Mean Connectome statistical decision theoretical problem statement

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1. Sample Space

Adjacency matrices such that:

$$A = \{0,1\}^{n \times n}$$

2. Model

The adjacency matrix A is initialized to a zero matrix. Each element of adjacency matrix A, which represents an edge, is drawn from a Bernoulli distribution of a particular parameter:

$$a_{ij} \sim Bernoulli(\theta_{ij})$$

3. Action Space

There either exists an edge or not, so the action space is:

$$\{0,1\}^{n\times n}$$

4. Decision Rule

The decision rule is to calculate the average:

$$\widehat{P} = \frac{1}{n^2} \sum_{ij \in \mathbb{R}^{n \times n}} a_{ij}$$

5. Loss Function

The loss function is the squared difference between the estimate and the actual:

$$l = (P - \hat{P})^2 = (P - \frac{1}{n^2} \sum_{ij \in R^{n \times n}} a_{ij})^2$$

6. Risk Function

The expected value of the loss function: