# Problem Set #2 - DATASCI W241

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## 1. FE, exercise 3.6

```
setwd("/Users/ceccarelli/MIDS/DATASCI_W241/Async Material and Sample Files/Chapter 3")
#clear variables
rm( list = ls() )
#read in tabular data
data <- read.csv("Clingingsmith_et_al_QJE_2009dta.csv", sep=",", header = TRUE)
#create shorthand reference
d<-data
##replicate the table on 3.2 in the book
prop.table(table(d$views,d$success),2)*100</pre>
```

```
##
##
                 0
##
    -12 0.0000000 0.1960784
    -9 0.2232143 0.0000000
##
##
    -8 0.0000000 0.1960784
##
    -6 0.4464286 0.1960784
##
    -5
        0.0000000 0.1960784
##
    -4 0.4464286 0.5882353
    -3 0.0000000 0.1960784
##
    -2 1.1160714 0.9803922
##
    -1
       1.5625000 2.7450980
##
    0 27.2321429 18.6274510
##
       18.3035714 13.1372549
##
       24.3303571 25.2941176
##
    3
        8.4821429 10.9803922
       5.8035714 9.6078431
##
##
    5
       3.3482143 3.9215686
        3.7946429 7.2549020
##
##
    7
        2.2321429 2.5490196
##
       0.8928571 1.3725490
        0.2232143 0.7843137
##
    9
##
    10
       0.4464286 0.0000000
##
    11
         0.6696429 0.1960784
         0.4464286 0.9803922
```

```
#Create two vectors to sample from based on the data
Y <- d$views #outcome
Z <- d$success #treatment

# number of RI iterations
numiter <- 10000
set.seed(1234567) # set random number seed (so that results can be replicated)</pre>
```

```
#compute ATE prior to randomization
denom <- var(Z)
ate <- cov(Y,Z)/denom
#iterate
tauRI <- rep(NA, numiter)</pre>
for (i in 1:numiter) {
Zri <- sample(Z)</pre>
tauRI[i] <- cov(Y,Zri)/denom</pre>
if (i %% round(numiter/10) == 0) cat("Iteration",i,"of",numiter,"\n")
}
## Iteration 1000 of 10000
## Iteration 2000 of 10000
## Iteration 3000 of 10000
## Iteration 4000 of 10000
## Iteration 5000 of 10000
## Iteration 6000 of 10000
## Iteration 7000 of 10000
## Iteration 8000 of 10000
## Iteration 9000 of 10000
## Iteration 10000 of 10000
```

How many of the simulated random assignments generate an ATE that is at least as large as the actual estimate of the ATE?

```
nrow(subset(as.data.frame(tauRI),tauRI>=ate))
```

## [1] 26

What is the implied one-tailed p-value?

```
pvalue <- round(nrow(subset(as.data.frame(tauRI),tauRI>=ate))/numiter,3)
pvalue
```

## [1] 0.003

How many of the simulated random assignments generate an ATE that is at least as large in absolute value as the actual estimate of the ATE?

```
#wrap tauRI in abs function
nrow(subset(as.data.frame(tauRI),abs(tauRI)>=ate))
```

## [1] 42

What is the implied one-tailed p-value?

```
#wrap tauRI in abs function
pvalue.twotailed <- round(nrow(subset(as.data.frame(tauRI),abs(tauRI)>=ate))/numiter,3)
pvalue.twotailed
## [1] 0.004
2. FE, exercise 3.8
setwd("/Users/ceccarelli/MIDS/DATASCI_W241/Async Material and Sample Files/Chapter 3")
#clear variables
rm( list = ls() )
require("foreign")
## Loading required package: foreign
#read in tabular data
data <- read.dta("Titiunik_WorkingPaper_2010.csv.dta")</pre>
#create shorthand reference
t<-data
  a. For each state, estimate the effect of having a two-year term on the number of bills introduced?
#Create two vectors for Texas
t.Y <- unlist(subset(t,t$texas0_arkansas1==0)[2]) #outcome</pre>
t.Z <- unlist(subset(t,t$texas0_arkansas1==0)[1])#treatment
#compute ATE for Texas
t.denom <- var(t.Z)
t.ate <- cov(t.Y,t.Z)/t.denom
cat("Texas ATE = ", t.ate)
## Texas ATE = -16.74167
##done another way
\#mean(unlist(subset(t,t\$texas0\_arkansas1==0\ \&\ t\$term2year==0)[2]))-mean(unlist(subset(t,t\$texas0\_arkansas1==0\ \&\ t\$term2year==0)[2]))
#Create two vectors for Texas
a.Y <- unlist(subset(t,t$texas0_arkansas1==1)[2]) #outcome
a.Z <- unlist(subset(t,t$texas0_arkansas1==1)[1])#treatment
#compute ATE for Arkansas
a.denom <- <pre>var(a.Z)
a.ate <- cov(a.Y,a.Z)/a.denom
```

## Arkansas ATE = -10.09477

cat("Arkansas ATE = ", a.ate)

b. For each state, estimate the standard error of the ATE?

```
t.obs.1 <- unlist(subset(t,t$texas0 arkansas1==0 & t$term2year==1)[2])
##calculate t.Var.hat.1 manually
sum((t.obs.1 - ((sum(t.obs.1)/length(t.obs.1))))^2)/(length(t.obs.1)-1)
## [1] 413.6952
##matches
var(t.obs.1)
## [1] 413.6952
t.se.hat \leftarrow sqrt((var(t.obs.0)/length(t.obs.0)) + (var(t.obs.1)/length(t.obs.1)))
cat("Texas STE of ATE = ", t.se.hat)
## Texas STE of ATE = 9.345871
##arkansas
a.obs.0 <- unlist(subset(t,t$texas0 arkansas1==1 & t$term2year==0)[2])
a.obs.1 <- unlist(subset(t,t$texas0_arkansas1==1 & t$term2year==1)[2])
a.se.hat \leftarrow sqrt((var(a.obs.0)/length(a.obs.0)) + (var(a.obs.1)/length(a.obs.1)))
cat("Arkansas STE of ATE = ", a.se.hat)
## Arkansas STE of ATE = 3.395979
  c. Estimate overall of ATE for both states combined
t.pe.count<-length(t.obs.0)+length(t.obs.1)</pre>
a.pe.count<-length(a.obs.0)+length(a.obs.1)
pooled.ate <- a.ate*(a.pe.count/sum(t.pe.count,a.pe.count))+t.ate*(t.pe.count/sum(t.pe.count,a.pe.count
cat("Pooled ATE = ", pooled.ate)
## Pooled ATE = -13.2168
  d. Explain why pooling the data leads to biased estimates of ATE? Quite simply, there are not the same
     number of observations in the treatment and control in each state
  e. Insert the estimated standard errors into equation 3.12 to estiamte standard error for overall ATE
pooled.se <- sqrt(a.se.hat^2*(a.pe.count/sum(t.pe.count,a.pe.count))^2+a.se.hat^2*(t.pe.count/sum(t.pe.
cat("Pooled SE for Overall ATE = ", pooled.se)
```

t.obs.0 <- unlist(subset(t,t\$texas0\_arkansas1==0 & t\$term2year==0)[2])</pre>

##texas

## Pooled SE for Overall ATE = 2.405726

f. use randomization inference to test sharp null

```
{\tt numiter} \ {\small \backprime} \ {\small 10000} \ {\it \# number of RI iterations}. \ {\it Use more for greater precision, fewer for greater speed}.
set.seed(1234567) # set random number seed (so that results can be replicated)
##texas
t.tauRI <- rep(NA, numiter)</pre>
for (i in 1:numiter) {
t.Zri <- sample(t.Z)</pre>
t.tauRI[i] <- cov(t.Y,t.Zri)/t.denom</pre>
if (i %% round(numiter/10) == 0) cat("Iteration",i,"of",numiter,"\n")
## Iteration 1000 of 10000
## Iteration 2000 of 10000
## Iteration 3000 of 10000
## Iteration 4000 of 10000
## Iteration 5000 of 10000
## Iteration 6000 of 10000
## Iteration 7000 of 10000
## Iteration 8000 of 10000
## Iteration 9000 of 10000
## Iteration 10000 of 10000
t.pvalue <- round(sum(t.tauRI >= t.ate)/numiter,3)
t.pvalue
## [1] 0.957
##arkansas
a.tauRI <- rep(NA, numiter)</pre>
for (i in 1:numiter) {
a.Zri <- sample(a.Z)</pre>
a.tauRI[i] <- cov(a.Y,a.Zri)/a.denom
if (i %% round(numiter/10) == 0) cat("Iteration",i,"of",numiter,"\n")
## Iteration 1000 of 10000
## Iteration 2000 of 10000
## Iteration 3000 of 10000
## Iteration 4000 of 10000
## Iteration 5000 of 10000
## Iteration 6000 of 10000
## Iteration 7000 of 10000
## Iteration 8000 of 10000
## Iteration 9000 of 10000
## Iteration 10000 of 10000
```

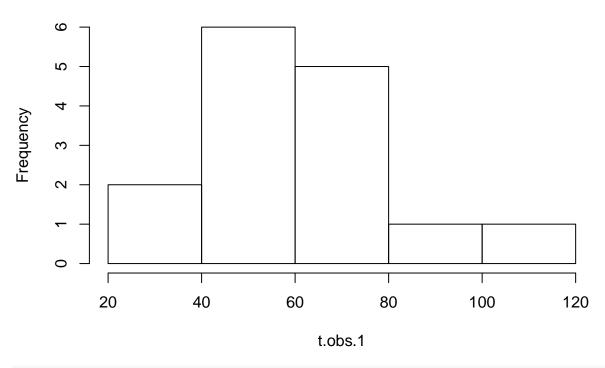
```
a.pvalue <- round(sum(a.tauRI >= a.ate)/numiter,3)
a.pvalue
```

## [1] 0.998

g. plotting histograms per instruction

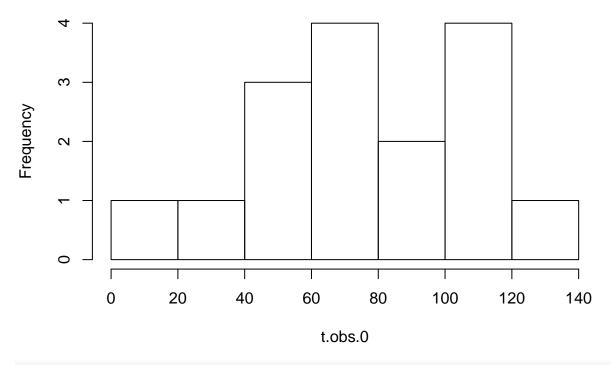
```
#texas treatment & control
hist(t.obs.1)
```

# Histogram of t.obs.1



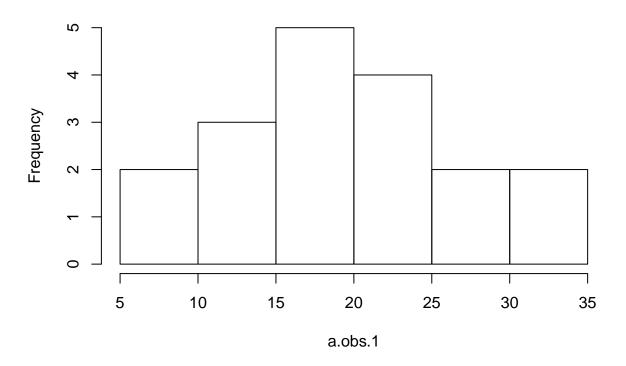
hist(t.obs.0)

# Histogram of t.obs.0



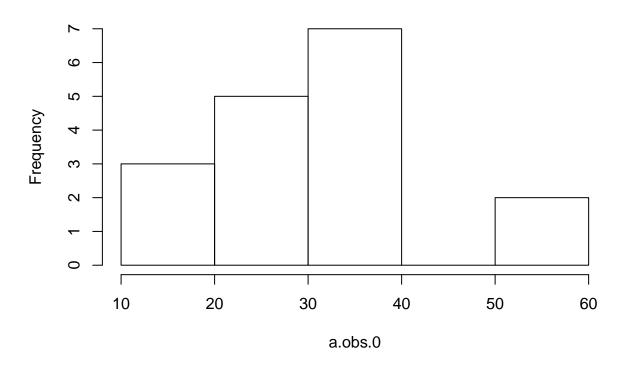
#arkansas treatment & control
hist(a.obs.1)

# Histogram of a.obs.1



hist(a.obs.0)

## Histogram of a.obs.0



## 3. FE, exercise 3.11

a.

```
setwd("/Users/ceccarelli/MIDS/DATASCI_W241/Async Material and Sample Files/Chapter 3")
#clear variables
rm( list = ls() )
#read in tabular data
data <- read.csv("GerberGreenBook_Chapter3_Table_3_3.csv",header = TRUE)
#create shorthand reference
c<-data
##View(c)
c$Y

## [1] 0 1 2 4 4 6 6 9 14 15 16 16 17 18

c$D

## [1] 0 0 1 2 0 0 2 3 12 9 8 15 5 17

cluster.ids<-c(1,1,2,2,3,3,4,4,5,5,6,6,7,7)
cluster.ids</pre>
```

**##** [1] 1 1 2 2 3 3 4 4 5 5 6 6 7 7

```
all.clusters <- unique(cluster.ids)</pre>
randomize.clustered <- function(){</pre>
  treat.cluster.ids <- sample(all.clusters, length(all.clusters)*(3/7))</pre>
  return(
    as.numeric(cluster.ids %in% treat.cluster.ids)
  )
}
randomize.clustered()
## [1] 1 1 0 0 0 0 1 1 1 1 0 0 0 0
cl1<-as.data.frame(cbind(c$Y,c$D,randomize.clustered()))</pre>
names(cl1)[1] <- "Y"</pre>
names(cl1)[2] <- "D"</pre>
names(cl1)[3] <- "Cluster"</pre>
cl1.0 <- unlist(subset(cl1,cl1$Cluster==0)[1])</pre>
cl1.1 <- unlist(subset(cl1,cl1$Cluster==0)[2])</pre>
se.ate \leftarrow sqrt((1/(7-1))*((4*var(cl1.0)/(7-4))+(3*var(cl1.1)/(4))+(2*cov(cl1.0,cl1.1))))
cat("SE for Cluster Assingment 1 = ", se.ate)
## SE for Cluster Assingment 1 = 4.487085
  a.
setwd("/Users/ceccarelli/MIDS/DATASCI W241/Async Material and Sample Files/Chapter 3")
#clear variables
rm( list = ls() )
#read in tabular data
data <- read.csv("GerberGreenBook_Chapter3_Table_3_3.csv",header = TRUE)</pre>
#create shorthand reference
c<-data
##View(c)
c$Y
   [1] 0 1 2 4 4 6 6 9 14 15 16 16 17 18
##
c$D
   [1] 0 0 1 2 0 0 2 3 12 9 8 15 5 17
cluster.ids<-c(1,2,3,4,5,6,7,7,6,5,4,3,2,1)
cluster.ids
## [1] 1 2 3 4 5 6 7 7 6 5 4 3 2 1
```

```
all.clusters <- unique(cluster.ids)

randomize.clustered <- function(){
   treat.cluster.ids <- sample(all.clusters, length(all.clusters)*(3/7))
   return(
      as.numeric(cluster.ids %in% treat.cluster.ids)
   )
}
randomize.clustered()</pre>
```

## [1] 1 0 1 0 0 1 0 0 1 0 0 1 0 1

```
cl1<-as.data.frame(cbind(c$Y,c$D,randomize.clustered()))
names(cl1)[1] <- "Y"
names(cl1)[2] <- "D"
names(cl1)[3] <- "Cluster"

cl1.0 <- unlist(subset(cl1,cl1$Cluster==0)[1])
cl1.1 <- unlist(subset(cl1,cl1$Cluster==0)[2])

se.ate <- sqrt((1/(7-1))*((4*var(cl1.0)/(7-4))+(3*var(cl1.1)/(4))+(2*cov(cl1.0,cl1.1))))
cat("SE for Cluster Assingment 2 = ", se.ate)</pre>
```

## SE for Cluster Assingment 2 = 4.699053

c. They lead to different standard errors because there is variability in cluster level means across clusters. For large experiments its crucial to look for ways to INCREASE the number of clusters to reduce this penalty.

## 4. Question 4

```
#problem set up
cws <- 1
advertising <- .10
users <- 1:1000000
profit <- 100
conversion <- .005</pre>
```

a. By how much does the ad campaign need to increase the probability of purchase in order to be "worth it" and a positive ROI?

```
options("scipen"=100, "digits"=10)
average.sales<- length(users)*conversion*profit
average.sales</pre>
```

## [1] 500000

```
cost<-length(users)*.10</pre>
required.conversion <- (average.sales+cost+1)/(length(users)*profit)
required.increase <- (required.conversion - conversion)/conversion*100</pre>
required.increase
## [1] 20.0002
  b.
measuredeffect<-.002
size<-length(users)/2
treatment <- (length(users)/2)*(measuredeffect+conversion)</pre>
control <- (length(users)/2)*(conversion)</pre>
#prop.test(x=c(treatment,control),n=c(size,size))
p<-(treatment+control)/(size+size)</pre>
р
## [1] 0.006
se \leftarrow sqrt(p*(1-p)*((1/size)+(1/size)))*size
## [1] 77.22693831
#confidence interval
conf<- c(treatment-se*1.96, treatment+se*1.96)</pre>
conf
```

- ## [1] 3348.635201 3651.364799
  - c. Is this confidence interval precise enough that you would recommend running this experiment? Why or why not? Yes, not only is it statistically significant (per a prop.test) but the interval does not dip behind the average expectation
  - d. What would be the width of the confidence interval for this experiment if only 1% of users were placed in the control group?

```
measuredeffect<-.002
size.treatment<-length(users)*(.99)
size.control<-length(users)*(.01)
treatment <- (size.treatment)*(measuredeffect+conversion)
control <- (size.control)*(conversion)
#prop.test(x=c(treatment,control),n=c(size.treatment,size.control))
p<-(treatment+control)/(size.treatment+size.control)
p</pre>
```

## [1] 0.00698

```
se <- sqrt(p*(1-p)*((1/size.treatment)+(1/size.control)))*size.treatment
se

## [1] 828.3698935

#confidence interval
conf<- c(treatment-se*1.96, treatment+se*1.96)
conf

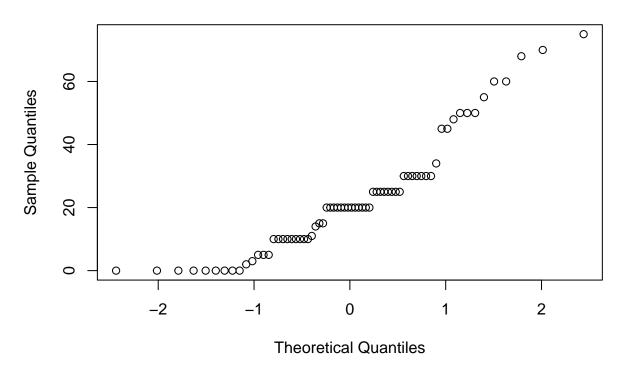
## [1] 5306.395009 8553.604991</pre>
```

## 5. Question 5

a. Compute a 95% confidence interval for the difference between the treatment mean and the control mean, using analytic formulas for a two-sample t-test from your earlier statistics course.

```
#Check if means statistically different
# From the qqplot, it's not clear if loggdp is normally distributed, doesn't look like it
qqnorm(r$bid)
```

## Normal Q-Q Plot



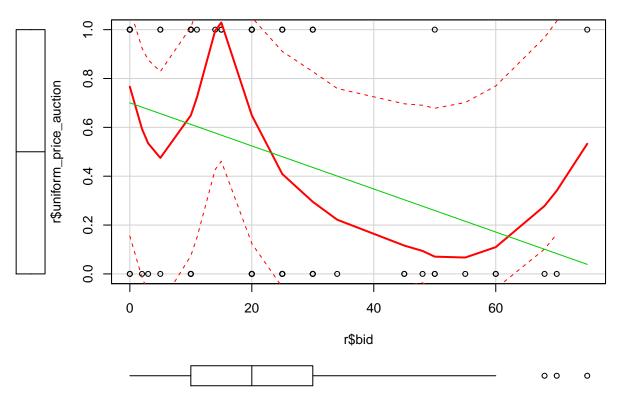
```
# The Shapiro test suggests that it's not
shapiro.test(r$bid)
```

```
##
##
   Shapiro-Wilk normality test
##
## data: r$bid
## W = 0.90139907, p-value = 0.00005581924
# But we have a large sample size, so we can rely on
# the central limit theorem and use a regular t.test
t.test(r$bid ~ r$uniform_price_auction, r, alternative = "two.sided")
##
##
   Welch Two Sample t-test
##
## data: r$bid by r$uniform_price_auction
## t = 2.8211439, df = 61.982867, p-value = 0.006420778
\#\# alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##
     3.557140637 20.854624069
## sample estimates:
## mean in group 0 mean in group 1
##
       28.82352941
                       16.61764706
```

b. In plain language, what does this confidence interval mean? The confidence interval (3.557140637 20.854624069) is the range of values that is likely to contain the population mean in this case.

c. Calculate the 95% confidence interval you get from the regression.

```
library(car)
#look at a quick scatter
scatterplot(r$bid,r$uniform_price_auction)
```



```
# Build regression model
model = lm(bid~uniform_price_auction, data = r)
summary(model)
```

```
##
## Call:
## lm(formula = bid ~ uniform_price_auction, data = r)
##
## Residuals:
##
                      1Q
                             Median
                                            3Q
                                                      Max
## -28.823529 -11.617647
                          -3.220588
                                      8.382353
                                               58.382353
##
## Coefficients:
                                                                    Pr(>|t|)
##
                           Estimate Std. Error t value
## (Intercept)
                          28.823529
                                      3.059348 9.42146 0.00000000000078066
##
  uniform_price_auction -12.205882
                                      4.326572 -2.82114
                                                                   0.0063148
##
## (Intercept)
## uniform_price_auction **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 17.83891 on 66 degrees of freedom
```

```
## Multiple R-squared: 0.1076119, Adjusted R-squared: 0.09409087
## F-statistic: 7.958853 on 1 and 66 DF, p-value: 0.006314796
estimate <- as.vector(model$coefficients[1])</pre>
standard.error <- abs(as.vector(model$coefficients[2]))</pre>
lower.bound <- estimate - standard.error * 1.96</pre>
upper.bound <- estimate + standard.error * 1.96
cat("Conf Interval = ", lower.bound, upper.bound)
## Conf Interval = 4.9 52.74705882
  d. On to p-values. What p-value does the regression report?
cat("p-value = ", summary(model)$coefficients[,4])
## p-value = 0.00000000000007806564729 0.006314795951
  e. Now compute the same p-value using randomization inference.
numiter <- 10000 # number of RI iterations. Use more for greater precision, fewer for greater speed.
set.seed(12) # set random number seed (so that results can be replicated)
# Compute RI Distribution
Y <- r$bid
Z <- r$uniform_price_auction</pre>
denom <- var(Z)
tau <- cov(Y,Z)/denom
tauRI <- rep(NA, numiter)</pre>
for (i in 1:numiter) {
Zri <- sample(Z)</pre>
tauRI[i] <- cov(Y,Zri)/denom</pre>
if (i %% round(numiter/10) == 0) cat("Iteration",i,"of",numiter,"\n")
## Iteration 1000 of 10000
## Iteration 2000 of 10000
## Iteration 3000 of 10000
## Iteration 4000 of 10000
## Iteration 5000 of 10000
## Iteration 6000 of 10000
## Iteration 7000 of 10000
## Iteration 8000 of 10000
## Iteration 9000 of 10000
## Iteration 10000 of 10000
```

```
pvalue <- round(sum(tauRI >= abs(tau))/numiter,3)
pvalue
```

## [1] 0.003

f. Compute the same p-value again using analytic formulas for a two-sample t-test from your earlier statistics course. (Also see part (a).)

```
# compare with traditional t-tests
t.test(Y~Z,var.equal=FALSE,alternative = "two.sided")
```

```
##
## Welch Two Sample t-test
##
## data: Y by Z
## t = 2.8211439, df = 61.982867, p-value = 0.006420778
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## 3.557140637 20.854624069
## sample estimates:
## mean in group 0 mean in group 1
## 28.82352941 16.61764706
```

g. Compare the two p-values in parts (e) and (f). Are they much different? Why or why not? How might your answer to this question change if the sample size were different?

The t-tests aren't very much different in real terms. They would converge as the sample size increased.