EN.580.694 ASSIGNMENT # 3

HEATHER PATSOLIC
STATISTICAL CONNECTOMICS

Write down a statistic decision theoretic for finding the mean human connectome.

(1) **Sample Space**: The set of adjacency matrices: $\mathcal{A} = \{0,1\}^{n \times n}$.
(2) **Model**: Here, $A \in \mathcal{A}$ will have a Bernoulli distribution matrix $\mathbb{P} \in [0,1]^{n \times n}$. For each element a_{uv} of the matrix A, representing an edge between neurons u and v, $a_{uv} \sim \text{Bernoulli}(p_{uv})$; that is, the probability that there is a synapse between nodes u and v is p_{uv}. Note: The expected value of A is $\mathbb{P}$.
(3) **Action Space**: The action space here is the same as the parameter space, $[0,1]^{n \times n}$.
(4) **Decision Rule**: The decision rule is $\hat{\mathbb{P}} = \dfrac{1}{m} \displaystyle\sum_{i=1}^{m} A^{(i)} + \dfrac{\epsilon}{m^2}(ee^T)$ where $0 < \epsilon \in \mathbb{R}$ and e is the vector of all 1's. So, $\hat{p_{uv}} = \dfrac{1}{m} \displaystyle\sum_{i=1}^{m} a_{uv}^{(i)} + \dfrac{\epsilon}{m^2}$.
(5) **Loss Function**: We can use the maximum likelihood estimator as the loss function $\ell : \mathcal{A}^m \times [0,1]^{n \times n} \leftarrow \mathbb{R}^+$ where we consider

$$\prod_{u,v} (\hat{p}_{uv})^{a_{uv}} \left(1 - \hat{p}_{uv}\right)^{1 - a_{uv}}.$$

Note: We could also use the mean square error as our loss function if we are looking for simplicity.
(6) **Risk Functional**: Our risk function will be the expected loss: $\mathbb{E}[\ell]$.

Date: March 2, 2015
Due: March 3, 2015.