Roots of Quadratic Equation

Students often wonder if a quadratic equation can have more than one solution? Are there any equations that don't have any real solution? Yes, it's possible with the root of quadratic equation concept.

The value of a variable for which the equation gets satisfied is called the solution or the root of quadratic equation. The number of roots of a polynomial equation is equal to its degree. Hence, a quadratic equation has 2 roots. Let α and β be the roots of quadratic equation in the general form: ax2 + bx + c = 0. The formulas for solving quadratic equations can be write as:

(-b-√b2-4ac)/2a   and   (-b+√b2-4ac)/2a

Here a, b, and c are real and rational. Hence, the nature of the roots α and β of equation ax2 + bx + c = 0 depends on the quantity or expression (b2 – 4ac) under the square root sign. We say this because the root of a negative number can't be any real number. Say x2 = -1 is a quadratic equation. There is no real number whose square is negative. Therefore for this equation, there are no real number solutions.

Formulas for solving quadratic equations

(-b±√b2-4ac)/2a

Hence, the expression (b2 – 4ac) is called the discriminant of the quadratic equation ax2 + bx + c = 0. Its value determines the nature of the roots of quadratic equation.

Nature of Roots of Quadratic Equation

Nature of Roots of Quadratic Equation Cases:

Let us recall the general formulas for solving quadratic equations, α = (-b-√b2-4ac)/2a and β = (-b+√b2-4ac)/2a

Case I: b2 – 4ac > 0

When a, b, and c are real numbers, a ≠ 0 and the discriminant is positive, then the roots α and β of the quadratic equation formula ax2+bx+ c = 0 are real and unequal.

Case II: b2– 4ac = 0

When a, b, and c are real numbers, a ≠ 0 and the discriminant is zero, then the roots α and β of the quadratic equation ax2+ bx + c = 0 are real and equal.

Case III: b2– 4ac < 0

When a, b, and c are real numbers, a ≠ 0 and the discriminant is negative, then the roots α and β of the quadratic equation formula ax2 + bx + c = 0 are unequal and not real. In this case, we say that the roots are imaginary.

Case IV: b2 – 4ac > 0 and perfect square

When a, b, and c are real numbers, a ≠ 0 and the discriminant is positive and perfect square, then the roots α and β of the quadratic equation ax2 + bx + c = 0 are real, rational and unequal.

Case V: b2 – 4ac > 0 and not perfect square

When a, b, and c are real numbers, a ≠ 0 and the discriminant is positive but not a perfect square then the roots of the quadratic equation formula ax2 + bx + c = 0 are real, irrational and unequal.

Here the roots α and β form a pair of irrational conjugates.

Case VI: b2 – 4ac > 0 is perfect square and a or b is irrational

When a, b, and c are real numbers, a ≠ 0 and the discriminant is a perfect square but any one of a or b is irrational then the roots of the quadratic equation ax2 + bx + c = 0 are irrational.