MATH 303 ODE AND DYNAMICAL SYSTEMS PROJECTS — SPRING 2025

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1. Projects

Below are some suggestions for project topics in MATH 303. For each of the projects I give a title, short description, and a reference to a book where you can find more information. These references should be considered as starting points and you are encouraged to explore more. If you have an idea for another topic related to ordinary differential equations or dynamical systems that is not included in this list do not hesitate to discuss it with me.

1. Hamiltonian Systems: Explore the elegant framework of Hamiltonian mechanics to understand the dynamics of energy-conserving systems. This project connects classical mechanics and modern physics, using phase space geometry to reveal deep insights into motion and stability. [3, Proj. 5.D] and [4, Ch. 2]

2. Index of a Vector Field and Dynamical Features: Learn how the index of a vector field at an isolated singularity reveals key dynamical properties of planar systems. Use topological tools like the Poincaré–Hopf theorem to analyze flow structure, classify critical points, and understand global behavior from local data. [6, Sec. 6.8]

3. Spherically Symmetric Solutions to Schrödinger's Equation for the Hydrogen Atom: Use differential equations to study the hydrogen atom in quantum mechanics. Learn how series solutions yield the allowed energy levels and reveal the mathematical beauty underlying atomic structure. [3, Proj. 8.B]

4. Hermite Polynomials and the Harmonic Oscillator: Dive into the quantum harmonic oscillator and see how Hermite polynomials naturally emerge from Schrödinger's equation. This project bridges special functions, orthogonality, and quantum physics. [3, Proj. 11.A]

5. Solitons and Korteweg-de Vries (KdV) Equation: Explore the KdV equation and its soliton solutions—stable, solitary waves that behave like particles. Learn how nonlinear and dispersive effects balance to produce remarkable wave phenomena. [3, Proj. 12.A]

6. Hysteresis in the Driven Pendulum and Josephson Junction: Examine nonlinear systems with multiple stable states and memory effects. Discover how periodic forcing in pendulums and superconducting circuits leads to hysteresis and complex dynamical behavior. [6, Sec. 8.5]

7. Liénard Systems: Delve into a class of second-order nonlinear systems that model real-world oscillators. Analyze limit cycles and relaxation oscillations, with applications from electronics to biology. [6, Sec. 7.4]

8. Synchronization in the Kuramoto model: Explore how populations of coupled oscillators spontaneously synchronize. This project sheds light on the mathematics of collective behavior, from fireflies to power grids. [5, Sec. 1-4]

9. Circle Maps: Investigate simple yet rich models of periodically forced systems. Circle maps reveal bifurcations, chaos, and the devil's staircase—a window into complex dynamics from simple rules. [1, Sec. 1.14]

10. Hénon Strange Attractor: Explore one of the most iconic examples of chaos in discrete dynamical systems. The Hénon map illustrates sensitivity to initial conditions and the emergence of strange attractors. [6, Sec. 12.2]

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11. Chaos and the Lorenz Attractor: Investigate one of the most iconic examples of deterministic chaos. Study the Lorenz system—a simplified model of atmospheric convection—and explore sensitive dependence on initial conditions, strange attractors, and the geometry of chaos in three dimensions. [6, Sec. 9.1–9.3]

12. The Logistic Map and the Period-Doubling Route to Chaos: Explore how a simple nonlinear recurrence, the logistic map, exhibits period doubling and chaos. Investigate the Feigenbaum constants, self-similarity, and how renormalization explains universal features in the transition to chaos. [6, Sec. 10.3–10.6]

13. External Synchronization of a van der Pol Oscillator: Study how external periodic forcing can entrain a nonlinear oscillator. The van der Pol equation provides a classic model for understanding resonance, synchronization, and entrainment zones. [4, Sec. 4.4]

14. Diffusion and Epidemics on Networks: Combine dynamical systems with network theory to model how diseases or information spread across social or biological networks. Investigate how structure influences outcomes and thresholds for outbreaks. [4, Sec. 5.3]

15. Spread of Staph Infections in Hospitals: Model how contagious bacteria spread in hospital settings using systems of differential equations. Possible analyze how different treatment policies and hygiene practices can impact infection rates, and explore strategies for containment and prevention. [3, Proj. 5.B, Proj. 12.E]

2. Written Report Structure

In each project you are supposed to work on understanding a topic and presenting it (orally and in writing) in a way that makes it clear to your fellow students. I expect that each written report will consist of the Introduction, the main part, and the Conclusions. A report of about 6-8 pages (roughly, 4000-5000 words) is sufficient. I strongly suggest referring to a book about mathematical writing such as [2] or [8] or, at least, the much shorter [7].

In the Introduction, you should describe the topic and try to motivate the reader. You can achieve this in many ways. For example, you can write about relevant applications, or about the topic's history, or about its significance within mathematics, or about its educational value as an interesting example of a more general but complicated theory. This section is also where you can briefly discuss what other work exists on this topic. However, you do not need to go into much detail about other work.

The main part may consist of one or more sections depending on the specifics of your project. The main thing I expect to see in this part is evidence of understanding of the topic and as much interesting information that fits in around 10 pages. Here you should try to explain the main results related to the topic, explore connections to other topics in the course or in applications. Understanding can be made evident by providing proofs or your own numerical computations (depending on the topic) and by ensuring that information is presented in a logically coherent way.

In the Conclusions, you should summarize the main points you presented in your report and discuss further extensions or lessons learned.

3. Rubric for Written Report

Introduction (15 points). The introduction will be evaluated according to how well it: defines the topic; motivates its study; explains the topic's importance; gives a clear overview of what the reader should expect in the rest of the report by providing an outline of what will be discussed; discusses, if relevant, the historical and scientific context of the topic. The role of the introduction is to convince the reader that it is worth spending their time to read the rest of the work.

Main Part (40 points). The main part will be evaluated according to how well it: describes the methods, or arguments, or numerical computations, used; presents the relevant results in a clear way; structures the given information in a logically coherent way; adopts a reflective and thoughtful approach to the topic; shows clear evidence of understanding; achieves some depth in the provided information.

Conclusions (10 points). This section will be evaluated according to how well it: summarizes the report; discusses possible extensions or variations of the topic.

Writing Style (10 points). The writing style of the report will be evaluated according to how well it: presents information in a clear and easy to read way; uses appropriate mathematical language and writing conventions. Utmost attention should be paid to spelling and grammar — especially to spelling mistakes that can be easily caught by spell checking.

Formatting (15 points). The report should have a separate title page; table of contents; clear section and subsection headings; numbered figures and tables with captions; references to numbered figures, tables, and equations; properly typeset mathematics formulas and symbols; consistently formatted literature citations. Note that all these are easy in LaTeX. You can check the Overleaf documentation for how these can be achieved.

Other (10 points). The points for the five aspects of the report mentioned above sum up to 90 points. This is the maximum score for a technically impeccable but otherwise perfunctory report. For additional points, you should go beyond that level and show that you have put some serious work into thinking about the report's topic, for example, by engaging at a deep level with the relevant literature and by exploring dimensions of the topic that I have not provided. Feel free to use GenAI to explore such dimensions but do not depend on it to write the report for you — I plan to check your understanding of the topic.

References

[1] R. L. Devaney. *An Introduction to Chaotic Dynamical Systems*. Benjamin-Cummings, 1986.

[2] S. G. Krantz. *A Primer of Mathematical Writing*. 2nd ed. American Mathematical Society, 2017.

[3] R. K. Nagle, E. B. Saff, and A. D. Snider. *Fundamentals of Differential Equations and Boundary Value Problems*. 7th ed. Pearson, 2017.

[4] D. D. Nolte. *Introduction to Modern Dynamics*. Oxford University Press, 2015.

[5] S. H. Strogatz. "From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators". In: *Physica D: Nonlinear Phenomena* 143.1–4 (2000), pp. 1–20. DOI: 10.1016/S0167-2789(00)00094-4.

[6] S. H. Strogatz. *Nonlinear Dynamics and Chaos*. 2nd ed. CRC Press, 2015.

[7] F. E. Su. "Some Guidelines for Good Mathematical Writing". In: *MAA Focus* 35.4 (2015), pp. 20–22.

[8] F. Vivaldi. *Mathematical Writing*. Springer Undergraduate Mathematics Series. Springer, 2014. DOI: 10.1007/978-1-4471-6527-9.

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