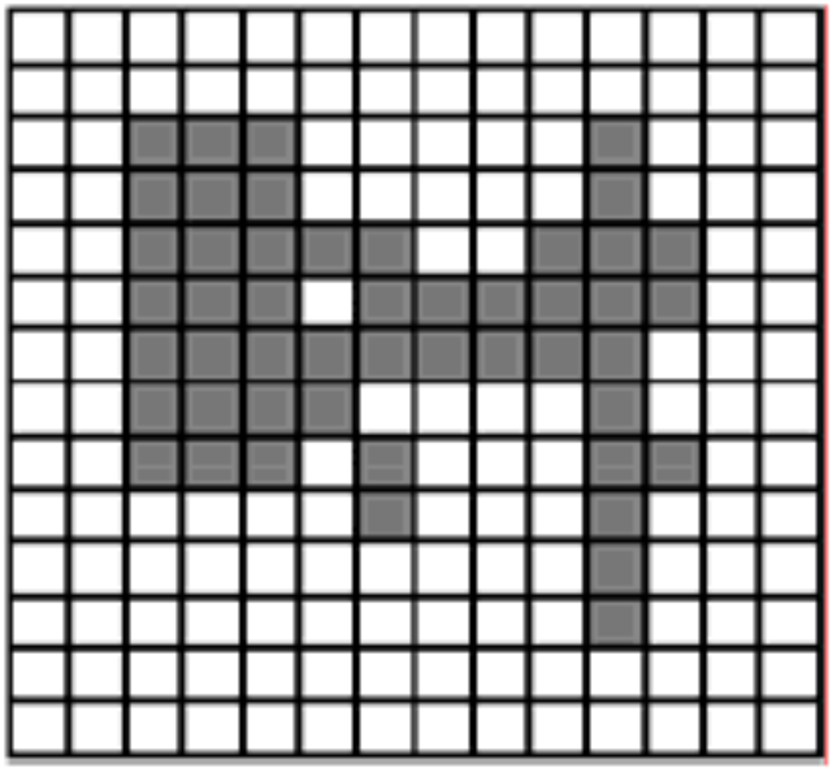


$A \oplus B$ $\quad A \bullet B$

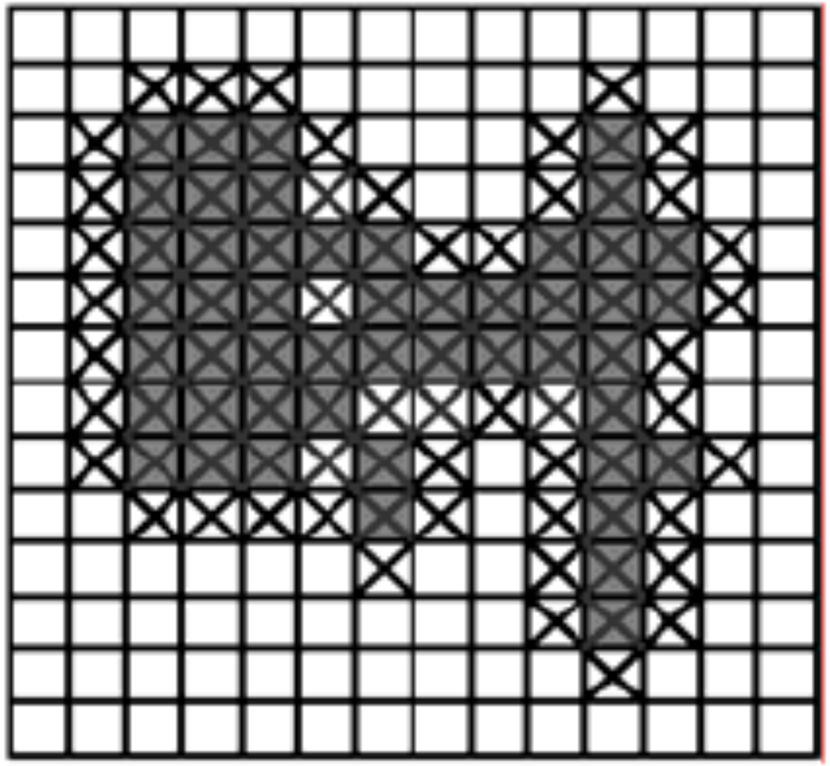
Closing

B=



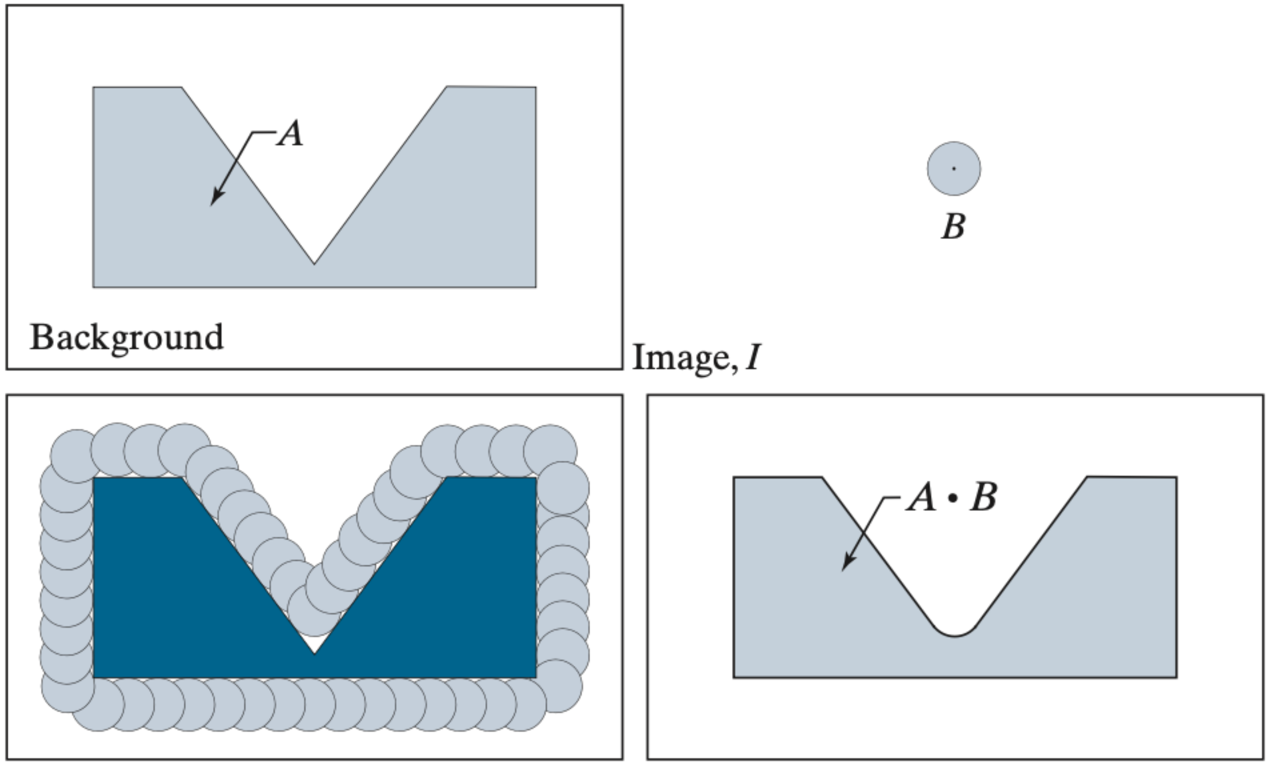


A



$A \oplus B$ $A \bullet B$

Closing



$$A \cdot B = (A \oplus B) \ominus B$$

Properties

- Opening
  - $A \circ B$ is a subset (subimage) of A
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  - $(A \circ B) \circ B = A \circ B$
- Closing
  - A is a subset (subimage) of $A \cdot B$
  - If C is a subset of D, then $C \cdot B$ is a subset of $D \cdot B$
  - $(A \cdot B) \cdot B = A \cdot B$

- Note: repeated openings/closings has no effect!

Duality

- Opening and closing are dual with respect to complementation and reflection

$$(A \circ B)^c = \left(A^c \cdot \hat{B}\right)$$

$$(A \cdot B)^c = \left(A^c \circ \hat{B}\right)$$

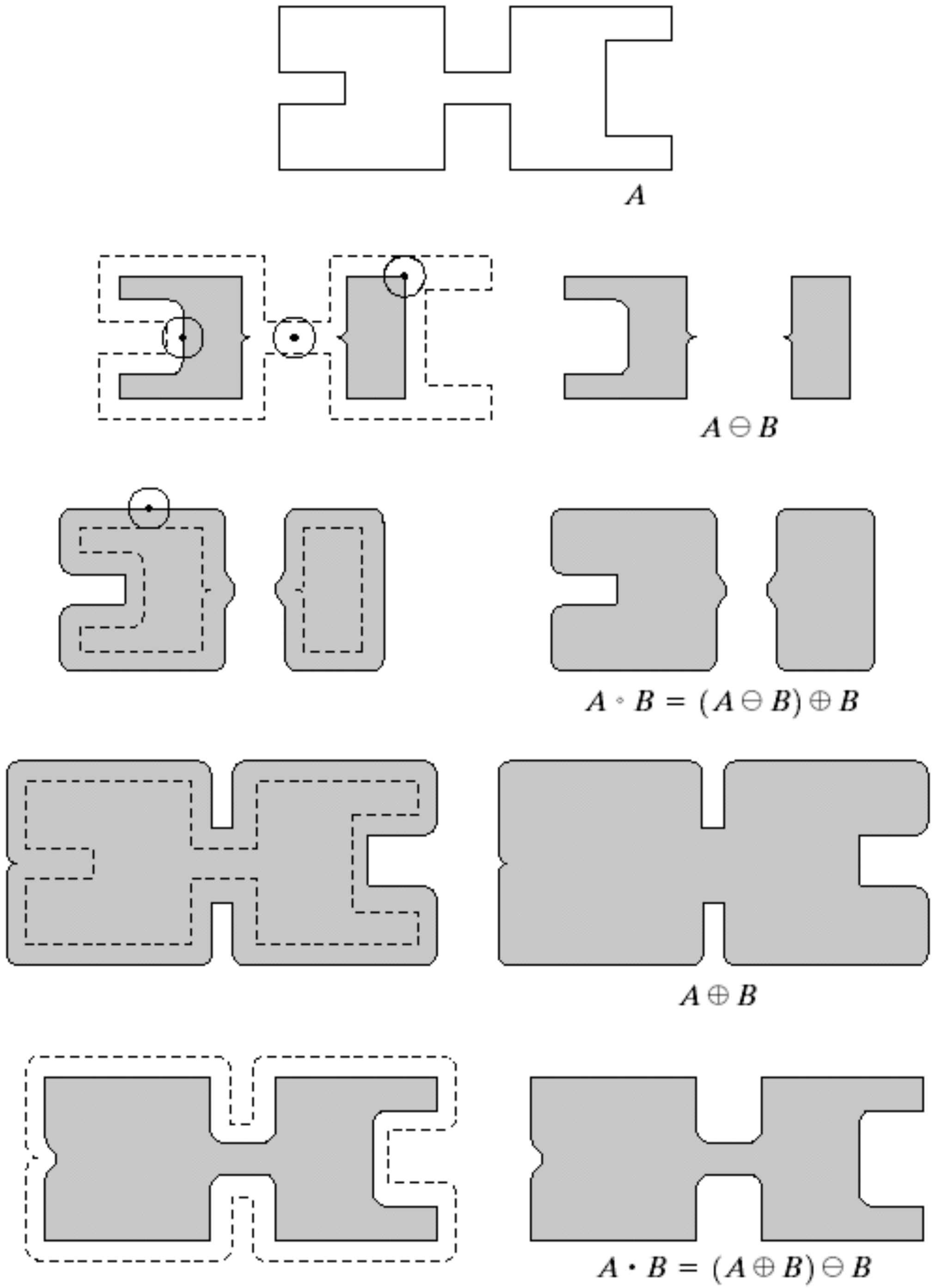


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

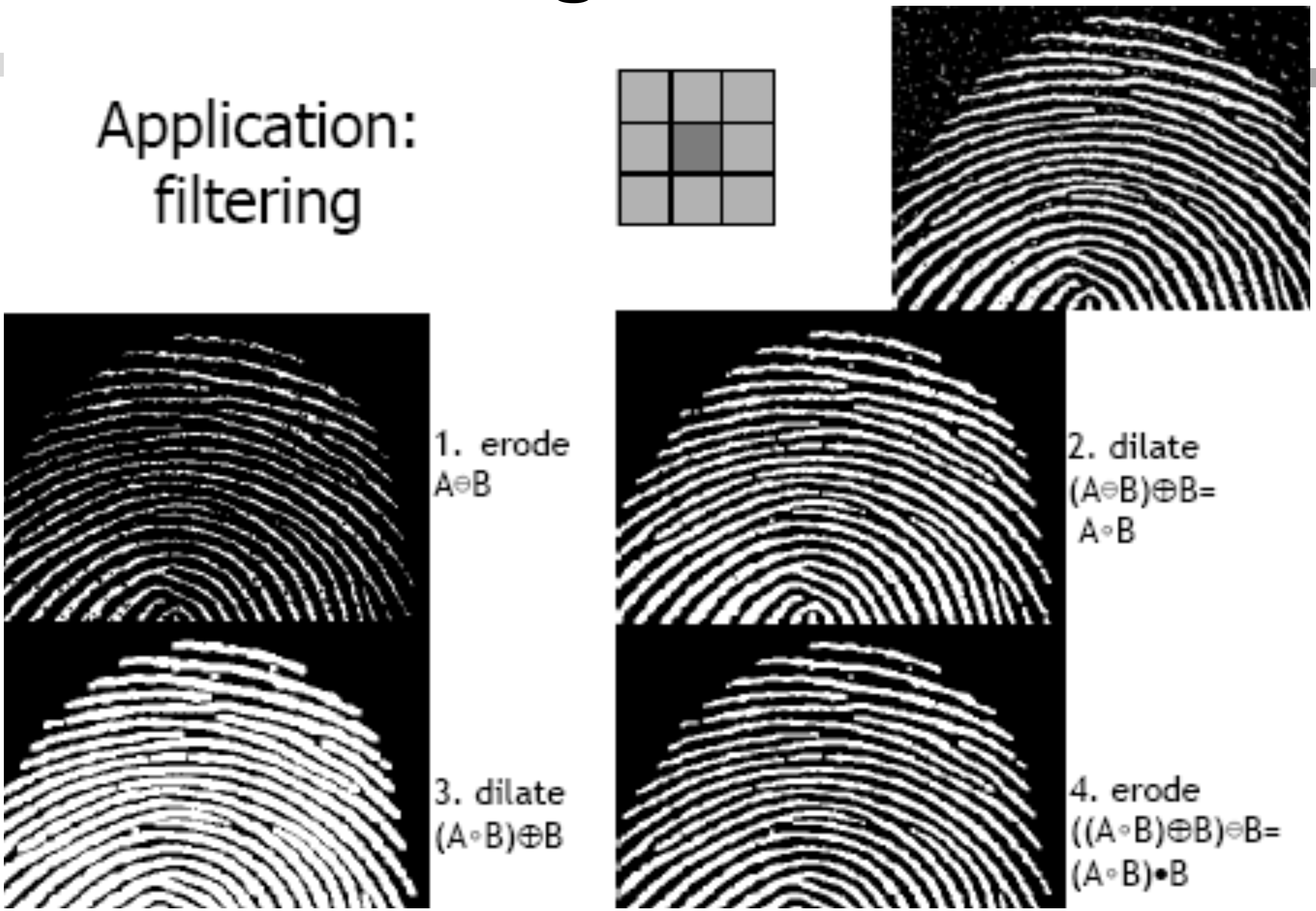
Useful: open & close

A

opening of A
→removal of small protrusions, thin connections, ...

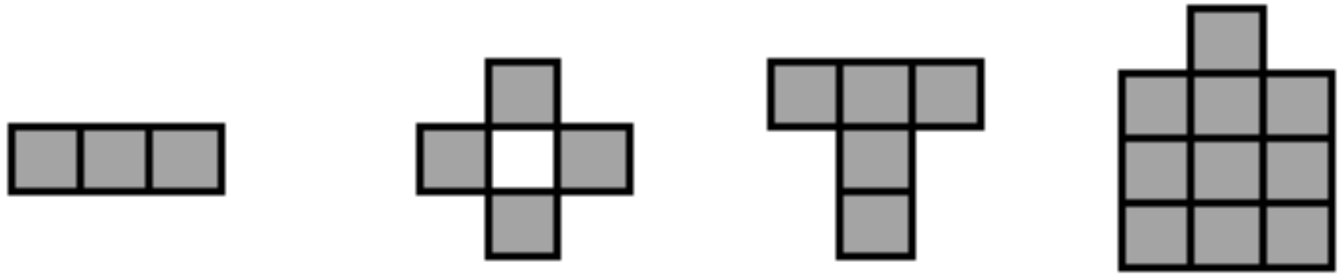
closing of A
→ removal of holes

Application: filtering



Hit-or-Miss Transformation $\circledast$ (HMT)

- Find location of one shape among a set of shapes: template matching

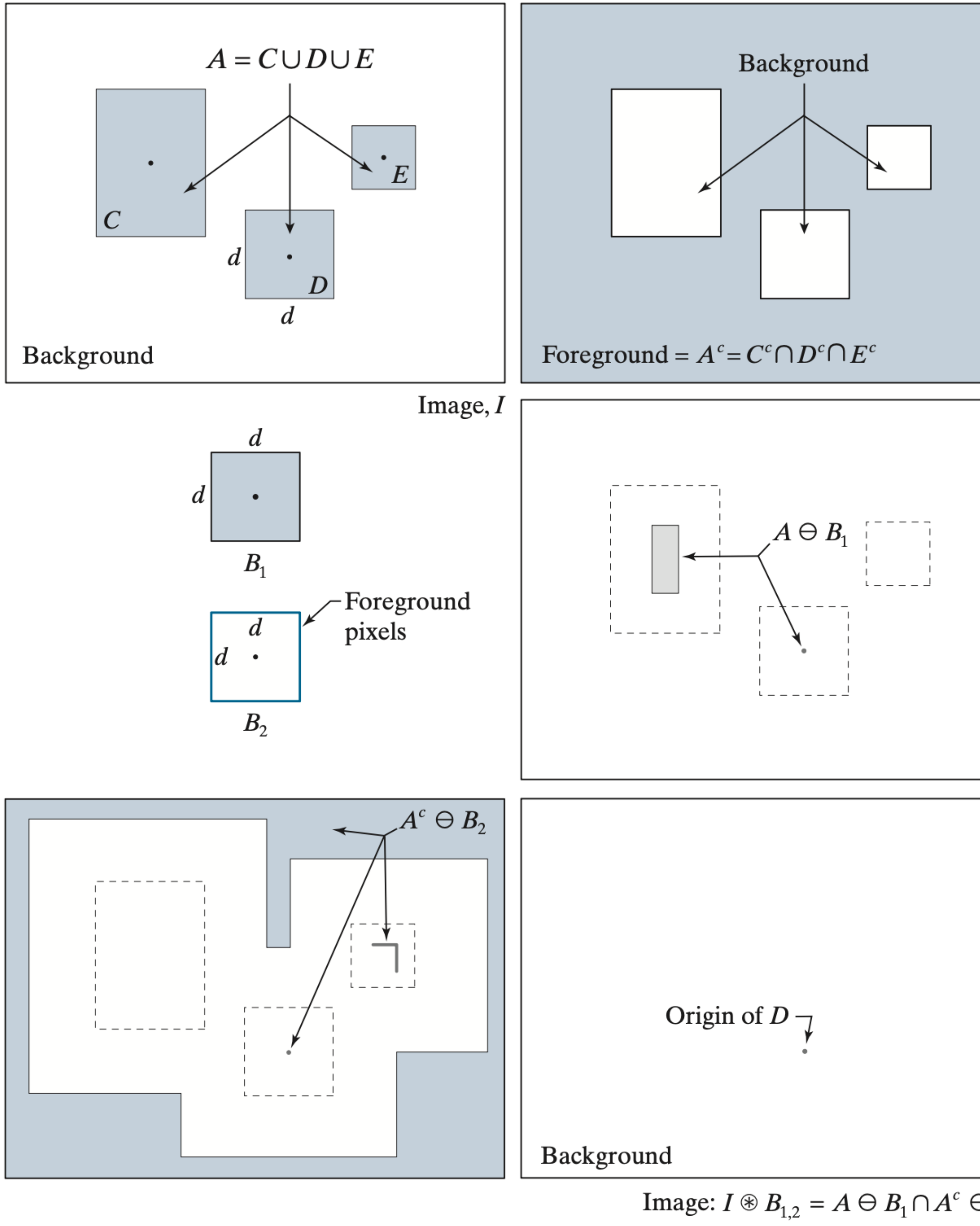


- Composite SE: object part (B1) and background part (B2)
- Does B1 fits the object while, simultaneously, B2 misses the object, i.e., fits the background?

HMT

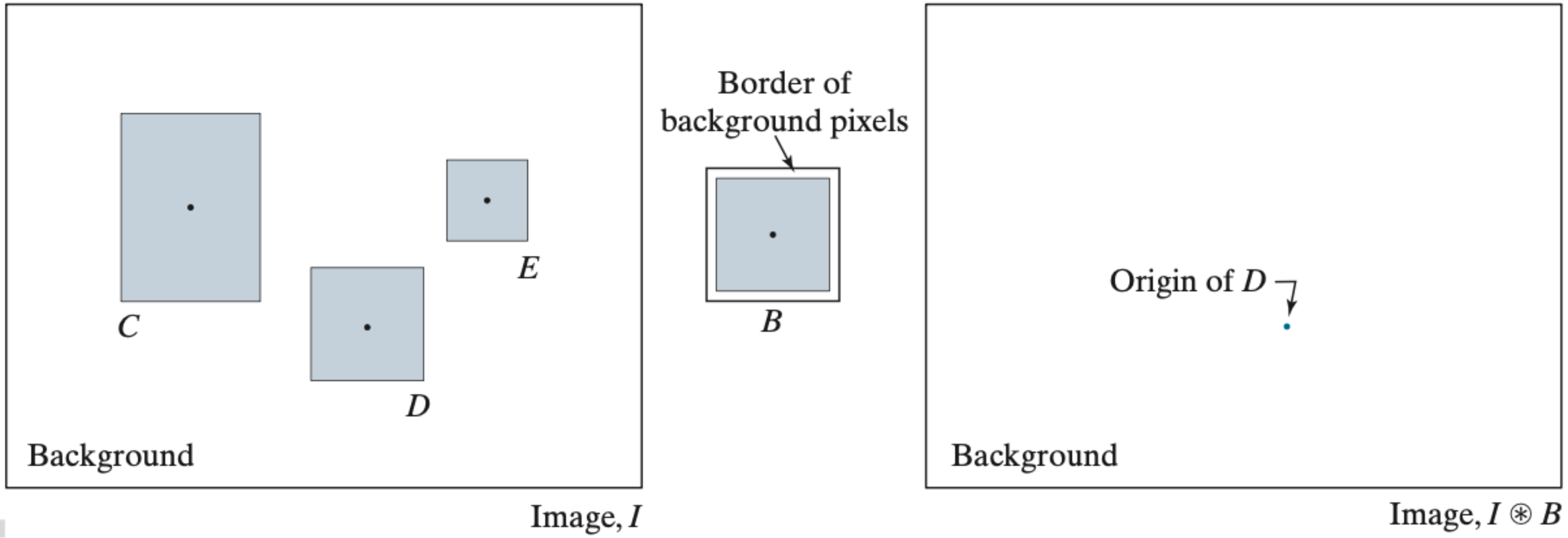
$$(A \ominus X) \cap [A^c \ominus (W - X)]$$

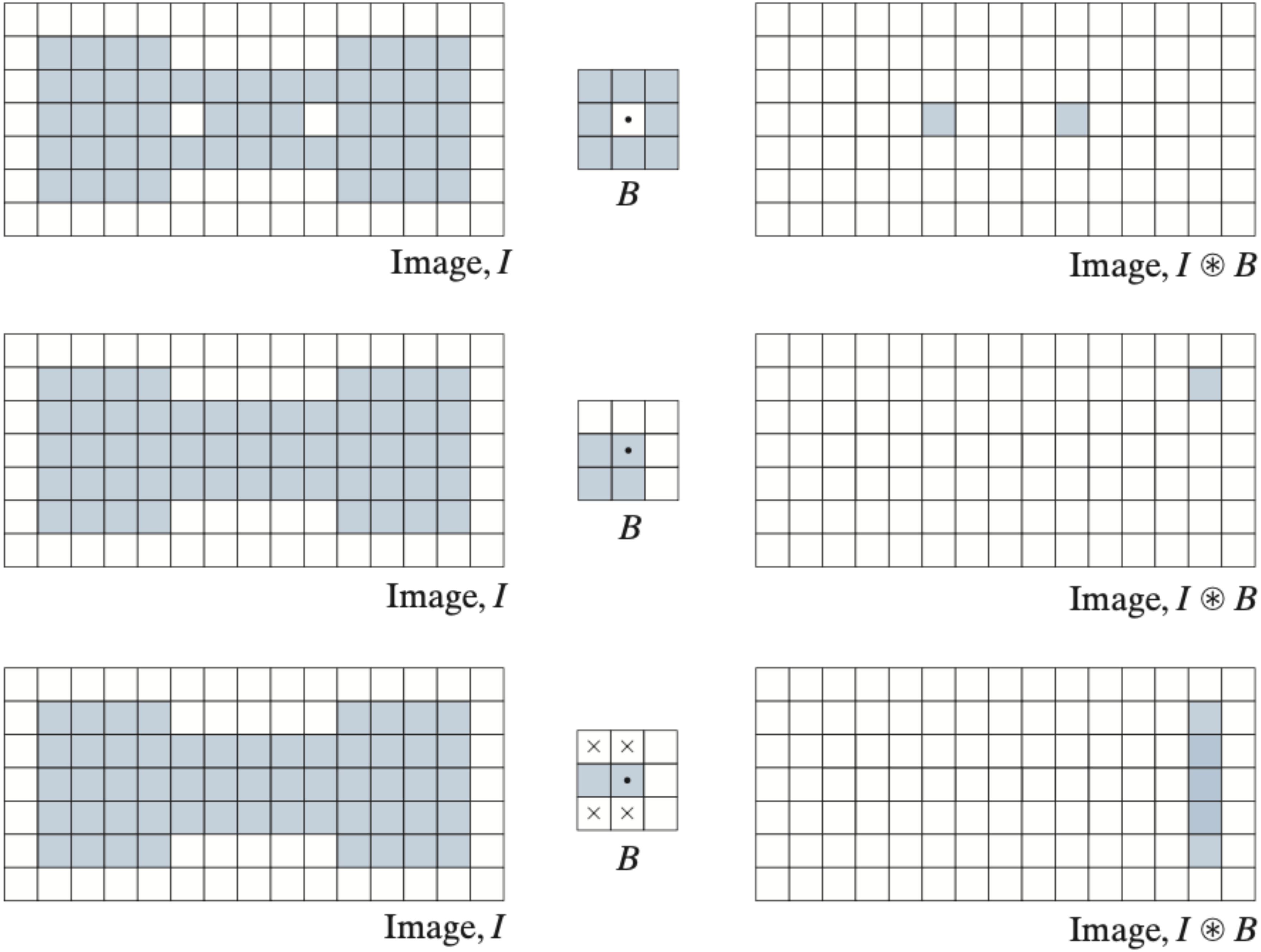
$$\begin{aligned} A \circledast B_{1,2} &= (A \ominus B_1) \cap [A^c \ominus B_2] \\ &= (A \ominus B_1) - \left(A \oplus \hat{B}_2\right) \end{aligned}$$



HMT

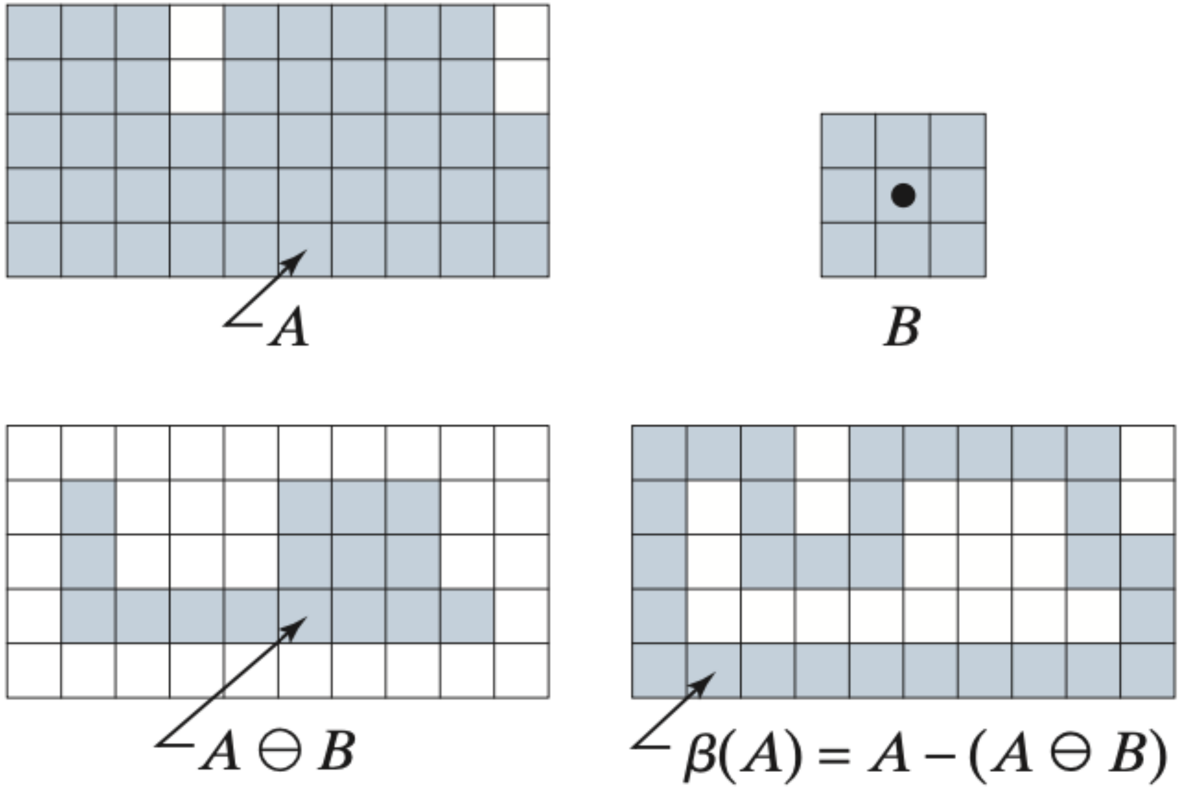
- Typical Hit and Miss Transformation
  - Making B contain pixels should be true and false at the same time
  - $I \circledast B = \{z|(B)z \subseteq I\}$
  - Meaning that if I arround z is the same as B, output is true





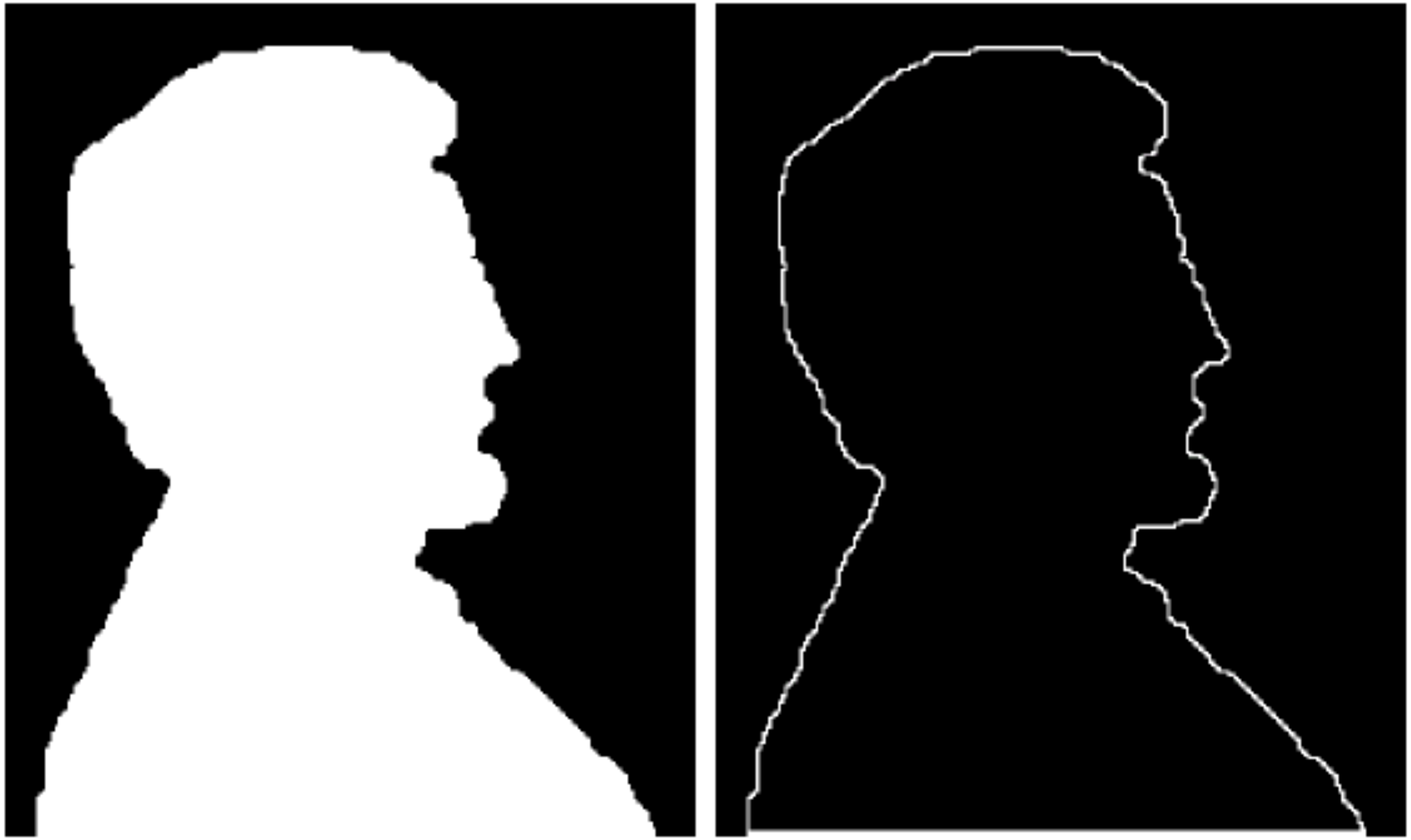
B
Image, I
Image, I ⊛ B
B
Image, I
Image, I ⊛ B
B
Image, I
Image, I ⊛ B

Boundary Extraction



$$\beta(A) = A - (A \ominus B)$$

Example



a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1,2,3,\cdots$$

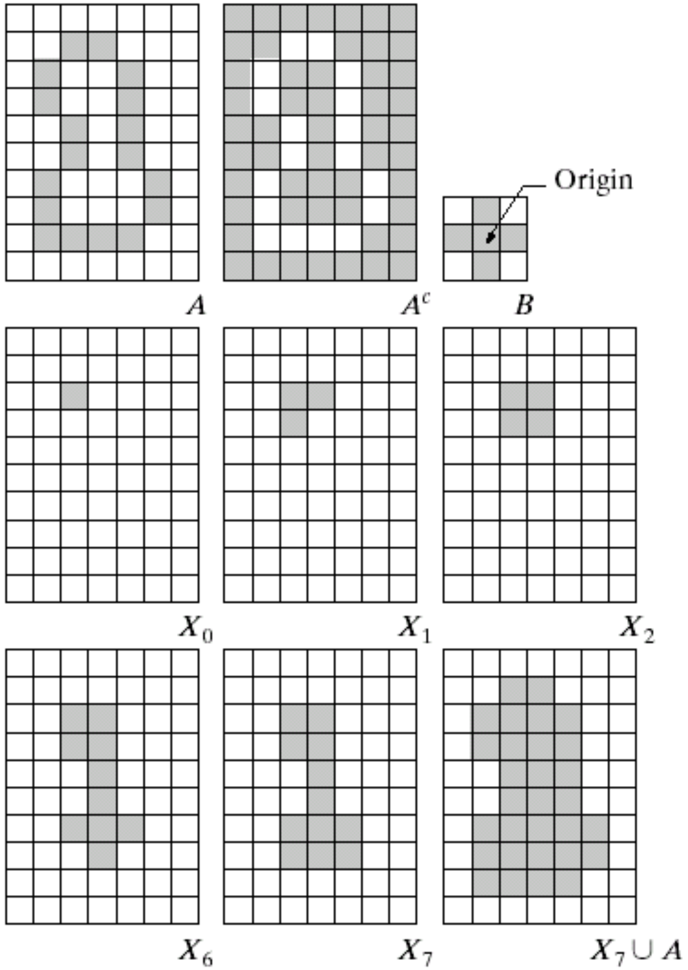
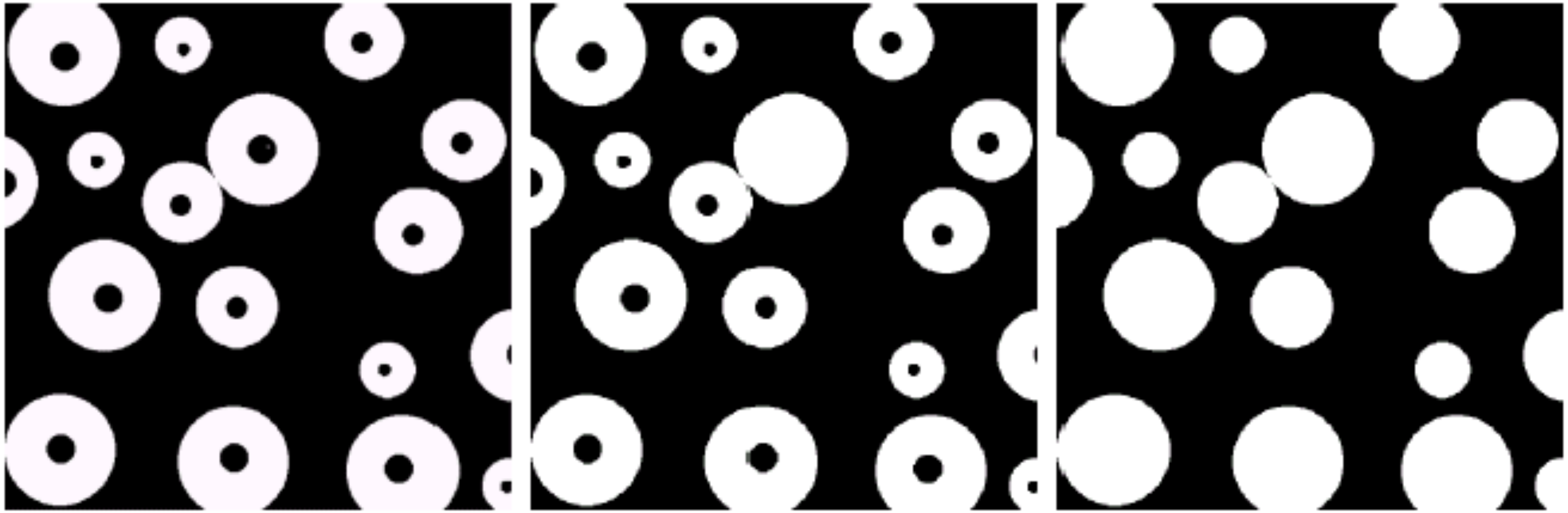


FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

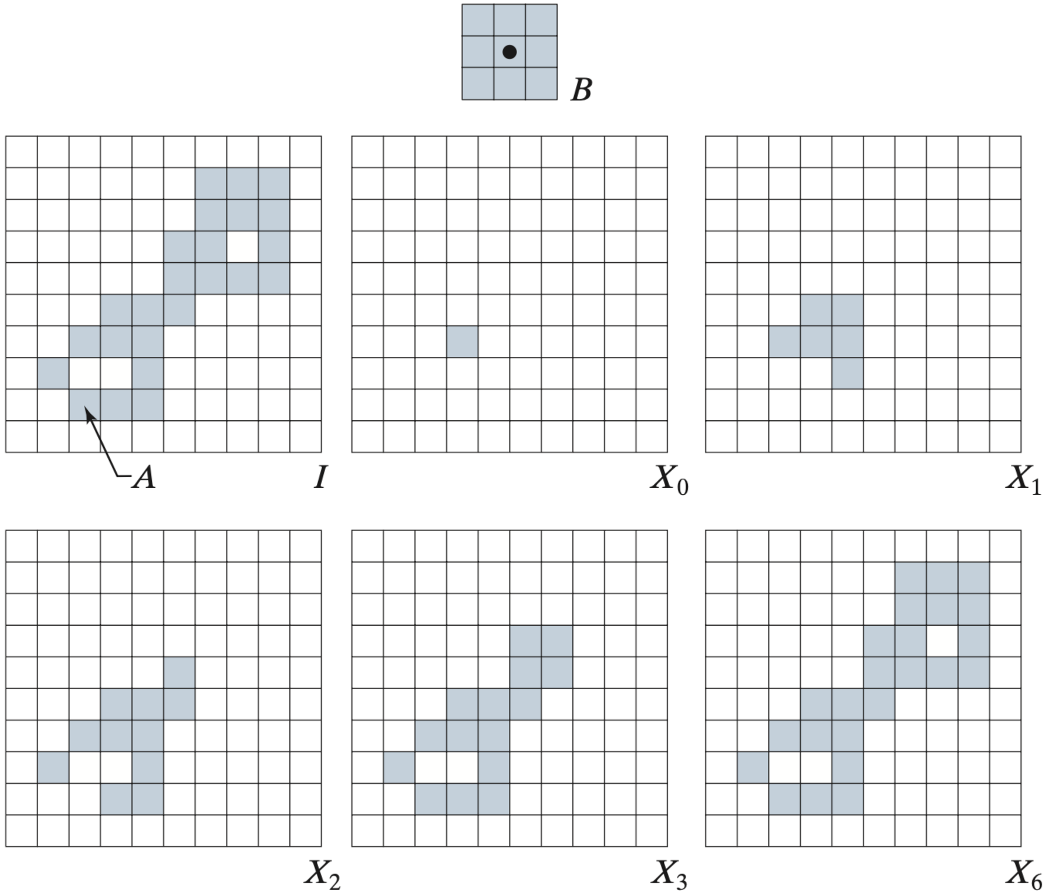
Example



a b c

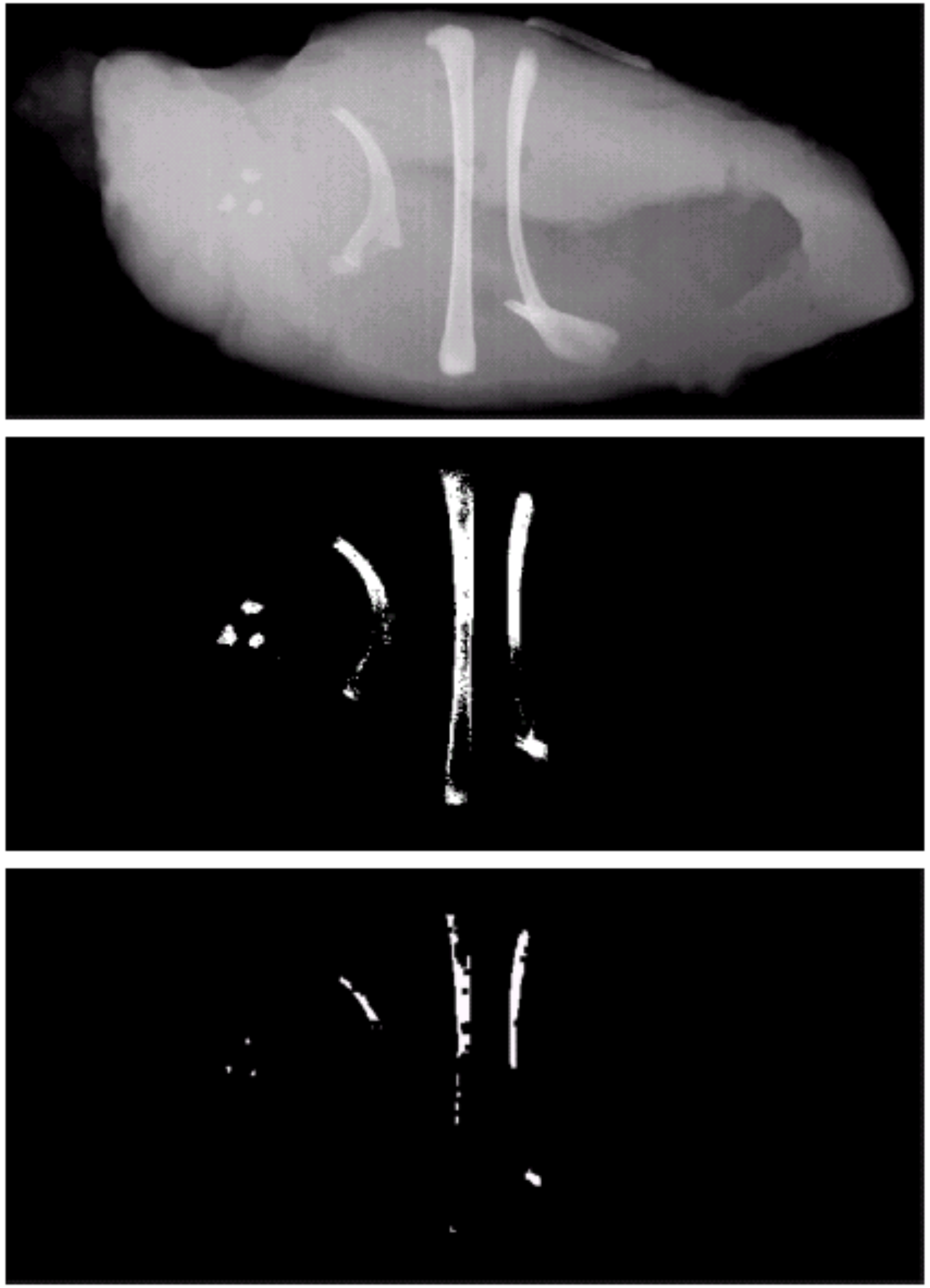
FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Extraction of Connected Components



$$X_k = (X_{k-1} \oplus B) \cap A, k = 1,2,3,\cdots$$

Example



a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex hull

- A set A is said to be convex if the straight line segment joining any two points in A lies entirely within A.

$$X_k^i = \left(X_{k-1}^i \circledast B^i\right) \cup X_{k-1}^i, k = 1,2,3,\cdots$$

$$C(A) = \bigcup_{i=1}^{4} D^i, where\ D^i = X_k^i$$

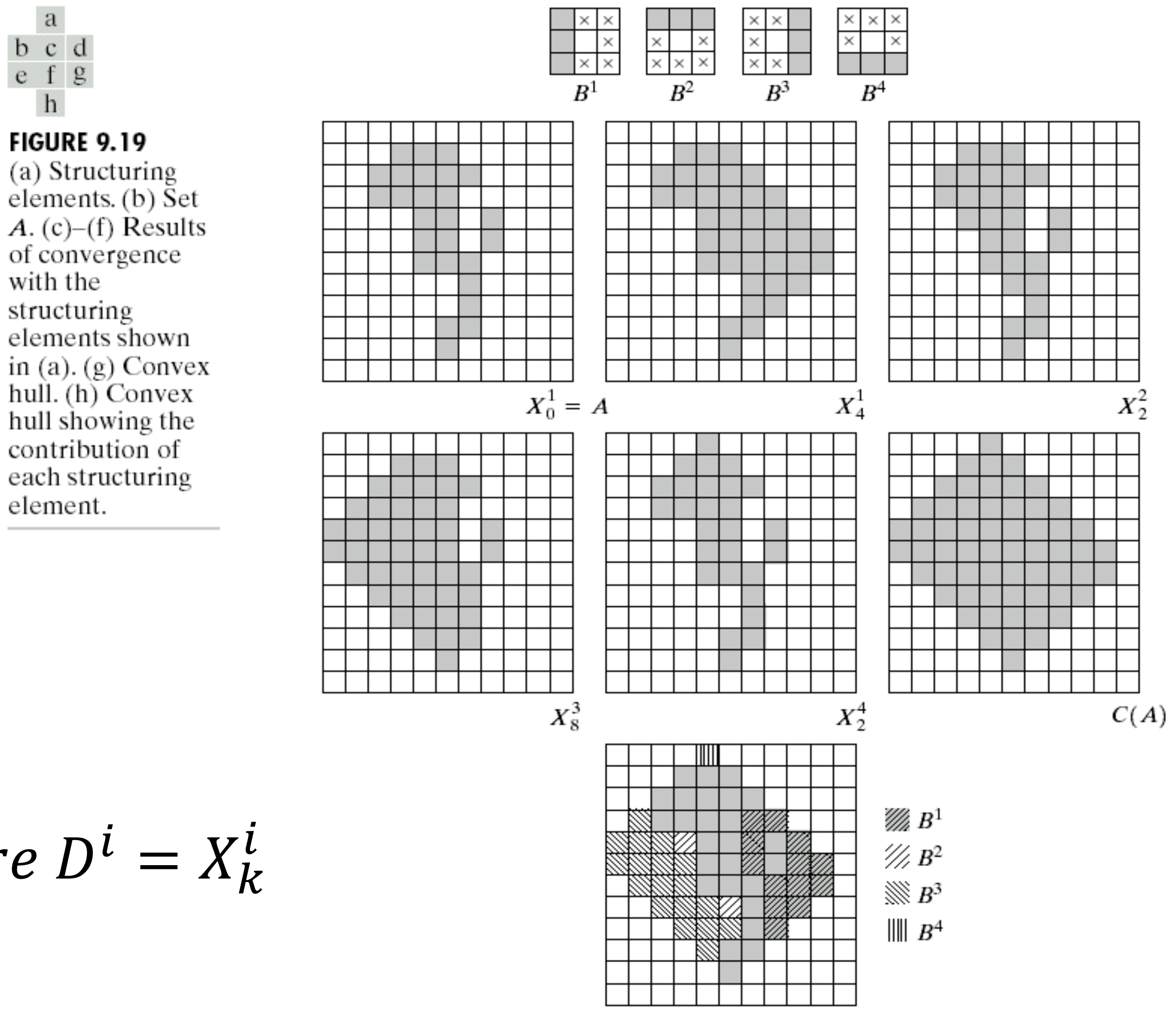


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

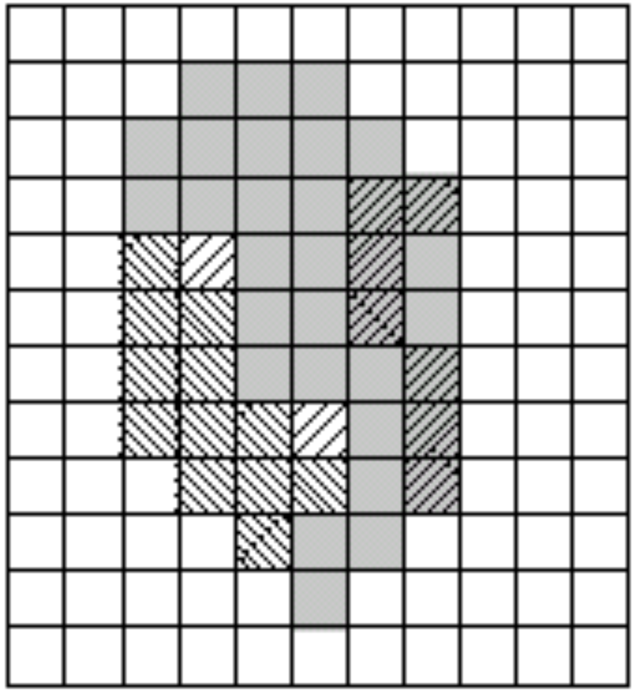
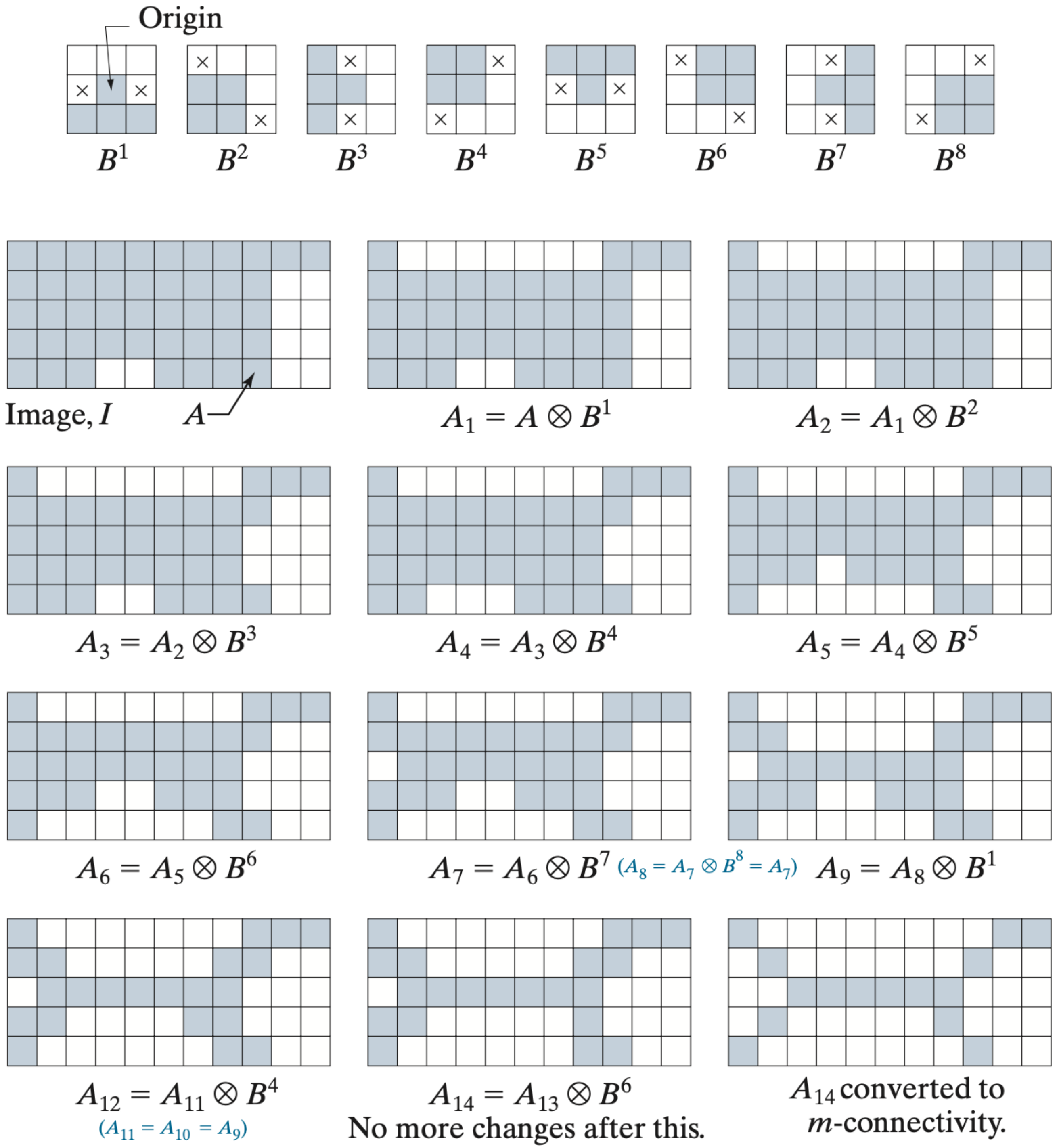


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

Thinning

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

$$A \otimes \{B\} = \left(\left(\cdots\left(\left(A \otimes B^1\right) \otimes B^2\right)\cdots\right) \otimes B^n\right)$$



Thickening

- Thickening is the morphological dual of thinning
- Defined as

$$A \odot B = A \cup (A \circledast B)$$

  - Note structural emement is complement of B
  - $A \odot \{B\} = \left(\left(\cdots\left((A \circledast B^1) \circledast B^2\right)\cdots\right) \circledast B^7\right)$

- Can be implemented using thinning

Thickening

$$A \odot B = A \cup (A \circledast B)$$

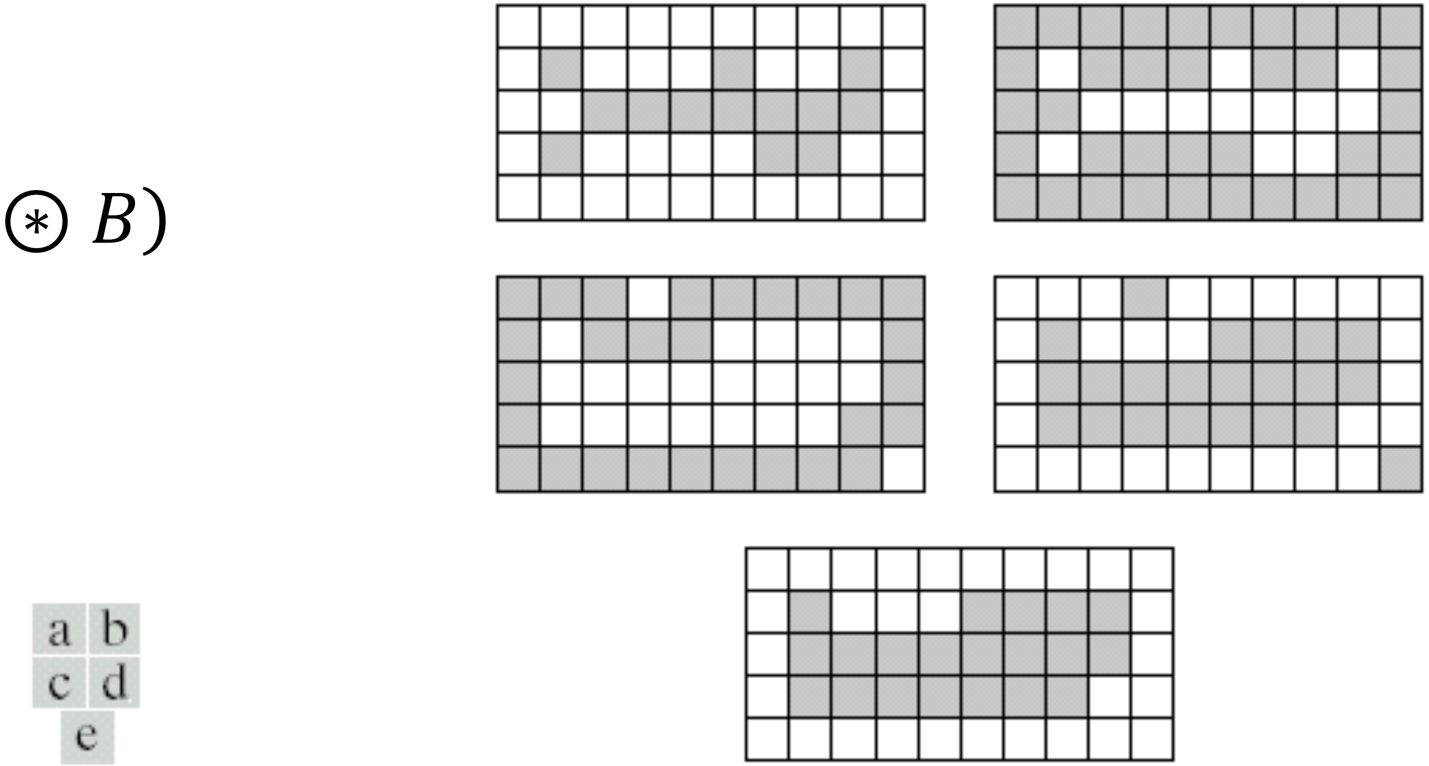
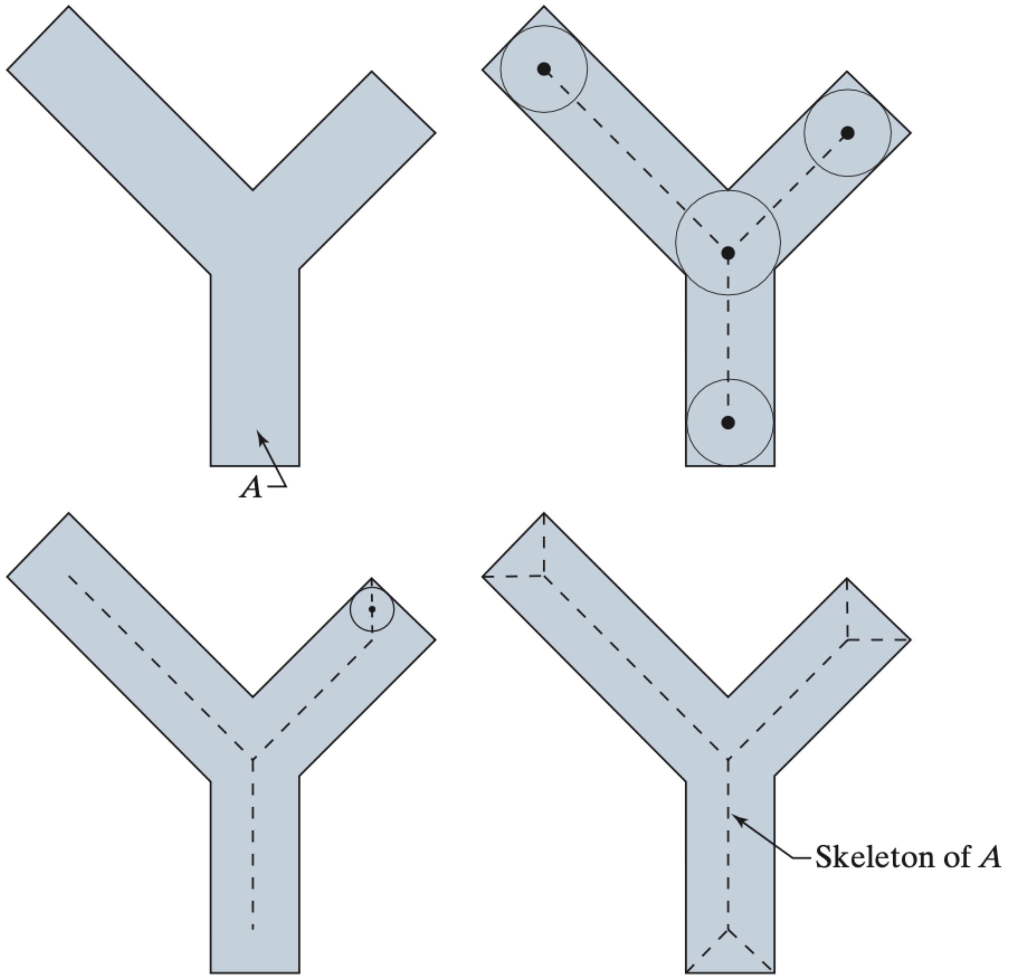


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

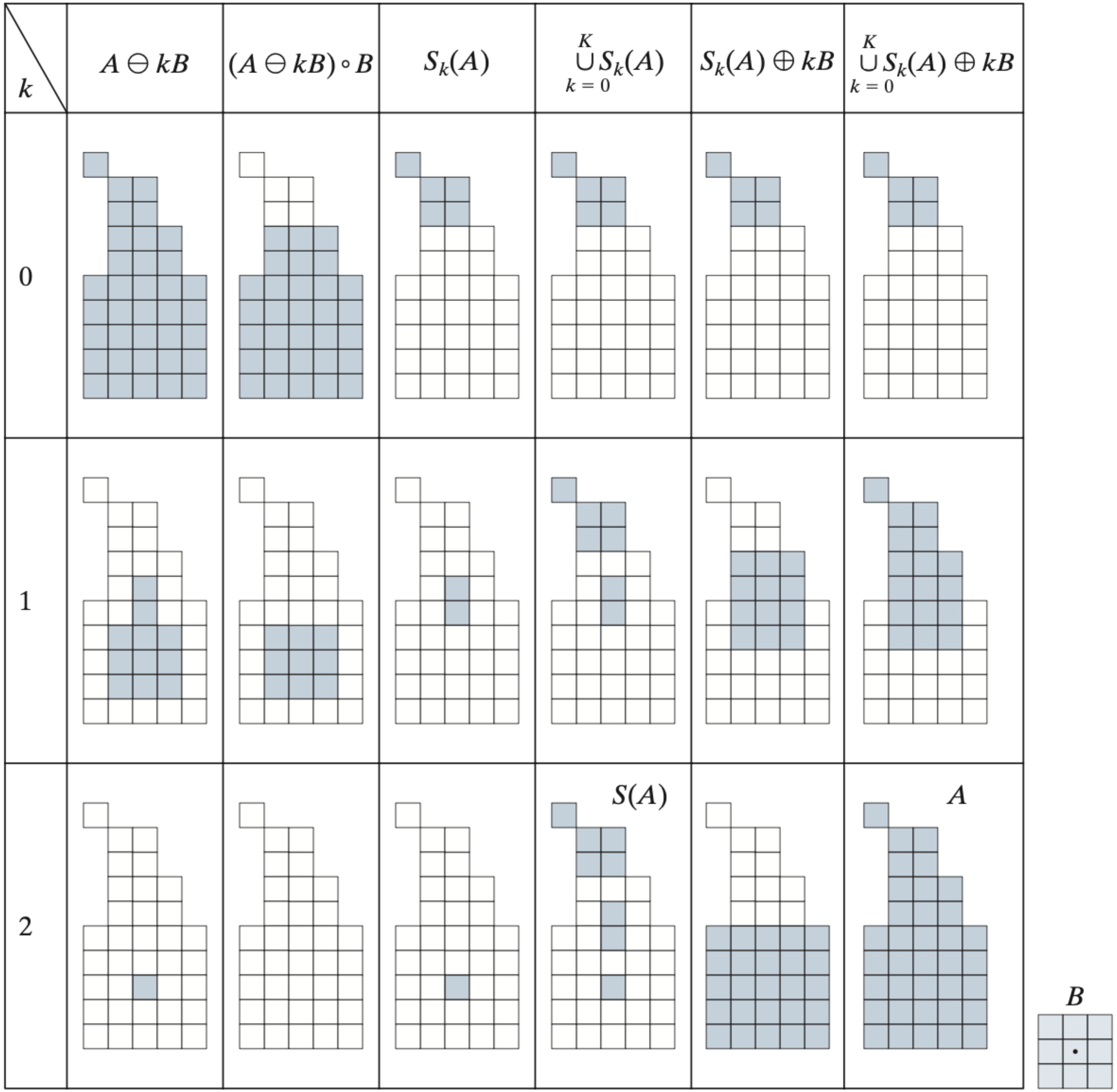
Skeletons



$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

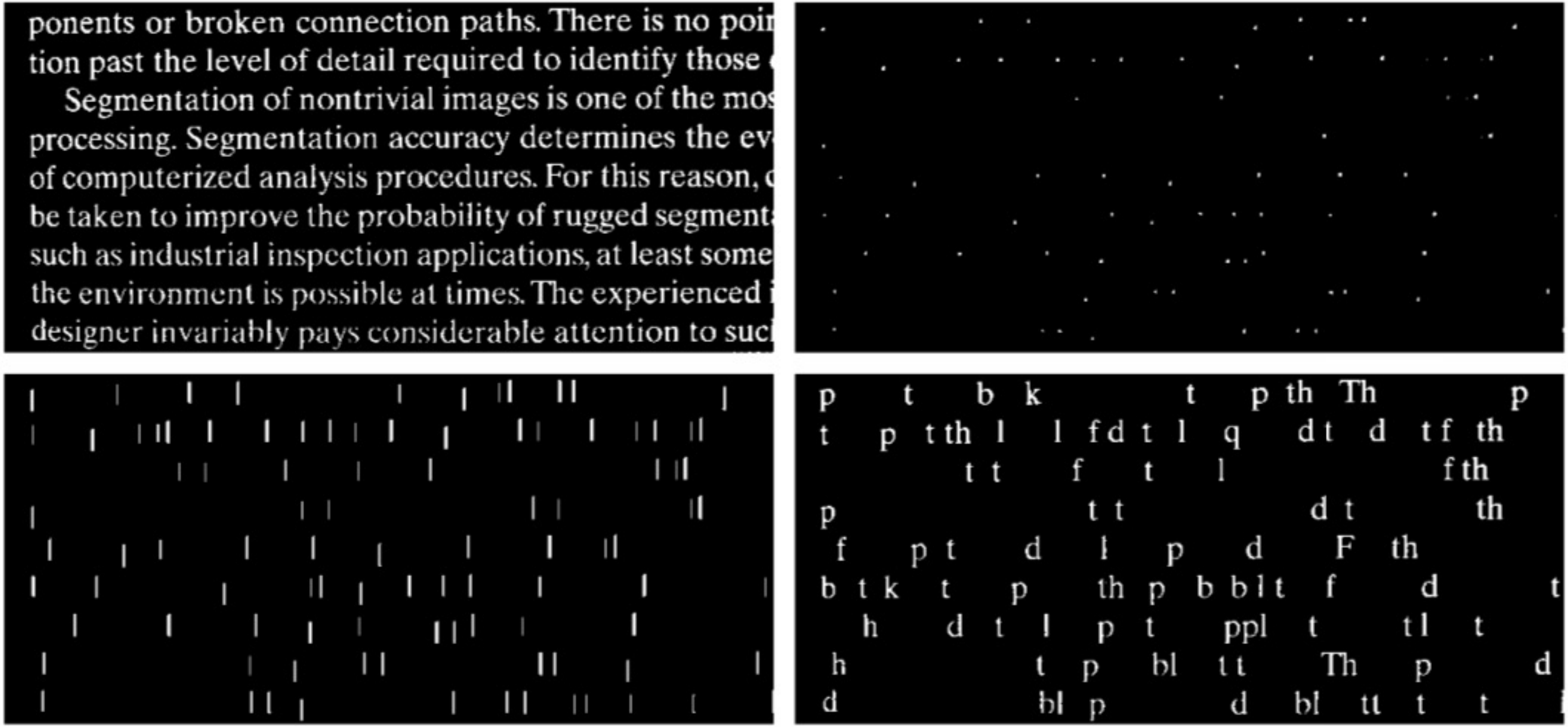
$$K = \max\{k \mid (A - kB) \neq \emptyset\}$$



$$\bigcup_{k=0}^{K}\left(S_k(A) \oplus kB\right)$$

$$\bigcup_{k=0}^{K}\Big(\big(\cdots\left(S_k(A) \oplus B\right)\big)\cdots \oplus B\Big)$$

Morphological Reconstruction Motivation



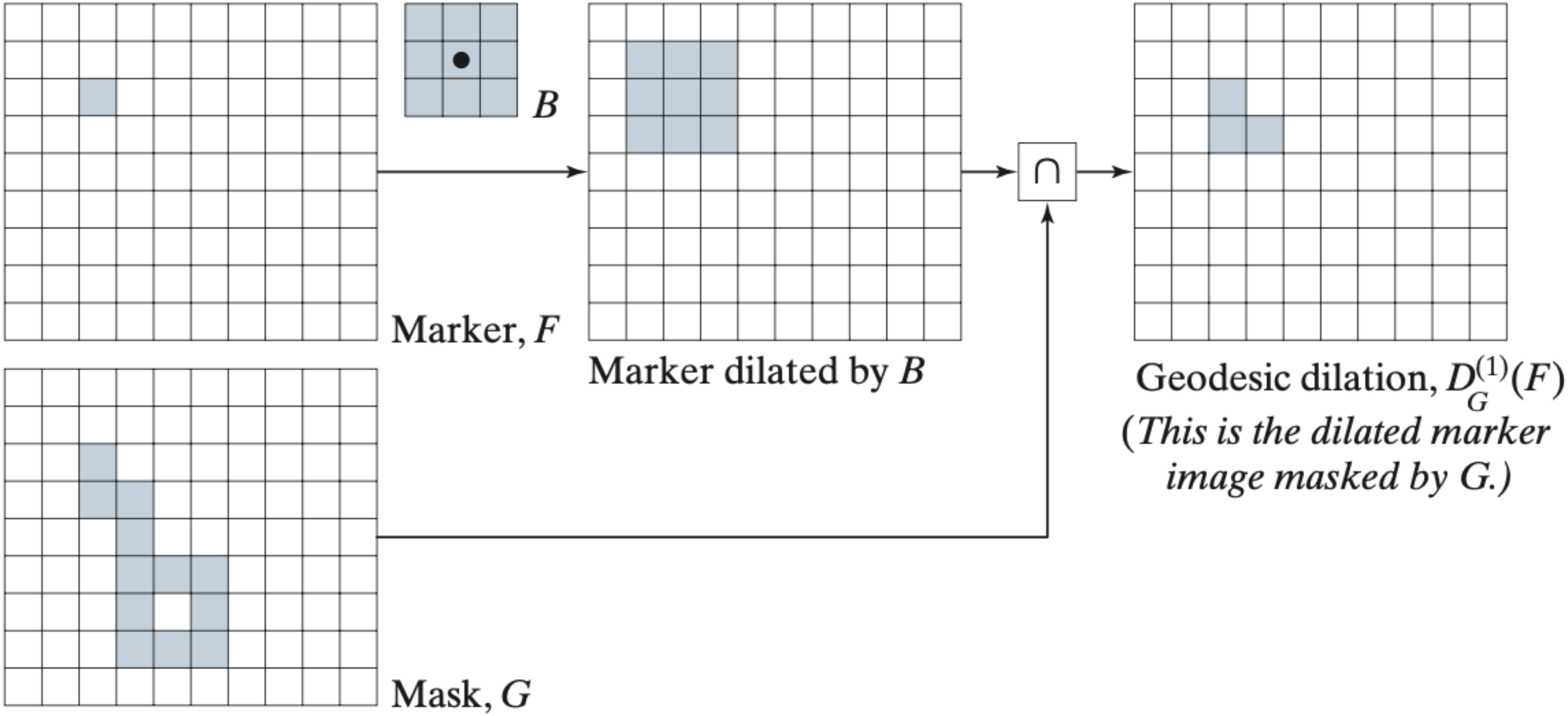
Morphological Reconstruction

- Notation
  - F: marker
  - G: mask ($F \subseteq G$)

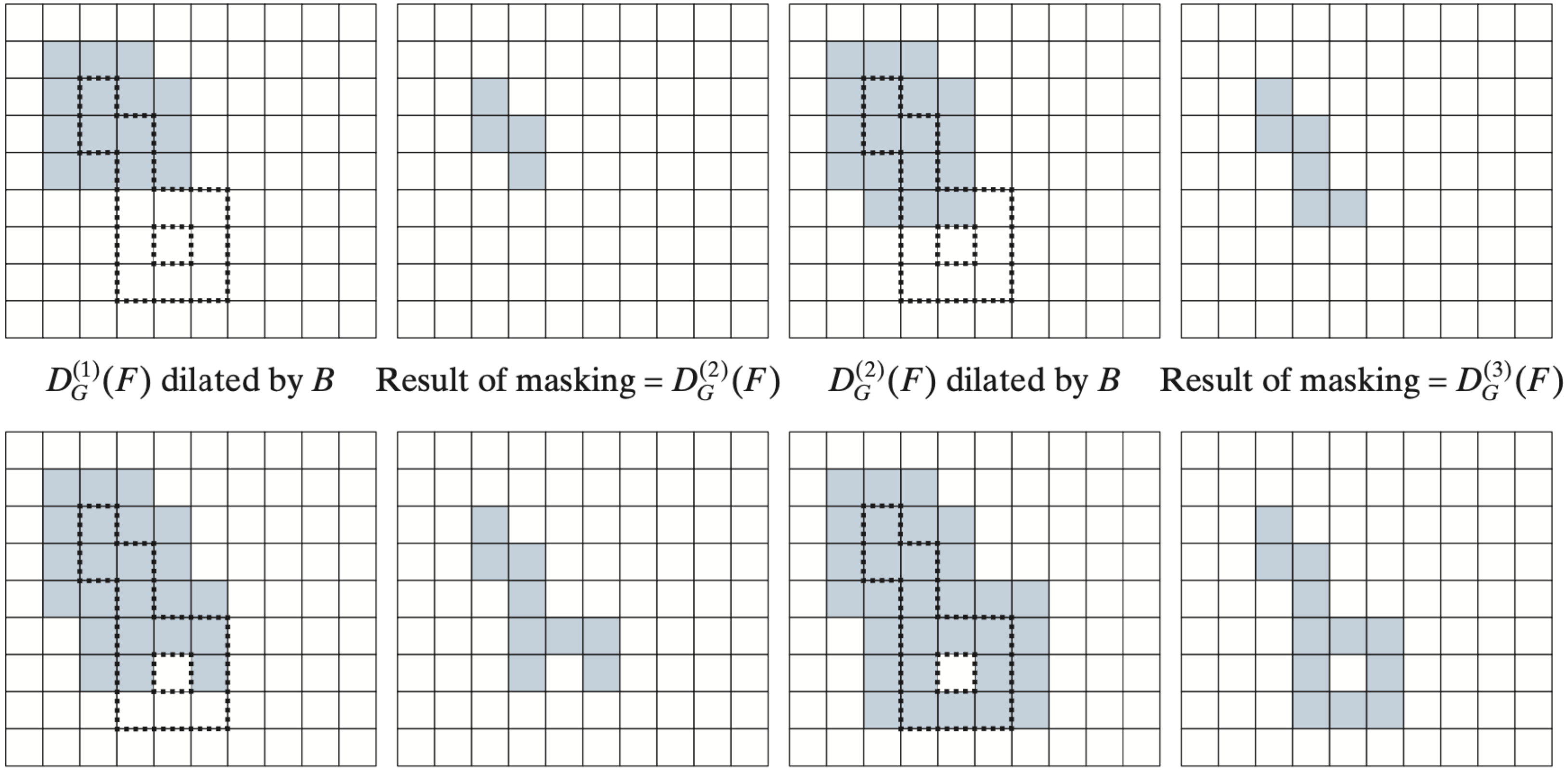
- Geodesic dilation
  - $D_G^{(1)}(F) = (F \oplus B) \cap G$
  - $D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$

- Geodesic erosion
  - $E_G^{(1)}(F) = (F \ominus B) \cup G$
  - $E_G^{(1)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$

Morphological Reconstruction



Morphological Reconstruction in Action

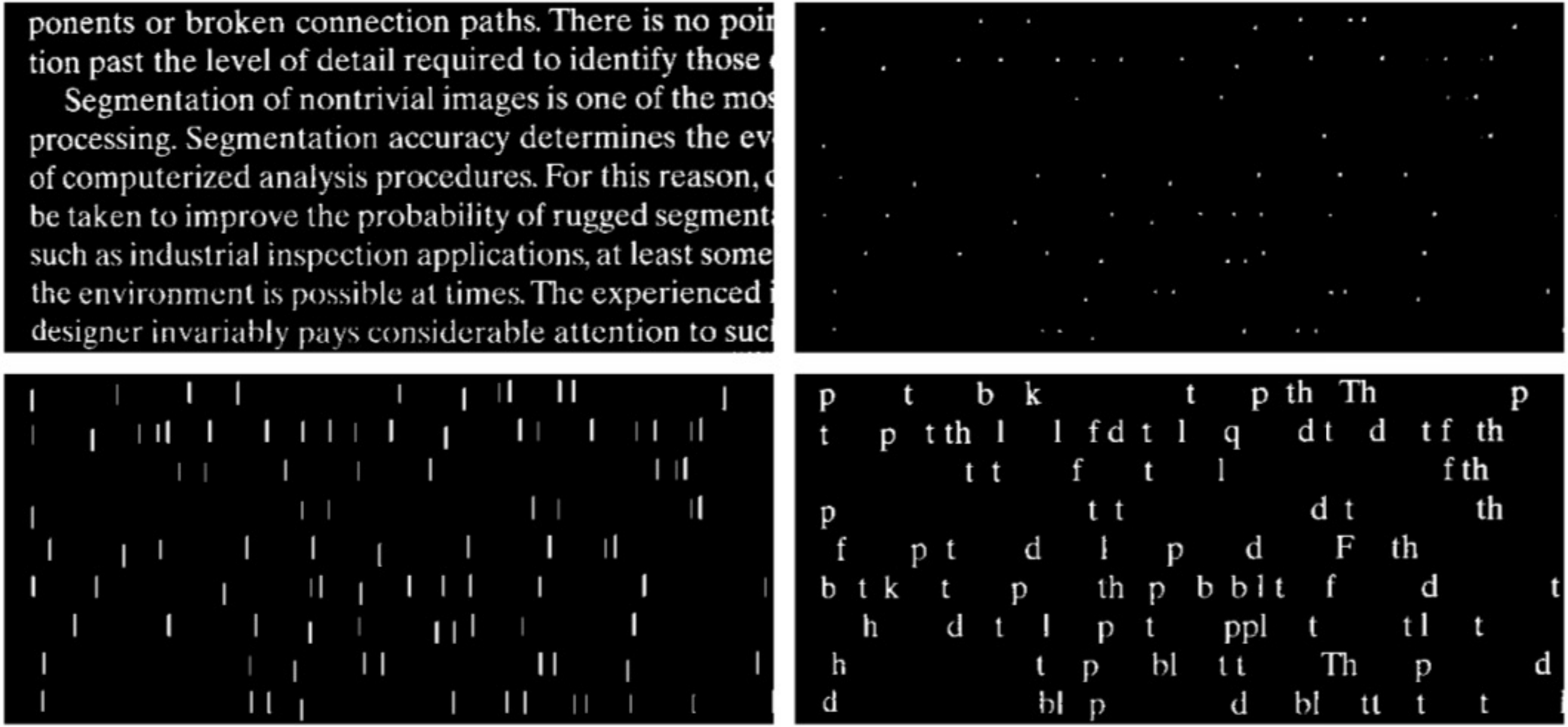


$D_G^{(1)}(F)$ dilated by B — Result of masking = $D_G^{(2)}(F)$ — $D_G^{(2)}(F)$ dilated by B — Result of masking = $D_G^{(3)}(F)$

$D_G^{(3)}(F)$ dilated by B — Result of masking = $D_G^{(4)}(F)$ — $D_G^{(4)}(F)$ dilated by B — Result of masking = $D_G^{(5)}(F)$

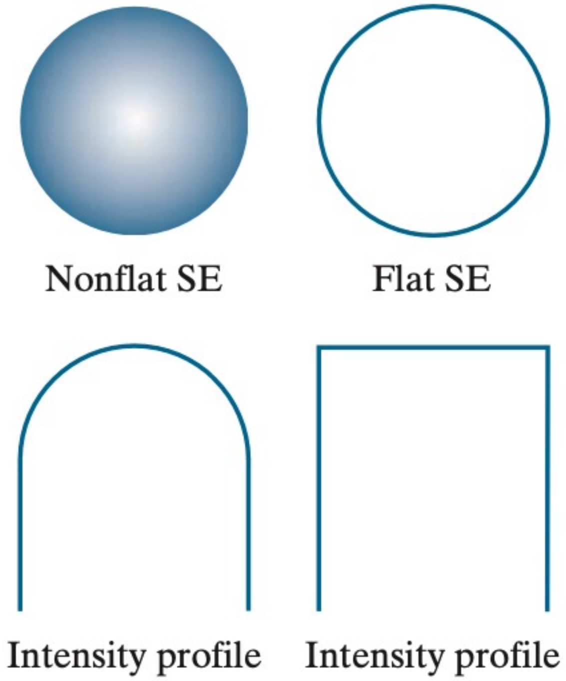
No changes after this point, so $R_G^D(F) = D_G^{(5)}(F)$

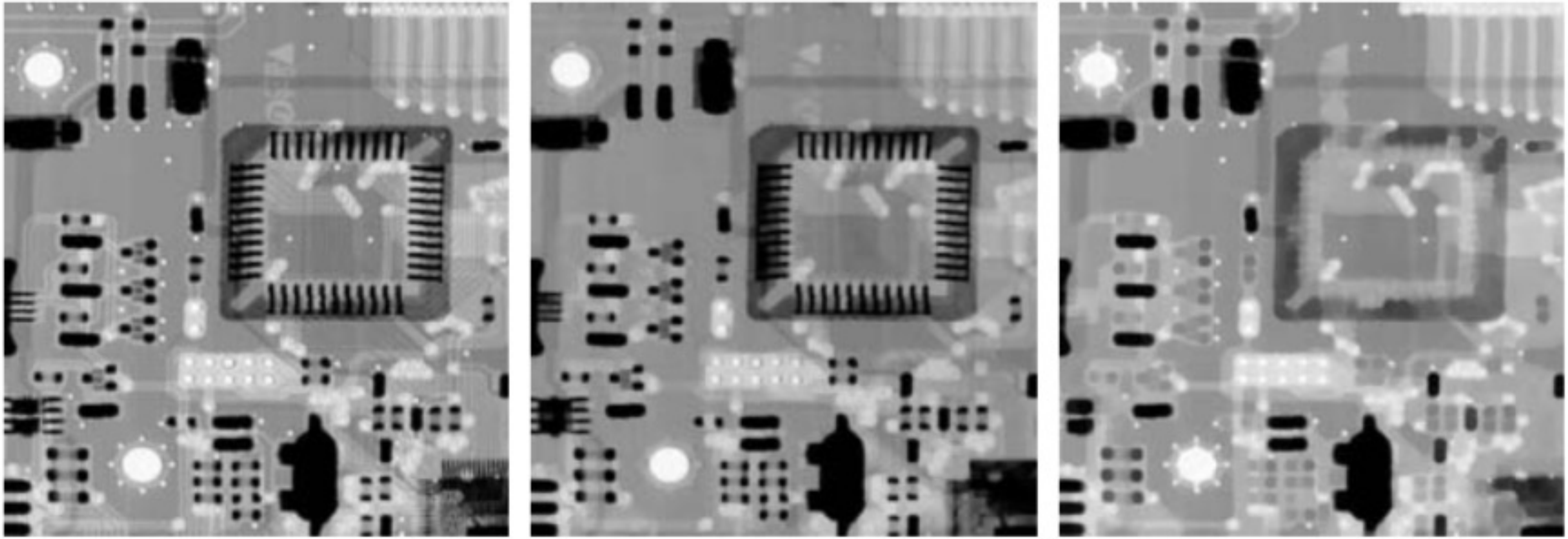
Example


ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

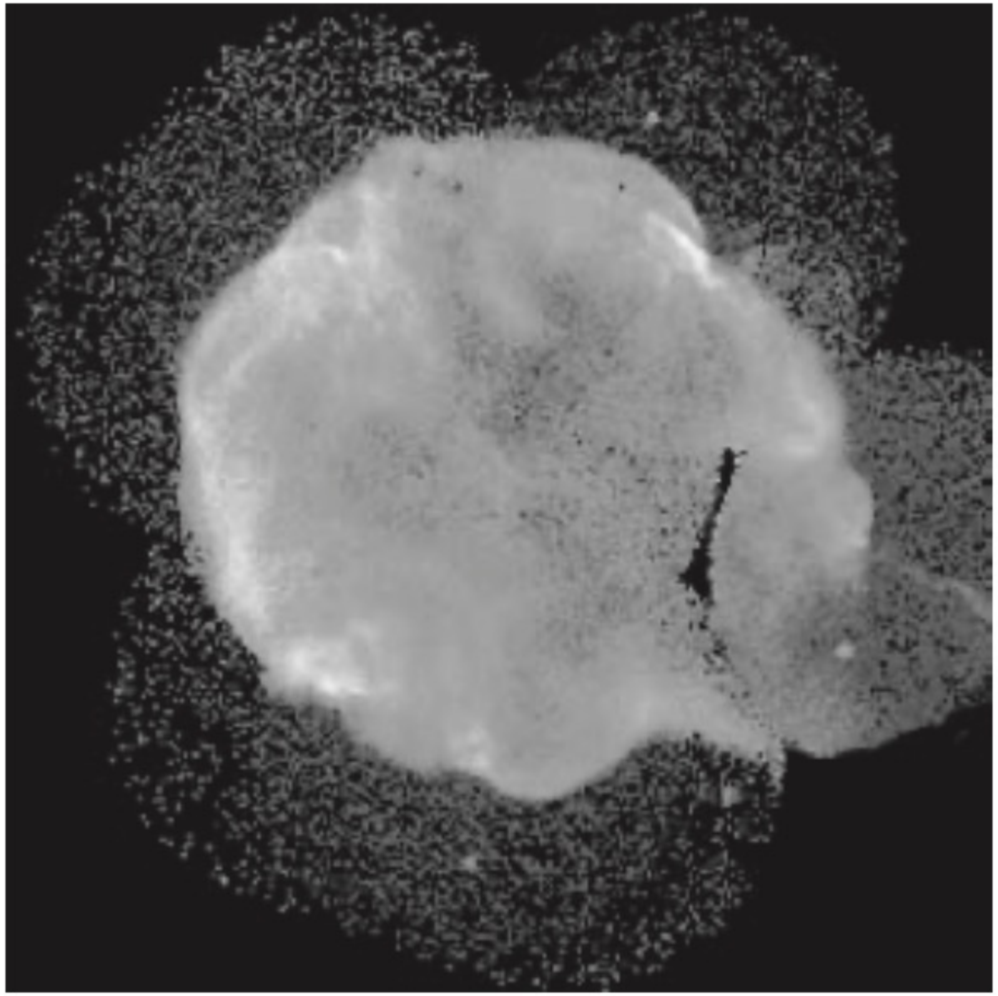
Gray-Scale Morphology

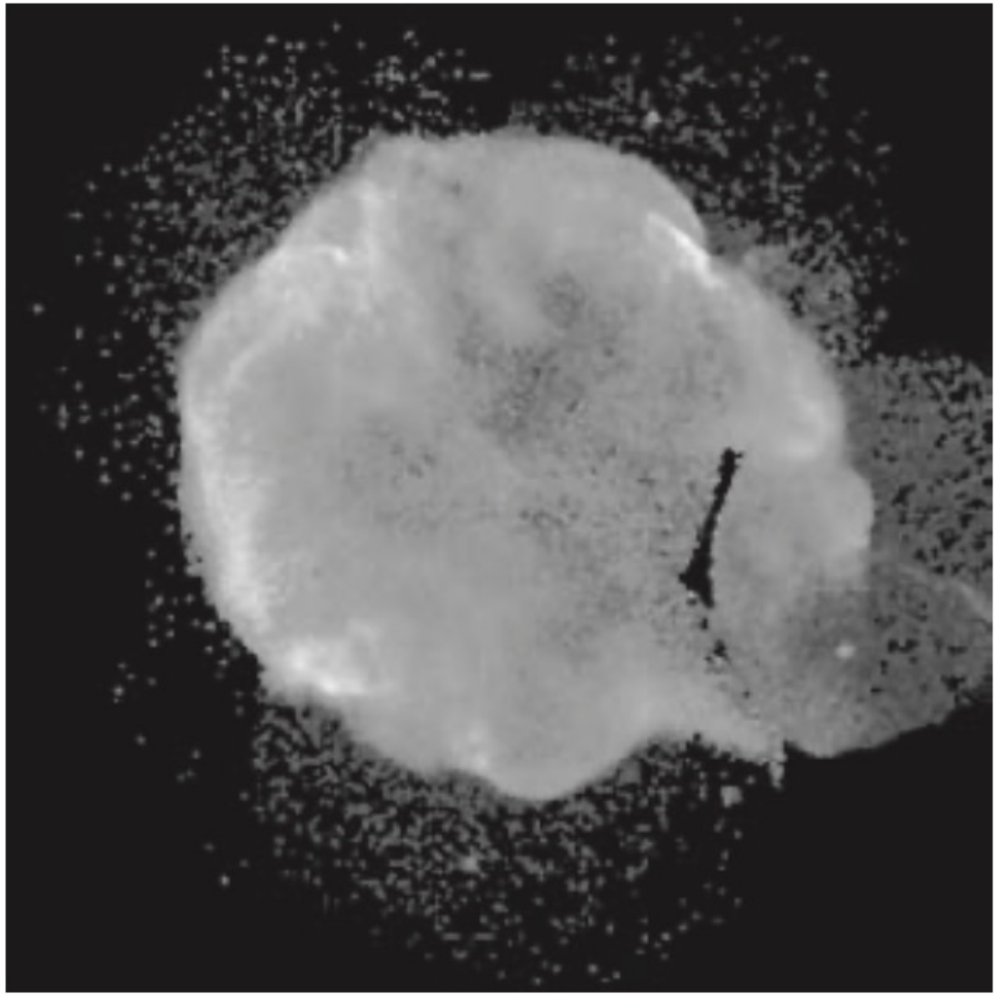
- Erosion
  - $[f \ominus b](x, y) = \min_{(s,t)\in b}[f(x+s, y+t)]$
- Dilation
  - $[f \ominus b](x, y) = \max_{(s,t)\in b}[f(x+s, y+t)]$
- Nonflat SE
  - $[f \ominus b](x, y) = \min_{(s,t)\in b}[f(x+s, y+t) - b_N(s, t)]$
  - $[f \ominus b](x, y) = \max_{(s,t)\in b}[f(x+s, y+t) - b_N(s, t)]$
- Duality
  - $(f \ominus b)^c = \left(f^c \oplus \hat{b}\right)$



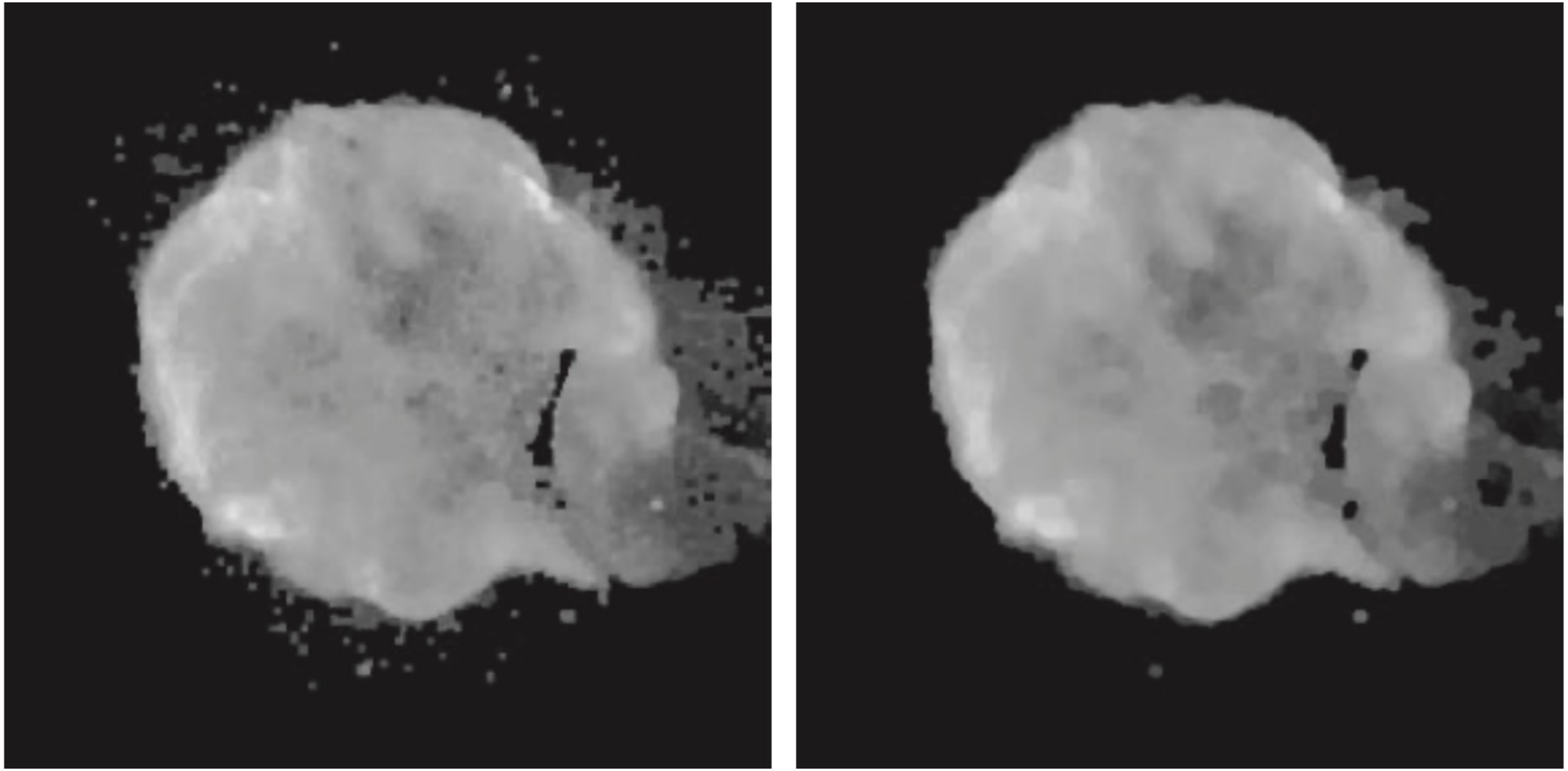


Morphological Smoothing



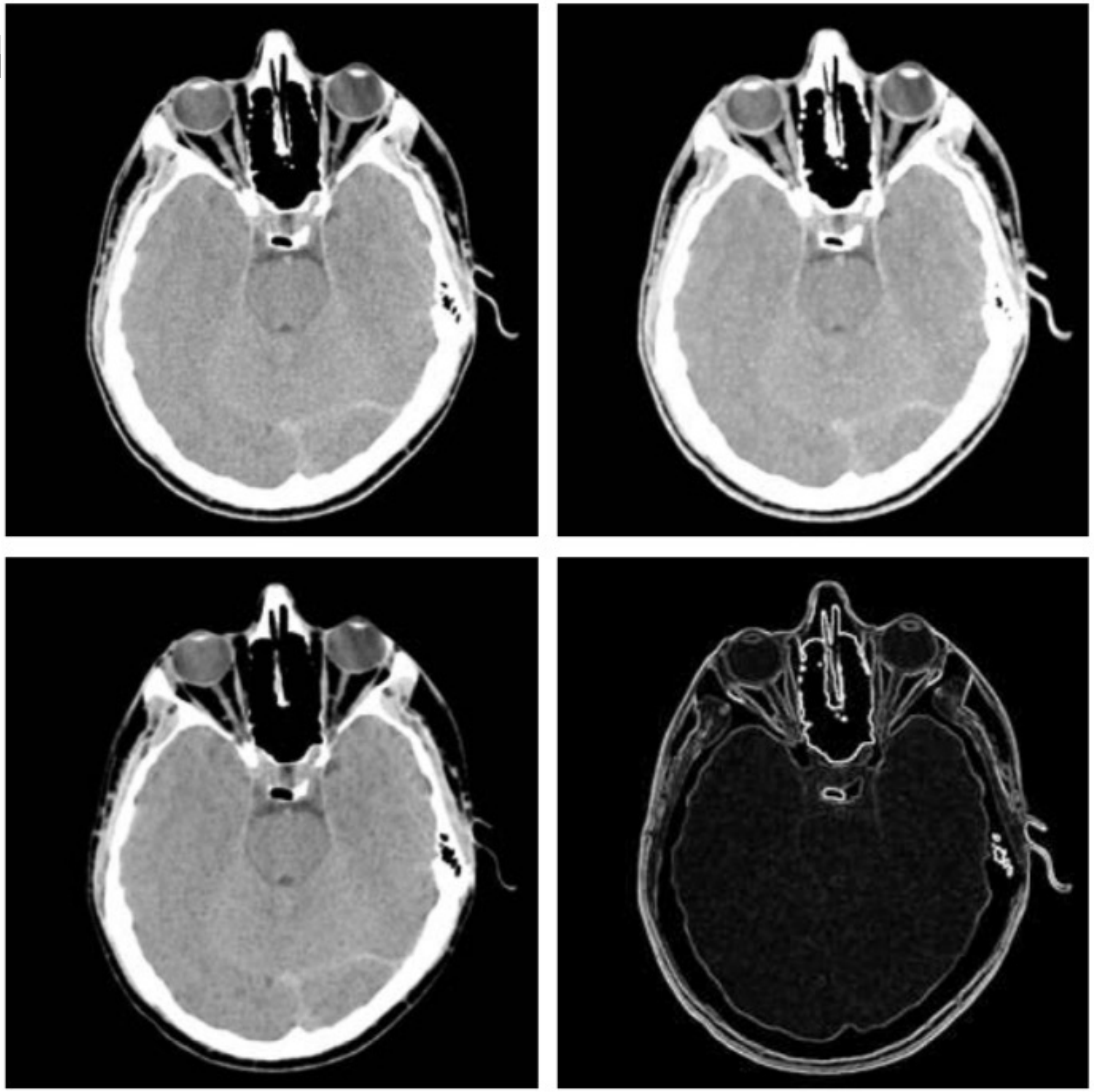


Morphological Smoothing



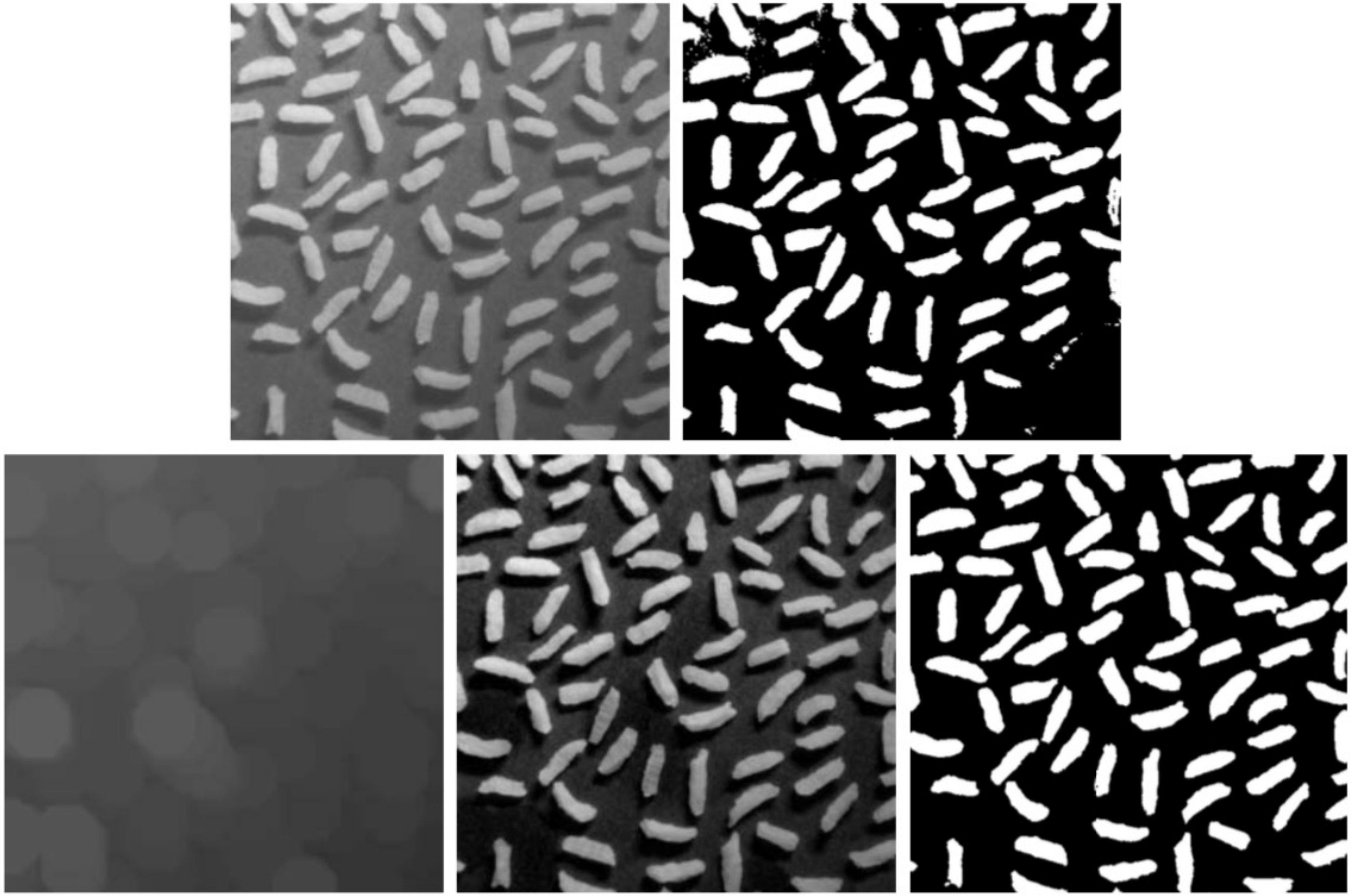
Morphological Gradient

- Difference between openning and closing
  - $g = (f \oplus b) - (f \ominus b)$

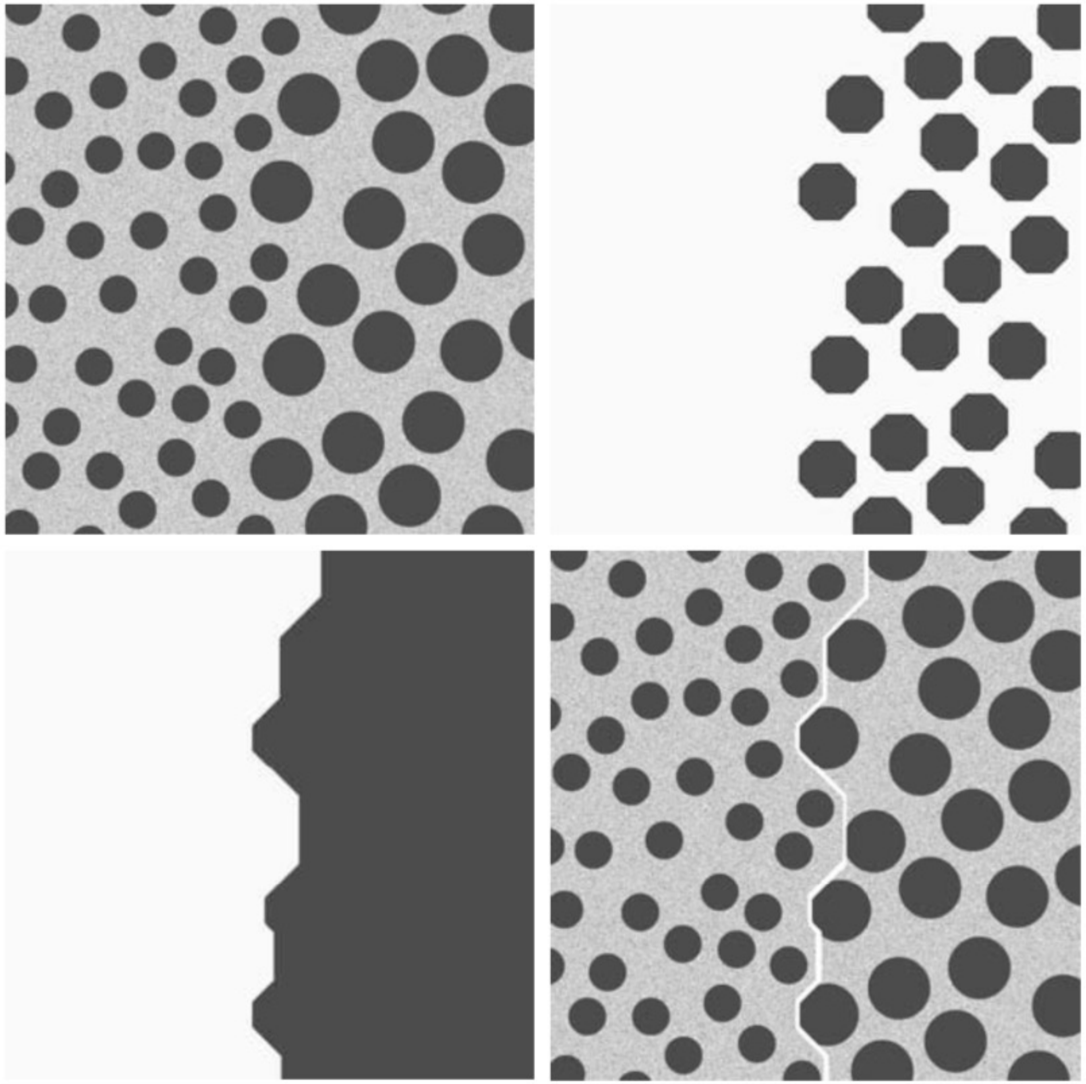


Top-Hat Transformation

- $T_{hat}(f) = f - (f \circ b)$



Textural Segmentation



a b
c d

FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

Gray-Scale Morphorogical Reconstruction

- Geodesic dilation of gray-scale images
  - $D_g^{(1)}(f) = (f \oplus b) \wedge g$ where $\wedge$ is pointwise minimum operator
  - $D_g^{(n)}(f) = D_g^{(1)}\left[D_g^{(n-1)}(f)\right]$
- Geodesic erosion
  - $E_g^{(1)}(f) = (f \ominus b) \vee g$ where $\vee$ is pointwise maximum operator
  - $E_g^{(1)}(f) = E_g^{(1)}\left[E_g^{(n-1)}(f)\right]$

Gray-scale Morphological Operation Application

- Opening by reconstruction using SE consisting of a horizontal line 71 pixels long
- Subtraction

scale Morphological Operation cation

- Opening by reconstruction using SE consisting of a vertical line 11 pixels long
- Dilation with the SE
- Top-hat by reconstruction