exact2x2: Exact Conditional Tests and Matching Confidence Intervals for 2 by 2 Tables

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1 Motivation for Package and an Example

Here is the problem which motivated this package. Consider the following $2x2$ table

```
> x <- matrix(c(6, 12, 12, 5), 2, 2, dimnames = list(c("Group A",
+     "Group B"), c("Event", "No Event")))
> x

        Event No Event
Group A     6       12
Group B    12        5
```

When we perform the two-sided Fisher's exact test on this table we get

```
> ft <- fisher.test(x)
> ft

        Fisher's Exact Test for Count Data

data:  x
p-value = 0.04371
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.03888003 1.05649145
sample estimates:
odds ratio
 0.2189021
```

The two-sided p-value is less than 0.05 but the 95 percent confidence interval on the odds ratio contains 1. What we want is a matching confidence interval that goes with the test, but what fisher.test outputs is a confidence interval that matches a different test. The confidence interval that fisher.test outputs matches with a two-sided Fisher's exact test whose p-value is twice the minimum one-sided p-value. We call that test the central Fisher's exact test, since the matching confidence interval is a central confidence interval (i.e., there is a maximum of $\alpha/2$ probability that the true odds ratio is lower than the lower limit and analogously for the upper limit).

The function exact2x2 gives the proper matching interval. Here are the two types of two-sided Fisher's exact test. We use the option "minlike" for the usual two-sided Fisher's exact test since it is based on the priciple of minimum likelihood.

```
> exact2x2(x, tsmethod = "minlike")
```

```
        Two-sided Fisher's Exact Test (usual method using minimum likelihood)

data:  x
p-value = 0.04371
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.0435 0.9170
sample estimates:
odds ratio
 0.2189021

> exact2x2(x, tsmethod = "central")

        Central Fisher's Exact Test

data:  x
p-value = 0.06059
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.03888003 1.05649145
sample estimates:
odds ratio
 0.2189021
```

Additionally, exact2x2 gives another exact conditional test proposed by Blaker (2000), which we call Blaker's exact test. This test is always more powerful than the central Fisher's exact test, but does not necessarily have central confidence intervals. It will often have the same p-value as the usual two-sided Fisher's exact test, as it does in this example:

```
> exact2x2(x, tsmethod = "blaker")

        Blaker's Exact Test

data:  x
p-value = 0.04371
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
 0.0423 0.9170
sample estimates:
odds ratio
 0.2189021
```

There are some theoretical nuiances since when you invert the two-sided Fisher's exact test (or Blaker's exact test) the resulting confidence set may not be an interval. See Fay (2009) for a complete discussion of these issues.

There is an option for paired data which gives an exact version of McNemar's test, see vignette("exactMcNemar") for a description.

References

Blaker, H. (2000). "Confidence curves and improved exact confidence intervals for discrete distributions" *Canadian Journal of Statistics* **28,** 783-798 (correction **29,** 681).

Fay, M.P. (2009). "Confidence Intervals that Match Fisher's Exact or Blaker's Exact Tests" (to appear Biostatistics. See Fay2009MatchingCI.pdf in doc directory of this package for earlier version which is essentially the paper plus the supplement).