Relationship Between Coefficients and Roots of Quadratic Equation

Let’s pick equation ax2 + bx + c = 0. It is one of the formulas for solving quadratic equations, where (a ≠ 0)

Assume α and β to be the roots of the equation ax2 + bx + c = 0

Sum of the roots = α + β = -b/a = (- coefficient of x/ coefficient of x2).

Product of the roots = α x β = c/a = (constant term/ coefficient of x2).

Difference of the roots =

(α – β)2= (α + β)2 – 4αβ = b2/a2 – 4c/a = (b2-4ac)/a2 = (√b2-4ac)/a = √D/a

The quadratic equation can always be expressed in terms of sum and product of roots = x2-(Sum of roots) x + product of roots

= x2 – (α + β)x + αβ = (x-α) (x-β)

Click here for examples on quadratic equations

6. Methods to Solve Quadratic Equations

Unique methods help in deriving solutions to complex sums and equations. While solving quadratic equations too, there are 3 algebraic methods and 1 graphical method that are beneficial. They are:

Factorization

Completing the square method

Quadratic Equation Formula

In addition to the three methods discussed here, we also have a graphical method. As you may have guessed, it involves plotting the given equation for various values of x. The intersection of the curves thus obtained with the real axis will give us the solutions.

Factorization

The first and simplest method of solving quadratic equations is the factorization method. Certain algebraic expressions can be factored. These factors, if done correctly, will give two linear equations in x. Hence, from these equations, we get the value of x.

Completing the Square Method/ Method of Completing the Square

Each quadratic equation has a square term. If we could get two square terms on two sides of the quality sign, we will again get a linear equation. Let us see an example first and understand how we go about solving quadratic equations.

Example: Let us consider the equation, 2×2=12x+54. Here's how to solve a quadratic equation, step by step by completing the square.

Solution: Let us write the equation 2×2=12x+54. In the standard form, we can write it as: 2×2 – 12x – 54 = 0. Next let us get all the terms with x2 or x in them to one side of the equation: 2×2 – 12 = 54

In the next step, we have to make sure that the coefficient of x2 is 1. So dividing through by the coefficient of x2, we have: 2×2/2 – 12x/2 = 54/2 or x2 – 6x = 27. Next, we make the left hand side a complete square by adding (6/2)2 = 9 i.e. (b/2)2 where 'b' is the new coefficient of 'x', to both sides as: x2 – 6x + 9 = 27 + 9 or x2 – 2×3×x + 32 = 36. Now we can write it as a binomial square:

(x-3)2 = 36; Take square root of both sides

x – 3 = ±6; Which gives us these equations:

x = (3+6) or x = (3-6) or x = 9 or x = -3

This is known as the method of completing the squares.

If you want to learn more about solving quadratic equations, click here.

7. Solving Quadratic Equations – Tips and Tricks

Many students dedicate hours in solving quadratic equations but still are not able to secure high marks in the exams. The primary reason for this is improper preparation strategy and too much focus on memorising quadratic equation formulas.

Students need a well-structured study plan that helps them achieve desired learning outcomes. Here are some tips and tricks that will help in solving quadratic equations of higher order within no time.