MEAN CONNECTOME

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1. Posed Statistical Decision Theoretic

This is an attempt to follow my notes from class and replicate what we discussed.

1.1. **Sample Space.** $\Omega = \mathcal{A}_n = \{0,1\}^{n\times n}$, the set of adjacency matrices $\mathcal{A}_n$ on graphs $\mathcal{G}_n = (V, E, Y)$.

1.2. **Model.** $P = SBM_n^k(\rho, \beta)$. We posit the probability of an edge between u, v is given by $P(u \sim v) = P_{uv}$ and the distribution of edges between any pair of vertices u, v is $a_{u,v} \sim Bern(P_{uv})$

1.3. **Action Space.** $A = (0,1)^n \subseteq \Theta$, where Θ is the parameter space. This is desirable since this is a parameter estimation problem.

1.4. **Decision Rule Class.** The decision rule is to minimize the loss defined by the loss function below.

1.5. **Loss Function.** $l : \Theta \times A \to \mathbb{R}_{>0}$ where $l(\theta, \delta(\Omega))$ is the cost of action δ under parameter θ. More explicitly, $l(\theta, \delta(\Omega)) = \prod_{uv} \hat{P_{uv}}^{a_{uv}}(1 - \hat{P_{uv}}^{1-a_{uv}})$, where $\hat{P_{uv}} = \frac{1}{m}\sum^m a_{uv} + \frac{\epsilon}{m^2}$ such that $\frac{\epsilon}{m^2} \leftarrow 0$ asymptotically. Since $P_{uv} \in [0,1]$, you have issues at the boundaries when $P_{uv} = 0$ or 1, as the whole estimation becomes zero probability. Thus the use of $\hat{P}$ is a necessary smoothing.

1.6. **Risk Function.** $R(\theta, \delta) = E_p(L(\theta, \delta(\Omega)))$. The risk function is taken as the expected loss.

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