EN.580.694 ASSIGNMENT # 3

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STATISTICAL CONNECTOMICS

We were working on the statistical theory from the Bock paper in class (February 19th I believe) and came up with a sample space and model, but were unable to determine a potential model for Z and a potential structure of $\rho(z)$ and $\beta(z)$.

(1) **Sample Space**: $\{0,1\}^{n\times n} \times \{0,1\}^n \times (0,2\pi)^n$ with $X = \{0,1\}^{n\times n}$ is an adjacency matrix representing the neural network, $Y = \{0,1\}^n$ is the vector of latent labels for the n neurons (either exhibitory or inhibitory), and $Z = (0,2\pi)^n$ is the preferred angle (or tuning) for each neuron. After class, Jason proposed modifying the sample space so that $Z = [-\pi,\pi]^{n^3}$. This is the model I am using.

(2) **Model**: The proposed model to use was a Stochastic Block Model on n vertices and with 2 blocks with parameters $\vec{p} \in \Delta_2$ and $\vec{\beta} \in (0,1)^{2\times 2}$.

In our model, we have $(X,Y,Z) \sim F_{XYZ}$ with XY modeled by the Stochastic blockmodel mentioned above. However, there is no mention of how Z contributes to the model. Note that $(X,Y,Z) \sim F_{XYZ} = F_{XY}F_{Z|XY}$. Essentially, the question asked in the Bock paper is with regard to how the tuning preferences are distributed given a particular model, so we are really interested in $F_{Z|XY}$. Now, we have a stochastic block model for F_{XY} and just need to determine $F_{Z|XY}$. We are interested in the difference of the angles between one exhibitory neuron and another exhibitory neuron when they are connected to the same inhibitory neuron. So, we can consider the distribution of the differences between pairs of exhibitory neurons which are adjacent to the same inhibitory neurons. In this case,

$$Z_{uvw} = \begin{cases} z_u - z_v & \text{if } u \sim w; v \sim w; u,v \in E; \text{ and } w \in V \\ 0 & \text{otherwise} \end{cases}$$

where $E = \{\text{exhibitory neurons}\}$ and $V = \{\text{inhibitory neurons}\}$. The goal would then be to check if $F_{Z_{uvw}|XY}$ is uniformly distributed or not.