Kristin Gunnarsdottir
March 3, 2015

Homework 4

The mean human connectome

1) Sample Space

$\Xi = \mathcal{A}_n = \{0,1\}^{nxn}$, where $\mathcal{A}_n$ is the set of nxn adjacency matrices.

2) Model

$\mathcal{P} = \{P_\theta : \theta \epsilon \Theta\}$

Using the Stochastic Block Model, $SBM_k^n(\vec{p}, \vec{\beta})$ where k is the number of regions of the brain and each vertex is a voxel.

The probability of an edge between nodes u and v is $P(u \sim v) = P_{uv}$, that is, each element a_{uv} in the adjacency matrix (representing an edge between nodes u and v) is $a_{uv} \sim Bernoulli(P_{uv})$.

3) Action space

The action space is the same as the parameter space, $(0,1)^{nxn} = \Theta$

4) Decision Rule

We want to use mean square error to minimize the loss.

$\hat{P} = \frac{1}{m}\sum_{i=1}^{m} A^{(i)} + \frac{\epsilon}{m^2} J_n$, where J_n is a matrix of ones of the same size as A, i.e.

$$\hat{p}_{uv} = \frac{1}{m}\sum_{i=1}^{m} a_{uv}^{(i)} + \frac{\epsilon}{m^2}$$

5) Loss function

$\ell : \mathcal{A}^m x \Theta \to \mathbb{R}_+$

$$\ell = \sum_{u,v} (\bar{a}_{uv} - \hat{p}_{uv})^2$$

where $\bar{a}_{uv}$ is the sample average and $\hat{p}_{uv}$ is the population average.

6) Risk function

The risk function is just the expected loss, $E[\ell]$